DESIGN SENSITIVITY IN MSC/NASTRAN

by

Robert S. Lahey*, The MacNeal-Schwendler Corporation, Los Angeles, CA

INTRODUCTION

The purpose of this paper is to describe the capabilities and theoretical basis of Design Sensitivity Analysis in MSC/NASTRAN.

Design Sensitivity Analysis (DSA) is a design tool for estimating effects of many interrelated design variables such as element properties and materials on the structural response. The primary functions of DSA are to compute the values of the design constraint functions and the design sensitivity coefficients. The design constraint functions are a set of upper/lower bounds on the structural response quantities such as grid point displacement, element force and stress, buckling load factor and natural frequency. The design sensitivity coefficients are defined as the gradients of the design constraint function with respect to the design variables at the current design point. The computation of the design sensitivity coefficients constitutes the major task of design sensitivity. The design sensitivity coefficients are useful in themselves as they give the designer a feel as to how the structure will respond to a proposed design change. In addition, these data are required by many optimization algorithms.

---

* Senior Project Engineer

The method of implementing design sensitivity analysis presented is based on E.J Haug and J.S. Arora's work*. This method is based on a first variation of the mechanical systems equilibrium equations with respect to the design variables. In this formulation the design variables and the system response (state variables) are treated explicitly. For the structural problems, the design variables are normally parameters associated with structural member sizes. These design variables may effect many element section properties along with complex linking of many individual finite elements. The constraints on the state variables include for the linear statics problem limits on nodal displacements, element stresses and forces, and for the normal mode and buckling problem limits on eigenvalues (buckling load factors).

---

* Arora, S.J., and Haug, E.J., "Methods of Design Sensitivity Analysis in Structural Optimization," AIAA Journal, Vol. 17, No. 9, pp. 970-974, 1979.

Design Sensitivity Analysis (DSA) in MSC/NASTRAN provides a method of determining the values of user defined constraints at each design point and the design sensitivity coefficient matrices. MSC/NASTRAN has three DSA solution sequences which provide constraint values and design sensitivity matrices for STATIC, MODAL and BUCKLING type analyses. DSA analysis is performed as an additional step after the primary analysis is complete.

Currently DSA allows for the computation of displacements, element stress, and element force constraint values in STATIC analysis. In MODAL and BUCKLING analyses, DSA allows for eigenvalue constraint values. These constraint values are determined from the relationships:

$$\Psi_i = \frac{(\text{Constraint value})^o}{|\text{LIMIT}|} - 1. * \text{sign(LIMIT)}$$

or

$$\Psi_i = 1. * \text{sign(LIMIT)} - \frac{(\text{Constraint value})^o}{|\text{LIMIT}|}$$

for upper or lower bounds respectively.

A design variable in MSC/NASTRAN DSA can be as simple as the cross-sectional area of a rod element or a complex linking of many individual element cross-sectional properties (such as area and second moments for a beam) combined with a complex linking of many individual finite elements of various types together. The design variable linking to element property values has the following algebraic form:

$$P_i = P_{o_i} + \sum_{K=1}^{N_B} \sum_{j=1}^{M_K} P_{ref_{ijK}} (B_K^{\alpha ijK} - 1)$$

where P_i is the resultant value of an individual element property value (section-property) due to all 'M_K' user-defined algebraic relations with all design variables, B_K. In the above equation, P_{ref} and α are user-defined constraints. A capability to study material properties is supported in addition to element section-properties.

