DIFFUSE ILLUMINATION WITH MSC/NASTRAN

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Presented at

MSC/NASTRAN Users Conference
Pasadena, CA

March 24-25, 1983

Abstract

The reflection of light from ideal Lambertian (diffuse) surfaces follows the same physical laws as the radiation of heat. Thus, the thermal radiation capabilities of MSC/NASTRAN can be used by analogy to analyze the behavior of illumination systems containing diffuse surfaces. The view factor calculation capability and the mesh generation capability within MSC/NASTRAN facilitate the analysis. A model (description included) of a simple illumination system are shown to agree closely with measured values.

Introduction

Simply stated, illumination systems collect light from a source and illuminate an object. These systems are found in a wide variety of optical systems including such products as microfilmers and copiers. Currently most designs of illumination systems are done empirically, although some analyses of specular illumination systems have been done using conventional ray tracing.(1) The development of analytic evaluation techniques will encourage the design of more effective and efficient illumination systems.

The purpose of this article is to describe how a subclass of illumination systems containing only Lambertian diffusers and sources can be analyzed using the MSC/NASTRAN general purpose finite element program.

Analysis

An ideal Lambertian (diffuse) surface accepts all incoming radiation (light/heat) regardless of direction and re-radiates it with the same spatial energy distribution as a radiating black body. A "flat" white surface is an example of a Lambertian surface.

A specular (polished) surface on the other hand reflects all incoming radiation such that the angle of incidence equals the angle of reflection. A mirror is a common example of a specular surface. Systems containing specular surfaces can be analyzed using ray tracing techniques.

An illumination system comprised of Lambertian emitters and diffusers is analogous to a radiative heat transfer problem in which the emission of all elements but the source is negligible, and in which the heat capacity of each element is insignificant. Irradiance (incident light) is by analogy equal to irradiation (incident heat).

Using the notation in the NASTRAN Theory Manual(2),

A = diagonal matrix of areas

E_e = diagonal matrix of emissivities

E_α = diagonal matrix of absorptivity

Q = vector of net heat flow

Q_i = vector of heat flow in

Q_o = vector of heat flow out

q_i = vector of irradiation

= power/unit area arriving at surface

q_o = vector of radiosity

The heat flow into the surface (irradiation or illumination) is:

$$Q_i = Aq_i$$

The heat flow out of the surface is the radiation plus reflection

$$Q_o = Aq_o = A\sigma E_e U^4 + A(I-E_a)q_i$$

where σ is the Stefan-Boltzman constant. If the diffusing surfaces do not radiate, but only reflect as initially assumed, the radiation term can be eliminated by setting all grid point temperatures to zero (i.e. using SPC boundary conditions in NASTRAN). Then

$$Q_o = Aq_i - AE_a q_i$$

The net heat flow into a grid point can be written

$$Q = Q_i - Q_o = Aq_i - Aq_i + AE_a q_i$$
$$Q = AE_a q_i$$

Since E_a and A are diagonal, the order can be transposed.

$$Q = E_a A q_i = E_a Q_i$$

To get the illumination Q_i

f = view factor matrix

F = radiation exchange matrix

U = vector of absolute temperatures.

The above vectors and matrices are grid point properties and thus have dimensions equal to the number of grid poins in the model.

Elements of the radiation exchange matrix are given by

$$F_{ij} = \int \int \frac{\cos\theta_i \cos\theta_j}{\pi v_{ij}^2} dA_i dA_j$$

where v_{ij} is the length of a line connecting two points on the surfaces, and θ_i and θ_j are the angles between the connecting line and the normals to the surfaces. The range of integration must be limited to regions which "see" each other. The F matrix has units of area and is symmetric. Its elements are related to the more commonly used form factors/shape factors/view factors f_{ij} by

$$F_{ij} = A_j f_{ij}$$

where f_{ij} is the fraction of power leaving element j which reaches element i.

