

# CRACK PROPAGATION ANALYSIS USING NASTRAN SUPERELEMENT CAPABILITY

Zvi Zaphir

Israel Aircraft Industries, Ltd.

Ben Gurion International Airport, Israel

## Abstract

Crack propagation in a Delta wing skin of an aircraft is analyzed. The main effort in this analysis is the NASTRAN finite element (FE) computation needed to evaluate the stress intensity factor  $K_I$ .  $K_I$  is computed using the energy release rate  $G$  obtained by a numerical differentiation of the strain energy with respect to the crack-length. The strain energy difference is evaluated from a finite element analysis of the wing structure carried out for different crack-lengths.

The FE calculation is performed with NASTRAN using the superelement option. The FE mesh consists mainly of the CQUAD4 isoparametric four-node elements. The crack origin and direction of propagation were chosen to conform with the Australian Mirage III wing test results. Good correlation is obtained when comparing the FE results of  $K_I$  with those of Rooke and Cartwright. Good correlation is also obtained between test results and the crack growth analysis, showing that this analytical approach is suitable for engineering purposes.

## 1. INTRODUCTION

The purpose of the present paper is to analyze the crack propagation in a Delta wing-lower skin of an aircraft. A similar crack has been observed by the Australians in the fatigue test of their Mirage III aircraft (1).

One of the main factors in this analysis is the stress intensity factor ( $K_I$ ). A major effort is therefore made here to calculate this parameter, as a function of the crack-length and geometry (2). Based on these  $K_I$  calculations, the crack growth history and the residual strength of the wing can be predicted later on in the analysis (3).

$K_I$  is evaluated by means of the energy release-rate method which is discussed and examined in (4-5). According to this method, the stress intensity factor is obtained as the numerical derivative of the strain energy referring to two slightly different crack lengths. The strain energy itself is obtained as a direct result of the finite-element analysis carried-out with NASTRAN (6-7).

The finite element analysis in this case involves generally a static analysis of a relatively large model which is geometrically modified several times in a relatively small region (the crack vicinity). Consequently the superelement method was found to be the most adequate for this case.

According to this method the structure is divided into superelements in such a way that the crack region which is geometrically modified during the runs, is assigned to the residual structure. Therefore, once the whole structure is computed, all the modifications in the crack area can be computed by analyzing only the residual structure which is relatively small.

## 2. DETERMINATION OF THE STRESS INTENSITY FACTOR USING THE ENERGY RELEASE RATE

The method is based on the relation between  $K_I$  and the energy release rate  $G$  (8). This relation may be written for the case of the plane stress as

$$K_I = \left(\frac{E}{t} G\right)^{1/2} \quad (2)$$

where  $E$  is the skin Young's modulus,  $t$  the thickness, and  $G$  the energy release rate obtained by a central difference numerical differentiation of the strain energy  $W$  with respect to the crack length,  $a$ , written as

$$G(\bar{a}) = \frac{dW}{da} = \frac{W_2 - W_1}{a_2 - a_1} \quad (3)$$

In this equation  $W_1$  and  $W_2$  are the strain energies computed by a FE method for the wing structure containing a crack of two different lengths  $a_1$  and  $a_2$  respectively, and

$$\bar{a} = 1/2 (a_1 + a_2)$$

## 3. THE FINITE ELEMENT MODEL AND SUPERELEMENT ANALYSIS

The FE model represents the whole wing including the leading edge and control surfaces (Figs. 1-2). The wing model is supported at the wing-fuselage fitting which is analogous to the assumption of a rigid fuselage.

The crack line is represented by a couple of coincident grid-lines (Fig. 3).

The loading consists mainly of the aerodynamic lift pressure and the inertial weight loads of the wing structure and its installations.

The plate elements modeling the skins are the isoparametric CQUAD4 and CTRIA3 of the NASTRAN element library. These elements have bending stiffness as well in order to avoid singularities due to motions normal to their planes.

Best results are expected to be when the elements are rectangular and the local axes are parallel to the major and minor stress axes. Since the stresses normal to the crack-edge are zero, the major stress axis is directed along the crack line. Therefore, an effort was made to orient one of the local axes parallel to the crack-lines as shown in Fig. 3.

