Resultant Forces and Moments in Static and Dynamic Analysis

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SUMMARY

The theory and DMAP instructions for obtaining resultant force and moment vectors of an arbitrary set of static or dynamic forces are presented. The resultant moment vector is defined with respect to any point specified by the analyst. DMIG cards are used to define the portion of the structure for which the resultant force and moment vectors are desired. Rigid body displacement vectors are used to calculate and sum the contributing moments and forces. The use of optional pre-processing programs for generation of the DMIG cards is discussed along with examples.

INTRODUCTION

Statically equivalent forces for a complex structure are often required at a cross section, that may be transversed by many individual elements. Summing the individual element forces by hand and calculating their moments about some point in the section plane is tedious and time consumming. The theory and DMAP instructions presented here perform the summation and moment calculations for any set of forces specified by the analysts.

THEORETICAL DEVELOPMENT

The structure is shown generally in figure 1. The forces and moments to be summed are to the right of the section shown. First, moments are to be calculated with respect to point '0' of figure 1. The familiar force and moment summation laws are:

$$M = \sum_{i=1}^{m} r_i \times F_i + \sum_{i=1}^{m} M_i \quad (1)$$

$$F = \sum_{i=1}^{m} F_i \quad (2)$$

$$\text{where:}\quad F_i = \begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix}_i \quad \text{and} \quad M_i = \begin{Bmatrix} m_x \\ m_y \\ m_z \end{Bmatrix}_i \tag{3}$$

r_i = the position vector to the i'th point where force F and moment M are applied

n = number of points in the sub-region for which resultant forces and moments are being calculated.

The cross product terms in equation (1) may be written as :

$$r_i \times F_i = \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix} \begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix}_i \tag{4}$$

where : x_i, y_i, z_i are the cartesian coordinates for point i and $\{f_x, f_y, f_z\}_i$ is the force vector applied at point i.

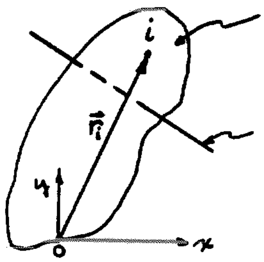


Figure 1

To form the first summation in equation (1) we use (4) and the definition of matrix multiplication:

$$\sum_{i=1}^{m} r_i \times F_i = [R_1 \;\; R_2 \;\; \dots \;\; R_m] \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ F_m \end{bmatrix} \tag{5}$$

$$\text{where :}\quad R_i = \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix} . \tag{6}$$

The second summation in (1) can be written as:

$$\sum_{i=1}^{m} M_i = [I \quad I \quad \cdots \quad I] \begin{bmatrix} M_1 \\ M_2 \\ \cdot \\ \cdot \\ \cdot \\ M_m \end{bmatrix} \tag{7}$$

where I is the three by three identity matrix. Therefore, given the forces and moments at the grid points in the form:

$$\{F_G\} = [\{f_x, f_y, f_z, m_x, m_y, m_z\}_1 \ \cdots \ \{f_x, f_y, f_z, m_x, m_y, m_z\}_m] \tag{8}$$

we can calculate the resultant force and moment substituting equations (7) and (5) in equations (1) and (2). Combining equations (1) and (2) into one equivalent matrix equation, we obtain:

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{bmatrix} I & 0 & & & I & 0 \\ & & \cdot & \cdot & \cdot & \\ R_1 & I & & & R_m & I \end{bmatrix} \{F_G\} \tag{9}$$

where 0 is a three by three null matrix.

