PREDICTION OF DAMPING IN STRUCTURES WITH VISCOELASTIC MATERIALS

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ABSTRACT

This paper describes an efficient method for finite element modeling of structures containing a viscoelastic material. Modal damping ratios are estimated from undamped normal mode results by means of the modal strain energy (MSE) method. Comparisons are given between results obtained by the MSE method implemented in MSC/NASTRAN, by various exact solutions for approximate governing differential equations, and by experiment. Results are in terms of frequencies, modal damping ratios, and mechanical admittances for simple beams, plates, and rings, as well as for an actual hardware application. Use of the finite element/MSE method in design of integrally damped structures is discussed.

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1.0. INTRODUCTION

Increasing the damping of a structure can often improve performance under dynamic load. A rational design process requires a method for predicting the damping values that can be expected for a given structural configuration. However, the prediction of damping has historically been largely an art, or at best, a specialized science requiring a greater investment of time and effort than is practical within most design projects.

Predictions of dynamic response in finite element analysis are most often made by the simple expedient of choosing a single structural loss factor to characterize all modes of vibration or, at most, assigning loss factors based on the natural frequency of each mode. The values themselves are obtained emperically or are otherwise based on the judgement and experience of the user. Uncertainties in loss factor are simply ignored or are accounted for by running the analyses for a range of values.

This approach is probably reasonable for situations where the damping is accidental, i.e. due to mechanisms not under the control of the designer. Phenomena such as hysteretic losses in metals, air pumping and friction in joints, or acoustic radiation invariably lead to some measurable amount of damping. However, it is often too little and too unpredictable to produce a satisfactory design. In these cases the addition of damping by the use of viscoelastic materials can be highly effective. Damping-by-design has proven valuable in many applications, particularly in the aerospace industry. Methods have recently been developed for predicting damping in structures with integral or add-on viscoelastic treatments.

Several mathematical techniques for dynamic analysis of structures containing viscoelastic materials (v.e.m.'s) have been implemented using finite elements. All make use of the correspondence principal of viscoelasticity. That is, the Young's modulus and shear modulus of the v.e.m. are treated as complex quantities. The ratio of the imaginary part to the real part is called the loss factor or loss tangent and is a measure of the materials ability to dissipate vibrational energy.

One technique in particular, the modal strain energy (MSE) method, has been developed by the authors with design work in mind. It has proven to be accurate and flexible in both theoretical studies and practical applications. This paper is an overview of the authors' work over the last several years in developing and using the method.

Layered dampers have played an important role as test cases in developing the MSE method. This parallels their practical importance where they have historically been one of the most weight-effective methods of using v.e.m.'s. A constrained layer damper is formed by sandwiching a thin layer of v.e.m. between two metal or composite face sheets. Bending of the sandwich then causes shearing strain in the core which dissipates the energy of vibration.

Since much of the difficulty of analyzing damping in structures stems from complicated geometries, it is natural to look to finite element methods for solutions, just as they are used for analysis of general undamped structures. In this paper, several approaches to damped structural design are examined in the context of implementation by MSC/NASTRAN. A modeling method suitable for three layer sandwiches and other configurations is described and details of its use are discussed.

Three methods are reviewed for forming and solving the equations of motion for structures with viscoelastic materials. Details of modeling layered structures with MSC/NASTRAN are discussed as well as solution methods. The MSE method is described in terms of examples where results are compared to known solutions. Finally, a case history is described and directions of ongoing work are discussed.

2.0 DISCRETIZED EQUATIONS OF MOTION

Three distinct decisions must be made in arriving at response predictions for a structure containing viscoelastic materials:

-- What form should the discretized equations of motion take?

-- What type of elements should be used in modeling the structure?

-- How should the equations of motion be solved?

Current methods for finite element analysis of damped structures can generally be placed in one of three categories depending on how the first of these questions is answered. The three methods are briefly described in this section along with the advantages and disadvantages of each for design purposes.

All three of the methods use, to some extent, the idea of treating the elastic constants of a viscoelastic material as complex quantities. This very useful notion is often misunderstood. It derives simply from the use of complex arithmetic to keep track of relative phase between stress and strain under deformations that vary sinusoidally in time. The idea is obviously extendable to non-sinusoidal motion for linear systems by the use of Fourier transform theory to represent arbitrary time histories as sums of sinusoids. Crandall [1,5] has shown that the notation can, if taken too literally, lead to implications that are physically impossible. The point is a subtle one and is mentioned here only in passing.

2.1 Complex Eigenvalue Method

Suppose the discretized equations of motion take one of the following two forms

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{l(t)\} \qquad (1)$$

or

$$[M]\{\ddot{x}\} + [K1]\{x\} + i[K2]\{x\} = \{l(t)\} \qquad (2)$$

where

$[M],[C],[K]$ = physical coordinate mass, damping and stiffness matrices (all real and constant)

$[K1],[K2]$ = real and imaginary parts of the stiffness matrix calculated with complex material constants

$\{x\},\{\dot{x}\},\{\ddot{x}\}$ = vectors of nodal displacements, velocities, and accelerations

$\{l\}$ = vector of applied node loads

The solution for either form of the equations of motion can be carried out in terms of damped normal modes [2-3]. Both the eigenvalues and eigenvectors will in general be complex but the method is nonetheless quite standard in that the modes obey an orthogonality condition and thus allow uncoupled equations of motion to be obtained.

The complex eigenvalue method has three important drawbacks. It is computationlly expensive; typically three times the cost of an undamped solution of the same order [4]. In MSC/NASTRAN, one also does not have the same sophisticated spill logic as for real eigenvalue analysis and thus the maximum problem size is smaller. Finally, for a structure to be described by Eq. (1) or (2), its materials, including any viscoelastics, must have dynamic stress-strain behavior of a certain type. Eq. (2) implies that both storage and loss moduli are constant. Eq. (1) requires that storage moduli be constant and loss moduli increase linearly with frequency [5]. Real viscoelastic materials simply do not behave in such accommodating ways. Storage moduli tend to increase monotonically while loss moduli exhibit a single, mild peak [6].

2.2 Direct Frequency Response Method

If the applied load varies sinusoidally in time, energy dissipation in the structure can be accounted for by treating the elastic constants of any or all of the materials as complex quantities which are functions of frequency and temperature. These material properties are presumably available from sinusoidal tests. If the structure is linear, its response will be sinusoidal with frequency equal to the driving frequency, and the steady-state equations of motion will have the form:

$$[-\underset{\sim}{M}\omega^2 + \underset{\sim}{K}_1(\omega) + i\underset{\sim}{K}_2(\omega)]\ \underset{\sim}{X}(\omega) = \underset{\sim}{L}(\omega) \qquad (3)$$

where

$\underset{\sim}{K}_1(\omega)$, $\underset{\sim}{K}_2(\omega)$ = stiffness matrices calculated using the real and imaginary parts of the material properties, respectively

ω = radian frequency of excitation

$L(\omega)$, $X(\omega)$ = complex amplitude vectors of applied node loads and responses, respectively

$i = \sqrt{-1}$

There are several drawbacks to the direct frequency response (DFR) method. It is computationally expensive because a general sinusoidal solution requires that the displacement impedance matrix (the bracketed quantity in Eq. (3)) be re-calculated, decomposed, and stored at each of many frequencies. Further, the method does not give information of direct use in improving a candidate design.

The costliness of the direct frequency response method indicated by Eq. (3) is a direct result of the restriction to physical coordinates (as opposed to modal coordinates). This restriction is caused by the form of the corresponding time domain representation. General convolution integral relations between forces and displacements must be admitted in order to accommodate the variation of [K1] and [K2] with frequency that is observed in real viscoelastic materials. Since not even the form, let alone the parameter values, of the convolution relation is generally known, it must be represented numerically in the frequency domain in terms of its Fourier transform. A tabular frequency representation can be arbitrarily accurate as long as the underlying stress-strain operator is linear. However, the use of such a data format is costly, particularly if a high level of frequency resolution is required.

The most reasonable application of the DFR method in design work is in making final predictions of frequency response after the materials and geometry of the damping treatment have been selected using the modal strain energy method. The ability to account exactly for the frequency dependence of material properties will generally yield some improvement in accuracy over the MSE method. This may justify its higher cost if only a small number of functions with moderate frequency resolution are required,

Direct frequency response analysis is possible in MSC/NASTRAN as explained in section 2.11-2 of the Applications Manual. A frequency-dependent complex shear modulus may be specified in terms of tables. These are prepared in a particular format to include both frequency dependent damping for each element material as well as additional damping which is constant with frequency.

2.3 Modal Strain Energy Method

The modal strain energy method is an approximation to the more expensive complex eigenvalue method. It has been developed by the authors with the intent of producing a tool which is accurate, flexible, and yet usable in day-to-day design analysis.

The essence of the method is that it does not attempt to calculate the damping matrix [C] of Eq. (1) nor the imaginary stiffness [K2] of Eq. (2). Rather, it avoids the use of complex matrices entirely by assuming that the real mode shapes obtained by suppressing the term [C]{x} in Eq. (1) or [K2]{x} in Eq. (2) are approximations to the true complex mode shapes. This leads to a simple approximate formula for calculating structural loss factor for each mode based on its shape [7]. Comparisons with true complex eigenvalue solutions, both differential and finite element, have shown the approximation to be reasonable even for values of material loss factor in excess of unity [7,8]. The damped structure is thus represented in terms of its undamped mode shapes with appropriate damping terms inserted into the uncoupled modal equations of motion. That is:

$$\ddot{\alpha}_r + \eta^{(r)}\omega_r\dot{\alpha}_r + \omega_r^2\alpha_r = \ell_r(t) \qquad (4)$$

$$\underset{\sim}{x} = \sum \underset{\sim}{\phi}^{(r)}\alpha_r(t) \qquad (5)$$

$$r = 1, 2, 3 \ldots$$

where

α_r = r'th modal coordinate

ω_r = natural radian frequency of the r'th mode

$\underset{\sim}{\phi}(r)$ = r'th mode shape vector of the associated undamped system

$\eta^{(r)}$ = loss factor of the r'th mode

It is implied that the physical coordinate damping matrix [C] of Eq. (1) need not be explicitly calculated but that it can be diagonalized, at least approximately, by the same real modal matrix that diagonalizes [K] and [M].