The couple of coincident grid-lines along the crack line, if not connected to each other, represents the condition of the free edges existing along the crack lips (i.e. represent a crack along the whole crack-line). In order to obtain shorter cracks, the points in each couple can be rigidly connected by applying the appropriate MPC condition which yields the condition of continuity (i.e. no crack).

This kind of idealization enables to analyze a set of crack geometries by changing only the boundary conditions without changing the definition of the structure geometry, line grid point locations and element connection. Such variations can be usually analyzed by NASTRAN in a single run.

The present superelement analysis, where the modifications exist only in the residual structure, was carried out as follows. Primarily, the whole structure was processed, except the residual structure, then a restart was carried out taking care of the residual structure. At the beginning, it was tried to consider all of the various crack lengths in a single multi-subcase restart run. However NASTRAN (Level 60) does not allow this exercise and it was therefore necessary to execute several restart runs representing each of the various crack lengths separately via the different MPC condition.

The total CPU time on the I.A.I. CDC CYBER 730 computer needed to obtain the results for the first crack geometry was approximately 3 hours. On the other hand, each additional run carried out for a crack geometry modification required approximately only 5 CPU minutes. If the same structure was computed in a regular NASTRAN run (without the superelements), it would require approximately only one CPU hour, but in that case, any additional run with modified crack geometry would require the same hour again.

Since the present analysis involves a large number of crack-geometry modifications, the advantage of the superelement method is obvious.

#### 4. RESULTS

Fig. 4 depicts the deformed structure in the crack region as computed by NASTRAN and plotted by the SMOG (10) interactive graphic system.

Stress intensity factors  $K_I$  obtained by this method for the wing lowerskin are plotted in Fig. 5 against crack length, and compared with results of (9) for eccentric cracks of similar geometry. Since the stress values vary in the wing-skin, the Rooke and Cartwright data are calculated for both the average stress in the crack region and the local stress near the crack tip (which varies with the crack-tip location). These stresses were compared along the crack-path but for the uncracked model.

It is seen from Fig. 5 that slow increase of  $K_I$  with crack length will lead to a large critical crack length, as was obtained from the wing test. An analysis of crack length against flight hours is performed for that aircraft using the Australian Mir. III spectrum (1). The calculation was carried out using the CRACKS4 computer program (11).

The analysis was carried out with several retardation models and coefficients. However it was found that the best correlation with the Australian test is obtained when the closure model is used with RETARD MODEL 3,  $C_{f0} = .4$  and  $C_{f-1} = .34$  (Fig. 6).

## 5. CONCLUSION

This work has shown that the FEM can be efficiently used to calculate stress intensity factors in structure of complex geometry, with various crack lengths, loads and boundary conditions. The method is a powerful tool for including fracture mechanics aspects in the analysis of structures.

## REFERENCES

1. Mann J.Y., "A Review of the Australian Investigation on Aircraft Fatigue During the Period April 1973 - March 1975", ICAF 14th Conf. Doc. No. 800, Lausan, May 1975 compiled by J. Branger and F. Berger.
2. Saltoun D. and Zaphir Z., "Stress Intensity Factor Calculation for a Crack in the Wing Lower Skin Near the Wing Root". I.A.I. Internal Report.
3. Zaphir Z., "Crack Growth Analysis in the Wing (in the Lower Skin Near the Wing Root )", I.A.I. Internal Report.
4. Zaphir Z., "Calculation of the Stress Intensity Factor by means of the Energy Release Rate". I.A.I. Internal Report.
5. Zaphir Z., "Stress Intensity Factors for a Central Crack Specimen, Calculated by the Finite Element Method". I.A.I. Internal Report.
6. McCormic C.W., "The MSC/NASTRAN User's Manual". The McNeal-Schendler Co., Los Angeles, Ca, May 1976 (Revised Aug. 1980).
7. Josef J.A., "MSC/NASTRAN Application Manual". The McNeal-Schendler Co., Los Angeles, Ca, Nov. 1972 (Revised March 1980).
8. Broek D., "Elementary Engineering Fracture Mechanics", Nordhoff, Leyden, 1974.
9. Rooke D.P. and Cartwright D.J., "Stress Intensity Factors", HMS, London, 1976.
10. Somekh E, Herz K. and Heimer L, "SMOG 80 User's Manual". I.A.I. Internal Report.
11. Bell P.D, Pardo H. and Schriro G.R., "User's Manual for the CRACKS4 Computer Program". Grumman Rep. No. SAR-79-6-6, Code 26512, Aug. 1970 (Revised June 16, 1980).

