STATIC AEROELASTIC ADDITION TO MSC/NASTRAN

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Abstract

The static aeroelastic problems of an elastic flight vehicle have traditionally been formulated in terms of flexibility influence coefficients or vibration modes. A new stiffness formulation has been developed and added to MSC/NASTRAN to solve the basic trim load problem and estimate the aeroelastic stability derivatives at subsonic speeds. The user options for symmetric and antisymmetric motions are illustrated in example problems.

Introduction

A design study for incorporation of an aeroelastic capability in NASTRAN® was prepared for NASA in the early part of 1971 under contract NAS1-9801. This design study (Reference 1) included flutter, dynamic aeroelastic response, and static aeroelastic response capabilities. Only the first two capabilities were implemented and the latter has been neglected since preparation of Reference 1. This paper is presented to update the design of the linear static aeroelastic capability as it is being incorporated into MSC/NASTRAN, Version 64.

A static aeroelastic problem may be defined as a problem involving the response of a flexible structure to aerodynamic loading in which loadings and accelerations of the structure are assumed to be independent of time. Thus, inertia forces, if they enter at all, are assumed to be constant in time. The three common types of static aeroelastic problems are

a. Calculation of static response, including loads and stresses in the structure.

b. Calculation of stability and control derivatives, i.e., the calculation of the changes in the aerodynamic loading (and, more particularly, of the changes in its resultants) due to small changes in the motions of the vehicle and of control surface deflections.

c. Static divergence, which is an idealized stability problem of a restrained vehicle in which the determinant of the stiffness matrix, including both structural and aerodynamic terms, vanishes. Divergence of an unrestrained vehicle is a dynamic phenomenon that is appropriately analyzed by the flutter solutions.

The first two static aeroelastic problems may be further classified as to whether the structure is supported or free to move. If it is free to move, the inertia forces due to (steady) accelerations must be taken into account.

An additional distinction occurs in the determination of the static response of a freely moving structure, with respect to the manner in which the velocity components and the control surface deflections of the structure are determined. In some cases the analyst constrains all the components of velocity and the control surface deflections and accepts whatever accelerations they produce. He may, on the other hand, constrain the accelerations to zero in order that some of the velocity components and control surface deflections are evaluated for a trimmed flight condition.

The general task of formulating and classifying static aeroelastic problems for solution in MSC/NASTRAN is described below and the system of equations to be solved by the solution sequences are derived using standard MSC/NASTRAN notation. It will be seen that all of the problem types mentioned above may be solved with the following rigid formats (SØL 21):

-- Static Aeroelastic Response
-- Aeroelastic Divergence

In particular, the calculation of stability and control derivatives is treated as a subcase of aeroelastic response.

The subdivision of static aeroelastic analysis into Functional Modules and Rigid Formats is summarized in Table 1 which presents a sequence of functional module calls for all forms of static aeroelastic analysis. It also provides a very brief statement of the operations performed by each module and indicates whether the module is an existing module or a new module.

The static part of MSC/NASTRAN is used to generate a structural stiffness matrix $[K_{aa}]$, referred to the displacement set $\{u_a\}$. It is also used to generate vectors of static loads $\{P_\ell^s\}$ and $\{P_r^s\}$ which may include, for example, gravity loads, pressure loads on structural panels, and loads due to thermal expansion. Gravity loads corresponding to different load factors can, incidentally, be specified by changing a single data entry. In addition, a number of data blocks ($[D]$, $[m_r]$, and $[M_{\ell\ell}D + M_{\ell r}]$) are generated which are used to treat inertia relief effects.

In MSC/NASTRAN terminology, the displacement set $\{u_\ell\}$ is a subset of the dynamic analysis set $\{u_a\}$, which excludes the "support" degrees of freedom $\{u_r\}$ that provide a determinate set of reactions for free bodies. Thus,

$$\{u_a\} = \begin{Bmatrix} u_\ell \\ \hline u_r \end{Bmatrix} \tag{1}$$

The vectors $\{P_\ell^s\}$ and $\{P_r^s\}$ are, respectively, the static structural loads applied to $\{u_\ell\}$ and $\{u_r\}$.

The 1971 system defined in References 1 and 2 describe the method of calculating the basic static aeroelastic matrix coefficients. However, the solution of the combined system had several deficiencies which were removed. These are:

1. In Reference 1, the free body motion of the system is described by the motions of the u_r (fictional support) points, and the flexible motion is defined as motion of the u_ℓ structure points relative to the u_r points. The aerodynamic extra points (u_e) are typically defined as motions of the inertial center of gravity, or more precisely, the motion of the rigid body (zero frequency) modes. Since the u_r points may typically move during motion of the flexible modes, the aerodynamic rigid motions (pitch, plunge, etc.) are measured relative to a deflecting system! The proper method requires that rigid body motion be defined relative to the

undeflected CG. The correction described below ensures that the flexible motion is constrained to be orthogonal to the rigid body modes.