The matrix pre-multiplying $\{F_G\}$ in equation (9) is the transpose of the rigid body mode matrix. Call this matrix Φ_{RB}. Then equation (9), which is equivalent to equations (1) and (2), may be written as:

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = [\Phi_{RB}] \ \{F_G\} \ . \tag{10}$$

In the above formulation, since the coordinates x_i ,y_i and z_i are measured with respect to the basic coordinate system, M in equation (10) is with respect to the origin of the basic coordinate system. We will denote this by placing a subscript '0' on the vector {F , M} . To calculate the force and moment about another point 'i' from the force and moment at point '0' we use a degenerate form of equation (9), but with the position vector reversed in sign (see figure 2).

$$\begin{Bmatrix} F \\ M \end{Bmatrix}_i = \begin{bmatrix} I & 0 \\ -R_i & I \end{bmatrix} \begin{Bmatrix} F \\ M \end{Bmatrix}_0 \tag{11}$$

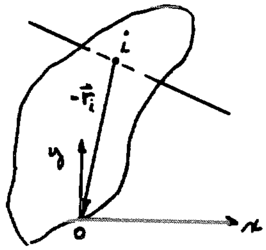


Figure 2

DMAP FORMULATION

It should be noted that if equation (10) is used with $\{F_G\}$ being the G size force vector and Φ_{AB} being the rigid-body mode matrix, then the forces and moments calculated are for the total structure and are with respect to the origin of the basic coordinate system. To calculate resultants for part of the structure, the F_G vector is "zeroed" out for those grid points whose forces and moments are not to be included in the resultant force and moment calculation. This is done by supplying NASTRAN with the grid points to be ignored via DMIG cards. For each grid to be ignored, a set of DMIG cards is supplied with -1's associated with the grids' six degrees of freedom. By doing this, a matrix is formed with -1 's on the diagonal elements corresponding to the grids' whose forces are not to be summed. This matrix is then added to a G size identity matrix . The resulting matrix pre-multiplies the F_G vector, forming the desired 'zeroed' vector. In detail the operation is:

$$\begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} = \left[\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & 1 \end{bmatrix} + \begin{bmatrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & \cdot & & & & & \\ & & & \cdot & & & & \\ & & & & \cdot & & & \\ & & & & & -1 & & \\ & & & & & & \cdot & \\ & & & & & & & \cdot \\ & & & & & & & & -1 \end{bmatrix} \right] \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ \cdot \\ F_m \end{bmatrix}$$

or: $\{FGGi\} = [\ IGG + IPICKi\ ]\ \{FGG\}$ (12)

The matrix IPICKi usually requires voluminous amounts of data but is easily generated by a simple FORTRAN program (see appendix I). To facilitate the generation of IPICKi, it is convenient to have the grid numbers for the structure in some orderly fashion, so that blocks of them may be defined for the FORTRAN program. The program can then read a copy of the BULK data, and generate DMIG cards for those grids in the range specified.

For each section about which resultants are to be calculated, an IPICKi matrix is required. The resulting {FGGi} vector is then used as the $\{F_G\}$ of equation (10). Since the moments in equation (10) are calculated with respect to the origin of the basic coordinate system, the {F,M} of equation (10) must be transformed to point i (see figure 2) using equation (11). The transformation matrix of equation (11) is also supplied using DMIG cards and is called [SKEWi]. It was generated here using the same program that generated [IPICKi]. The resultant forces and moments about a point i, for a sub-region i, are then given by the DMAP equivalent of equation (11):

$$\{FORSECi\} = [\ SKEWi\ ][\Phi_{as}]\{FGGi\} \qquad (13)$$

EXAMPLE PROBLEM 1 , STATIC ANALYSIS

The sample problem is a cantilever beam as shown in figure 3.