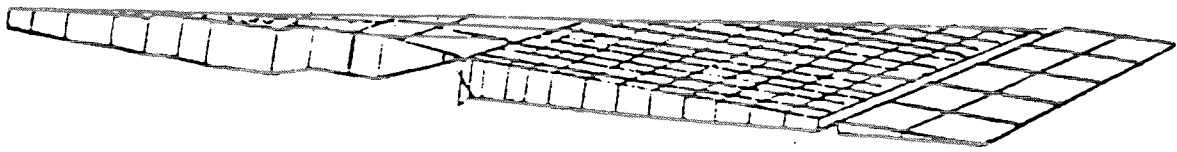


Fig. 1 - Overall Wing Finite Element Model

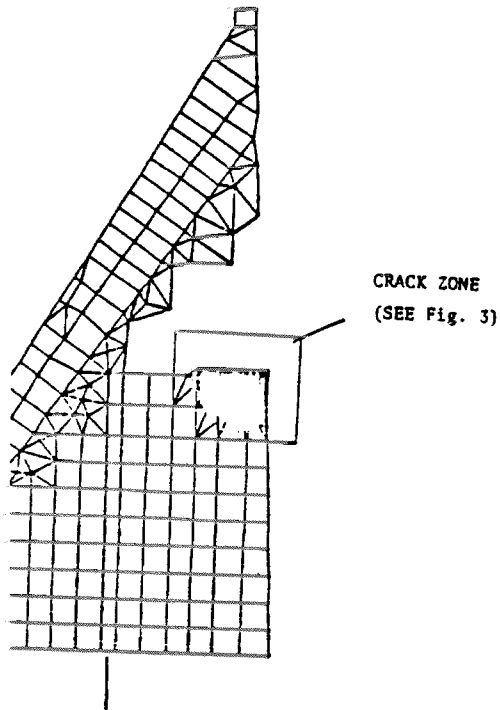


Fig. 2 - Wing Lower Skin  
Finite Element Model

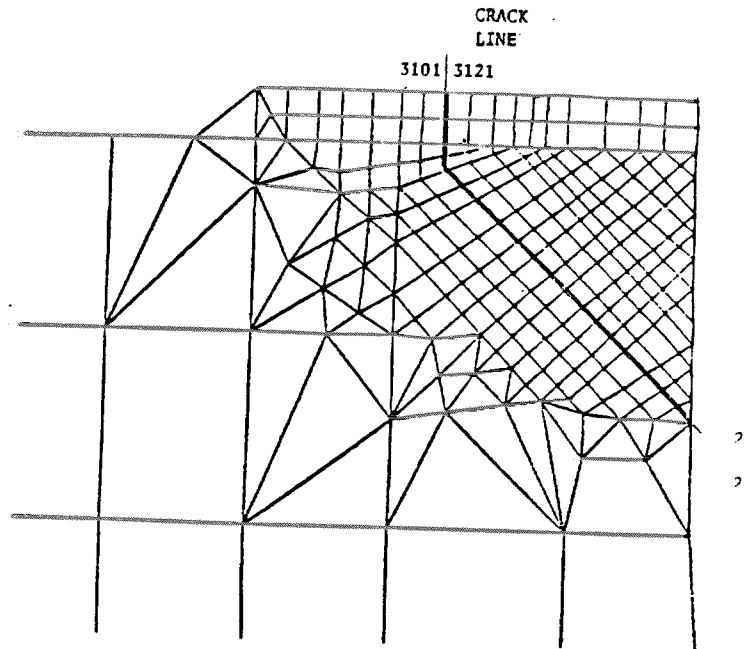


Fig. 3 - Crack-Zone Finite Element Model