MSC/NASTRAN calculates the elements of F_{ij} internally in the VIEW module from the data description of the geometry of the model. The technique is documented in the MSC/NASTRAN Application Manual(3), section 3.3 (be careful of a change of notation in the definition of F in the Application Manual from the Theory Manual).

$$Q_i = E_a^{-1}Q$$

where Q is standard NASTRAN output of net heat flow into an element and E_a^{-1} is a diagonal matrix of reciprocal absorptivities.

For gray bodies with no transmission, the absorptivity α is equal to the emissivity ϵ, according to Kirchoff's law. Also for a gray surface, the reflectivity ρ can be written(4)

$$\rho = 1-\epsilon = 1-\alpha$$

Thus, the illumination of any element can be found from

$$Q_i = \frac{\text{net heat flow into an element}}{\text{emissivity of surface}}$$

Modelling

The modelling procedure can be divided into the following steps:

1) The MSGMESH(5) preprocessor is recommended as an efficient technique to generate a finite element mesh of GRIDs and HBDY elements on all surfaces of the model. The user must take care to define the proper active (illuminated) side for the simply shaped HBDY radiation elements. As in all finite element models, the mesh size must be small enough to obtain the desired solution accuracy.

2) Light sources, such as light bulbs or flash units, should be modelled as HBDY elements of the proper geometry with an emissivity of 1.0 for perfect emitters. Fix the temperature of the emitting grid points to +1.0 with SPC and TEMP boundary condition cards.

3) Model all diffuse reflecting surfaces as HBDY radiation elements with the proper emissivity on the PHBDY card. A very white, flat surface might have a reflectivity of 0.98, or an emissivity of 0.02 since

$$\varepsilon = \alpha = 1-\rho.$$

Fix all GRID temperatures to zero via SPC cards to prevent radiation; allowing only reflection.

4) Model completely black surfaces or holes as HBDY elements with a reflectivity of zero, or an emissivity of 1. Again, fix all GRID temperatures to zero on SPC cards.

5) Utilize the VIEW module in MSC/NASTRAN to calculate the radiation exchange matrix between all HBDY elements in the model. The subdivision or contour integration is under user control for desired accuracy. Shaded surface calculations are possible to account for baffles and obstructions.

6) Call the non-linear steady state heat transfer solution sequence SOL 74 in MSC/NASTRAN.

7) Request flux output for the HBDY elements in question and divide by the element emissivity to get the illumination of the desired elements.

8) Note that all degrees of freedom have been fixed via SPC's in this model. To fake NASTRAN into a solution add a couple of dummy GRIDS and elements so the conductivity matrix K is not null.

Example Problem

An experiment was conducted as described in Reference 6 to verify the analysis. The illumination in a white box containing a tungsten filament lamp as shown in Figures 1, 2 and 3 was measured experimentally at 48 points and compared to a NASTRAN model given in the Appendix of this paper. The differences between the measured and the predicted patterns vary from +2 to -5%. The standard deviation of the differences is 1.8%. The experimental errors in the measurements have standard deviations of 1 to 2%, and thus the differences between predicted and measured errors are about the same as the experimental errors.

Figure 4 gives a pictorial representation of the relative irradiance pattern predicted by NASTRAN for a white box with a black top. Figure 5 shows the irradiance pattern when the top is changed to a white top. Figure 6 shows the difference in Figures

4 and 5. This could represent the type of study conducted for a copier with the platten cover open and closed.

The model is shown in Figures 2 and 3. The associated NASTRAN input deck and output is given in the Appendix.

Discussion

The finite element radiation heat transfer capability can be used effectively to analyze illumination systems with ideal Lambertian diffusers. Some materials, such as Kodak White Reflectance Coating closely approximate such ideal diffusers in their behavior. Like any analysis system, this technique allows parametric studies to be conducted to optimize the design of illumination systems before prototypes are constructed. These techniques should prove valuable to much of the photo/optical industry.