To illustrate the usefulness of design variable linking, consider the design of a rectangular cross sectional beam (see Figure 1). Choosing the design variables to be the depth (B_d) and width (B_w) of the beam, the beam sectional properties are expressed as follows:

$$w = w_0 B_w \quad \text{(Width)}$$

$$d = w_0 B_d \quad \text{(Depth)}$$

$$A = A_0 + A_0(B_w - 1) + A_0(B_d - 1)$$

$$I_1 = I_{1_0} + I_{1_0}(B_w - 1) + I_{1_0}(B_d^3 - 1)$$

$$I_2 = I_{2_0} + I_{2_0}(B_w^3 - 1) + I_{2_0}(B_d - 1)$$

$$\begin{aligned} J &= I_1 + I_2 \\ &= J_0 + I_{1_0}(B_w - 1) + I_{2_0}(B_w^3 - 1) + I_{1_0}(B_d^3 - 1) + I_{2_0}(B_d - 1) \end{aligned}$$

where d_0 and w_0 are the initial beam depth and width dimensions. These design variable relationships can be directly transferred into MSC/NASTRAN inputs. Note for the beam section property 'J', there are two terms for each design variable. Table 1 shows the user input data which defines the design variable to element section property linking for the beam.

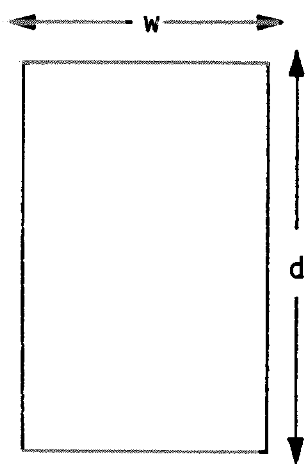
The output for DSA in MSC/NASTRAN consists of the combined Design Sensitivity/Constraint Matrix which contains both the normalized constraint parameter value matrix and the design sensitivity coefficient matrix. This matrix has two parts: the constraint values for each physical load (or selected eigenvector) and the design sensitivity coefficient for each physical load.

The design sensitivity coefficient matrix, in general, contains the gradients of all design constraints with respect to all design variables. A designer may, however, be interested only in a few selected design constraints and the gradients of this with only a few selected design variables. MSC/NASTRAN provides the user controls which will cause only those terms of interest to be generated, thus avoiding unnecessary costs.

	B_w		B_d	
	P_{ref}	α	P_{ref}	α
Area	A_o	1.0	A_o	1.0
I_1	I_{1_o}	1.0	I_{1_o}	3.0
I_2	I_{2_o}	3.0	I_{2_o}	1.0
J	I_{1_o}	1.0	I_{1_o}	3.0
	I_{2_o}	3.0	I_{2_o}	1.0

DSA USER INPUT DATA

TABLE 1



RECTANGULAR BEAM

FIGURE 1

DSA provides constraint values and design sensitivity matrices for three types of superelement solution sequences using an additional run:

Analysis Solution	Design Sensitivity Solution
SØL 61 SE STATICS	SØL 51 DS STATICS
SØL 63 SE MODES	SØL 53 DS MODES
SØL 65 SE BUCKLING	SØL 55 DS BUCKLING

Constraint

DSA allows in STATIC analysis for the computation of:

- Displacement Constraint Values
- Element Stress Constraint Values
- Element Force Constraint Values

In MODAL and BUCKLING analysis, the constraint values computed are for:

- Eigenvalue Constraint Values

All information pertaining to a specific constraint is input on the DSCØNS bulk data card.

Design variables

A design variable in DSA can be as simple as the cross-sectional area of a rod element or the modulus of elasticity of a specific element to a complex linking of many individual element cross-sectional properties (such as the area and second moments for beam type elements) along with a complex linking of many individual finite elements together.

A specific design variable is identified on a DVAR bulk data card.

No matter how simple or how complex the design variable relationship is, the relationship must refer back to the various element property cards and material cards. This is the purpose of the VID fields on the DVAR card. These fields are pointers to the DVSET bulk data card.

The DVSET card then points to a specific property card. The simplest DVAR, DVSET combination would be when an individual finite element section property is a design variable.

DSA Control

The design constraints and design variables used in any given DSA run are controlled by the case control card SENSITY.

If SENSITY = ALL is requested, the value of every design constraint is computed. Also, the gradient of each and every design constraint with respect to each and every design variable is computed.

A designer may, however, be interested only in a few selected design constraints and the gradients of these constraints with only a few selected design variables. The MSC/NASTRAN case control cards SET and SET2 provides him with this capability.