The modal loss factors are calculated by using the undamped mode shapes and the material loss factor for each material. This general approach was first suggested by Ungar and Kerwin [9] in 1962. Its application by finite element methods was first suggested by Rogers [8]. The material loss factor of the metal face sheets of a sandwich structure is very small compared to that of the viscoelastic core. In this situation the modal loss factor is found from [7]

$$\eta^{(r)} = \eta_v \frac{V_v^{(r)}}{V^{(r)}} \tag{6}$$

where

η_v = material loss factor of viscoelastic core evaluated at the r'th calculated resonant frequency

$\frac{V_v^{(r)}}{V^{(r)}}$ = fraction of elastic strain energy attributable to the sandwich core when the structure deforms in the r'th mode shape

The calculation of the modal energy distributions fits quite naturally within finite element methods and is a standard option in MSC/NASTRAN. The basic advantages of the method are that only undamped normal modes need be calculated and that the energy distributions are of direct use to the designer. They lead to an optimum choice of viscoelastic material as well as the optimum location for it. The disadvantage is that some approximation is required to accommodate frequency-dependent material properties.

3.0 FINITE ELEMENT ANALYSIS OF THREE LAYER SANDWICHES

3.1 Choice of Elements

Regardless of the solution method to be employed, modeling of sandwich structures requires that the strain energy due to shearing of the core be accurately represented. To be practical, a modeling method must do this without incurring an unacceptable increase in computation cost relative to a uniform, single layer model. A modeling method is described in this section which is reasonably efficient and has the important advantage of being readily implemented with MSC/NASTRAN.

Figure 1 shows the arrangement for modeling of a three layer sandwich. The face sheets are modeled with quadrilateral or triangular plate elements producing stiffness at two rotational and three translational degrees of freedom per node. The viscoelastic core is modeled with solid elements producing

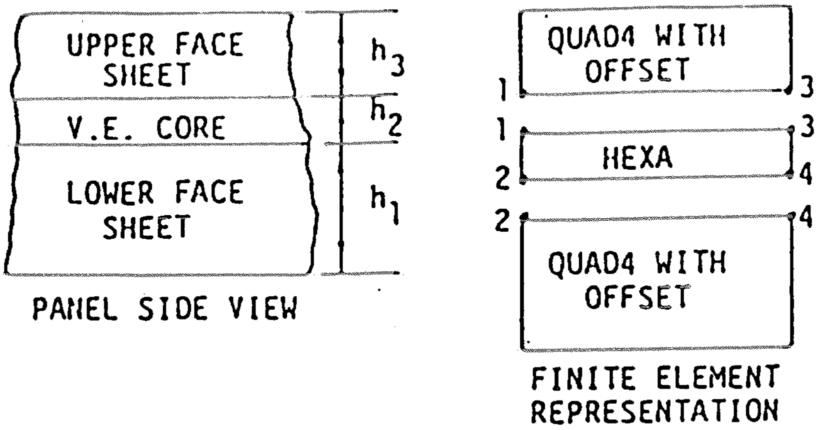


Figure 1 Finite element modeling of a sandwich panel with viscoelastic core.

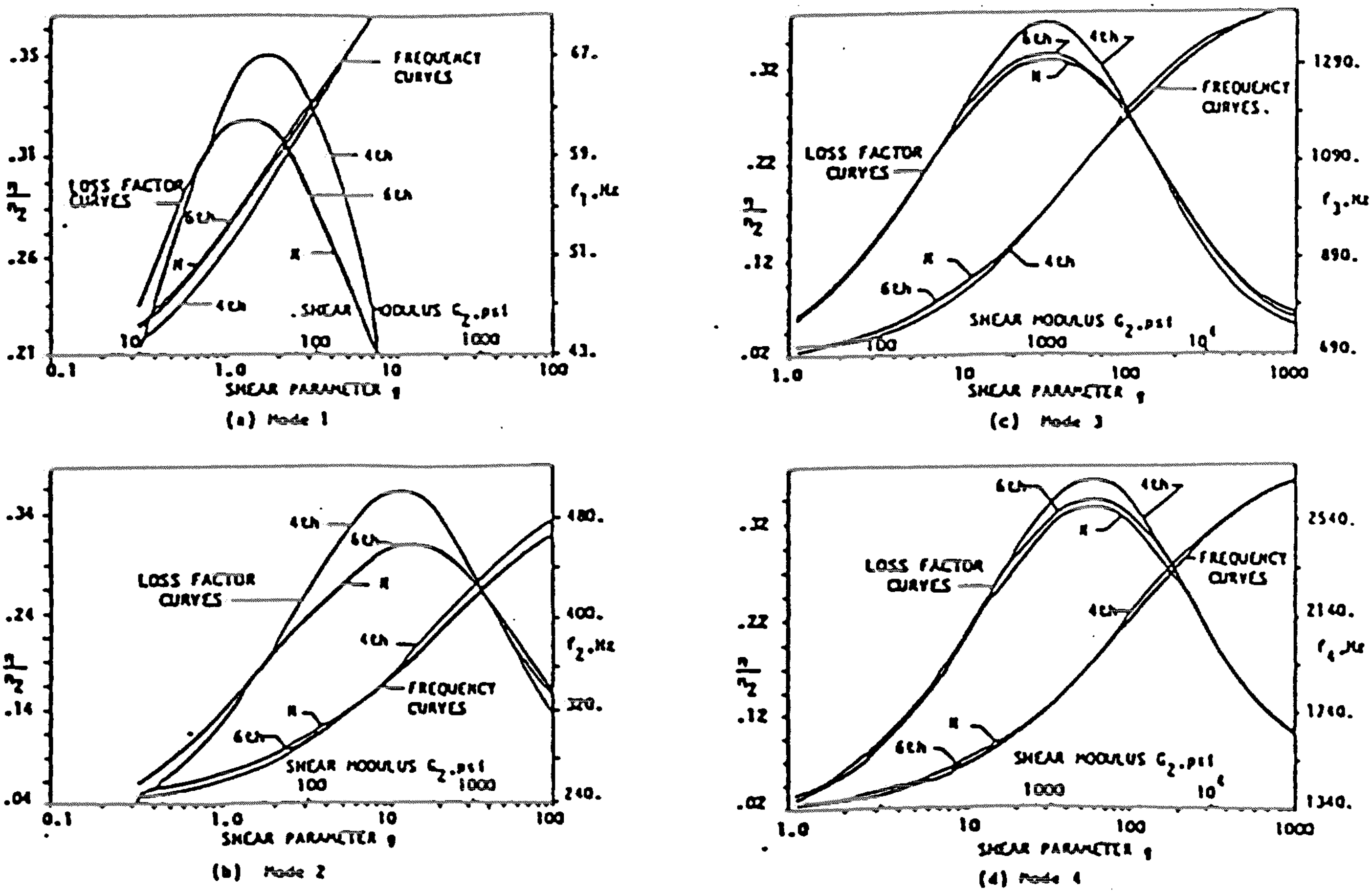


Figure 2 Modal loss factors and natural frequencies of sandwich cantilever beam as computed by 4th Order Theory, 6th Order Theory, and NASTRAN

stiffness at three translational degrees of freedom per node. All nodes are at element corners. The plate elements used are TRIA3's, TRIA6's, QUAD4's, and QUAD8's, and the solid elements are PENTA's and HEXA's. A key feature of the plate elements in the present application is their ability to account for coupling between stretching and bending deformations [10]. This allows the plate nodes to be offset to one surface of the plate, coincident with the corner nodes of the adjoining solid elements. In this way a three layer plate can be modeled with only two layers of nodes. The technique is explained in the Applications Manual under the TRIA3 and QUAD4 elements. Alternately, one may use the PCOMP property card normally employed for layered composites. A two-layer composite is defined but with a near-zero modulus specified for one layer. The other layer then represents the offset plate element.

Very large aspect ratios (in-plane dimension/thickness dimension) may be used for the solid elements in the core layer because the strain field is usually quite simple. Values as high as 5000/1 have been used successfully and are in fact necessary since core layers are often only a few mils thick. Poisson's ratio for the core material is taken as 0.49 for all examples described here.

3.2 Reduction of Equations of Motion

In all but the smallest problems, some form of dynamic reduction must be employed to reduce the number of degrees-of-freedom in the analysis. Either Guyan reduction or generalized dynamic reduction may be used. In Guyan reduction, the d.o.f. to be retained are selected by the user prior to calculation of eigenvalues. As usual in vibration analysis, some care is warranted in this selection. Some displacements should be retained for both face sheets, although it is not necessary to keep both upper and lower face displacements at any single in-plane location on the model. This result is somewhat surprising in that virtually all sandwich panel theories assume the transverse displacments of the face sheets to be equal. If out-of-plane displacements of only one face sheet are kept, the results for natural frequency as well as core-to-total energy ratio show a pronounced dependence on the Poisson's ratio of the core. Although such a dependence is real for some cases, such as doubly curved shells, it should not occur for simpler cases such as straight sandwich beams--and in fact does not occur if the rule given above is observed in performing the reduction.

3.3 Solution Method

Once the model is assembled, either direct frequency response or modal strain energy analysis can be performed. In the latter, a standard normal mode extraction run is made with all material constants treated as real and constant. The elastic strain energy in each element for each mode is calculated as well as the energy fraction in the viscoelastic core for each mode. These fractions multiplied by the core material loss factor give the modal loss factors which are input via a damping vs. frequency table for use in subsequent forced response calculations.

When the modal strain energy method (or any normal mode method) is used, the modal properties are obtained from system matrices which are assumed to be constant. Viscoelastic materials, however, have storage moduli which vary significantly with frequency. There is no theoretically correct way of resolving this contridiction. Nonetheless, normal mode methods have great practical advantages, both for making response predictions and for suggesting design improvements. Real normal modes, uncoupled but with damping, have been found to be reasonably accurate if a simple correction is made to the modal loss factors obtained by Eq. (6). The correction is obtained as follows.

