ABSTRACT

When blades of reasonable length are attached to a disk, the characteristics of the resulting vibratory system are more complex. The modes of the combined system do not, in general, correspond to the modes of the components of the system, that is, the blades and the disk.

Very often 'long' turbine blades are shrouded "harmonically." It is important to analyze the situation when the number of blade groups in a row do not coincide with the resonant harmonic. In this study, typical blade-disk assemblies are analyzed using the NASTRAN Finite Element Program. Because of the repetitive pattern of the blade groups, substructural modal synthesis and cyclic symmetry approaches are used. The agreement in output illustrates the validity of both methods. The results are then discussed.

I. INTRODUCTION

Rotating turbine blades are subjected to fluctuating steam forces. These force variations, being static in space, can be expanded in a Fourier Series in the circumferential space coordinate. The Nth term in this expansion will exert N complete force cycles on each rotating blade in one shaft revolution. When a blade group natural frequency of vibration is such as to also give N complete cycles of vibration in each shaft revolution, a resonant condition exists.

The vibration characteristics of a stationary free disk are well known. For a rotating disk, nodal diameter type vibration generates forward traveling waves and backward traveling waves relative to the disk. When an N nodal diameter mode natural frequency is such as to give precisely N vibration cycles in one shaft revolution, the backward traveling wave will cause the vibration to appear stationary in space, permitting the mode to be excited by the Nth Fourier term in the expansion of the circumferential steam force variation. The speed at which this occurs is called a disk critical speed.[1]

When blades of reasonable length are attached to a disk, the characteristics of the resulting vibratory system are complex. The modes of the combined system do not, in general, correspond to the modes of the components of the system; that is, the blades and the disk. However, the calculation procedure we follow currently is to assume a sinusoidal disk mode shape in the circumferential direction. For every nodal diameter mode, several blade modes are calculated. Analyses are only performed on those modes which have the same number of disk nodal diameters as the number of the resonant harmonic.[2]

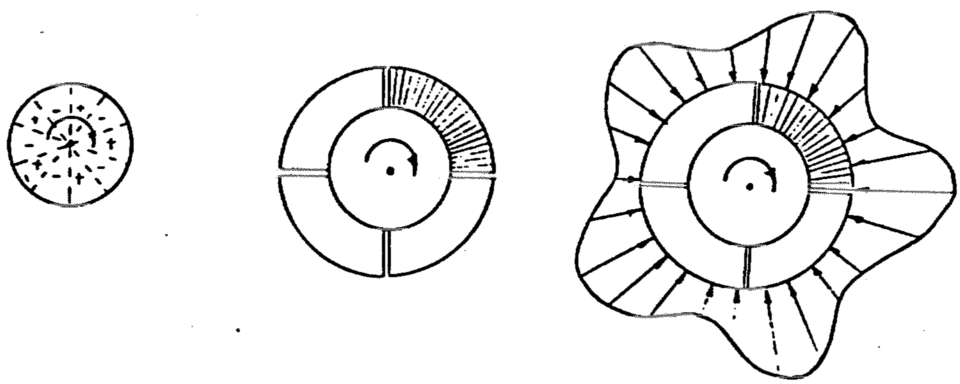
Very often, 'long' turbine blades are shrouded "harmonically."[3] It is important to analyze the situation when the number of blade groups does not coincide with the resonant harmonics (and with disk mode nodal diameters). Therefore, to model the disk-blade properly, the following factors have to be considered:

A. For a single disk only, the modal shapes along the periphery are sinusoidal.

B. The blades are shrouded harmonically.

C. Force distribution on the blades is a function of cos $(n\theta)$ or sin $(n\theta)$.

The factors are illustrated in Figure 1.



A. B. C.

Figure 1

II. FINITE ELEMENT MODEL

A typical disk-blade assembly generally consists of more than a hundred blades in 4 to 20 banded groups. Traditional finite element methods or other numerical solutions have become too expensive to be applicable. However, because of the repetitive pattern of the blade groups, the modal synthesis (or dynamic substructuring) method seems to be a feasible solution method.