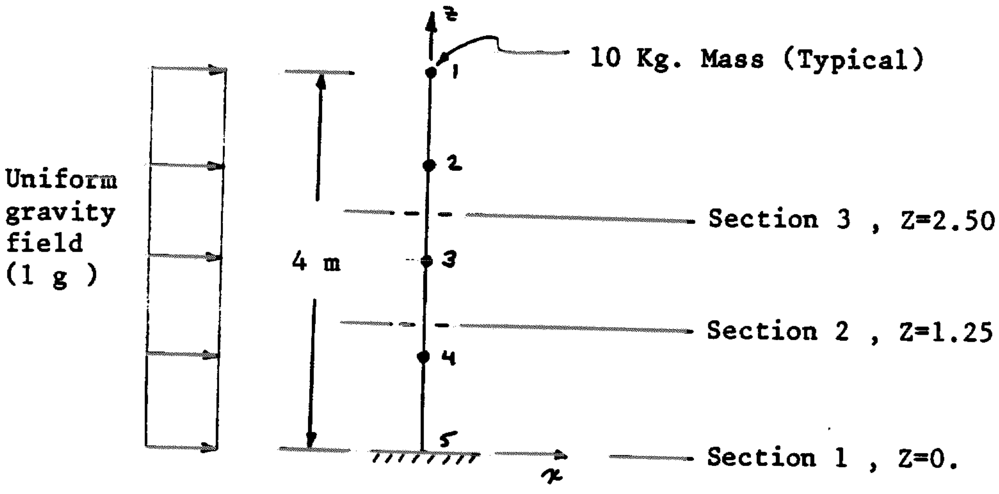


Figure 3

The FORTRAN program IPICGEN is presented in appendix I with the associated input and output (DMIG cards are output). The DMIG cards are the same for both static and dynamic analysis examples.

The input data deck and DMAP procedure for the static analysis are presented in appendix II. The output obtained using the alter on this sample problem is presented in appendix III. For each section we obtain a six by k matrix, where k is the number of load vectors in PGG. A simple calculation verifies the results obtained.

EXAMPLE PROBLEM 2 , DYNAMIC ANALYSIS

The only difference between this alter (presented in appendix II) and the previous alter is that instead of operating on the force vector PGG we use the modal forces given by:

$$F = [\ MGG\ ]\ [\ PHIG\ ] \qquad (14)$$

and we operate on the mass matrix with the FPICKi matrices. In this example, the output matrices must be interpreted as "modal forces", and must pre-multiply the generalized accelerations to obtain actual forces. That is:

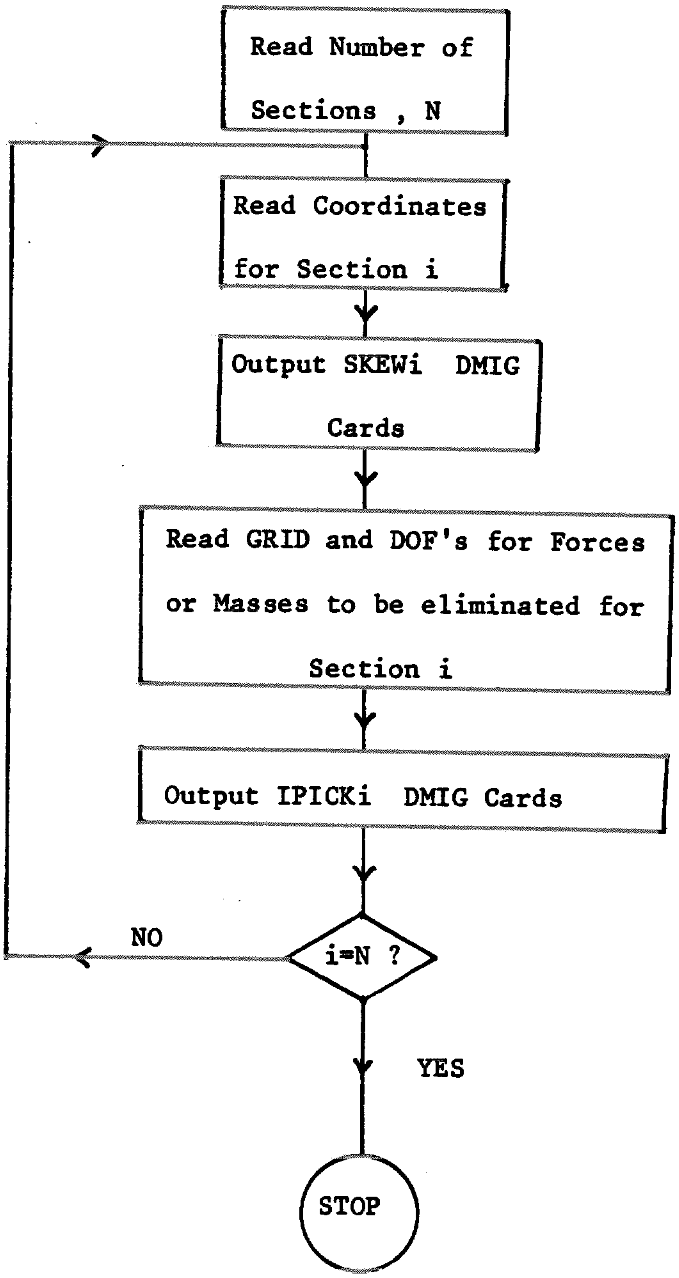
$$F_d = [\ FORTSECTi\ ]\ \{\ \ddot{q}\ \} \qquad (15)$$

