NONLINEAR PERTURBATION METHODS IN DYNAMIC REDESIGN

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ABSTRACT

Many mechanical systems have poorly placed natural frequencies and undesirable mode shapes. Current methods of dynamic redesign work well for small changes in the eigenproblem. Practical dynamic redesign, however, often involves moderate changes in stiffness. The present research includes all nonlinear terms in an inverse perturbation procedure. Two new approaches allow moderate and large change. One method uses a penalty function with a minimum weight objective and a penalty term containing error in equilibrium for a given vibration mode. The other is a predictor-corrector approach based on an energy balance in a truncated set of modes. Both methods use the finite element code MSC/NASTRAN and require only one finite element analysis--that of the original (baseline) problem. The methods are more powerful than competing Taylor series methods, which have limited radii of convergence.

I. INVERSE PERTURBATION RESEARCH AT THE UNIVERSITY OF MICHIGAN

A research group at The University of Michigan has been developing inverse perturbation methods for use in dynamic redesign. The work is implemented through MSC/NASTRAN, the most capable of the large, commercial finite element codes. The project involves a strong commitment on the part of the research team because of the complexity of the code. It has required a major effort in learning DMAP (Direct Matrix Abstraction Program) procedures.

The University of Michigan was the first participant in the MacNeal-Schwendler program for University use of MSC/NASTRAN. The code was installed in April, 1981 and has been used for many dynamic perturbation studies since then [1].

Two professors and three doctoral students are currently working on perturbation methods. In addition, a visiting scholar from the People's Republic of China (Prof. Bingchen Zhi, Tsinghua University, Beijing) has joined the research group.

The primary goal of this research is to develop methods for dynamic redesign, system parameter identification and model correlation of structural systems. The work is implemented in DMAP (Direct Matrix Abstraction Program) where possible; this should make the results attractive to industry.

Two types of dynamic redesign have been studied. The first is a nonlinear perturbation scheme using a penalty function formulation and conjugate gradient solution [2]. The second method is a nonlinear perturbation using an incremental formulation with a predictor-corrector solution at each increment [3].

System parameter identification studies have been carried out for linear inverse perturbation for offshore marine towers [4]. Current work is underway for structural parameter identification in civil structures, due to earthquake damage. This work will not be discussed here.

II. INTRODUCTION TO PERTURBATION METHODS

The general eigenvalue problem arising in the dynamic analysis of structures is:

$$\underset{n\times n}{[k]}\ \underset{n\times n}{[\Phi]} = \underset{n\times n}{[m]}\ \underset{n\times n}{[\Phi]}\ \underset{n\times n}{[\lambda]} \tag{1}$$

where [k] and [m] are the stiffness and mass matrices and $[\Phi]$,$[\lambda]$ are the eigenvectors and eigenvalues. The matrices are all nxn in size.

Premultiplying Equation 1 by $[\Phi]^T$, an energy balance is obtained,

$$[\Phi]^T[k][\Phi] = [\Phi]^T[m][\Phi][\lambda] \tag{2}$$

If the masses and stiffnesses are changed, the frequencies and mode shapes also change. The equilibrium equation for such a perturbed eigensystem is:

$$\underset{n\times n}{[k']}\ \underset{n\times n}{[\Phi']} = \underset{n\times n}{[m']}\ \underset{n\times n}{[\Phi']}\ \underset{n\times n}{[\lambda']} \tag{3}$$

and the energy equation is:

$$[\Phi']^T[k'][\Phi'] = [\Phi']^T[m'][\Phi'][\lambda'] \tag{4}$$

The perturbed terms are:

$$[k'] = [k] + [\Delta k] \tag{5a}$$

$$[m'] = [m] + [\Delta m] \tag{5b}$$

$$[\Phi'] = [\Phi] + [\Delta\Phi] \tag{5c}$$

$$[\lambda'] = [\lambda] + [\Delta\lambda] \tag{5d}$$

Equations 3-4 can be expanded to show the nonlinearity in the perturbed terms.

$$([k]+[\Delta k])([\Phi]+[\Delta\Phi]) = ([m]+[\Delta m])([\Phi]+[\Delta\Phi])([\lambda]+[\Delta\lambda]) \tag{6}$$

$$([\Phi]+[\Delta\Phi])^T([k]+[\Delta k])([\Phi]+[\Delta\Phi]) = ([\Phi]+[\Delta\Phi])^T([m]+[\Delta m])([\Phi]+[\Delta\Phi])([\lambda]+[\Delta\lambda]) \tag{7}$$

The equilibrium equation 6 is of "third order" in perturbation terms while the energy equation 7 is of "fourth order." (This ordering of terms is not very meaningful--the expansion is not an expansion in terms of a single independent variable, but, rather is a mixture of increments in independent and dependent variables of unequal numerical size.)