2. The solution of the system requires nonsymmetric decomposition of large-order matrices for each type of results. Furthermore, the burden of obtaining the critical output quantities such as stability coefficients is placed on the user. The basic approach apparently was to require the user to use MSC/NASTRAN like a wind tunnel test and vary each aerodynamic motion independently in a separate run.

The present solution provides the matrix reduction scheme shown below which produces small solution matrices with individual terms that are scalar functions of the stability coefficients.

TABLE 1. Simplified Flow Diagram for Static Aeroelastic Analysis

STEP	MODULE	STATUS	FUNCTION
1	Static Part of MSC/NASTRAN	(Existing)	Forms structural mass and stiffness matrices, rigid body properties and nonaerodynamic loads.
2	Aerodynamic Pool Distributor	(Existing with Modifications)	Forms tables of aerodynamic data. 1. Defines boundaries of aerodynamic elements. 2. Locates and orients displacement components at aerodynamic control points.
3	Aerodynamic Plotter	(Existing)	Plots aerodynamic elements and control point displacement directions in 3-D projections.
4	Geometry Interpolator	(Existing)	Forms the matrix relating displacements at aerodynamic control points to structural displacements.
5	Aerodynamic Downwash Generator	(New)	Forms the downwash matrices based on user defined perturbations and the steady angle of attack vector for each aerodynamic theory.
6	(A) → Aerostatic Loop Driver	(New)	Sets parameters to control the aerostatic subcase loop.
7	Aerodynamic Matrix Generator	(Existing with Modifications)	Forms the basic aerodynamic matrices.
8	Aerodynamic Matrix Processor	(DMAP)	Forms composite aerodynamic matrices.
9	Aerodynamic Matrix Assembler	(DMAP)	Assembles matrices required for divergence analysis, untrimmed loads analysis or trimmed loads analysis.

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Continue 1

TABLE 1. Simplified Flow Diagram for Static Aeroelastic Analysis (Cont'd.)

STEP	MODULE	STATUS	FUNCTION
	Continue 1		
10	Stability and Control Derivative Printer	(New)	Calculates and prints the non-dimensional stability and control derivatives.
11	Divergence Analysis	(DMAP)	Finds divergence speed(s) and eigenvector(s).
12	Aerostatic Load Generator	(DMAP)	1. Reduces angle of attack distribution to structural grid point loads. 2. Combines air loads with other loads.
13	Aerostatic Solution Generator	(New)	Solves for displacements in trimmed condition.
14	Aerodynamic Data Recovery	(New)	1. Recovers aerodynamic displacements, downwashes, pressures and forces at control points (all optional). 2. Recovers net forces and moments on vehicle for untrimmed condition (optional).
	(A) Bottom of Subcase Loop		
15	Static Data Recovery Modules	(Existing)	1. Recovers dependent displacements. 2. Finds internal loads and stresses (optional).
16	Output File Processor	(Existing)	Organizes output data for printing and plotting.
17	Deformed Structures Plotter	(Existing)	1. Plots divergence mode(s) in 3-D projection (optional). 2. Plots deformed structure due to loads (optional).

Generation of Aerodynamic Matrices

The matrices generated by the Aerostatic Matrix Generator (see Section 5.13 of Reference 1) are denoted by symbols identical to those used in dynamic analysis (see Section 5.5 of Reference 1), with the addition of the experimental corections introduced here. Thus,

$$\{F_k^a\} = [S_{kj}]\{f_j^a\} \tag{2}$$

$$\{f_j^a\} = \bar{q}[W_{kk}][A_{jj}]^{-1}\{w_j\} + \{f_j^{a(e)}\} \tag{3}$$

and

$$\{w_j\} = [D_{jk}]\{u_k\} + [D_{jx}]\{u_x\} + \{w_j^g\} \tag{4}$$

where

$\bar{q} = \frac{1}{2}\rho V^2$ is the dynamic pressure.

$\{F_k^a\}$ is the vector of aerodynamic forces at aerodynamic control points.

$[W_{kk}]$ is a matrix of empirical correction factors to adjust the theoretical aerodynamics to agree with experimental data. Reference 3 suggests one way for obtaining these factors.

$\{f_j^a\}$ is a vector of pressure coefficients; one or more for each aerodynamic element.

$\{w_j\}$ is a vector of aerodynamic degrees of freedom (e.g., angles of attack).

$\{f_j^{a(e)}\}$ is a vector of experimental pressure coefficients.

$\{u_x\}$ are "extra aerodynamic points" used to describe aerodynamic control surface deflections, and overall rigid body motions.

$\{w_j^g\}$ represents the static aerodynamic excitation. It includes, primarily, the static incidence distribution which may arise from an initial angle of attack, camber, washout, or thermal distortion.