If the user duplicate output requests, DSA removes this duplication internally.

Design Sensitivity in Design

The component Λ_i of the design sensitivity matrix are called the sensitivity coefficient of the constraint Ψ_i with respect to the corresponding design variables. These derivatives represent the effect of a design change on a constraint. Specifically, if the component Λ_{ij} is positive, an increase in b_j will increase Ψ_i. If Λ_{ij} is negative, an increase in b_j will decrease Ψ_i. Also, the order of magnitude of the various sensitivity coefficients Λ_{ij} determine which design variables have a significant effect on Ψ_i.

Design Sensitivity General Limitations and Requirements

The following are general limitations or requirements to design sensitivity in MSC/NASTRAN:

1) All constraints and design variables must reside only in the residual superelement.

2) In Normal Modes, all elements associated with design variables must have structural mass (non-zero mass density).

3) In Buckling, the gradients are generated as if the loading was induced by enforced displacements. This restriction in the DSA theory can be partially relaxed by the user control parameter DSNØKD (see Section 3.1 of the MSC/NASTRAN User's Manual).

4) The composite element properties are not supported in DSA (PCØMP only).

APPROACH

A brief discussion of the approach used to implement DSA STATIC analysis in MSC/NASTRAN will help bring into perspective the work involved in DSA solution. The DSA problem in MSC/NASTRAN is considered to be the additional task required after the solution of primary analysis.

DSA in a STATICS analysis is based on solving for $\{\Delta u_g\}$ in the first order variation of the nodal equilibrium equation:

$$[K_{gg}^{o}]\{\Delta u_g\} = \{\Delta P_g\} - [\Delta K_{gg}]\{u_g^{o}\}$$

The DSA task of solving for $\{\Delta u_g\}$ is equivalent to solving for a second set of loading conditions in a superelement static analysis. DSA reuses much of the data previously calculated in STATIC analysis (SØLution 61). This task only involves the recovery of the solution vector using a previously computed forward solution of the system equations (only the backward pass operation is required). The computation involved is a function of the product of the number of design variables and loading conditions. The following DSA tasks are required in addition to solving the system equations:

- DSA Data Organization
- DSA Data Assembly
- DSA Data Recovery

These tasks are functions of the triple product of the number of design variables, design constraints and loading conditions. For large DSA problems, the data organization, assembly and recovery tasks are the dominant users of computer resources.

A major design consideration was to support all structural finite element types in MSC/NASTRAN. Due to the fact that a large number of the elements developed are semi-imperical, the determination of consistent element derivative formulations cannot be practically accomplished. Due to these reasons, a method was developed to calculate element derivatives by a differencing scheme about the current design point. This method involved the calculation of the element matrix at the design point plus or minus the user specified design variable increment. This element data is differenced with the data at the design point to determine the corresponding element derivatives. For example, the following shows the change in element stiffness due to a change in the design variable.

$$[\Delta K_{gg}] = [K_{gg+\Delta B}] - [K^{o}_{gg}]$$

Similar expressions were developed to compute the various terms in Equations (13), (17), and (23) whose development will be subsequently shown.

Another benefit of differencing about the design point is that it avoids most potential numerical problems. This is because the evaluation of the perturbated element data is computed near a design point which has already been determined to be numerically acceptable in the primary analysis.

Theory

The calculation of the static design sensitivity coefficients is based on the first order variations of the system equations:

$$[K_{gg}]\{u_g\} - \{P_g\} = 0 \quad (1)$$

where b_r are the design parameters and u_g are displacements associated with the static solution. In Equation (1), $[K_{gg}]$ is the stiffness matrix, $\{u_g\}$ is the displacement vector, and $\{P_g\}$ is the applied load vector.