For broadband excitation, most of the response of a given mode occurs within a narrow band around the mode's natural frequency. It is natural then to require that the energy distribution used to compute the loss factor for a given mode be obtained using a stiffness matrix evaluated for material properties taken at that mode's frequency. Because the natural frequencies themselves depend on material properties, an iterative solution of two simultaneous relations (the eigenvalue problem for each mode number and the material property vs. frequency relation) is required. The procedure is not difficult and is explained in Ref [8]. However, a further problem remains. The final modal coordinate representation of the structure must come from a single stiffness matrix evaluated using a single value of storage modulus for the core material. Natural frequencies, mode shapes, and modal masses will be correct for, at most, one mode. A further correction of the modal loss factor has been found to give some improvement.

Each modal equation of motion has the form given in Eq. (4). At resonance the first and last terms on the left cancel each other. The response magnitude is inversely proportional to the product $\eta^{(r)}\omega_r$ which is the coefficient of the modal velocity. If $\eta^{(r)}$ is altered to correct for the error in ω_r, an improvement in peak response may be expected although resonance will still occur at a slightly shifted frequency and some small error will remain due to ℓ_r which depends on modal mass. In test cases run for sandwich beams [8], it was found that taking ω_r to be proportional to the square root of G2 (G2 = core shear modulus) would improve the agreement between the MSE method and the direct frequency response method. This is of course an approximation since ω_r depends on properties of the face sheets as well as the core. The modal damping ratios are adjusted according to

$$\eta^{(r)'} = \eta^{(r)} \sqrt{\frac{G_2(f_r)}{G_{2,ref}}} \qquad (7)$$

where

$\eta^{(r)'}$ = adjusted modal damping ratio for the r'th mode

$\eta^{(r)}$ = modal damping ratio for the r'th mode obtained by iteration

$G_{2,ref}$ = core shear modulus used in final normal modes calculation to obtain modal frequencies, shapes, and masses

$G_2(f_r)$ = core shear modulus at $f = f_r$ where f_r is r'th mode frequency calculated with $G_2 = G_{2,ref}$

4.0 EXAMPLES

The modal strain energy method implemented in MSC/NASTRAN has been applied to a number of simple structural elements for which other solutions are available. Tests cases for sandwich beams, rings, and plates are described in this section.

4.1 Sandwich Beams

Sandwich beams have been analyzed by a number of authors. DiTaranto [11] derived a sixth order differential equation for vibration of a general three-layer beam. Rao [12] obtained complex eigenvalue solutions of this equation for a complex shear modulus of the core with various boundary conditions. It is convenient to use these results as test cases in that the complex eigenvalue solution yields modal loss factors which can be compared directly to those obtained by the MSE method. The assumptions of the sixth order derivation are consistent with those of the finite element model except, in the differential analysis, the rotations and out-of-plane displacements of the upper and lower face sheets are taken to be equal. This is quite reasonable for a thin core layer which tightly couples the face sheets.

A second, more widely known analytical solution is also available for three-layer beams [13]. It is based on the usual fourth order differential equation for flexural vibration of a uniform beam but with the sandwich construction accounted for in terms of an equivalent complex bending stiffness. Core shear is not explicitly retained as a dependent variable and therefore cannot be prescribed at boundaries. Mode shapes are simply assumed to be sinusoidal and general boundary conditions are not considered.

Figure 2 shows a comparison between results for a cantilever sandwich beam as obtained from sixth order theory, fourth order theory, and the MSE method using a NASTRAN model having 20 elements in the lengthwise direction. The 17.78 cm (7 in.) long cantilever beam has equal aluminum face sheets 1.52 mm (0.060 in.) thick, and a viscoelastic core 0.127 mm (0.005 in.) thick. Results are given in terms of loss parameter (composite loss factor normalized on the material loss factor of the core) and in terms of natural frequency for each of the first four modes. The loss parameter is obtained from the finite element MSE results simply as the ratio of core-to-total elastic strain energies (Eq. 6) and thus does not require specification of core material loss factor. The quantity η_2 is also called η_v in Eq.(6).

The shear parameter g, shown as the abscissa in Figure 2, is a normalized shear modulus for the core material. It is a real quantity defined by:

$$g = \frac{G_2^*}{(1 + j\eta_2)} \frac{A_2 L^2}{t_2^2} \frac{(E_1 A_1 + E_3 A_3)}{E_1 A_1 E_3 A_3} \qquad (8)$$

where

G_2^* = complex shear modulus of core material

η_2 = loss modulus of core material (η_v)

A_2 = cross-sectional area of core

t_2 = thickness of core

L = beam length

E_1, E_3 = elastic moduli of face sheets

A_1, A_3 = cross-sectional area of face sheets

The quantity $g(1+j\eta_2)$ occurs as a coefficient in the nondimensional sixth order differential equation of motion [12].

A value of core material loss factor much smaller than unity (η_2 =0.01) was used in obtaining the fourth and sixth order results of Figure 2. Figure 3 illustrates the effect of core material loss factor on composite loss factor for the fairly high values of η_2 which one would wish to use in practice. It may be seen that η increases almost linearly with η_2. The MSE theorem implies an exactly linear relationship which is close enough for most practical purposes. Figure 4 illustrates a test case where results obtained by the MSE and DFR methods are compared for the case of the sandwich cantilever beam of Figure 2. The dashed curve was obtained by the direct frequency response method using constant material properties. The solid line was calculated using the first six modes with modal damping ratios obtained via the modal strain energy method (Eq. (6)). A fairly high value of material loss factor was used in both methods. It may be seen that the difference is quite small even in the neighborhood of resonance.

Figure 5 shows the strain energy distribution in the cantilever beam of Figure 2 for various values of core shear modulus. In this case, both the top and bottom face sheets are restrained at the root of the beam and, therefore, the core shearing strain is zero at this location. A plot of strain energy distribution is very helpful to the designer since it indicates where the energy dissipation is occurring for each mode and thus where the viscoelastic is doing some good.

A comparison of results from the MSE method and the complex eigenvalue method (constant complex stiffness), both implemented in MSC/NASTRAN, is shown in Table 1 for the beam geometry of Figure 2 and a core shear stiffness of .623 MPa (65 psi). It indicates that $\eta^{(r)}$ and η_v are almost exactly proportional over a wide range of η_v and agree quite well with MSE results.

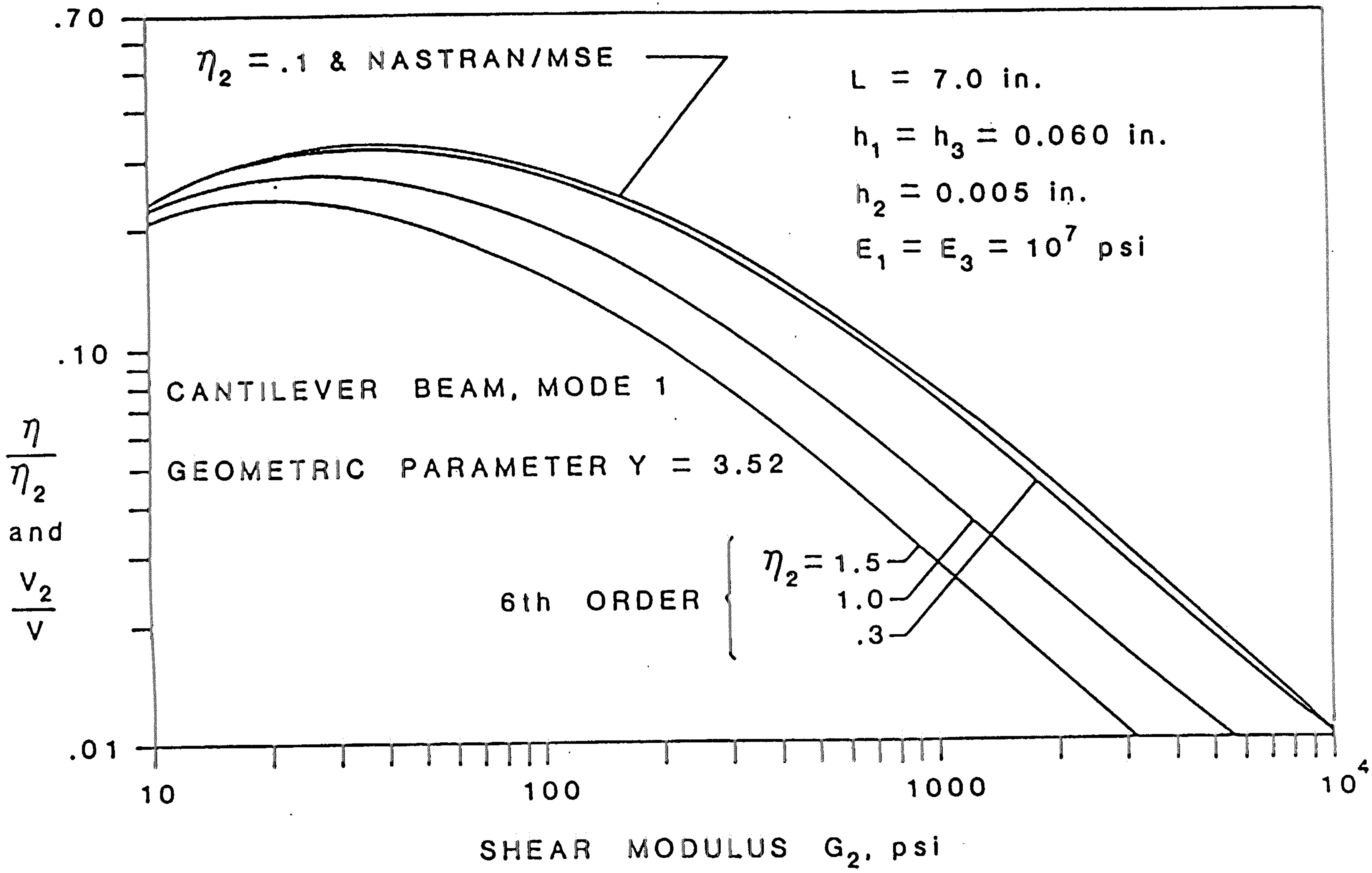


Figure 3 Composite loss factor for various core loss factors as computed by Modal Strain Energy Method (NASTRAN) and 6th Order Beam Theory.