$\{u_k\}$ is a vector of structural displacements (deformations).

The matrices, $[W_{kk}]$, $[S_{kj}]$, $[A_{jj}]$, $[D_{jk}]$ and $[D_{jx}]$, are matrices of real constants, which may be functions of Mach number or other parameters. $[W_{kk}]$ is supplied by the user. Each aerodynamic theory provides separate procedures for calculating the remaining matrices. $[A_{jj}]$ must be nonsingular.

As an example to illustrate the use of aerodynamic extra points, let $\{u_x\}$ be written in the partitioned form

$$\{u_x\} = \left\{ \begin{array}{c} u_\alpha \\ \hline u_q \end{array} \right\} \tag{5}$$

in which $\{u_\alpha\}$ is the set of incidence variables, e.g., angle of attack and elevator deflection, and $\{u_q\}$ is the set of rate variables, e.g. $q\bar{c}/2V$. In the basic trim problem, the rate variables are specified by the user and the incidence variables are determined by the trim solution. u_x may include the following motion: angle of attack, sideslip, roll rate, pitch rate, yaw rate, The forces on the system are assumed to be applied directly to the structure and affect the aerodynamic motion only indirectly.

The aerodynamic forces applied to structural degrees of freedom can be written in the form

$$\begin{Bmatrix} F_\ell^a \\ \hline F_r^a \\ \hline F_x^a \end{Bmatrix} = - \left[\begin{array}{c|c|c} K_{\ell\ell}^a & K_{\ell r}^a & K_{\ell x}^a \\ \hline K_{r\ell}^a & K_{rr}^a & K_{rx}^a \\ \hline 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} u_\ell \\ \hline u_r \\ \hline u_x \end{Bmatrix} + \begin{Bmatrix} P_\ell^a \\ \hline P_r^a \\ \hline 0 \end{Bmatrix} \qquad (6)$$

The partitions $[K_{\ell r}^a]$ and $[K_{rr}^a]$ of the aerodynamic stiffness matrix are not required because the rigid body displacements $\{u_r\}$ are assumed to be zero in calculating the loads $\{P_\ell^a\}$ and $\{P_r^a\}$. The non-zero partitions may be written as

$$\left[\begin{array}{c|c} K_{\ell\ell}^a & K_{\ell x}^a \\ \hline K_{r\ell}^a & K_{rx}^a \end{array}\right] = - \bar{q} \left[\begin{array}{c|c} Q_{\ell\ell} & Q_{\ell x} \\ \hline Q_{r\ell} & Q_{rx} \end{array}\right] = - \bar{q}[Q] \qquad (7)$$

In general, the $[Q]$ matrix is formed by means of the formula

$$[Q] = [G_k]^T[W_{kk}][S_{kj}][A_{jj}]^{-1}[D_{jk}][G_k] \qquad (8)$$

The matrix $[G_k]$ is the splining matrix formed in the Geometry Interpolator to connect structural grid points to the aeroelastic points.

Expressions for the initial static aerodynamic load vector are

$$\{P^a\} = \bar{q}\Big([Q_j]\{w_j^g\} + \{Q^e\}\Big) \qquad (9)$$

where $\{w_j^g\}$ are "downwash" terms supplied via input data and

$$[Q_j] = [G_k]^T[W_{kk}][S_{kj}][A_{jj}]^{-1} \tag{10}$$

Formulation of the Coupled Free System

In References 1 and 2, the structure displacements are defined by the u_a (u_ℓ and u_r) displacement sets. The motions of the rigid body and aerodynamic surfaces are defined by the u_x set in Reference 2 (using u_e as in Reference 1 was considered to be a conflict with the old EPOINT data). The structural and aerodynamic matrices calculated by the matrix routines are:

Structural Matrix Partitions

$$K_{\ell\ell},\ K_{\ell r},\ K_{rr},\ M_{\ell\ell},\ M_{\ell r},\ M_{rr}$$

Aerodynamic "Stiffness" Matrices

$$K_{\ell\ell}^a,\ K_{\ell r}^a,\ K_{\ell x}^a,\ K_{r\ell}^a,\ K_{rx}^a$$

(Note the non-symmetry and the fact that we ignore forces on the u_x points. These forces are obtained from the structural points.)

The total displacements of the system are:

$$u\ (\text{Total}) = u_a\ (\text{Structure}) + u_x\ (\text{Aerodynamic})$$

First we must constrain the structural motion, u_a, to be orthogonal to the rigid body modes, i.e.,

$$\Phi_o^T M_{aa} u_a = 0 \tag{11}$$

Since Φ_o is a linear combination of the rigid body support shapes, it may be replaced by the rigid body transformation matrix D in the form:

$$[\Phi_o] = \left[\begin{array}{c} D_{\ell r} \\ \hline I \end{array} \right] [\alpha_{ro}] \tag{12}$$

where $[\alpha_{ro}]$ is a nonsingular matrix of constants and:

$$[D_{\ell r}] = -[K_{\ell\ell}]^{-1}[K_{\ell r}] \tag{13}$$

The columns of $[D_{\ell r}]$ represent the rigid motions and is the normal output from the free body operation. Since only one form of $[D_{\ell r}]$ exists, it is abbreviated as $[D]$ below.