References

1. T.S. Chou, Optical Engineering 13, 299 (1979).

2. R.H. MacNeal, Ed., NASTRAN Theoretical Manual, NASA SP-221(01), (1972).

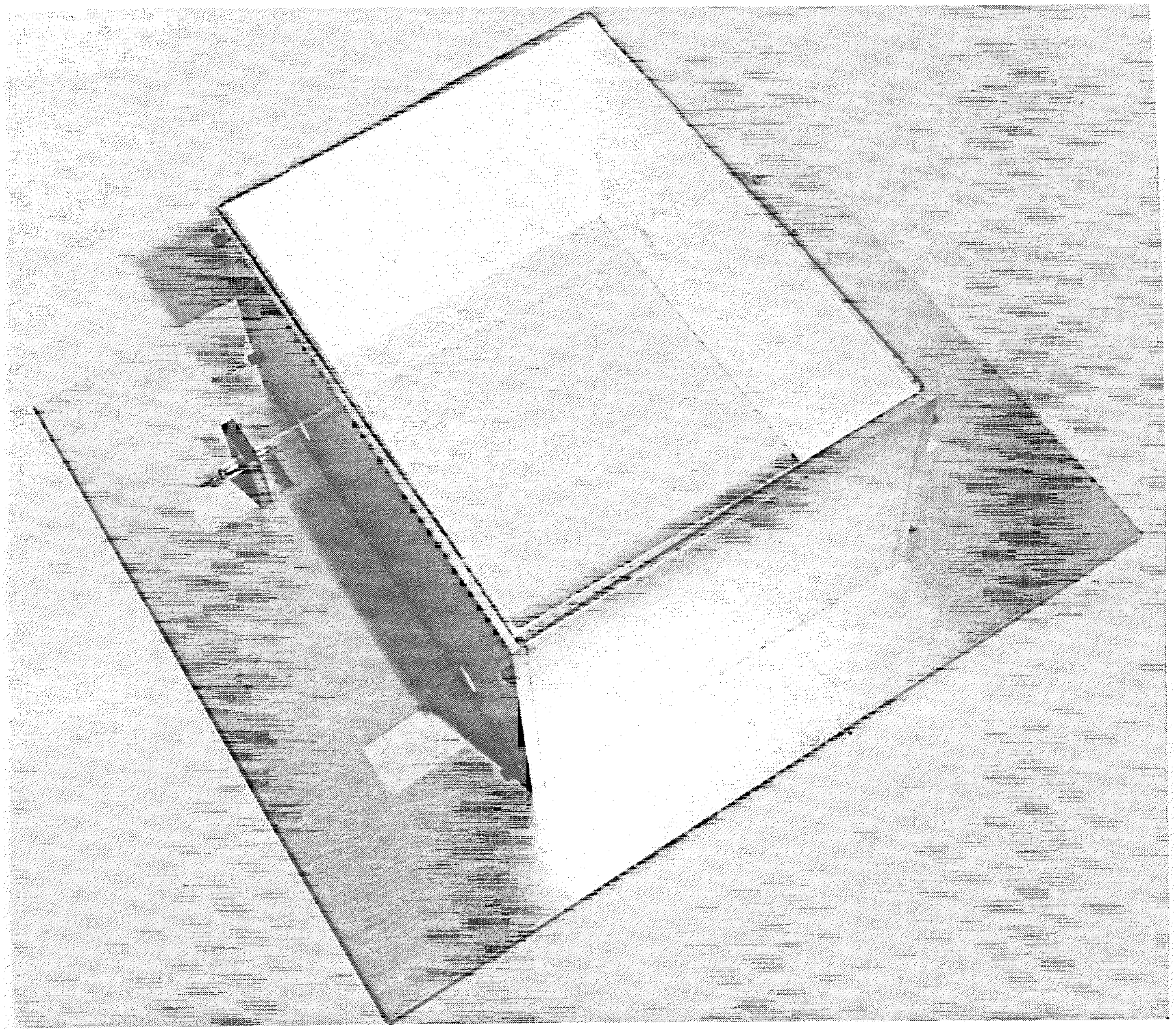
3. J.A. Joseph, Ed., MSC/NASTRAN Application Manual, MacNeal-Schwendler Corp., Los Angeles, CA (1982).