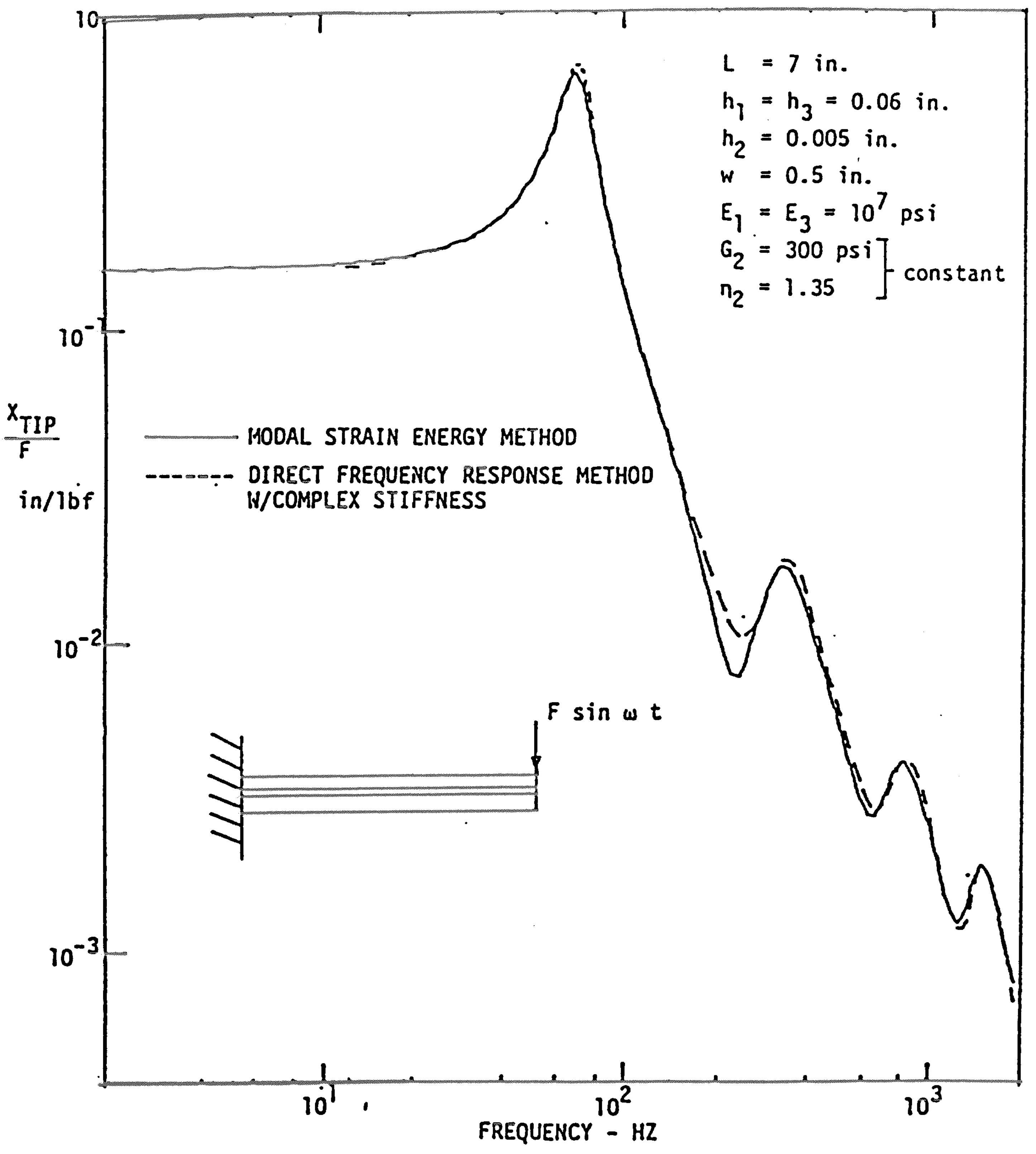


Figure 4 Frequency response of sandwich beam calculated by direct and modal strain energy methods.

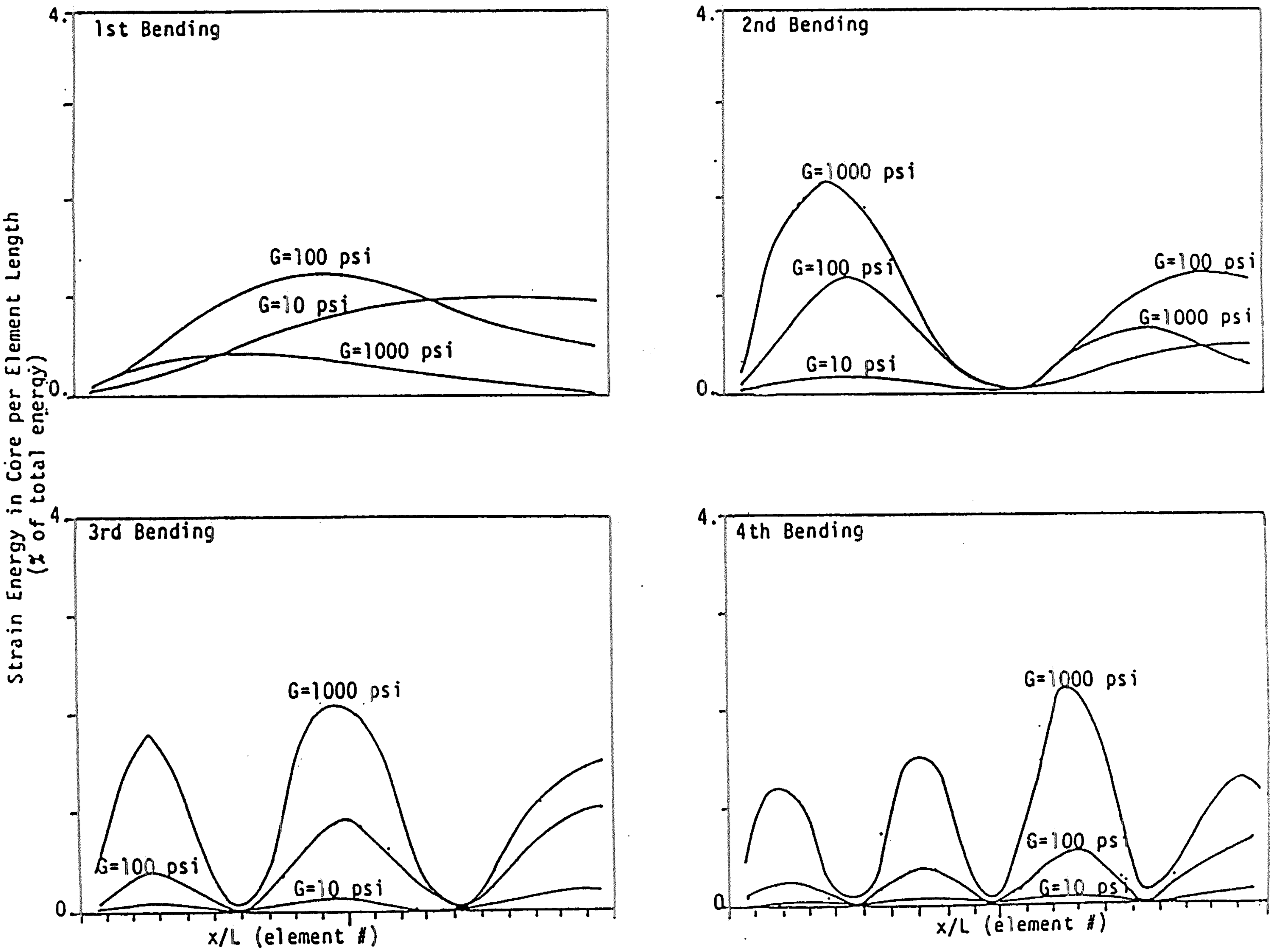


Figure 5 Strain energy distribution in a cantilever beam for various values of core shear modulus.

Computation cost for eigenvalue extraction in the complex stiffness, complex eigenvalue method was found to be about 5 times greater than for real eigenvalue extraction in the corresponding MSE run. The Hessenberg method was used in the former and the Givens method in the latter.

TABLE 1
COMPARISON OF LOSS PARAMETER FOR A SANDWICH BEAM CALCULATED BY THE MODAL STRAIN ENERGY AND COMPLEX STIFFNESS EIGENVALUE METHODS

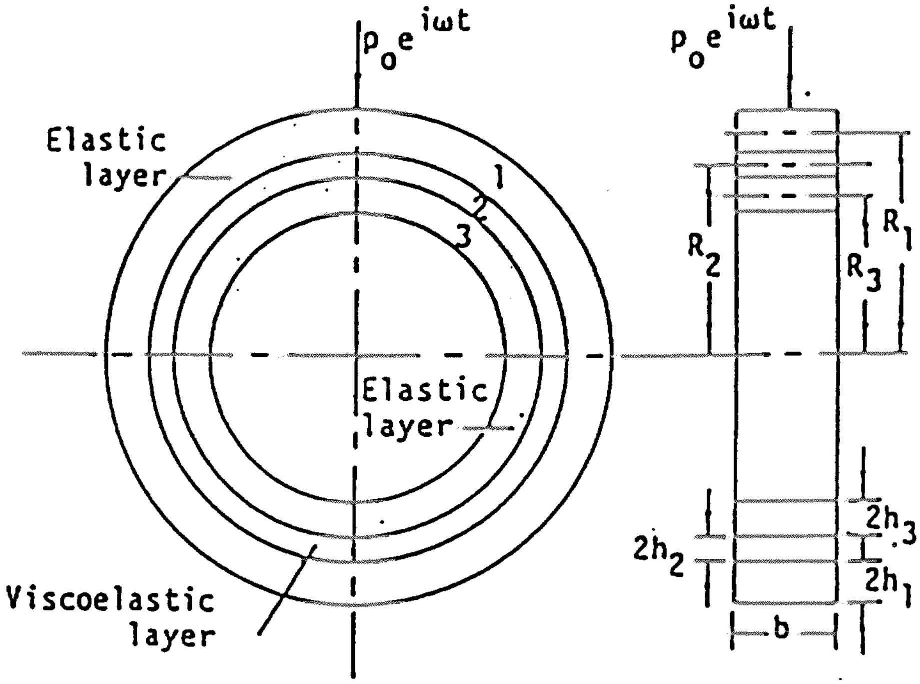
MODE	TYPE	MSE	CE	
			$\eta_v=0.3$	$\eta_v=1.3$
1	1B	0.214	0.216	0.210
2	2B	0.280	0.287	0.276
3	1T	0.025	0.025	0.025
4	3B	0.213	0.223	0.218
5	4B	0.134	0.142	0.141
6	2T	0.026	0.026	0.026
7	5B	0.081	0.093	0.092
8	3T	0.031	0.031	0.031
9	6B	0.057	0.062	0.062

Similar calculations were made for a variety of beam section geometries and boundary conditions with similar results. The sixth order and MSC/NASTRAN results for damping and frequencies were, for practical purposes, identical for the first six modes. Results from the fourth order theory differed somewhat for certain boundary conditions as would be expected based on the assumptions of the theory.