An alternate method is obtained by partitioning Eq (11) and substituting Eq (12) to produce the following constraint:

$$[D^T M_{\ell\ell} + M_{\ell r}^T] \; \{u_\ell\} + [D^T M_{\ell r} + M_{rr}] \; \{u_r\} = \{0\} \tag{14}$$

Using the Lagrange multiplier method of applying constraints, the "inertially constrained" structure system is:

$$\begin{bmatrix} K_{\ell\ell} & K_{\ell r} & M_{\ell\ell}D+M_{\ell r} \\ K_{\ell r}^T & K_{rr} & M_{\ell r}^T D+M_{rr} \\ D^T M_{\ell\ell}+M_{\ell r}^T & D^T M_{\ell r}+M_{rr} & 0 \end{bmatrix} \begin{Bmatrix} u_\ell \\ u_r \\ \lambda \end{Bmatrix} = \begin{Bmatrix} P_\ell \\ P_r \\ 0 \end{Bmatrix} \tag{15}$$

The λ displacements actually correspond to accelerations because of their units. The u_ℓ and u_r motions defined by Eq (15) are measured relative to the actual CG motion (inertial reference system).

Note that Eq (15) is nonsingular if the $K_{\ell\ell}$ partition is nonsingular and the mass terms have at least one mass per direction. This matrix formulation provides the same effect as the FLEXSTAB flexibility approach (Reference 4).

Another form of Eq (15) may be obtained by multiplying the first row partitions by $[D]^T$, adding them to the second row of partitions and replacing the second row by the results. The system then becomes:

$$\begin{bmatrix} K_{\ell\ell} & K_{\ell r} & M_{\ell\ell}D+M_{\ell r} \\ 0 & 0 & m_r \\ D^T M_{\ell\ell}+M_{\ell r}^T & D^T M_{\ell r}+M_{rr} & 0 \end{bmatrix} \begin{Bmatrix} u_\ell \\ u_r \\ \lambda \end{Bmatrix} = \begin{Bmatrix} P_\ell \\ P_r+D^T P_\ell \\ 0 \end{Bmatrix} \tag{16}$$

where $[m_r] = [M_{rr} + M_{r\ell}D + D^T M_{\ell r} + D^T M_{rr} D]$ is the "total" mass matrix attached to the u_r points.

Note that the zero terms occur because $[K_{rr} + D^T K_{\ell r}] = 0$ when the u_r set properly spans the rigid body motion. The second row of partitions in Eq (16) now obviously define the rigid body accelerations, $\lambda = \ddot{u}_r$, in terms of the loads. The next step is to include the aerodynamic forces in terms of their matrix partitions.

Adding the aerodynamic matrix terms, $[K_{\ell\ell}^a]$, etc., and the extra degree of freedom, u_x, and rearranging the system of Eq (16), we obtain:

$$\left[\begin{array}{c|c|c|c} K_{\ell\ell}+K_{\ell\ell}^a & K_{\ell r}+K_{\ell r}^a & M_{\ell\ell}D+M_{\ell\ell} & K_{\ell x}^a \\ \hline D^T M_{\ell\ell}+M_{r\ell} & D^T M_{\ell r}+M_{rr} & 0 & 0 \\ \hline D^T K_{\ell\ell}^a+K_{r\ell}^a & D^T K_{\ell r}^a+K_{rr}^a & m_r & D^T K_{\ell x}^a+K_{rx}^a \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} u_\ell \\ u_r \\ \ddot{u}_r \\ u_x \end{Bmatrix} = \begin{Bmatrix} P_\ell \\ 0 \\ D^T P_\ell + P_r \\ P_x \end{Bmatrix} \tag{17}$$

This matrix corresponds to Eq (27) on page (B-9) of Reference 1 (with $u_x = u_e^a + u_o^a$) . If direct input matrices, $[K^2]$, are included, they are added automatically to Eq (17).

Note that the transformed and lower null partitions are deliberate. This indicates a singularity of order N_x which must be satisfied by constraining various combinations of $\ddot{u}_r$ and u_x . This may be done more easily on a reduced set of equations to be obtained below.

Reduction and Solution of the Coupled System

The system of equations defined by Eq (17) may be reduced by the following scheme:

(1) Eliminate the variables, $[u_1] = [u_\ell, u_r]$, by solving the first two rows of Eq (17) for u_ℓ and substituting into the third row.

(2) Combine the remaining variables, $[u_2] = [\ddot{u}_r, u_x]$, for matrix calculations, then partition the sets for the final results.