4. F. Krieth, Principles of Heat Transfer 2nd Ed., International Textbook Co., Scranton, PA (1965).

5. L. Peterson, Ed., MSGMESH Analyst's Guide, MacNeal-Schwendler Corp., Los Angeles, CA (1982).

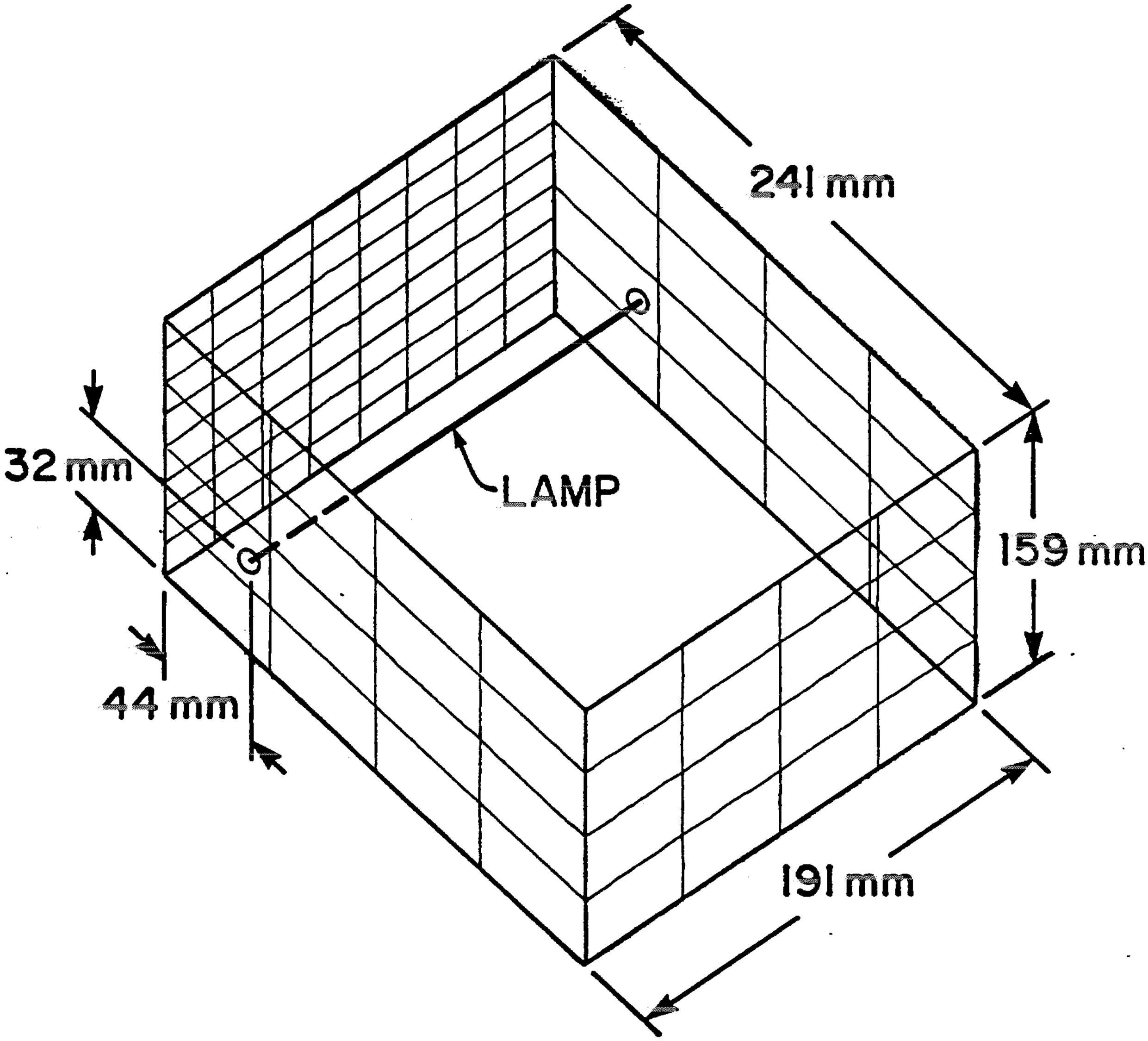
6. D.E. Oinen and V.L. Genberg, "Finite Element Analysis of Diffuse Illumination Systems", Applied Optics, V21, No. 24, 4453-4455 (Dec. 15, 1982).