4.2 Sandwich Rings

The modal strain energy method has been applied to several sandwich ring configurations for which Lu, et al. [14] have given a closed form solution for frequency response. Since the NASTRAN QUAD4's and HEXA elements are both capable of modeling curved surfaces, there is no basic difference in method between beams and rings. Some loss of accuracy could probably be expected at small radius/thickness ratios although this was not investigated.

Figure 6 gives the dimensions and material properties for specimen 1 of Ref. [14]. It may be noted that the viscoelastic material properties vary significantly over the



R_1 = 13.02 cm $\quad$ $E_1 = E_3 = 2.88 \times 10^5$ MPa

$h_1 = h_3$ = .635 cm $\quad$ $\nu_1 = \nu_3 = 0.3$

h_2 = .549 cm $\quad$ b = 2.54 cm

$\rho_1 = \rho_3 = 7.75$ gm/cm^3 $\quad$ $\rho_2 = 2.13$ gm/cm^3

$$G_2 = \exp[.50 \ln(f) + 6.299] \text{ psi}$$
$$\eta_2 = \exp[.288 \ln(f) - 2.402]$$

Figure 6 Sandwich ring used for test case.

50-5000 Hz analysis band.

Finite element results obtained by three methods are shown in Figure 7:

(1) the direct frequency response method (Eq. (3)) which accounts exactly for material property variation with frequency
(2) the modal strain energy method with constant material properties which allows for no variation
(3) the modal strain energy method with adjusted damping ratios which allows an approximate accounting for property variations

The agreement between the direct frequency response and adjusted MSE methods is quite good. It is significant because the MSE method is substantially cheaper and is more readily used in the design process. The simple correction applied to damping ratios is adequate, at least in this case, to account for the frequency dependence of core material properties.

A comparison between finite element/MSE results and the closed form solution of Lu, et al can be made directly in terms of natural frequencies and modal loss factors. The loss factors are obtained from the admittance funtion by the usual half power bandwidth method. That is

$$\eta^{(r)} = \frac{f_r' - f_r''}{f_r} \qquad (9)$$

where

$\eta^{(r)}$ = loss factor of r'th mode

f_r = resonant frequency of r'th mode

f_r', f_r'' = frequencies slightly above and below f_r at which the magnitude of the admittance function is reduced by 3 dB

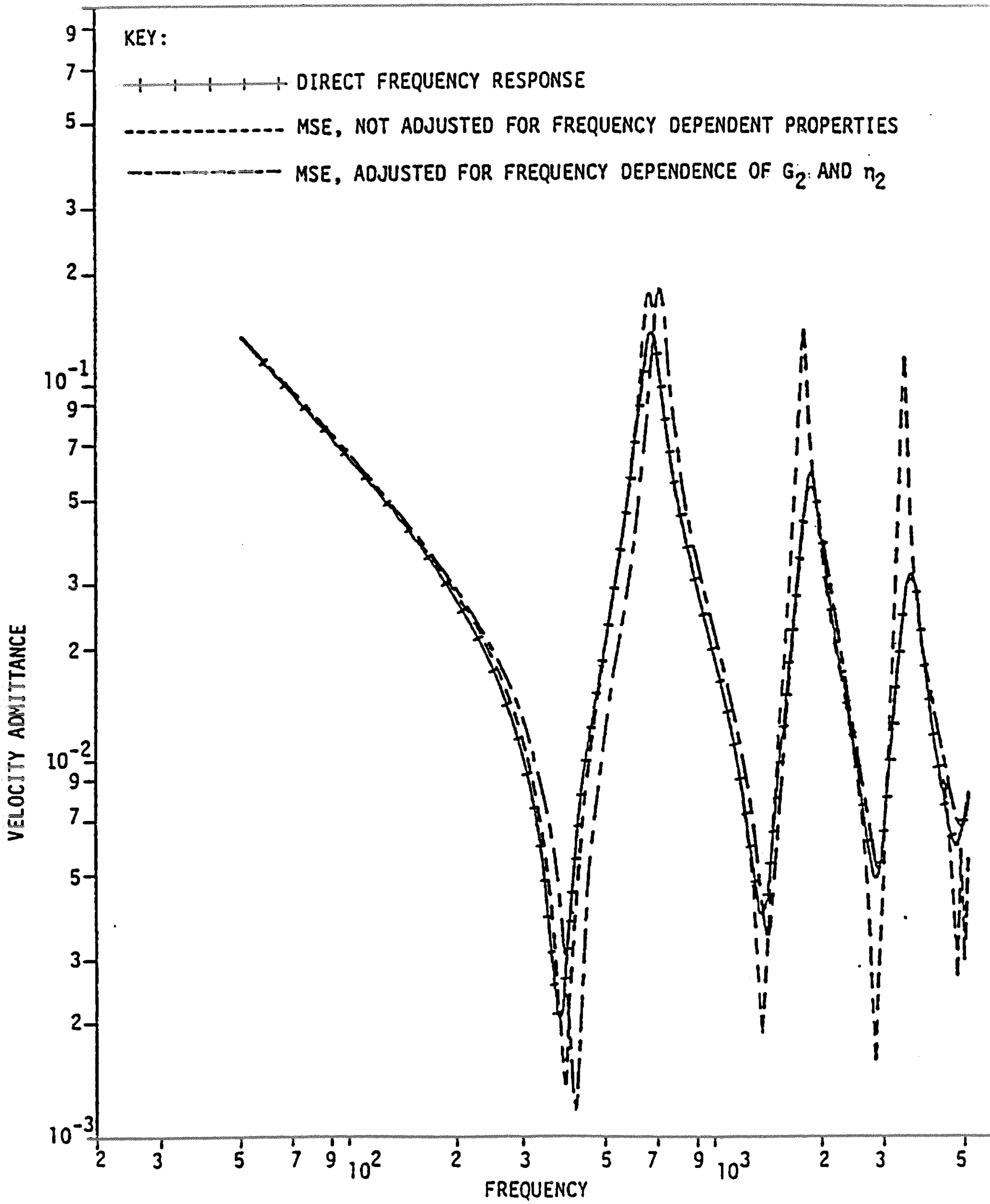


Figure 7 Driving point velocity admittance of sandwich ring

Loss factors are obtained from the undamped finite element results by (Eq. (6)) as usual. A comparison is given in Table 2 for the ring geometry shown in Figure 6. Both solutions are for a hypothetical case of constant core material properties (G_2 = 83.9 MPa (12164.0 psi), η_2 = 0.5420).

TABLE 2

NATURAL FREQUENCIES AND MODAL LOSS FACTORS FOR A DAMPED SANDWICH RING

Mode	ANALYTICAL SOLUTION (Lu, et al.[14]) Frequency (Hz)	Loss Factor	NASTRAN/MSE Frequency (Hz)	Loss Factor
1	660.7	.0946	649.2	.0974
2	1752.0	.0446	1746.8	.0501
3	3289.8	.0271	3307.1	.0320

4.3 Sandwich Plate

The closed form solution of Abdulhadi [15] for natural frequencies and modal loss factors of rectangular sandwich plates has been employed as a test case. The formulation in Ref. [15] is valid for a simply supported plate with core shear unrestrained at the plate edges.

Figure 8 shows a comparison of modal loss factor as a function of shear parameter as calculated by the closed form solution and by the modal strain energy method using MSC/NASTRAN. A grid of twelve elements in each in-plane direction was used. The dimensionless format of the plots is similar to that of Figure 2 for beams. The dimensionless parameters for a three layer plate are