First define the following partitions:

$$[K_{11}] = \left[\begin{array}{c|c} K_{\ell\ell}+K_{\ell\ell}^a & K_{\ell r}+K_{\ell r}^a \\ \hline D^T M_{\ell\ell}+M_{r\ell} & D^T M_{\ell r}+M_{rr} \end{array}\right] \tag{18}$$

$$[K_{r1}] = [D^T K_{\ell\ell}^a + K_{r\ell} \mid D^T K_{\ell r}^a + K_{rr}^a] \tag{19}$$

$$[K_{12}] = \left[\begin{array}{c|c} M_{\ell\ell} D + M_{\ell r} & K_{\ell x}^a \\ \hline 0 & 0 \end{array}\right] = [M_{1r} \mid K_{1x}] \tag{20}$$

$$\{P_1\} = \left\{\begin{array}{c} P_\ell \\ \hline 0 \end{array}\right\} \tag{21}$$

$$[M_{r2}] = [m_r \mid D^T K_{\ell x}^a + K_{rx}^a] \tag{22}$$

In terms of the new variables sets, Eq (17) becomes:

$$\left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{r1} & M_{r2} \end{array}\right] \left\{\begin{array}{c} u_1 \\ \hline u_2 \end{array}\right\} = \left\{\begin{array}{c} P_1 \\ \hline P_r - D^T P_\ell \end{array}\right\} \tag{23}$$

Solving the upper rows for u_1 and substituting into the lower rows produces the system:

$$[K_{r2}] \ \{u_2\} = \{P_r + D^T P_\ell - K_{r1} \ K_{11}^{-1} \ P_1\} = \{P_r^s\} \tag{24}$$

where

$$[K_{r2}] = [M_{r2} - K_{r1} \ K_{11}^{-1} \ K_{12}] \tag{25}$$

By constraining the $\ddot{u}_r$ and u_x generalized coordinates, the rectangular matrix $[K_{r2}]$ may be used directly in Eq (24) to obtain solutions. For direct solutions, the user selects N_x or more components of u_2 to be fixed values. These columns are then moved to the right hand side and treated as loads. It is also interesting to examine the partitions of $[K_{r2}]$ in the alternate form below:

$$[M_{rr}^{s}] \ \{\ddot{u}_r\} + [K_{rx}^{s}] \ \{u_x\} = \{P_r^{s}\} \tag{26}$$

where the matrices may be calculated by either method below:

a) Evaluate Eqs (22) and (25) and partition.

$$[M_{rr}^{s} \ \vdots \ K_{rx}^{s}] = [K_{r2}] \tag{27}$$

or b) Substitute for the parts of Eq (25) and calculate directly.

$$[M_{rr}^{s}] = [m_r - K_{r1} K_{11}^{-1} M_{1r}] \tag{28}$$

$$[K_{rx}^{s}] = [D^T][K_{\ell x}^{a}] - [K_{r1}][K_{11}]^{-1}[K_{1x}] + [K_{rx}^{a}] \tag{29}$$

If $\{u_x\}$ is partitioned as discussed in Eq (5) above

$$\{u_x\} = \left\{ \begin{array}{c} u_\alpha \\ \hline u_q \end{array} \right\}$$

then $[K_{rx}^{s}]$ is also partitioned

$$[K_{rx}^{s}] = [K_{r\alpha}^{s} \ \vdots \ K_{rq}^{s}] \tag{30}$$

and Eq (26) may be rewritten

$$[M_{rr}^{s}]\{\ddot{u}_r\} + [K_{r\alpha}^{s}]\{u_\alpha\} + [K_{rq}^{s}]\{u_q\} = \{P_r^s\} \tag{31}$$

Two trim options then follow: the basic trim assumes the rate variables to be specified, the accelerations to be zero (since gravity loads are included) in $\{P_r^s\}$, and solves for the incidence variables.

$$\{u_\alpha\} = [K_{r\alpha}^{s}]^{-1}(\{P_r^s\} - [K_{rq}^{s}] \{u_q\}) \tag{32}$$

The steady rate trim option assumes the attitude variables to be specified, e.g., an aileron deflection, and solves for the rate variables with the accelerations again set to zero and also with the external loads set to zero

$$\{u_q\} = -[K_{rq}^{s}]^{-1}[K_{r\alpha}^{s}] \{u_\alpha\} \tag{33}$$

Note that for any of these cases the full u_2 vector is obtained and the u_1 vector may be recovered from Eq (23). Structural data are then recovered as in a statics problem.