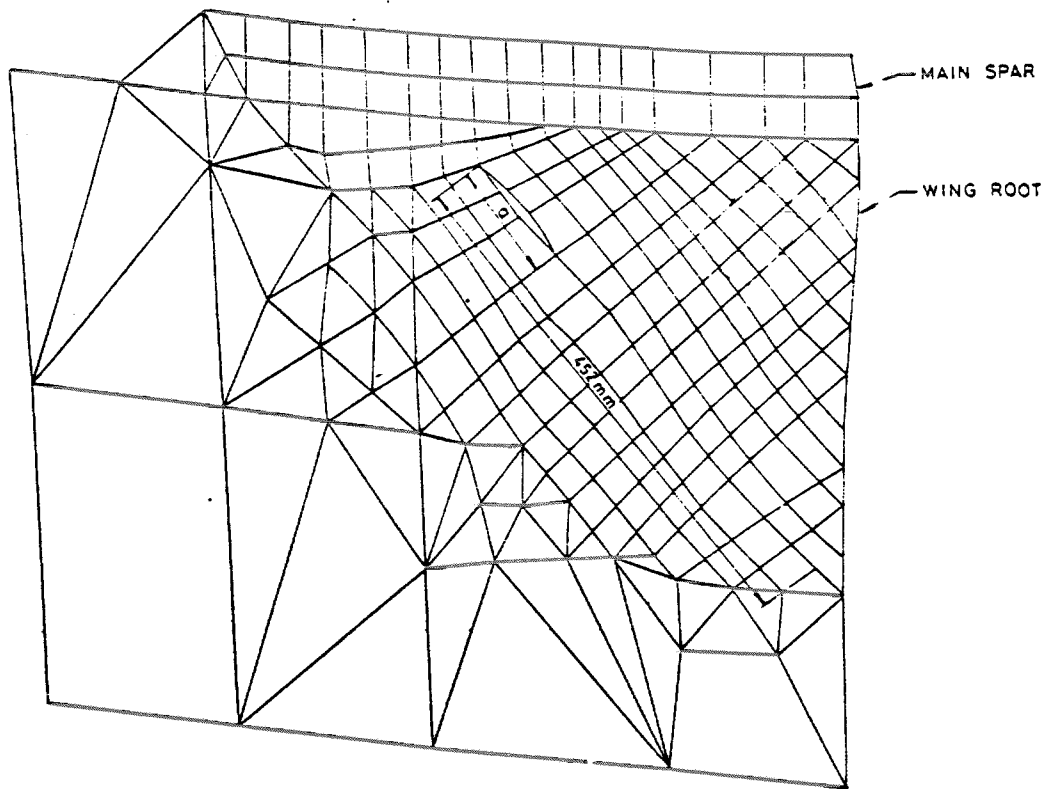


Fig. 4 - Deformed Structure in the Crack Region

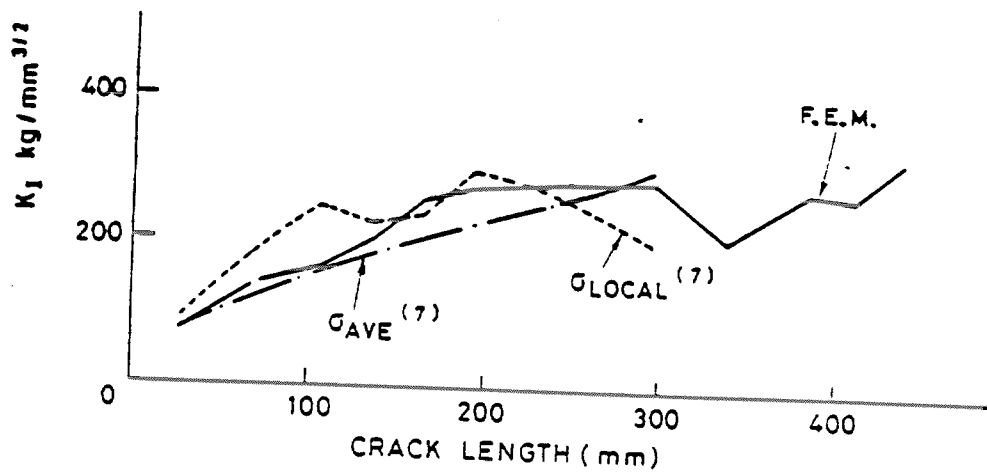


Fig. 5 - Stress Intensity Factor vs. Crack Length Calculated by the Finite Element Method and Compared with Ref. 9.