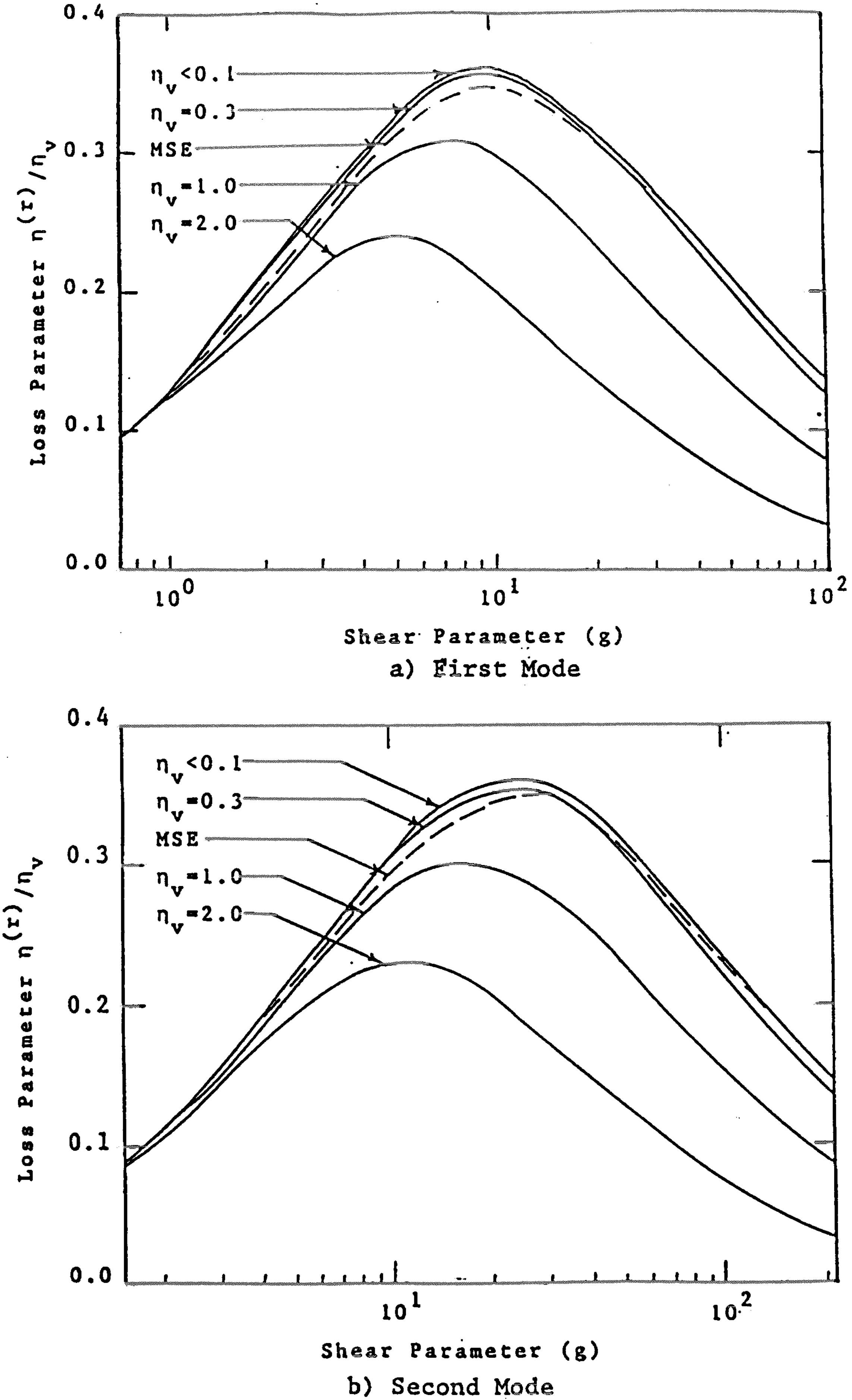


Figure 8 Modal loss factors for sandwich plate calculated by modal strain energy method and closed form solution.

η_v = core material loss factor

g = shear parameter

$$= \frac{\bar{G}}{T_2} \left(\frac{1}{E_1 T_1} + \frac{1}{E_3 T_3}\right) a^2 (1-\nu^2) \quad (10)$$

Y = geometry parameter

$$= \frac{(T_1+T_3+2T_2)^2}{4D(1-\nu^2)} \left[\frac{E_1 T_1 E_3 T_3}{E_1 T_1 + E_3 T_3}\right] \quad (11)$$

Δ_{xy} = in-plane aspect ratio

$$= b/a \quad (12)$$

where

T_1, T_3 = thicknesses of the face sheets

T_2 = thickness of the core layer

$\bar{G}$ = real part of the complex shear modulus $[\bar{G}(1+i\eta_v)]$ of the viscoelastic material

E_1, E_3 = Young's moduli of the face sheets

a,b = in-plane dimensions of the plate

D = sum of the flexural stiffnesses of the upper and lower face sheets, each about its own center plane

ν = Poisson's ratio of the face sheets

Figure 8 shows that the closed form solution and the modal strain energy method agree closely for small values of the material loss factor. Some divergence is seen for larger values, on the order of unity or greater. The agreement also depends on the value of the shear parameter g. It is best for g equal to or less than the value giving highest damping.

Fortunately, most practical constrained layer treatments tend to fall in this range [16].

5.0 DESIGN OF DAMPED STRUCTURES

Figures 9 shows a simplified flow chart of the logic that might be used in designing a viscoelastically damped structure using MSC/NASTRAN. The initial design effort (upper left part of Figure 9) is usually concerned with basic issues such as static stiffness, strength, and insuring that the normal modes of vibration will be acceptable, once adequate damping is obtained. When these issues are settled, the model is modified to account for additive damping, either in the form of damping inserts or add-on layered dampers. The geometry and material properties of the treatment are selected by an iterative process using the results of a modal strain energy analysis to guide each succeeding trial.

Once the engineer believes his damped design is close to optimum, he performs his final response calculations with NASTRAN using either the adjusted MSE method or the DFR method. These simulations are based on the best available estimate of the in-service dynamic load. The choice of method would depend on the type of load (transient, periodic, or random), the number of load cases, the frequency range, the properties of the viscoelastic, and the required accuracy.

5.1 Case Study

Figure 10 shows a structural part for which an add-on damping treatment was designed by the modal strain energy method. The part is an annular hollow plate which forms a divider between two modules of a high energy laser. The plate has many internal passages which carry coolant at high velocity. Gas under pressure flows radially outward from the hub and exhausts through the cavity between the lips at the outer circumference of the plate. Calculations indicated that unstable flow induced vibration, or flutter, could occur in the lips of the plate if the damping of certain modes was below a critical value.

The plate was tested in its original undamped form and the damping was found to be well below the desired value. The authors were asked to design a damping treatment subject to the following requirements:

(1) The damping of the critical mode had to be increased to at least 1.0% (structural).

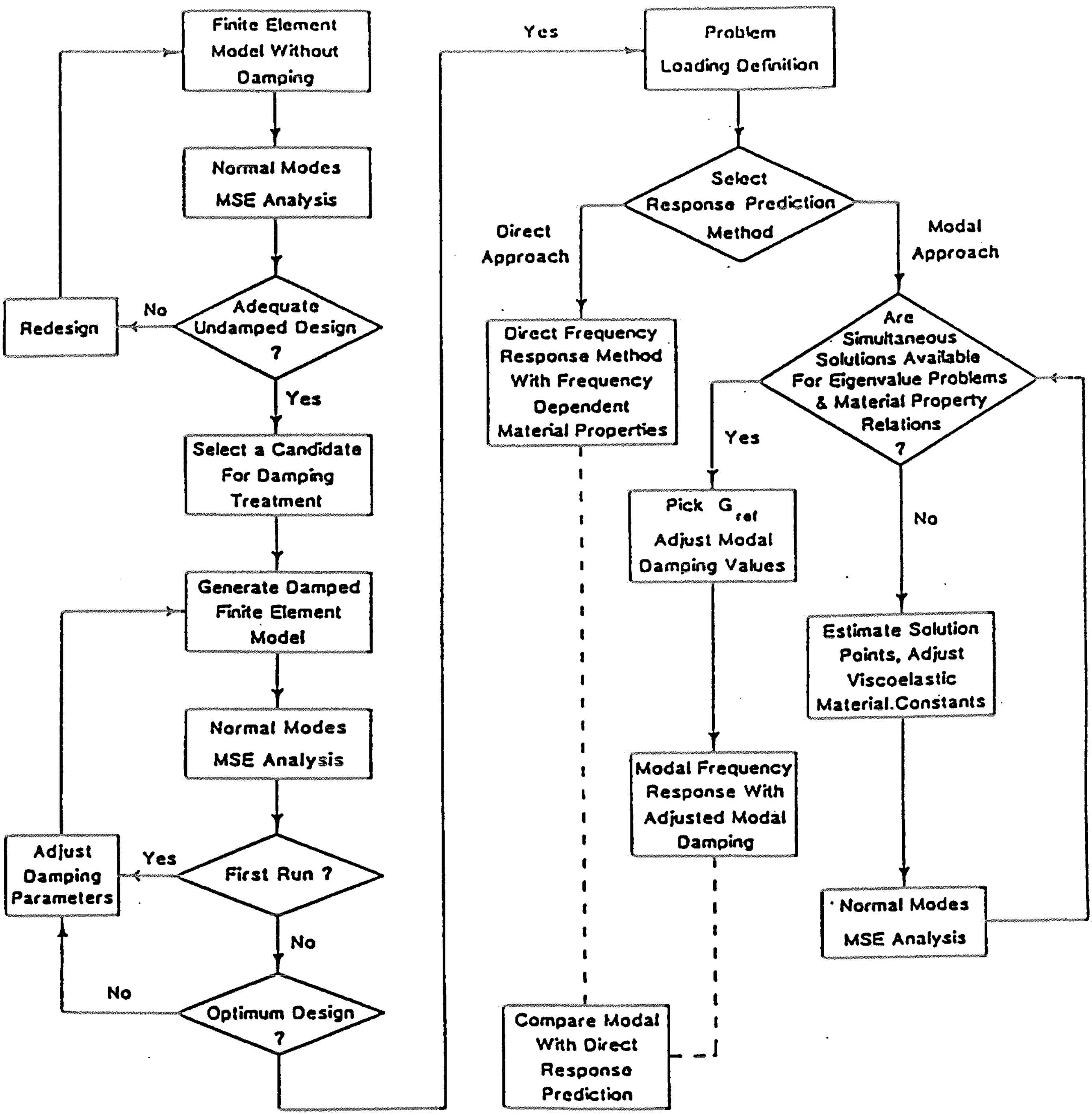


Figure 9   Damping design using modal strain energy method as implemented in NASTRAN