FIGURE 1



WHITE BOX FOR ILLUMINATION MEASUREMENT

FIGURE 2



Perspective representation of the theoretical white box model giving its dimensions and showing its division into finite elements.

FIGURE 3

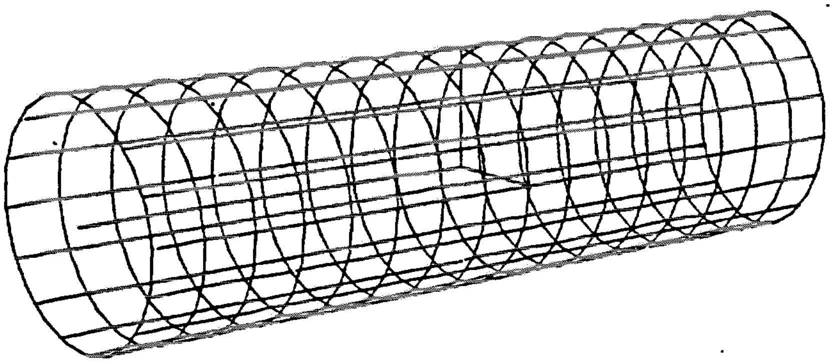


FIGURE 4

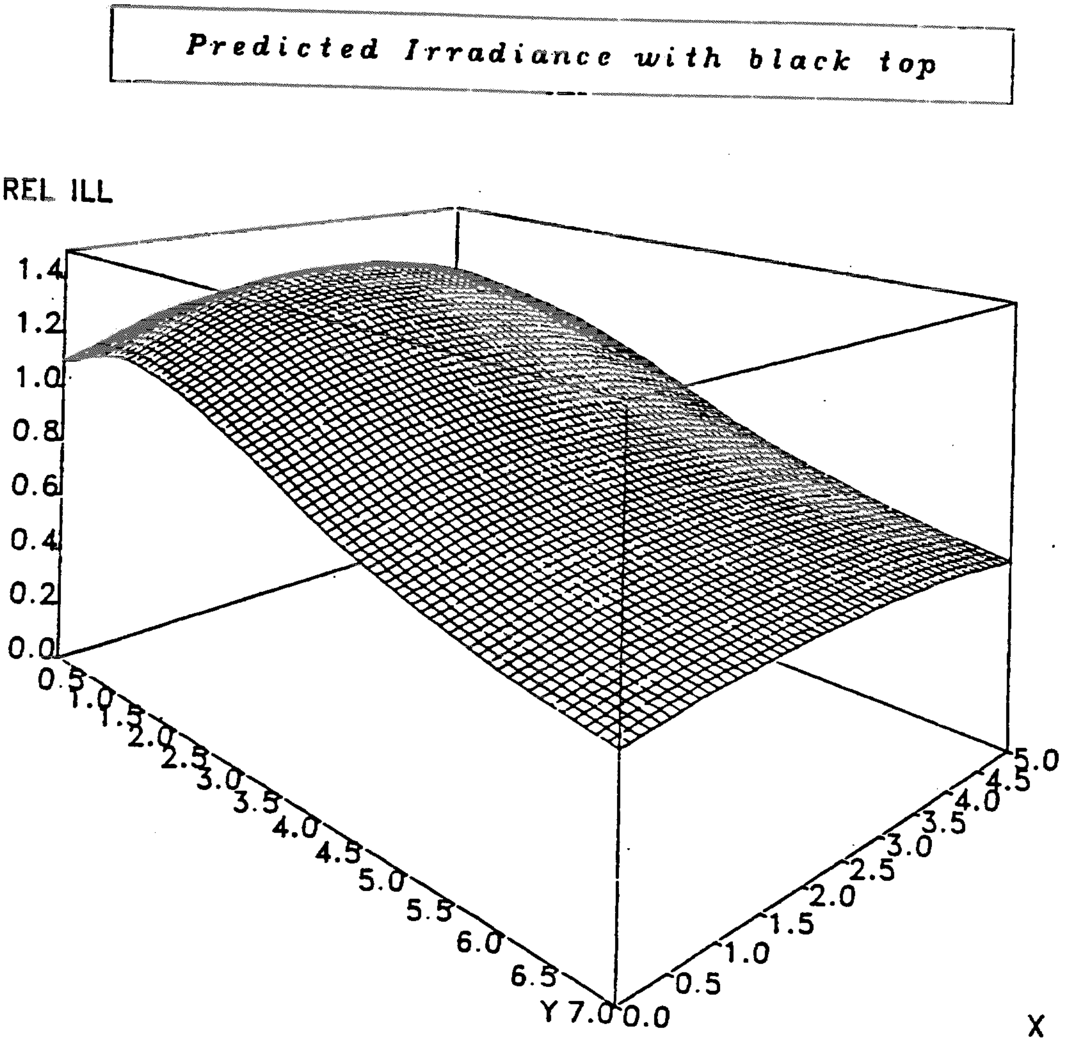

Predicted Irradiance with black top
REL ILL
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
Y 7.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
X