where : F_d is the resultant dynamic force
$\ddot{q}$ is the vector of generalized accelerations.

In this case we get a 6 by m matrix for each section, m being the number of eigenvectors calculated.

APPENDIX I

Flowchart for IPICGEN



APPENDIX I (continued)

Output from IPICGEN

DMI	SKEWR2	0	1	1			6	6
DMI	SKEWR2	1	1	1.	5		-1.250006	0.0000
DMI	SKEWR2	2	2	1.	4		1.250006	0.0000
DMI	SKEWR2	3	3	1.	4		0.000005	0.0000
DMI	SKEWR2	4	4	1.				
DMI	SKEWR2	5	5	1.				
DMI	SKEWR2	6	6	1.				
DMIG	IPICK2	0	1	1				
DMIG	IPICK2		4	1		4	1	-1.
DMIG	IPICK2		4	2		4	2	-1.
DMIG	IPICK2		4	3		4	3	-1.
DMIG	IPICK2		4	4		4	4	-1.
DMIG	IPICK2		4	5		4	5	-1.
DMIG	IPICK2		4	6		4	6	-1.
DMIG	IPICK2		5	1		5	1	-1.
DMIG	IPICK2		5	2		5	2	-1.
DMIG	IPICK2		5	3		5	3	-1.
DMIG	IPICK2		5	4		5	4	-1.
DMIG	IPICK2		5	5		5	5	-1.
DMIG	IPICK2		5	6		5	6	-1.
DMI	SKEWR3	0	1	1			6	6
DMI	SKEWR3	1	1	1.	5		-2.500006	0.0000
DMI	SKEWR3	2	2	1.	4		2.500006	0.0000
DMI	SKEWR3	3	3	1.	4		0.000005	0.0000
DMI	SKEWR3	4	4	1.				
DMI	SKEWR3	5	5	1.				
DMI	SKEWR3	6	6	1.				
DMIG	IPICK3	0	1	1				
DMIG	IPICK3		3	1		3	1	-1.
DMIG	IPICK3		3	2		3	2	-1.
DMIG	IPICK3		3	3		3	3	-1.
DMIG	IPICK3		3	4		3	4	-1.
DMIG	IPICK3		3	5		3	5	-1.
DMIG	IPICK3		3	6		3	6	-1.
DMIG	IPICK3		4	1		4	1	-1.
DMIG	IPICK3		4	2		4	2	-1.
DMIG	IPICK3		4	3		4	3	-1.
DMIG	IPICK3		4	4		4	4	-1.
DMIG	IPICK3		4	5		4	5	-1.
DMIG	IPICK3		4	6		4	6	-1.
DMIG	IPICK3		5	1		5	1	-1.
DMIG	IPICK3		5	2		5	2	-1.
DMIG	IPICK3		5	3		5	3	-1.
DMIG	IPICK3		5	4		5	4	-1.
DMIG	IPICK3		5	5		5	5	-1.
DMIG	IPICK3		5	6		5	6	-1.