Modal synthesis has been used in the aerospace industry for many years. Until recently,[4] this method has not been applied to turbine blade analysis. The basic concept of modal synthesis is to divide the structure into several components. Each one can be analyzed by a separate computer program or by experimental methods. By combining the modal properties of the components with the constraint equations between the components, the total system response is obtained. Depending on the assumptions made at the boundaries in between the substructures, this method can be separated into two approaches -- fixed constraint and free constraint. Each one has its advantages and disadvantages. Good discussions can be found in the papers by Klosterman[5], Hurty[6], and MacNeal[7].

Although the matrix calculation procedures are complicated, the modal synthesis techniques have already been developed for use with large scale finite element programs such as the NASTRAN Program[8]. In this Paper, two blade-disk assemblies are modeled by NASTRAN BEAM and QUAD4 elements, as shown in Figure 2.

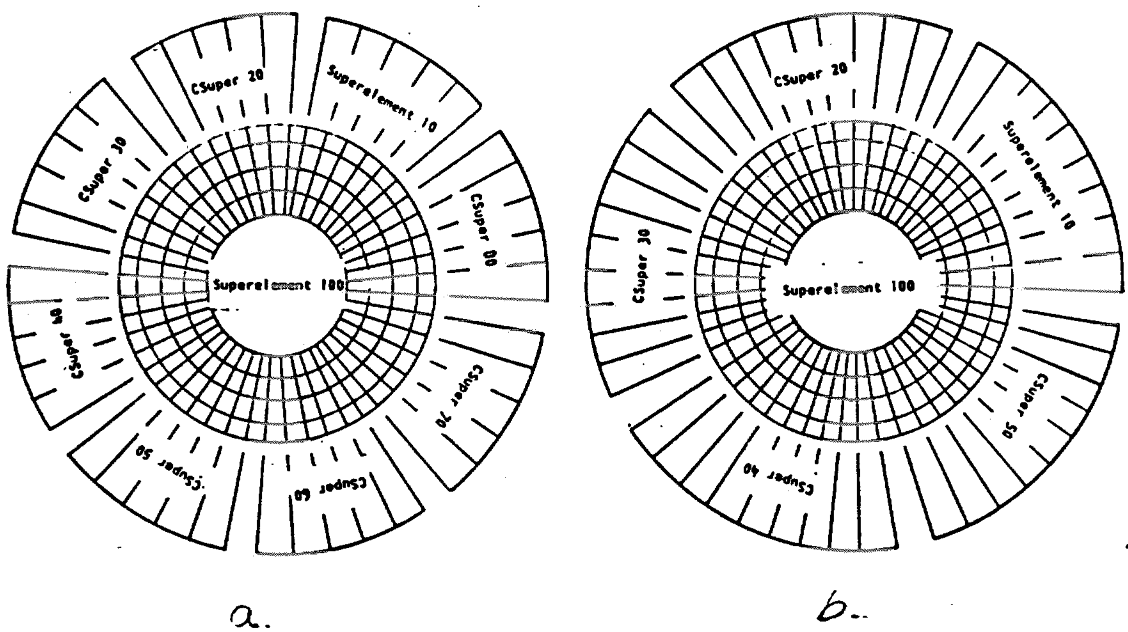


Figure 2. Finite Element Model

Two primary superelement, i.e., the disk (superelement 100) and a shrouded blade group (superelement 10), are used in each of the problems. The remaining identical blade groups are modeled by the "secondary superelements" (CSUPER 20, for example). To locate the secondary superelements at the proper locations, new coordinates have to be defined for each blade group. The blade groups are finally connected to the disk by the RBAR elements. Two different boundary conditions are specified at the outer disk diameter to further test the superelement capability. Therefore, the interfacing boundary conditions between the blade superelements and disk superelement may be either fixed-fixed or fixed-free. The data deck follows that in the example in Ref. 8.

Because of the repetitive patterns of structure in the θ direction, the blade disk assemblies can also be modeled by the cyclic symmetry option in NASTRAN[9]. To investigate the pros and cons of either method, the structure in Figure 2a is also analyzed by both methods. Eight segments of ROT symmetry are used in the analysis.