The Aerodynamic Stability Derivatives

The stability derivatives are aerodynamic coefficients nondimensionalized by reference geometry appropriate to the specific derivative. In the longitudinal case, e.g., in which angle of attack, α , and elevator deflection, δ_e, are the two incidence variables,

$$\{u_\alpha\} = \begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix} \tag{34}$$

so

$$[m_r][M_{rr}^{s}]^{-1}[K_{r\alpha}] = -\bar{q}S \begin{bmatrix} C_{z_\alpha} & C_{z_\delta} \\ C_{m_\alpha}\bar{c} & C_{m_\delta}\bar{c} \end{bmatrix} \tag{35}$$

where $\bar{q}$ is the dynamic pressure and S and $\bar{c}$ are the reference area and chord, respectively. The factor $[m_r][M_{rr}^{s}]^{-1}$ introduces the inertial relief effects into the unrestrained derivatives. For the case of longitudinal pitching with pitch rate, q,

$$\{u_q\} = q\bar{c}/2V \tag{36}$$

and

$$[m_r][M_{rr}^{s}]^{-1}[K_{rq}^{s}] = -\bar{q}S \begin{Bmatrix} C_{z_q} \\ C_{m_q}\bar{c} \end{Bmatrix} \tag{37}$$

Similar relationships are available for the lateral-directional stability derivatives. The rigid stability derivatives are appropriately found from the user's selection of a small value of $\bar{q}$ and do not need special consideration, other than ignoring the inertial derivatives which are finite at small values of $\bar{q}$.

The so-called intercept or initial aerodynamic coefficients which arise from incidence, camber, twist, and thermal distortion enter the aerodynamic problem through the downwash $\{w_j^g\}$ in Eq (4) of Appendix B of Reference 1. They are contained here, along with gravity loads, in $\{p_r^s\}$. In the longitudinal case, they may be found from

$$\bar{q}S \begin{Bmatrix} C_{z_0} \\ C_{m_0}\bar{c} \end{Bmatrix} = \lim_{g \to 0} \{p_r^s\} \tag{38}$$

where g is the acceleration of gravity.

Formulation for a Restrained System

Many users prefer aerodynamic coefficients for a restrained vehicle inasmuch as sensor locations are known with respect to structural axes attached to the body, rather than with respect to the principal axes implicit in the preceding development. In this case, we delete the second row and column of Eq (7) of Reference 1.

$$\left[\begin{array}{c|c|c} K_{\ell\ell} + K^a_{\ell\ell} & M_{\ell\ell}D + M_{\ell r} & K^a_{\ell x} \\ \hline D^T K^a_{\ell\ell} + K^a_{r\ell} & m_r & D^T K^a_{\ell x} + K^a_{rx} \\ \hline 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} u_\ell \\ \ddot{u}_r \\ u_x \end{Bmatrix} = \begin{Bmatrix} P_\ell \\ D^T P_\ell + P_r \\ P_x \end{Bmatrix} \tag{39}$$

Eliminating $\{u_\ell\}$ from the above leads to

$$\begin{aligned} [m_r]\{\ddot{u}_r\} = & \left([D]^T - [D^T K^a_{\ell\ell} + K^a_{r\ell}][K_{\ell\ell} + K^a_{\ell\ell}]^{-1}\right)\{P_\ell\} + \{P_r\} \\ & -\left([D^T K^a_{\ell x} + K^a_{rx}] - [D^T K^a_{\ell\ell} + K^a_{r\ell}][K_{\ell\ell} + K^a_{\ell\ell}]^{-1}[K^a_{\ell x}]\right)\{u_x\} \\ & -[D^T K^a_{\ell\ell} + K^a_{r\ell}][K_{\ell\ell} + K^a_{\ell\ell}]^{-1}[M_{\ell\ell}D + M_{\ell r}]\{\ddot{u}_r\} \end{aligned} \tag{40}$$

In Eq (40), the last term has been isolated because it provides the inertial aerodynamic stability derivatives.

As before, the various stability derivatives may be identified using the longitudinal case as the example.

$$\bar{q}S \begin{bmatrix} C_{z_\alpha} & C_{z_\delta} & C_{z_q} \\ C_{m_\alpha}\bar{c} & C_{m_\delta}\bar{c} & C_{m_q}\bar{c} \end{bmatrix} = [D^T K^a_{\ell x} + K^a_{rx}] - [D^T K^a_{\ell\ell} + K^a_{r\ell}][K^a_{\ell\ell}]^{-1}[K^a_{\ell x}] \tag{41}$$

$$\bar{q}S \begin{Bmatrix} C_{z_0} \\ C_{m_0}\bar{c} \end{Bmatrix} = \lim_{g\to 0} \left([D]^T - [D^T K^a_{\ell\ell} + K^a_{r\ell}][K_{\ell\ell} + K^a_{\ell\ell}]^{-1}\right)\{P_\ell\} + \{P_r\} \tag{42}$$

In this restrained case, the inertial stability derivatives are also obtained.