FIGURE 5

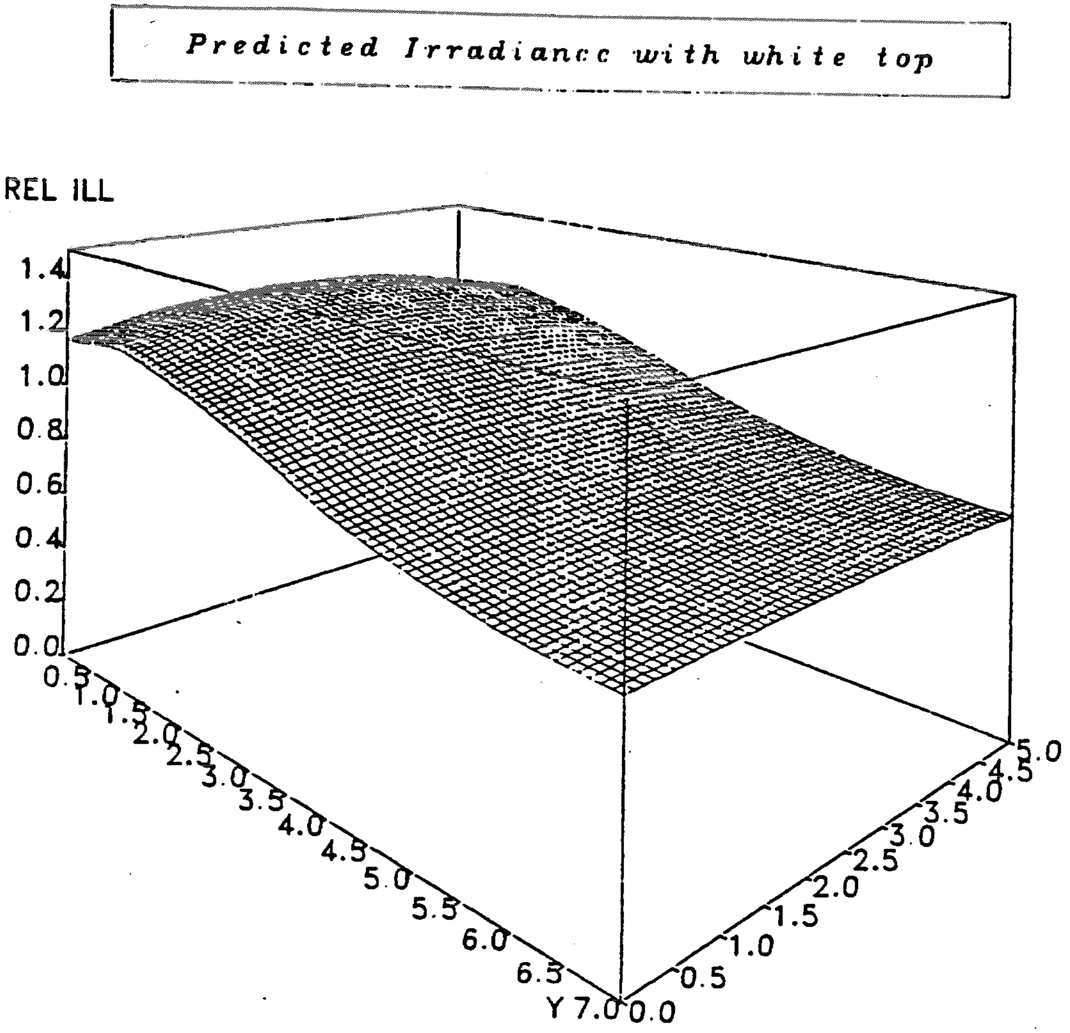
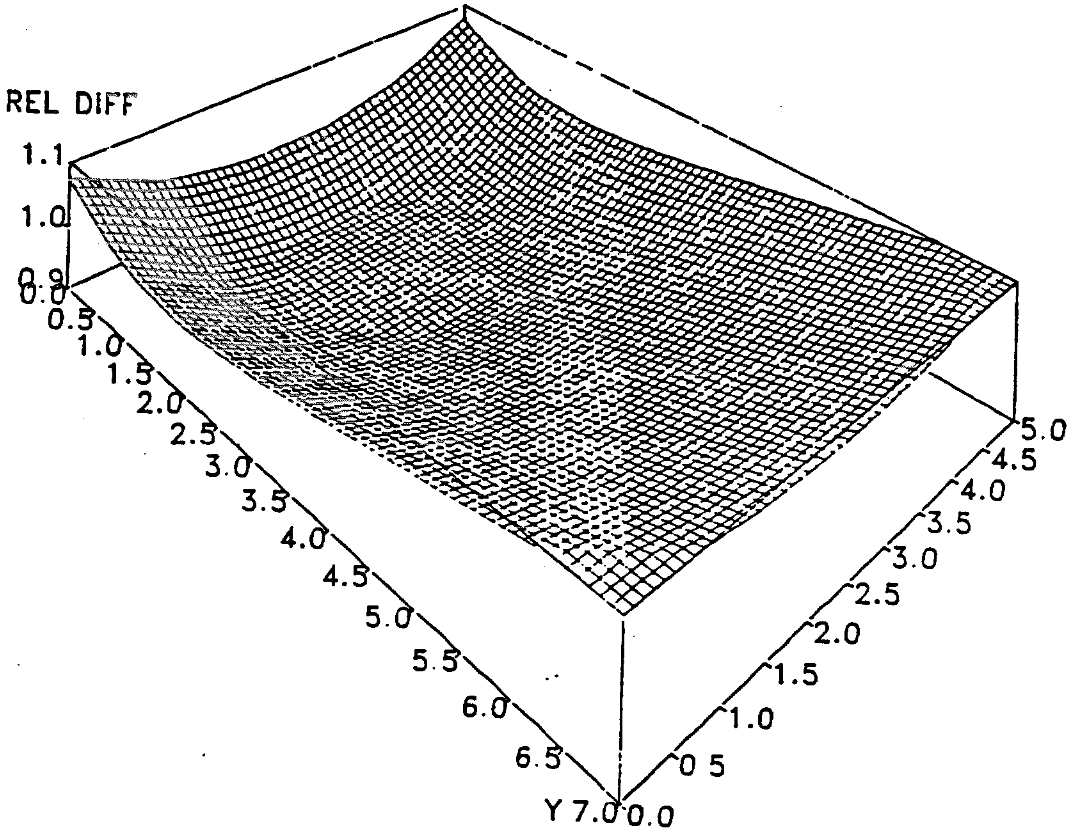

Predicted Irradiance with white top
REL ILL
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
Y 7.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