III. RESULT ANALYSIS

The first twn eigenvalues of the above different analyses are compared in Table I:

TABLE I

MODE No.	Superelem, Solution** Outer Disk Fixed (Hz)	Superelem. Solution** Outer Disk Free (Hz)	Cyclic Symm** Solution (Hz)	Superelem.* Solution (Hz)
1	680.3	680.2	680.1	678.5
2	1125.66	1125.65	1125.43	1138.29
3	1125.66	1125.65	1125.43	1138.29
4	1249.50	1249.20	1249.12	1242.26
5	1261.99	1261.99	1261.81	1268.23
6	1261.99	1261.99	1261.81	1268.23
7	1345.78	1345.78	1345.61	1432.64
8	1345.78	1345.78	1345.61	2126.28
9	1375.18	1375.18	1375.01	2126.28
10	1444.46	1443.74	1442.51	2125.75

Since the cyclic symmetry option in NASTRAN has been extensively tested, and is mathematically vigorous, it can be seen from the first three columns of this Table that either approach can provide reliable eigenvalues. The interfacing boundary conditions between the superelements do not influence the final results.

Although the superelement approach may be more expensive than the cyclic symmetry method, it provides additional information for detailed blade-disk vibration analysis. For example, the contribution of each superelement component mode to the synthesized response can be extracted from the NASTRAN output. Tables II and III show the normalized modal participation fraction of the individual modes to the final blade-disk modal shapes.

TABLE II

Super elem. No.		Mode 1	Mode 2		Mode 3		Mode 4		Mode 5		Mode 6	
	Blade Mode No.	1	2	4	2	4			1	4		
10		-0.014	1.0	-0.42	0.19	-0.08	0.008	-0.01	-1.	0.47	-0.34	0.16
20		-0.014	0.84	-0.35	-0.57	0.24	0.008	-0.01	0.34	-0.16	-1.	0.47
30		-0.014	0.19	-0.08	-1.0	0.42	0.008	-0.01	1.	-0.47	0.34	-0.16
40		-0.014	0.57	-0.24	-0.84	0.35	0.008	-0.01	-0.34	0.16	1.	-0.47
50		-0.014	1.0	0.42	-0.19	0.08	0.008	-0.01	-1.	0.47	-0.34	0.16
60		-0.014	-0.84	0.35	0.57	-0.24	0.008	-0.01	0.34	-0.16	-1.	0.47
70		-0.014	-0.19	0.08	1.0	-0.42	0.008	-0.01	1.	-0.47	0.34	-0.16
80		-0.014	-0.57	0.24	0.84	-0.35	0.008	-0.01	-0.34	0.16	1.	-0.47
	Disk Mode No.	1	3	4	3	4	1		8		9	
Disk		1.0	0.22	0.32	0.32	0.22	1.0		0.01		0.01	

TABLE III

Super elem. No.		Mode 1	Mode 2		Mode 3		Mode 4		Mode 5		Mode 6	
	Blade Mode No.		1	6	1	6	1	6	1	6	1	6
10		-0.019	0.25	-0.11	1.0	-0.43	0.011	-0.013	1.0	-0.48	0.16	-0.08
20		-0.019	1.0	-0.43	0.06		0.011	-0.013	-0.91	0.43	0.46	-0.22
30		-0.019	0.36	-0.16	-0.96	0.55	0.011	-0.013	0.46	-0.22	-0.91	0.42
40		-0.019	-0.78	0.33	-0.65	-0.28	0.011	-0.013	0.16	-0.08	1.0	-0.48
50		-0.019	-0.84	0.36	-0.56	0.24	0.011	-0.013	-0.72	-0.34	-0.72	0.34
	Disk Mode No.	1	3	4	3	4	1		8	9	8	9
Disk		1.0	0.17	0.29	0.22	0.13	1.0		0.00	-0.007	0.007	0.003

It can be concluded from the above Tables that when long shrouded blades are attached to the disk, the modes of the combined system do not correspond to the modes of the components. The contributions of each mode depend on how the blades are shrouded. In general, the mode shapes of the blade-disk assembly may not be single harmonics of sine or cosine. Therefore, each mode can respond to several harmonics of a forcing function.

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