The resultant first variations at the current design state, "o", are:

$$\left[[K^o_{gg}]\left\{\frac{\partial u_g}{\partial b_r}\right\} - \left[\frac{\partial K_{gg}}{\partial b_r}\right]\{u^o_g\} - \left\{\frac{\partial P_g}{\partial b_r}\right\}\right]\delta b_r = \{0\} \quad (2)$$

In the above equation, $\frac{\partial K_{gg}}{\partial b_r}$ are the element forces at the nodes and $\frac{\partial P_g}{\partial b_r}$ is the static load vector. They combine to form the change in nodal equilibrium due to a change in the design variable, b_r. Let the change in nodal equilibrium defined as $\left[\frac{\partial E_g}{\partial b_r}\right]$, the nodal equilibrium matrix.

$$\left[\frac{\partial E_g}{\partial b_r}\right] = \left[\frac{\partial P_g}{\partial b_1}, \ldots, \frac{\partial P_g}{\partial b_r}\right] - \left[\frac{\partial K_{gg}}{\partial b_1}u^o_g, \ldots, \frac{\partial K_{gg}}{\partial b_r}u^o_g\right] \quad (3)$$

Then $\frac{\partial u_g}{\partial b_r}$ may be obtained from the relationship:

$$[K_{gg}]\left[\frac{\partial u_g}{\partial b_r}\right] = \left[\frac{\partial E_g}{\partial b_r}\right] \quad (4)$$

Let $\Psi_i(b_j,u_g)$ be a set of design constraints which are functions of b_j design variables and the displacements u. The design constraints are expressed as

$$\Psi_i(b_r,u_g) < 0 \quad (5)$$

These constraints for example may be a set of bounds on the displacements, a set of tension and compression allowables. When $\Psi_i = 0$, the constraint is active. The variation in Ψ_i is given as

$$\delta\Psi_i = \left\lfloor \frac{\partial\Psi_i}{\partial b_r} \right\rfloor \{\delta b_r\} + \left\lfloor \frac{\partial\Psi_i}{\partial u_g} \right\rfloor \{\delta u_g\} \tag{6}$$

Next consider $\{u\} = \{u(b_r)\}$; then

$$\{\delta u_g\} = \left[\frac{\partial u_g}{\partial b_r} \right] \{\delta b_r\} \tag{7}$$

The variation in Ψ_i then becomes

$$\delta\Psi_i = \left\lfloor \left\lfloor \frac{\partial\Psi_i}{\partial b_r} \right\rfloor + \left\lfloor \frac{\partial\Psi_i}{\partial u_g} \right\rfloor \left[\frac{\partial u_g}{\partial b_r} \right] \right\rfloor \{\delta b_r\} \tag{8}$$

Define the design sensitivity for the i-th constraint and the k-th load case as

$$\lfloor \Lambda_i^{(k)} \rfloor = \left\lfloor \left\lfloor \frac{\partial\Psi_i}{\partial b_r} \right\rfloor + \left\lfloor \frac{\partial\Psi_i}{\partial u_g^{(k)}} \right\rfloor \left[\frac{\partial u_g^{(k)}}{\partial b_r} \right] \right\rfloor \tag{9}$$

The j-th element of $\lfloor \Lambda_i^k \rfloor$ is a total derivative of Ψ_i with respect to the j-th design variable b_j.

Before proceeding further, one important observation regarding $\{\Lambda_i^{(k)}\}$ should be made. In equation (9) $\left\lfloor \frac{\partial\Psi_i}{\partial b_r} \right\rfloor$ represent the variation in Ψ_i while holding u_g fixed. If the constraint Ψ_i is a displacement constraint, then $\left\lfloor \frac{\partial\Psi_i}{\partial b_r} \right\rfloor$ is identically zero. In other words the $\left\lfloor \frac{\partial\Psi_i}{\partial b_r} \right\rfloor$ term applies only to "element" constraints such as stress or stress resultant. For the case of displacement constraints, then, the design sensitivity vector takes the form

$$\lfloor \Lambda_i^{(k)} \rfloor = \left\lfloor \frac{\partial\Psi_i}{\partial u_g} \right\rfloor \left[\frac{\partial u_g}{\partial b_r} \right] \tag{10}$$

The design sensitivity vector for each design constraint is then obtained by use of Equations (9) or (10).