Two approaches can be used. First, one can perturb the stiffness and mass of the system and solve for frequencies and mode shape changes which result. This is called a "forward" perturbation. Second, one can specify desired changes in frequencies and mode shapes and determine the changes to the stiffness and mass required to cause the change. This is an "inverse" perturbation. In this study, inverse perturbation will be considered.

Stetson, et al [5] used a linearized version of Equation 7 in their perturbation formulation. We have shown, however [2], that the linear theory is error prone in meeting mode shape goals. It is now believed that nonlinear perturbation equations are preferable for typical dynamic redesign, because practical problems often involve moderate stiffness changes where $[\Delta k][\Delta\phi]$ is numerically important.

Structural changes

The structural changes can be decomposed into L element changes. In sheet metal or die-cast systems, many elements are required to change together for manufacturability. In this case, L is the number of groups of perturbed elements.

$$[\Delta k]_{system} = \sum_{e=1}^{L} [\Delta k_e] \tag{8}$$

$$[\Delta m]_{system} = \sum_{e=1}^{L} [\Delta m_e] \tag{9}$$

Furthermore, each element change can be expressed as a fractional change from the original system (or a sum of terms as needed to separate bending, stretching, and torsion).

$$[\Delta k_e] = [k_e]\alpha_e^k \tag{10}$$

$$[\Delta m_e] = [m_e]\alpha_e^m \tag{11}$$

These relations may be linear or nonlinear. For example, the effect of plate thickness on stretching stiffness is linear, while the effect on bending is of third order. The relations for mass changes are linear. The range of the structural changes can be specified by inequality constraints.

Characteristic changes

The perturbed eigenvalue

$$\lambda_i' = \lambda_i + \Delta\lambda_i \tag{12}$$

is defined

$$\equiv \omega_i^2 + \Delta(\omega_i^2) \tag{13}$$

The perturbed eigenvector in Equation 5 is discussed here in terms of a single eigenvector change

$$\{\Delta\phi\} = \begin{matrix} \Delta\overset{\circ}{\phi} \\ \Delta\bar{\phi} \end{matrix} \tag{14}$$

where $\{\Delta\overset{\circ}{\phi}\}$ and $\{\Delta\bar{\phi}\}$ are the specified and the unspecified degrees of freedom respectively. Usually, the the number of specified terms $\{\Delta\overset{\circ}{\phi}\}$ are fewer than $\{\Delta\bar{\phi}\}$. Since iteration schemes are used to determine the design variables, an initial guess for $\{\Delta\bar{\phi}\}$ is needed.

In some perturbation schemes, the perturbed mode can be represented as a linear combination of q mode shapes obtained in the analysis of the baseline system:

$$\underset{n\times 1}{\{\Delta\phi\}} = \underset{n\times q}{[\Phi]}\ \underset{q\times 1}{\{C\}} \tag{15}$$

This formal representation of the perturbed eigenvector, using a truncated set of eigenvectors as a basis, is purely a static relation. The perturbed eigenvectors may lack orthogonality.

Nonlinear iteration approaches are to be followed. The linear versions of the perturbation theory with the transformation in Equation 15 can be used to find initial values of the design variables.

III. NONLINEAR INVERSE PERTURBATION WITH PENALTY FUNCTION FORMULATION

This method has already shown promise for small systems of equations [2] and is now being generalized through dynamic reduction. The basic equations are the perturbed equations of motion 6 where all terms are included in the analysis. The unknowns are the mass and stiffness perturbations $[\Delta m]$, $[\Delta k]$ needed to create desired mode and frequency changes $[\Delta\Phi]$, $[\Delta\lambda]$.