FIGURE 6

Difference between black and white top



```
NASTRAN PREFOPT = 2

ID VIEW, BOX
SOL 74
TIME 50
CEND

TITLE = ILLUMINATION STUDY
SEALL = ALL
SPC = 1
TEMP(ESTIMATE) = 1
FLUX = ALL
OUTPUT(PLOT)
   SET 1 = ALL
   AXES MY, X, Z
   FIND SCALE, ORIGIN 1, SET 1
   PLOT SET 1
   SET 2 = 200 THRU 225, 300 THRU 325
   PLOT SET 2, ORIGIN 1, LABEL ELEMENTS
   SET 3 = 400 THRU 425, 500 THRU 525
   PLOT SET 3, ORIGIN 1, LABEL ELEMENTS
   SET 4 = 100 THRU 150
   PLOT SET 4, ORIGIN 1, LABEL ELEMENTS

BEGIN BULK
MESHOPT, , , , , YES
$ CREATE THE MODEL (GRIDS AND ELEMENTS)
$ LIGHT SOURCE
GRID, 10, , 0. 0254, 0. , -3. 75
GRID, 11, , 0. 0254, 0. , 0.
GRID, 12, , 0. 0254, 0. , 3. 75
CHBDY, 10, 10, REV, 10, 11, , , 10
CHBDY, 11, 10, REV, 11, 12, , , 10
$ BOX
EGRID, 1, , -2. 0, -1. 0, -3. 75
EGRID, 2, , -2. 0, 5. 25, -3. 75
EGRID, 3, , -2. 0, 5. 25, 3. 75
EGRID, 4, , -2. 0, -1. 0, 3. 75
EGRID, 5, , 7. 5, -1. 0, 3. 75
EGRID, 6, , 7. 5, 5. 25, 3. 75
EGRID, 7, , 7. 5, 5. 25, -3. 75
EGRID, 8, , 7. 5, -1. 0, -3. 75
$ TOP
GRIDG, 1, , , 6, -3, -2, -7, , +G1
+G1, 8, -6
CGEN, AREA4, 100, 1, 1, , , , , +C1
+C1, , , , , , , , 1
$ SIDE 2
GRIDG, 2, , , 4, -1, -2, -3, , +G2
+G2, 4, -4
CGEN, AREA4, 200, 2, 2, , , , , +C2
+C2, , , , , , , , 2
$ SIDE 3
GRIDG, 3, , , 4, -4, -3, -6, , +G3
+G3, 6, -5
CGEN, AREA4, 300, 2, 3, , , , , +C3
+C3, , , , , , , , 2
$ SIDE 4
GRIDG, 4, , , 4, -5, -6, -7, , +G4
+G4, 4, -8
```

```
CGEN, AREA4, 400, 2, 4, , , , , +C4
+C4, , , , , , , , 2
$ SIDE 5
GRIDG, 5, , , 4, -8, -7, -2, , +G5
+G5, 6, -1
CGEN, AREA4, 500, 2, 5, , , , , +C5
+C5, , , , , , , , 2
$ PROPERTIES
PHBDY, 1, , , 1.0
PHBDY, 2, , , .02
PHBDY, 10, , , 1.0
$ LOADS
TEMP, 1, 10, 1.0, 11, 1.0, 12, 1.0
TEMPD, 1, 0.
$ CONSTRAINTS
SPC1, 1, 1, 10, 11, 12
SPC1, 1, 1, 10000, THRU, 50605
$ SHADING AND SUBELEMENT INFO FOR VIEWFACTOR CALCULATION
VIEW, 1, 0, 0, 2, 2
VIEW, 2, 0, 0, 2, 2
VIEW, 10, 0, 0, 4, 16
$ PARAMETER NEEDED FOR RADIATION CALCULATION
PARAM, SIGMA, 1.0
$ STRUCTURAL ELEMENTS REQUIRED BY NASTRAN TO ALLOW PROCESSING
GRID, 1, , -1., 0., -5.
GRID, 2, , -1., 0., 0.
GRID, 3, , -1., 0., 5.
CONROD, 1, 1, 2, 1, 1.
CONROD, 2, 2, 3, 1, 1.
MAT4, 1, 1., 1.
SPC, 1, 1, 123456
TEMP, 1, 1, 1.0, 2, 1.0, 3, 1.0
ENDDATA
```