DMAP ALTER for Statics

```
$
$     FILE : FORSEC1
$
$
$     ALTER FOR CALCULATING FORCES AT VARIOUS SECTIONS , SOL 24
$
ALTER 160 $
$
$     GENERATE RIGID BODY AND AN IDENTY G SIZE MATRIX
$
VECPLOT,      ,BGPDT,EQEXIN,CSTM,,/PHIRB2///4 $
DIAGONAL      MGG/IGG/SQUARE/0. $
$
$     SECTION 1
$
MPYAD         PHIRB2,PGG,/FORSEC1/ $
MATPRN        FORSEC1// $
$
$     SECTION 2
$
MTRXIN,       ,MATPOOL,EQEXIN,SIL,/IPICK2,,/V,N,LUSET/1 $
ADD           IGG,IPICK2/IGG2/ $
SMPYAD        SKEWR2,PHIRB2,IGG2,PGG,,/FORSEC2/4 $
MATPRN        FORSEC2// $
$
$     SECTION 3
$
MTRXIN,       ,MATPOOL,EQEXIN,SIL,/IPICK3,,/V,N,LUSET/1 $
ADD           IGG,IPICK3/IGG3/ $
SMPYAD        SKEWR3,PHIRB2,IGG3,PGG,,/FORSEC3/4 $
MATPRN        FORSEC3// $
```

DMAP ALTER for Dynamics

```
$
$    FILE : FORSEC2
$
$
$    ALTER FOR CALCULATING FORCES AT VARIOUS SECTIONS , SOL 25
$
ALTER 163 $
$
$    GENERATE RIGID BODY AND AN IDENTY G SIZE MATRIX
$
VECPLOT,     ,BGPDT,EQEXIN,CSTM,,/PHIRB2///4 $
DIAGONAL     MGG/IGG/SQUARE/0. $
$
$    SECTION 1
$
SMPYAD       PHIRB2,MGG,PHIG,,,/FORSEC1/3 $
MATPRN       FORSEC1// $
$
$    SECTION 2
$
MTRXIN,      ,MATPOOL,EQEXIN,SIL,/IPICK2,,/V,N,LUSET/1 $
ADD          IGG,IPICK2/IGG2/ $
SMPYAD        IGG2,MGG,IGG2,,,/MGG2/3 $
SMPYAD       SKEWR2,PHIRB2,MGG2,PHIG,,/FORSEC2/4 $
MATPRN       FORSEC2// $
$
$    SECTION 3
$
MTRXIN,      ,MATPOOL,EQEXIN,SIL,/IPICK3,,/V,N,LUSET/1 $
ADD          IGG,IPICK3/IGG3/ $
SMPYAD        IGG3,MGG,IGG3,,,/MGG3/3 $
SMPYAD       SKEWR3,PHIRB2,MGG3,PHIG,,/FORSEC3/4 $
MATPRN       FORSEC3// $
```

APPENDIX III

Output Obtained Using Statics DMAP ALTER

```
     MATRIX FORSEC1  (GINO NAME 101 ) IS A REAL          1 COLUMN X      6 ROW RECTANG  MATRIX.

COLUMN      1       ROWS      1 THRU      5     --------------------------------------------------
   ROW

    1)    4.9050E+02  0.          0.          0.          9.8100E+02

THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =       2
THE DENSITY OF THIS MATRIX IS  33.33 PERCENT.


     MATRIX FORSEC2  (GINO NAME 101 ) IS A REAL          1 COLUMN X      6 ROW RECTANG  MATRIX.

COLUMN      1       ROWS      1 THRU      5     --------------------------------------------------
   ROW

    1)    2.9430E+02  0.          0.          0.          5.1502E+02

THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =       2
THE DENSITY OF THIS MATRIX IS  33.33 PERCENT.


     MATRIX FORSEC3  (GINO NAME 101 ) IS A REAL          1 COLUMN X      6 ROW RECTANG  MATRIX.

COLUMN      1       ROWS      1 THRU      5     --------------------------------------------------
   ROW

    1)    1.9620E+02  0.          0.          0.          1.9620E+02

THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =       2
THE DENSITY OF THIS MATRIX IS  33.33 PERCENT.
```