$$\frac{-\bar{q}S}{g}\begin{bmatrix} C_{z_{\ddot{z}}} & C_{z_{\ddot{\theta}}}\bar{c}/2 \\ C_{m_{\ddot{z}}}\bar{c} & C_{m_{\ddot{\theta}}}\bar{c}^2/2 \end{bmatrix} = [D^T K_{\ell\ell}^a + K_{r\ell}^a][K_{\ell\ell} + K_{\ell\ell}^a]^{-1}[M_{\ell\ell}D + M_{\ell r}] \tag{43}$$

Eq (17) is the equation of motion of the principal axes, whereas Eq (40) is the equation of motion of structural axes, i.e., axes through the support system. Eq (40) is therefore not invariant with the choice of support system as discussed in Reference 5. Since sensor signals are related to their displacements relative to the structural axes (as well as to the principal axes), the relative angle between the longitudinal principal and structural axes is a required addition to Eq (40), as also discussed in Reference 5. The displacements $\{u_p\}$ (translations and rotations) of the principal axis relative to the structural axis are given by

$$\{u_p\} = [m_r]^{-1}[D^T M_{\ell\ell} + M_{r\ell}]\ \{u_\ell\} \tag{44}$$

It should be noted that the angle of attack of the principal axis is the R2 component of the solution of Eq (40), i.e.,

$$\alpha_p = \text{R2 component of} \iint \{\ddot{u}_r\} dt\ dt \tag{45}$$

The derivatives of Reference 5, $\frac{\partial\alpha_p}{\partial\alpha_s}$, $\frac{\partial\alpha_p}{\partial\delta}$, and $\frac{\partial\alpha_p}{\partial q}$, are found from the R2 component of $\{u_p\}$ by letting

$$\{u_\ell\} = -\ [K_{\ell\ell} + K_{\ell\ell}^a]^{-1}[K_{\ell x}^a]\{u_x\} \tag{46}$$

in Eq (44), and selecting unit values of each u_x, in turn. The derivatives of Reference 5, $\frac{\partial \alpha_p}{\partial \ddot{z}}$ and $\frac{\partial \alpha_p}{\partial \ddot{\theta}}$, are found by letting

$$\{u_\ell\} = - [K_{\ell\ell}^a + K_{\ell\ell}^a]^{-1}[M_{\ell\ell}D + M_{\ell r}] \{\ddot{u}_r\} \qquad (47)$$

in Eq (44), and choosing nonzero values of each $\ddot{u}_r$ in turn (for plunge, $\ddot{u}_r = g$, and for pitch, $\ddot{\theta} = 2g/\bar{c}$) . Finally, the angle α_{p_0} of Reference 5 is found from the R2 component of $\{u_p\}$ by letting

$$\{u_\ell\} = \lim_{g \to 0} [K_{\ell\ell} + K_{\ell\ell}^a]^{-1}\{P_\ell\} \qquad (48)$$

Example Problems

Two example problems are solved to illustrate the new MSC/NASTRAN aeroelastic capability. The first example is the forward-swept-wing airplane (FSW) considered in Reference 5 in symmetrical flight. The second example is the BAH* Wing in rolling maneuvers.

An extremely idealized FSW configuration is shown in Figure 1. The wing has an aspect ratio of 4.0, no taper, no twist or camber, an incidence angle of 1.0° relative to the fuselage, and a forward sweep angle of 30°; the canard has an aspect ratio of 1.0 and no taper, twist, camber, incidence, or sweep. The chords of the wing and canard are both 10.0 ft; the reference chord is chosen as $\bar{c}$ = 10.0 ft. and the reference area is S = 400 sq. ft. The half-model of the wing is divided into 32 equal boxes, as shown on the left wing of Figure 1, for the doublet-lattice aerodynamic analysis and the canard is divided into 8 boxes as shown also in Figure 1. Aerodynamic forces on the fuselage are neglected. The right wing and fuselage in Figure 1 show the

*This is the example problem of a jet transport wing considered throughout Reference 6 that was used as a NASTRAN demonstration problem in Reference 7.

structural idealization. Four weights are located at the one-quarter and three-quarter span and chord positions, and are assumed to be connected to the 50% chord elastic axis by rigid streamwise bars; the weights are assumed to be 600 lbs. forward and 400 lbs. aft, giving a wing centroid at 45% of the wing chord. The wing is assumed to be uniform with equal bending (EI_x) and torsion (GJ) stiffnesses of 25 x 10^6 lb-ft^2 and connected at its root to the fuselage. The fuselage is assumed to have the same bending stiffness as the wing and is shown with four equal and equidistant weights (1500 lbs each per side); the fuselage length is 30.0 ft. The total weight per side is 8000 lbs., the center of gravity is 12.82 ft. forward of the intersection of the fuselage and wing elastic axis, and the centroidal moment of inertia in pitch per side is Iy = 892,900 lb-ft^2. The airplane is assumed to be flying at a Mach number M = 0.9 at sea level ($\bar{q}$ = 1200 psf).