Approach

The problem is to determine the various terms of Equation (9) without requiring explicit functions of design parameters in Equation (3) and for $\left[\frac{\partial \Psi_i}{\partial b_r}\right]$. The solution for this problem would allow MSC/NASTRAN element routines to remain untouched. Consider:

$$[K_{gg} + \Delta K_{gg}]\{u_g + \Delta u_g\} = \{P_g + \Delta P_g\} \tag{11}$$

Expand this equation as a first order variation and use the fact that $[K_{gg}]\{u_g\} - \{P_g\} = \{0\}$ to get

$$[K_{gg}]\{\Delta u_g\} = \{\Delta P_g\} - [\Delta K_{gg}]\{u_g\} \tag{12}$$

In these equations, ΔK_{gg}, Δu_g, and ΔP_g are caused by small variations in the design parameters. We may identify the right-hand side of Equation (12) as an incremental form of Equation (3). Namely

$$[\Delta E_{gr}] = [\Delta P_g(\Delta b_1),\ldots,\Delta P_g(\Delta b_r)] - [\Delta K_{gg}(\Delta b_1)u_g,\ldots,\Delta K_{gg}(\Delta b_r)u_g] \tag{13}$$

Then

$$[K_{gg}][\Delta u_{gr}] = [\Delta E_{gr}] \tag{14}$$

and

$$\left[\frac{\partial u_g}{\partial b_r}\right] \simeq \left[\frac{\Delta u_{gr}}{\Delta b_r}\right] = \left[\frac{\Delta u_{g1}}{\Delta b_1},\ldots,\frac{\Delta u_{gr}}{\Delta b_1}\right] \tag{15}$$

In a similar fashion

$$\left[\frac{\partial \Psi_i}{\partial b_r}\right] \simeq \left[\frac{\Delta \Psi_i}{\Delta b_r}\right] = \left[\frac{\Delta \Psi_i}{\Delta b_r},\ldots,\frac{\Delta \Psi_i}{\Delta b_r}\right] \tag{16}$$

Then for example for stress constraints

$$\lfloor \Lambda_i^{(k)} \rfloor \simeq \left[\frac{\Delta \Psi_i}{\Delta b_1},\ldots,\frac{\Delta \Psi_i}{\Delta b_r}\right] \pm \lfloor S_{ig} \rfloor \left[\frac{\Delta u_{g1}}{\Delta b_1},\ldots,\frac{\Delta u_{gr}}{\Delta b_r}\right] \tag{17}$$

Theory

The calculation of the normal modes and buckling design sensitivity are based on the first order variation of the following system equations:

$$[K_{gg}]\{\phi_g\} = \lambda[M_{gg}]\{\phi_g\} \qquad \text{(Normal Modes)}$$
$$[K_{gg}]\{u_g\} = -\lambda[K^d_{gg}]\{u_g\} \qquad \text{(Buckling)} \tag{18}$$

In Equation $(18)_1$, $[M_{gg}]$ is the mass matrix and $\{\phi_g\}$ is the mode shape corresponding to the eigenvalue λ. In Equation $(18)_2$, $[K^d_{gg}]$ is the differential stiffness.

In what follows the internal loads used in the differential stiffness are restricted to remain constant. With this restriction in mind, we may consolidate Equation (18) into the equation

$$[K_{gg}]\{\phi_g\} = \lambda[M_{gg}]\{\phi_g\} \tag{19}$$

Here, ϕ_g represents either the dynamic mode shape or the buckling mode shape, M_{gg} represents either the mass matrix or the differential stiffness, and λ is either plus or minus depending on whether a normal mode or buckling analysis is being considered.