Let us concentrate on the ith mode $\{\phi\}_i$. Any approximate solution to the perturbed equilibrium equation has residual force error

$$\{R\} \equiv [k']\{\phi'\}_i - [m']\{\phi'\}_i\lambda'_i \approx 0 \tag{16}$$

The penalty function is taken as

$$F(\{\alpha\},\{\Delta\bar{\phi}\}_i,\Delta\lambda_i) = f(\{\alpha\}) + \mu P(\{\alpha\},\{\Delta\bar{\Phi}\}_i,\Delta\lambda_i) \tag{17}$$

A good choice for the objective function $f(\{\alpha\})$ is

$$f(\{\alpha\}) = \Sigma W_j\alpha_j \tag{18}$$

corresponding to least weight, where the design variable α_j has associated weight W_j. An alternate objective function is

$$f(\{\alpha\}) = \Sigma\alpha_j^{\,2}$$

which provides the least change from the baseline structure.

The penalty term $P(\{\alpha\},\{\Delta\bar{\phi}\}_i,\Delta\lambda_i)$ has been found [2] to be best chosen as a weighted norm of force unbalance at the nodal degrees of freedom:

$$P \equiv \{R\}^T[\Gamma]_i\{R\} \tag{19}$$

where the weighting matrix $[\Gamma]_i$ is diagonal and its j^{th} component is:

$$(\Gamma_{jj})_i = \phi_{ji}^2 \qquad \text{(The jth component of the ith eigenvector)} \tag{20}$$

The penalty function acts to minimize the error in energy in the particular mode of vibration. The method can be generalized to include more than one mode by including the corresponding error in the penalty term.

One shortcoming of the present method is the need to include unconstrained eigenvector degrees of freedom as unknowns. The total list of unknown variables is $\{\alpha\},\{\Delta\bar{\phi}\}_i$ and λ_i. The unconstrained terms $\{\Delta\bar{\phi}\}_i$ are the terms which cause trouble. A condensation method is currently being employed to reduce the eigenvector degrees of freedom. A dynamic reduction technique is available in MSC/NASTRAN which will allow the reduction of unknowns $\{\Delta\bar{\phi}\}_i$ from thousands to perhaps 20 to 100.

Much needs to be learned in the use of modal condensation methods in dynamic redesign. How does modal truncation effect the results? What are the sources of error, and how does error propagate in the design-redesign cycle? Some preliminary work using dynamic reduction has been done and is discussed in the case study of Section V.

IV. NONLINEAR INVERSE PERTURBATION WITH INCREMENTAL PREDICTOR-CORRECTOR SOLVER

An alternative algorithm to that above is based on an energy formulation. The energy equations for the baseline and objective systems are repeated:

$$[\Phi]^T[k][\Phi] = [\Phi]^T[m][\Phi][\lambda] \tag{2}$$

$$[\Phi']^T[k'][\Phi'] = [\Phi']^T[m'][\Phi'][\lambda'] \tag{4}$$

Relationships between the two systems can be defined by expanding Equation 4 and cancelling the baseline solution (terms of zero order). The result is the general perturbation equation:

$$[\Phi']^T[\Delta k][\Phi'] - [\Phi']^T[\Delta m][\Phi']([\omega^2]+[\Delta(\omega^2)]) = [\delta] \tag{21}$$

where

$$[\delta] \equiv [\Phi']^T[m][\Phi']([\omega^2]+[\Delta(\omega^2)]) - [\Phi']^T[k][\Phi']$$

This equation is nonlinear in terms of the modal quantities $[\Phi']$ and $[\Delta\lambda]$ but is linear with respect to the desired structural changes $[\Delta k]$ and $[\Delta m]$.

Solution of Equation 21 will provide the required structural changes to meet the modal objectives. The direct solution is usually not possible, because it would require the user to specify all objective eigenvectors.

Several approaches have been used to solve Equation 21. The work of Stetson [5] and Sandstrom, et. al. [4] involved a first-order form of the equations. It has been found, however, that first order solutions are only accurate for small modal changes. This limitation motivated the development of this algorithm.

In this work, the solution of the general perturbation equation is to be based on an incremental predictor-corrector technique. The predictor phase uses the solution procedure for the first-order equations to provide an approximation to the element changes [Δk] and [Δm]. Using these approximate element changes, approximate perturbed eigenvectors are calculated. In the corrector phase, these eigenvectors are used in the general (nonlinear) perturbation equation 21 to correct the element changes. The solution is diagrammed in Figure 1. The predictor-corrector solution requires only linear equation solvers when least weight (a linear relation) is used as an objective.