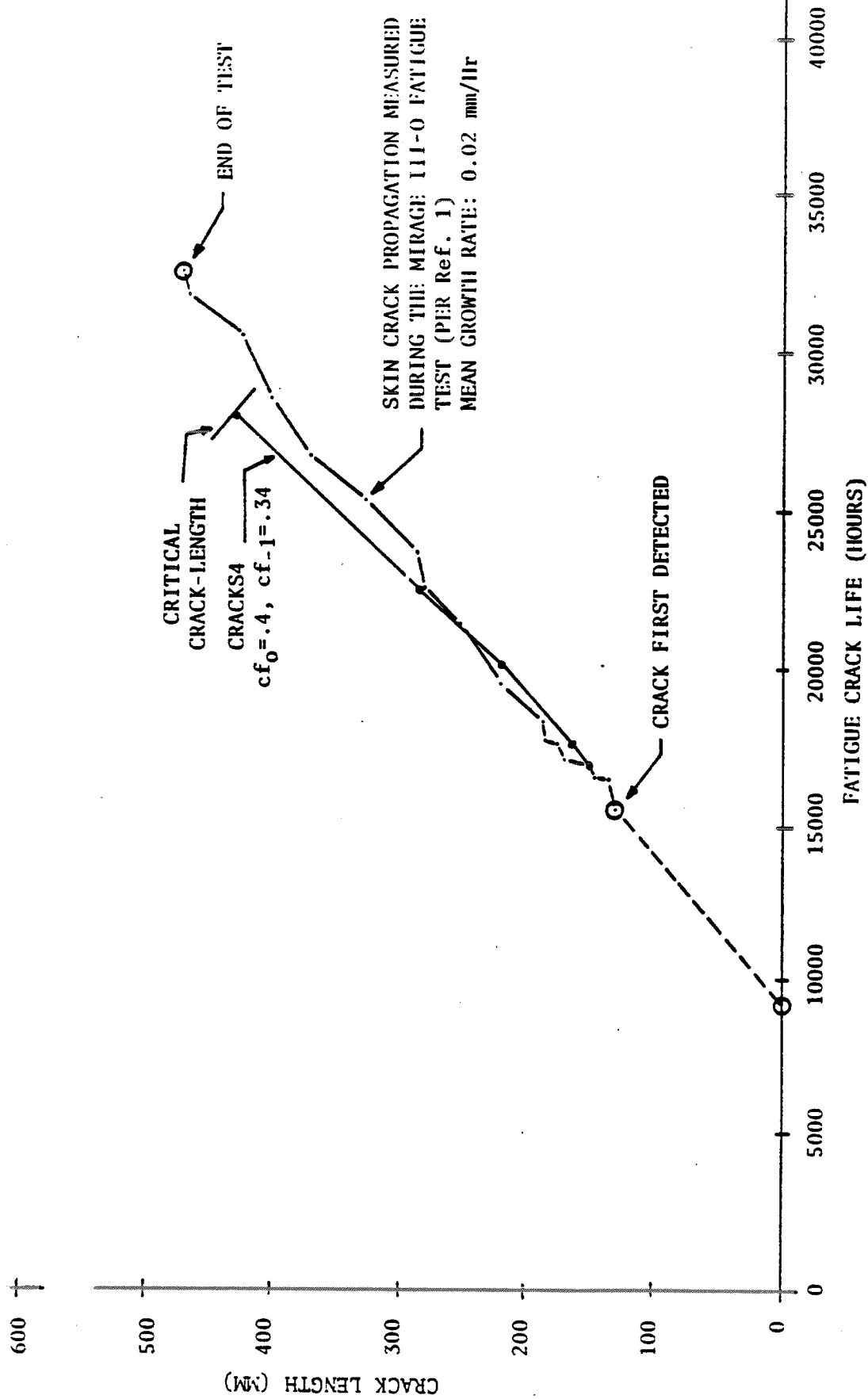


Fig. 6 - Crack Growth Results Obtained in Present Work by CRACKS4 Compared to Ref. 1.