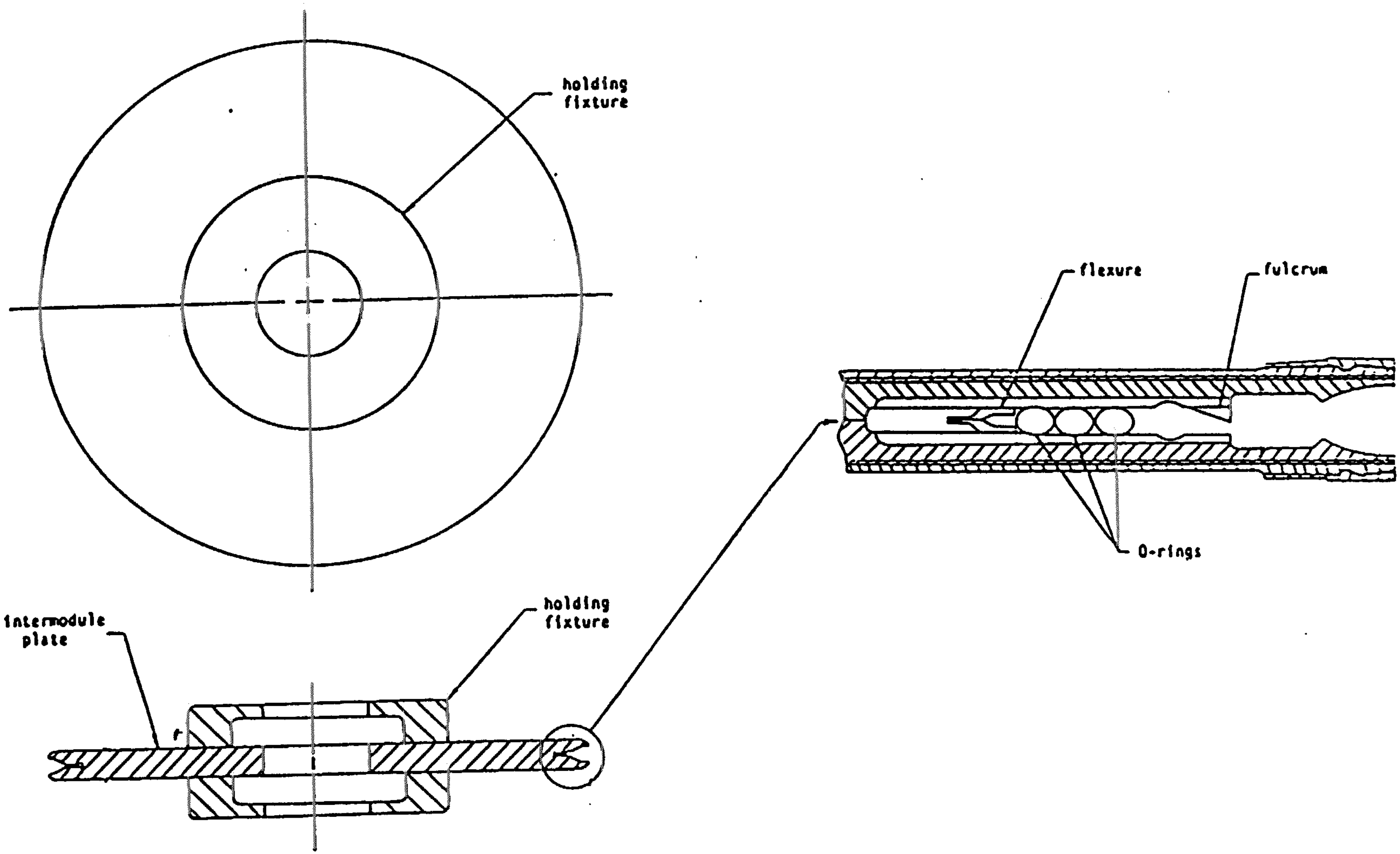


Figure 10 Plan and cross-section views of intermodule plate in holding fixture (not to scale)

(2) Static stiffness at the plate rim in the axial direction could be increased by no more than 10%.
(3) The damping treatment had to be effective over a temperature range of 75 deg F. to 400 deg F. and was to withstand short term exposure to 650 deg F.
(4) The treatment had to be designed, fabricated, and applied within a few weeks with no major machining of the original plate.

Based on these requirements, it was decided that a viscoelastic insert installed between the lips of the plate was the most desirable approach. The insert was to take the form of an O-ring which would be forced into the space between the lips of the plate around its entire circumference. The design was later changed to include several O-rings as shown in Figure 10.

One of the finite element models used in the design process is shown in cross section in Figure 11. The structure and symmetry of the plate were such that modeling of only a small azimuthal sector of one lip was necessary along with half of the O-ring(s) as shown. The model was composed entirely of solid elements. Calculations of modal properties, including energy distributions, were made for a range of values of material stiffness of the O-ring. Based on these calculations, a value was found that would allow the plate to meet both the damping and static stiffness requirements.

The next step was finding a viscoelastic material with the required properties. After some searching, a form of flourosilicon rubber was identified as a likely candidate. Arrangements were made with a munufacturer to have some fabricated into the required O-ring shape. The finished product, resembling a strand of spaghetti, was installed and the plate was retested.

Figure 12 shows, for the single O-ring configuration, predicted damping as a function of temperature along with measured values at two temperatures. The predicted natural frequency of the critical mode and the increase in static stiffness are also shown. Agreement between predicted and measured damping was excellent and the value was well above that needed to assure stability.

Figure 13 shows measured driving point frequency response functions between acceleration response and force input at the plate rim in the axial direction. The upper trace is for the plate with no damping insert and the lower trace is with two O-rings installed. The critical mode is at 2353 Hz. (without O-rings) and is characterized by out-of-phase motion of the plate lips, i.e. they move in opposite directions as they vibrate. The increase in damping due to the viscoelastic