Next multiply Equation (19) by $\lfloor\phi_g\rfloor$ and define the following quadratic form

$$\eta(b_r,\phi_g,\lambda) = \lfloor\phi_g\rfloor[K_{gg}]\{\phi_g\} - \lambda\lfloor\phi_g\rfloor[M_{gg}]\{\phi_g\} = 0 \tag{20}$$

Then considering $\eta(b,\phi,\lambda)$ to be a function of b_r (the design variables), ϕ_g (the eigenvalues), and λ (the eigenvalue), the first variation becomes

$$\begin{aligned}\delta\eta(b_r,\phi_g,\lambda) = 2\lfloor\delta\phi_g\rfloor[[K_{gg}]\{\phi_g\} - \lambda[M_{gg}]\{\phi_g\}] \\ + [\lfloor\phi_g\rfloor\frac{\partial[K]}{\partial b_1}\{\phi_g\},\ldots,\lfloor\phi_g\rfloor\frac{\partial[K]}{\partial b_r}\{\phi_g\}]\{\delta b_r\} \\ - \lambda[\lfloor\phi_g\rfloor\frac{\partial[M]}{\partial b_1}\{\phi_g\},\ldots,\lfloor\phi_g\rfloor\frac{\partial[M]}{\partial b_r}\{\phi_g\}]\{\delta b_r\} \\ - \lfloor\phi_g\rfloor[M]\{\phi_g\}\delta\lambda = 0\end{aligned} \tag{21}$$

In the above equation, $\frac{\partial}{\partial b}$ operates only on $K_{gg}(b)$ and $M_{gg}(b)$ and use has been made of the symmetry properties of [K] and [M]. From Equation (19), the $\lfloor\delta\phi_g\rfloor$ coefficients are zero. Define $m = \lfloor\phi_g\rfloor[M_{gg}]\{\phi_g\}$. Then Equation (21) becomes

$$\begin{aligned}\delta\lambda = \frac{1}{m}\,[\lfloor\phi_g\rfloor\frac{\partial[K]}{\partial b_1}\{\phi_g\},\ldots,\lfloor\phi_g\rfloor\frac{\partial[K]}{\partial b_r}\{\phi_g\}] \\ -\lambda[\lfloor\phi_g\rfloor\frac{\partial[M]}{\partial b_1}\{\phi_g\},\ldots,\lfloor\phi_g\rfloor\frac{\partial[M]}{\partial b_r}\{\phi_g\}]]\{\delta b\}\end{aligned} \tag{22}$$

We must emphasize that there is an equation of the form of Equation (22) for each eigenvalue λ.

As was done in Equation (3), we may define the change in nodal equilibrium matrix as

$$\left[\frac{d\bar{E}_g}{db_r}\right]_i = \left[\frac{\partial K}{\partial b_1}\phi_i,\ldots,\frac{\partial K}{\partial b_r}\phi_i\right] - \lambda_i\left[\frac{\partial M}{\partial b_1}\phi_i,\ldots,\frac{\partial M}{\partial b_r}\phi_i\right] \tag{23}$$

for the i-th mode shape.

To express the final results symbolically it is convenient to also define the diagonal matrices

$$[\bar{m}] = \begin{bmatrix} 1/m_1 & & 0 \\ & \ddots & \\ 0 & & 1/m_q \end{bmatrix}$$

and (24)

$$[\Phi] = \begin{bmatrix} [\phi_1] & & 0 \\ & \ddots & \\ 0 & & [\phi_q] \end{bmatrix}$$

Then symbolically, the contribution to the design sensitivity for modal or buckling may be expressed as

$$\lfloor \bar{\Lambda}_i^{(k)} \rfloor = \left\lfloor \frac{\partial \Psi_i}{\partial \lambda} \right\rfloor [\bar{m}][\Phi] \begin{bmatrix} (d\bar{E}_g/db_r)_1 \\ \vdots \\ (d\bar{E}_g/db_r)_q \end{bmatrix} \tag{25}$$