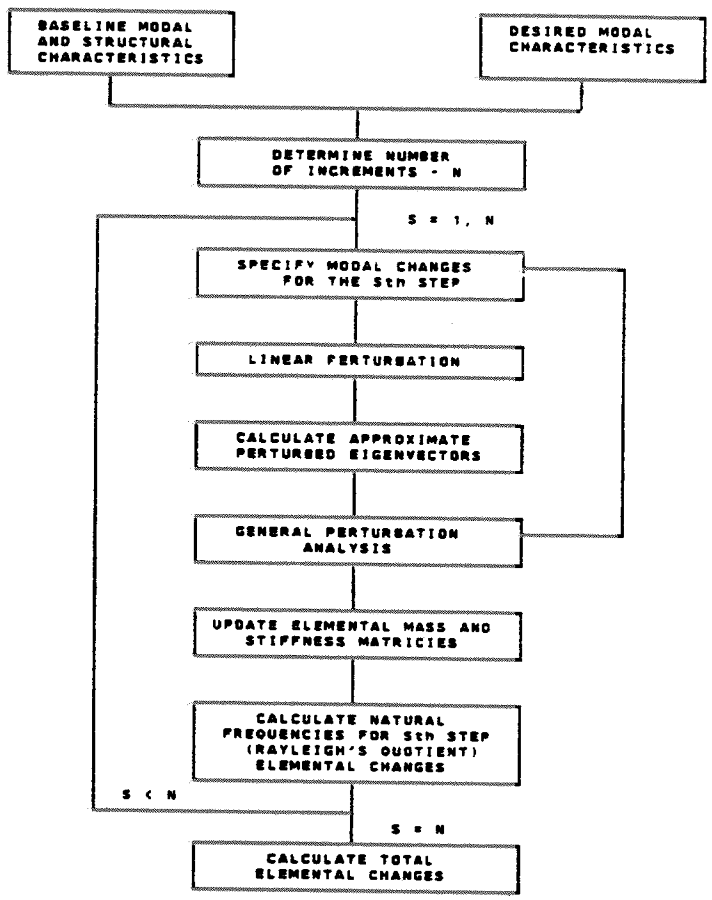

FIGURE 1. Incremental Predictor-Corrector Solution.

Preliminary results with this method look very promising. The method has been applied to a stiffened plate structure discussed in the next section.

V. CASE STUDY
FREQUENCY CONTROL FOR A CASTING

The University of Michigan has asked its faculty to carry out small demonstration projects aimed at revitalizing Michigan industry. The program was sponsored under the Center for Robotics and Integrated Manufacturing (CRIM). The research team proposed to try our perturbation schemes on a small aluminum casting used in the electronics industry. As a result, $10,000 was awarded for the frequency optimization of the mainframe casting for an Irwin-Olivetti micro-Winchester disk drive (Figure 2). The