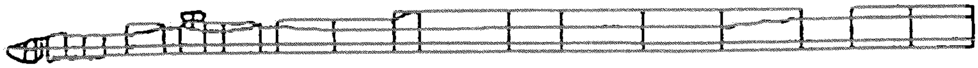


Figure 11   Single O-ring model, undeformed shape

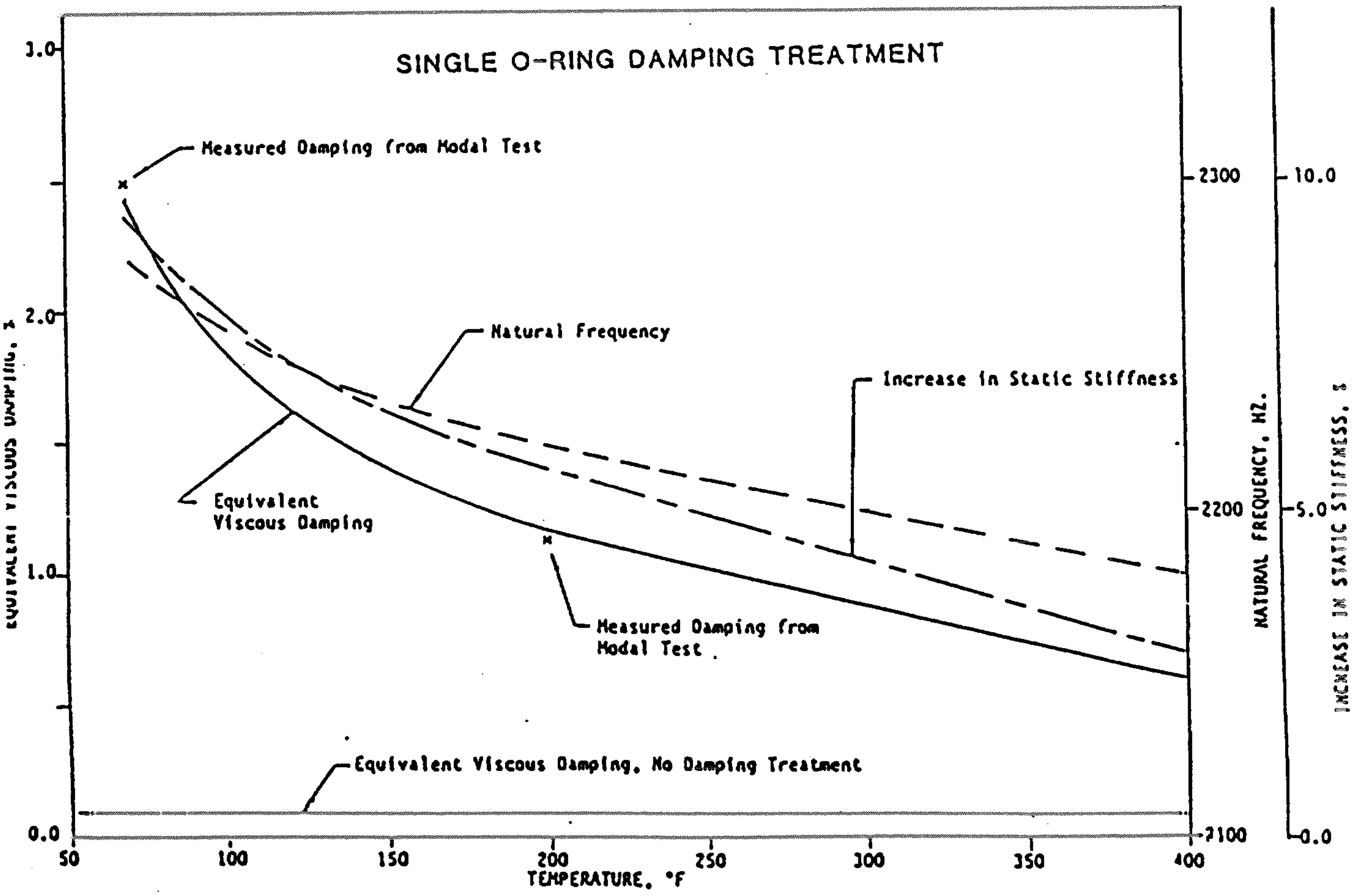
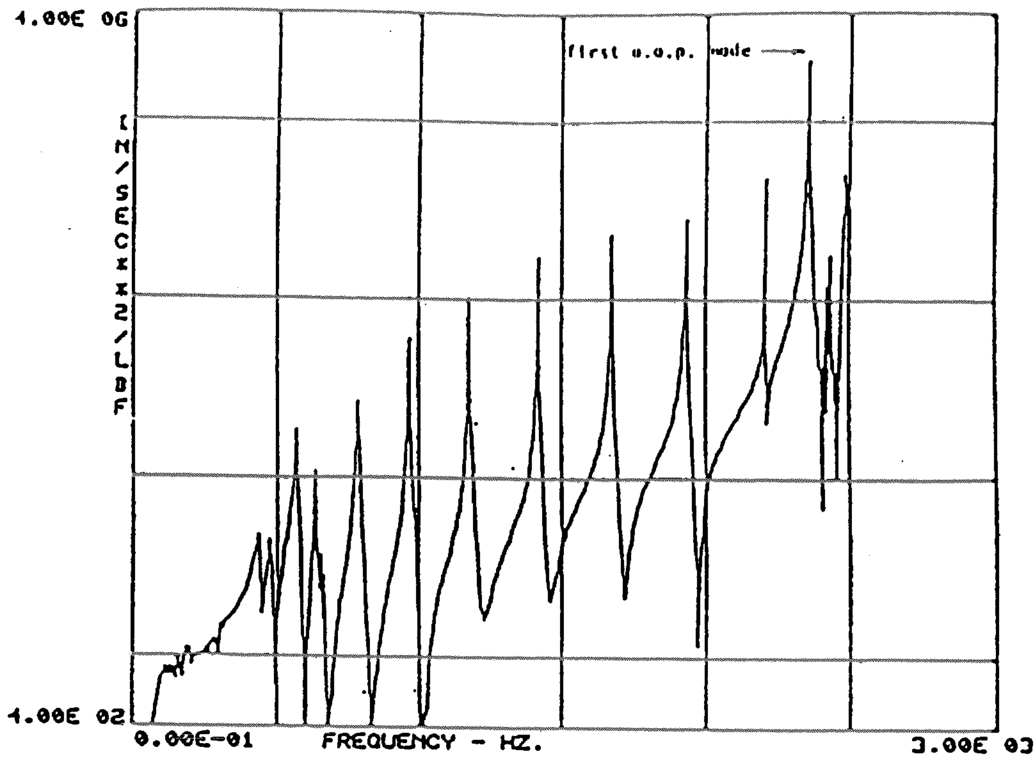
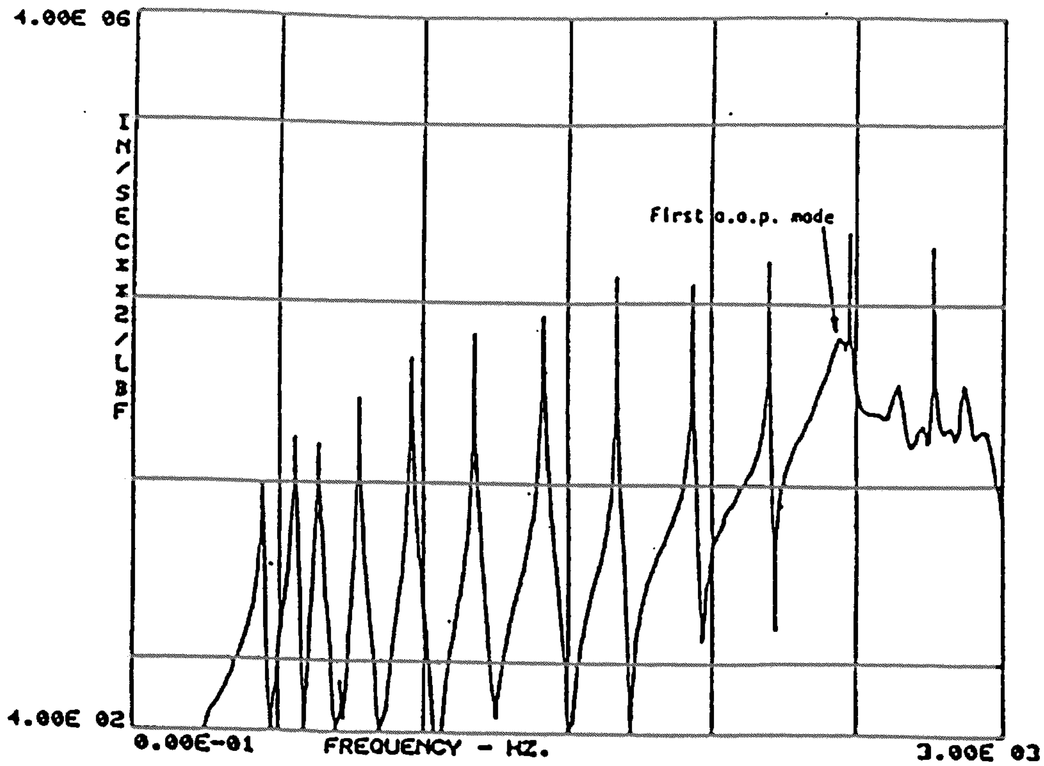


Figure 12   Properties of the Intermodule Plate with single O-ring damper



a) without added damping



b) with two O-ring inserts

Figure 13 Measured acceleration/force frequency response of intermodule plate.

inserts is quite dramatic and results in a decrease of response at resonance of about 30 dB. The numerous modes of vibration that occur below 2353 Hz. are only slightly affected by the O-ring. They are in-phase modes in which the lips of the plate move together and therefore produce no strain in the viscoelastic.

6.0 Summary and Conclusions

Three mathematical methods for analysis of viscoelastically damped structures have been reviewed in the context of applications to design using MSC/NASTRAN.

1. The complex eigenvalue method based on complex stiffnesses is generally too expensive and limited with respect to problem size. It is useful primarily as a benchmark for testing less expensive approximate methods.

2. The modal strain energy method is an approximation to the complex eigenvalue method. It is much less expensive and can be applied to larger problems because it uses only real normal modes. It is also well suited to design work because it leads to optimum choices for both viscoelastic material and geometry of the damping treatment. It can account for variations in material properties with frequency in an approximate but simple way.

3. The direct frequency response method can account exactly for material property variations with frequency but at a substantial cost penalty. It is useful primarily for final predictions of response once a design has been selected and load conditions have been defined.

The well-developed technology of viscoelastic materials is an important asset for the design of structures which must function under dynamic load. However, careful analysis is required to make use of these materials. The modal strain energy method is particularly well suited to design of add-on or integral damping.

The authors are actively engaged in ongoing research and development work in damping design methods. Current efforts concern the problem of curved sandwich panels. It has been found by painful experience that these are substantially more difficult to analyze than are flat sandwich plates. The difficulty is believed to stem at least in part from the out-of-plane normal stresses that exist in the interior of curved panels under bending. These stresses result in dilatational strain of the viscoelastic core as well as shearing. The strain energy of the core thus depends on the

Poisson's ratio of the viscoelastic material. That quantity is often not known with precision and seems to be a function of frequency and temperature just as are the storage and loss moduli. Current work is directed at resolving these questions and arriving at a practical analysis method for curved sandwiches.

REFERENCE

1. Crandall, S.H., "The Role of Damping in Vibration Theory," J. Sound and Vibration, Vol. 11, No.1, Jan. 1970, pp. 3-18.

2. Foss, K.A., "Coordinates Which Uncouple the Equations of Motion in Damped Linear Dynamic Systems," Journal of Applied Mechanics, Vol. 25, 1958, pp. 361-364.

3. Duncan, W.J., Elementary Matrices, MacMillan Co., New York, 1946.

4. Joseph, J.A. (Ed.) MSC/NASTRAN Applications Manual, Vol II, Section 7.2.6.2, MacNeal-Schwendler Corp., Los Angeles, Calif., April, 1974.

5. Crandall, S.H., "Dynamic Response of Systems with Structural Damping," Air, Space, and Instruments, Draper Anniversary Volume, 1963, S. Lees, Editor, McGraw-Hill, also M.I.T. AFOSR No. 1561, Oct. 1961.

6. Rogers, L., (Ed.), "Conference on Aerospace Polymeric Viscoelastic Damping Technology for the 1980's," Air Force Flight Dynamics Laboratory, TM 78-78-FBA, Feb. 1978.

7. Johnson, C.D. and Kienholz, D.A., "Finite Element Prediction of Damping in Structures with Constrained Viscoelastic Layers," AIAA Journal, Vol. 20, No. 9, Sept 1982.

8. Johnson, C.D., Kienholz, D.A. and Rogers, L.C., "Finite Element Prediction of Damping in Beams with Constrained Viscoelastic Layers," Shock and Vibration Bulletin, No. 51, May 1981, pp. 71-82.

9. Ungar, E.E. and Kerwin, E.M. Jr., "Loss Factors of Viscoelastic Systems in Terms of Energy Concepts," Acoustic Society of America, Vol. 34, July 1962, pp. 954-957.

10. Joseph, J.A. (Ed.), MSC/NASTRAN Applications Manual, Vol. 1, Section 2.22.4.3, MacNeal-Schwendler Corp. Los Angeles, Calif., March 1977.

11. DiTaranto, R.A., "Theory of Vibratory Bending for Elastic and Viscoelastic Layered Finite Length Beams," Journal of Applied Mechanics, Vol. 32, Dec. 1965, pp. 881-886.

12. Rao, D.K., "Frequency and Loss Factors of Sandwich Beams Under Various Boundary Conditions," Journal Mech. Eng. Science, Vol. 20, No. 5, May 1978, pp. 271-282.

13. Ross, D., Ungar, E.E., and Kerwin, E.M., "Damping of Plate Flexural Vibrations by Means of Viscoelastic Laminates," Section 3 of Structural Damping, ASME, 1959.

14. Lu, Y.P., Douglas, B.E. and Thomas, E.V., "Mechanical Impedance of Damped Three-Layer Sandwich Rings," AIAA Journal, Vol. 11, No.3, March 1973, pp. 300-304.

15. Abdulhadi, F. "Transverse Vibrations of Laminated Plates with Viscoelastic Layer Damping," Shock and Vibration Bulletin, No. 40, part 5, Dec. 1969.

16. Kienholz, D.A., Johnson, C.J., and Parekh, J. "Design Methods for Viscoelastically Damped Plates," Proc. 24th Structures, Structural Dynamics, and Materials Conference, Lake Tahoe, Nevada, May 2-4, 1983.