Finally, the eigenvalue constraint equation will be of the form

$$\Psi = \frac{\lambda_i}{\lambda^u} - 1 < 0$$

or (26)

$$\Psi = 1 - \frac{\lambda_i}{\lambda^L} < 0$$

for upper and lower limits on the constraint, respectively. Thus,

$$\left\lfloor \frac{\partial \Psi}{\partial \lambda_i} \right\rfloor = [0,\ldots,\pm 1/\lambda_{1\,\mathrm{limit}},\ldots,0] \tag{27}$$

which is a vector containing all zeros except at the i-th location. Then Equation (25) becomes

$$\lfloor \bar{\Lambda}_i^{(k)} \rfloor = \frac{\pm 1}{m_i \lambda_{i_{\mathrm{limit}}}} \lfloor \phi_i \rfloor \left[\frac{\Delta \bar{E}_{g1}}{\Delta b_1},\ldots,\frac{\Delta \bar{E}_{gr}}{\Delta b_r} \right] \tag{28}$$

Approach

If we consider a small variation in design parameter, we may define in a similar fashion as we did for Equation (11)

$$[K_{gg} + \Delta K_{gg}]\{\phi_g + \Delta\phi_g\} = (\lambda + \Delta\lambda)[M_{gg} + \Delta M_{gg}]\{\phi_g + \Delta\phi_g\} \tag{29}$$

Expand this equation as a first order variation and use Equation (19) to get

$$[\Delta K_{gg}]\{\phi_g\} - \lambda[\Delta M_{gg}]\{\phi_g\} = [M_{gg}]\{\phi_g\}\Delta\lambda + [K_{gg} - \lambda M_{gg}]\{\Delta\phi_g\} \tag{30}$$

The left-hand side of Equation (30) may be identified as an incremental form of Equation (23). Namely, for a specific λ

$$[\Delta\bar{E}_{gr}] = [\Delta K(\Delta b_1)\phi, \ldots, \Delta K(\Delta b_r)\phi - \lambda[\Delta M(\Delta b_1)\phi, \ldots, \Delta M(\Delta b_r)\phi] \tag{31}$$

Then

$$[M_{gg}]\{\phi_g\}\Delta\lambda = [\Delta\bar{E}_{gr}] - [K_{gg} - \lambda M_{gg}]\{\Delta\phi_g\} \tag{32}$$

Multiply Equation (32) by $\lfloor\phi_g\rfloor$ and use the symmetry of K_{gg} and M_{gg} and Equation (19) to obtain

$$\lfloor\phi_g\rfloor[M_{gg}]\{\phi_g\}\Delta\lambda = [\phi][\Delta\bar{E}_{gr}] \tag{33}$$

or

$$\Delta\lambda = \frac{1}{m}[\phi][\Delta\bar{E}_{gr}] \tag{34}$$

Then any component of $d\lambda/db$ is computed from $\Delta\lambda/\Delta b_i$.

Finally the eigenvalue constraint equation will be of the form

$$\Psi = \frac{\lambda_i}{\lambda^u} - 1 \leq 0$$

or (35)

$$\Psi = 1 - \frac{\lambda_i}{\lambda^L} \leq 0$$

for the upper and lower limits on the constraint, respectively. Thus,

$$\left\lfloor \frac{\partial\Psi}{\partial\lambda_q} \right\rfloor = \lfloor 0, \ldots, \pm 1/\lambda_{1imit}, \ldots, 0 \rfloor \tag{36}$$

which is a vector containing all zeros except at the i-th location. Then Equation (25) becomes

$$\lfloor\bar{\Lambda}_i^{(k)}\rfloor = \frac{\pm 1}{m_i \lambda_{i_{limit}}} \lfloor\phi_i\rfloor \left[\frac{\Delta\bar{E}_{g1}}{\Delta b_1}, \ldots, \frac{\Delta\bar{E}_{gr}}{\Delta b_r}\right] \tag{37}$$

CONCLUSION

In conclusion, Design Sensitivity Analysis, as implemented in MSC/NASTRAN, gives the user a very general and convenient tool to solve the design sensitivity problem. The design sensitivity coefficients generated in DSA are useful to the designer as they show how the structure will respond to a proposed design change. In addition, these data are required by many optimization algorithms. Design Sensitivity Analysis is The MacNeal-Schwendler Corporation's first development step toward automated structural design in MSC/NASTRAN.