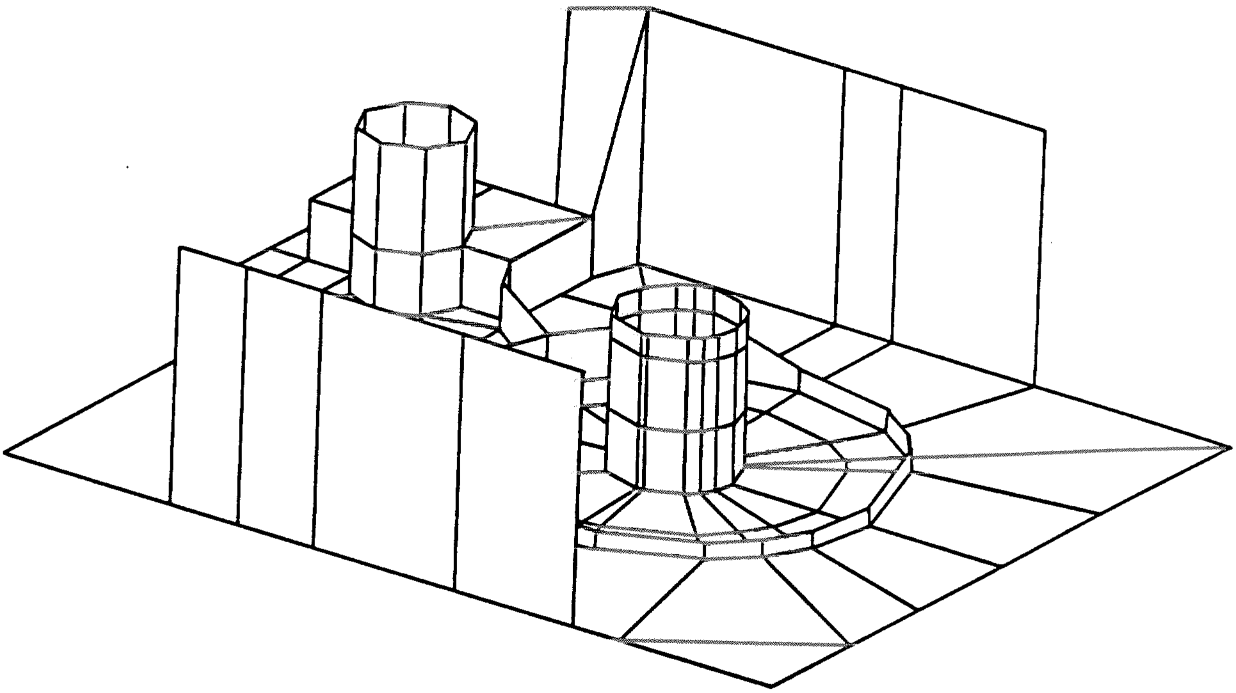

FIGURE 2. Casting for micro-Winchester disk drive.

work has successfully raised the lowest natural frequency of the casting by 30%, through redistribution of material [3]. The modal analysis of the casting was done with QUAD4, TRI3 and BAR elements with a total of 312 elements and 1075 d.o.f. The thicknesses of the plate elements were the structural design variables. The BAR elements were held constant during the redesign. Design variable linkage is used to cause regions of the casting to have the same thickness changes, for manufacturability (Figure 3).

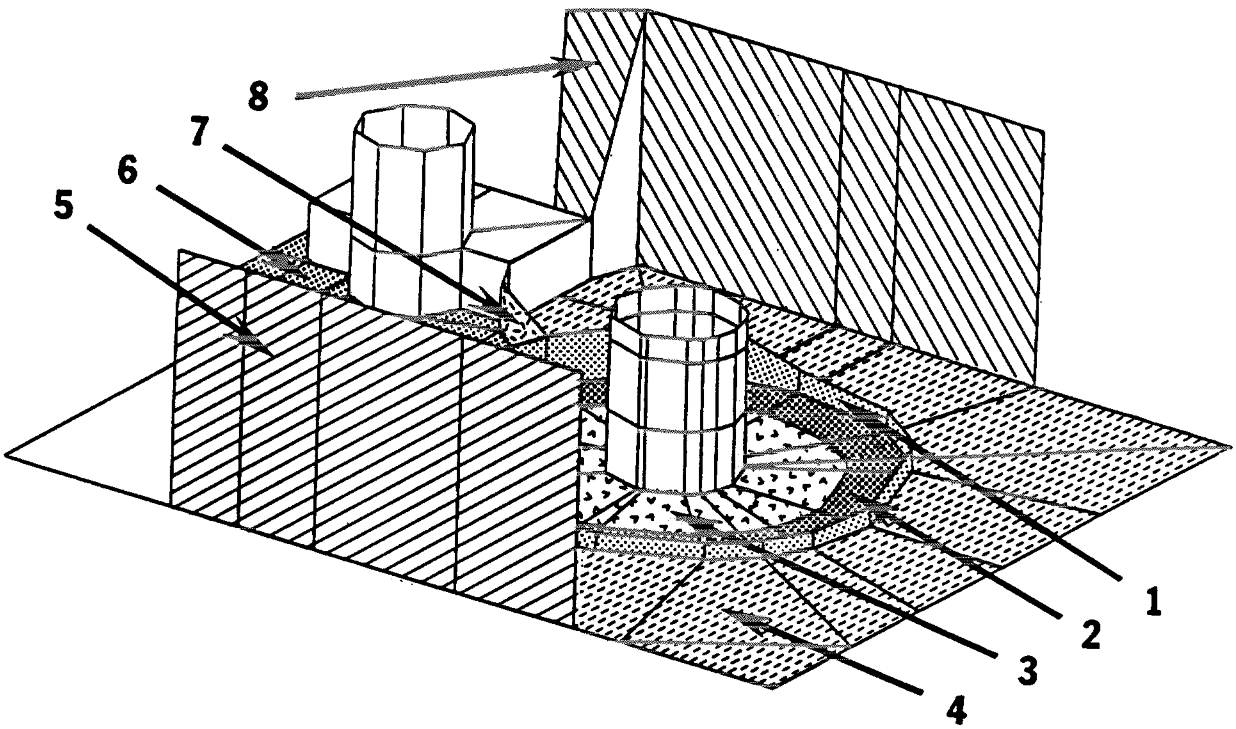

FIGURE 3. Regions with linked plate thicknesses [3].

The goal of the study was to raise the fundamental frequency of the casting from 351 to 456 hz. The baseline fundamental mode is given in Figure 4. This is a twisting mode, where the casting is approximately a channel section and is weak in torsion.

For a problem of this size (or larger), the conjugate gradient solver requires dynamic reduction to reduce the number of d.o.f. to 100 or less for a modest cost solution. At the time of this conference, the nonlinear inverse perturbation with conjugate gradient solver is achieving only 80% of the desired frequency change. The cost is low, with the optimization postprocessing adding only 50% to the cost of the baseline solution. Nevertheless, the algorithm needs to be "tinkered with" for more accuracy. The gain in frequency was combined with a minimum-weight goal, and weight was held almost constant under the redesign.

The nonlinear inverse perturbation method with incremental predictor-corrector method has also been applied to the casting, and is giving mixed results [3]. For unconstrained structural design variables, the frequency goal has been met with less than 1% error in the 105 hz

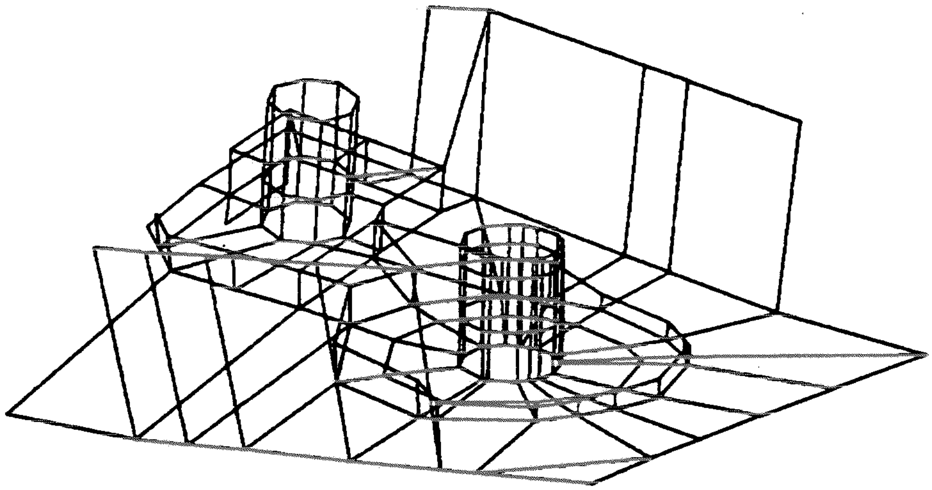

FIGURE 4. Fundamental mode shape of casting. Twisting mode.

change desired. This solution was done with a least change objective. When a severely constrained (real-world) redesign was carried out, the optimization led to achievement of 80% of the frequency goal (surprisingly similar to the results by the other method). The real-world redesign involved extensive design variable linkage, including a condition that regions must stay within 25% of the same thickness so that the casting would cool evenly. It is believed that much of the error in this method is due to an approximation in the eigenvector.

VII. CONCLUDING REMARKS

Efficient algorithms are being developed for the penalty-function approach and the predictor-corrector approach to dynamic redesign. Preliminary results for simple problems (with frequency control of plate-like structures only) show the new methods to have more accuracy for moderate change than linear approaches.

Control of propagation of error through these algorithms is important. For the design problem, error can be due to round-off, ill-conditioned matrices and unstable algorithms. For the identification problem and model correlation, there also is experimental error of the measured frequencies and modes in the perturbed structure.

Development of the algorithms above depended on the MSC/NASTRAN DATA PROCESSOR (NDP), which allows manipulation of element stiffnesses, masses and connectivities. At the present time, two software packages are available from The University of Michigan for dynamic redesign. These are NDP (MSC/NASTRAN Data Processor) and LDRUM (Linear Dynamic Redesign, University of Michigan). The NDP program extracts information from MSC/NASTRAN and reformats it for easier processing. The LDRUM program is a linear inverse perturbation redesign package. Both codes are written in Fortran IV for installation on the AMDAHL (IBM compatible). The source codes can be purchased for $80 and $120, respectively, by writing to Professor Anderson.

VII. ACKNOWLEDGMENT

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