Two sets of aerodynamic stability derivatives are obtained. One is for the restrained vehicle (restrained at GRID 100, the intersection of the fuselage and wing elastic axes), and the other is for the unrestrained vehicle, i.e., as defined in Reference 4 in the FLEXSTAB computer program. The derivatives are summarized in Table 2; the pitching moment coefficients are referred to an axis through GRID 100. The rigid values of the coefficients (found by setting $\bar{q} = 10^{-6}$) are shown for reference. Note that there are no inertial derivatives in the unrestrained case since inertial relief effects have been included in the basic derivatives. Table 2 also includes the principal axis rotations required by the equations of motion of Reference 5 when restrained derivatives are utilized. Finally, the MSC/NASTRAN level flight trim solution results in an angle of attack α at GRID 100 = -0.729^0 and a canard rotation $\delta = +2.060^0$.

Additional outputs for displacements and stresses are also available but are not presented because of space limitations. They will be presented in the update to the MSC/NASTRAN Aeroelastic Supplement.

The second example is the BAH Wing in a steady roll. The idealization of the wing/aileron combination is shown in Figure 2 which is taken from Reference 7 where all the physical and geometric characteristics of the wing are discussed. The characteristics of interest in this summary are the wing area S = 81,250 sq. in. per side, the total span b = 1000 in. Only the half-airplane value of S and is needed since only half the airplane is modeled in the MSC/NASTRAN analysis; the total span is required by the conventional definition of lateral-directional aerodynamic stability derivatives.

The case of steady roll has been analyzed in Reference 7 as a transient maneuver by the Fourier transform method, and the rolling helix angle was found to be $pb/2V\delta = 0.16$ at a sea level airspeed of $V = 475$ mph, assuming $M = 0.0$ aerodynamics. However, a more recent solution (based on more reduced frequencies on the MKAERØ card) resulted in a more accurate value of $pb/2V\delta = 0.21$. The present solution is shown in Figure 3 as a function of dynamic pressure $\bar{q}$ and the revised Fourier transform solution is also shown for $V = 475$ mph ($\bar{q} = 4.0075$ psi, for the units used in the analysis). The aileron reversal dynamic pressure is found to be $\bar{q} = 11.17$ psi which corresponds to $V = 793$ mph at sea level based on the $M = 0.0$ aerodynamics. The stability derivatives are shown in Table 3 for $\bar{q} = 4.0075$ psi, along with the rigid values for comparison. The steady roll solution at $V = 475$ mph is $pb/2V\delta = - C_{l_\delta}/C_{l_p} = 0.2151$, in agreement with the revised Fourier transform solution.

TABLE 2. Derivatives for Example FSW Airplane

Derivative	Rigid Value	Restrained Value	Unrestrained Value
C_{L_o}	0.8422	0.10638	0.1337
C_{m_o}	0.06624	0.08523	0.10630
C_{L_α}	5.071	6.413	8.109
C_{m_α}	4.736	5.951	7.201
C_{L_δ}	0.2461	0.3177	0.4512
C_{m_δ}	0.9407	1.0663	1.1101
C_{L_q}	-3.140	-4.786	-7.480
C_{m_q}	-6.050	-7.649	-9.592
$C_{L_{\ddot{z}}}$	0.0	-0.002850	-
$C_{m_{\ddot{z}}}$	0.0	-0.005123	-
$C_{L_{\ddot{\theta}}}$	0.0	-0.008810	-
$C_{m_{\ddot{\theta}}}$	0.0	-0.02103	-
α_{p_o}	0.0	-0.0001383	-
α_{p_α}	1.0	1.0659	1.0
α_{p_δ}	0.0	0.07378	-
α_{p_q}	0.0	0.1915	-
$\alpha_{p_{\ddot{z}}}$	0.0	-0.003321	-
$\alpha_{p_{\ddot{\theta}}}$	0.0	-0.01680	-

TABLE 3. Rolling Derivatives for BAH Wing

Derivative	Rigid Value	Value at V = 475 mph at Sea Level
C_{l_p}	-0.4412	-0.5193
C_{l_δ}	0.1446	0.1117

Concluding Remarks

A stiffness formulation of various quasi-static aeroelastic problems at subsonic speeds has been presented as an automated option in MSC/NASTRAN. The applications include the trim solution, aerodynamic stability derivatives for the restrained and unrestrained vehicle, aerodynamic loads, structural deflections and element stresses. Although the basic formulation is linear, some provisions have been made to account for nonlinear aerodynamic behavior through optional additional experimental loads and correction factors for the aerodynamic influence coefficients. Two examples have been presented to illustrate symmetrical and antisymmetrical trim solutions. The highlights of the solutions have been discussed and the complete details of the solutions will be published in a forthcoming update of the MSC/NASTRAN Aeroelastic Supplement.

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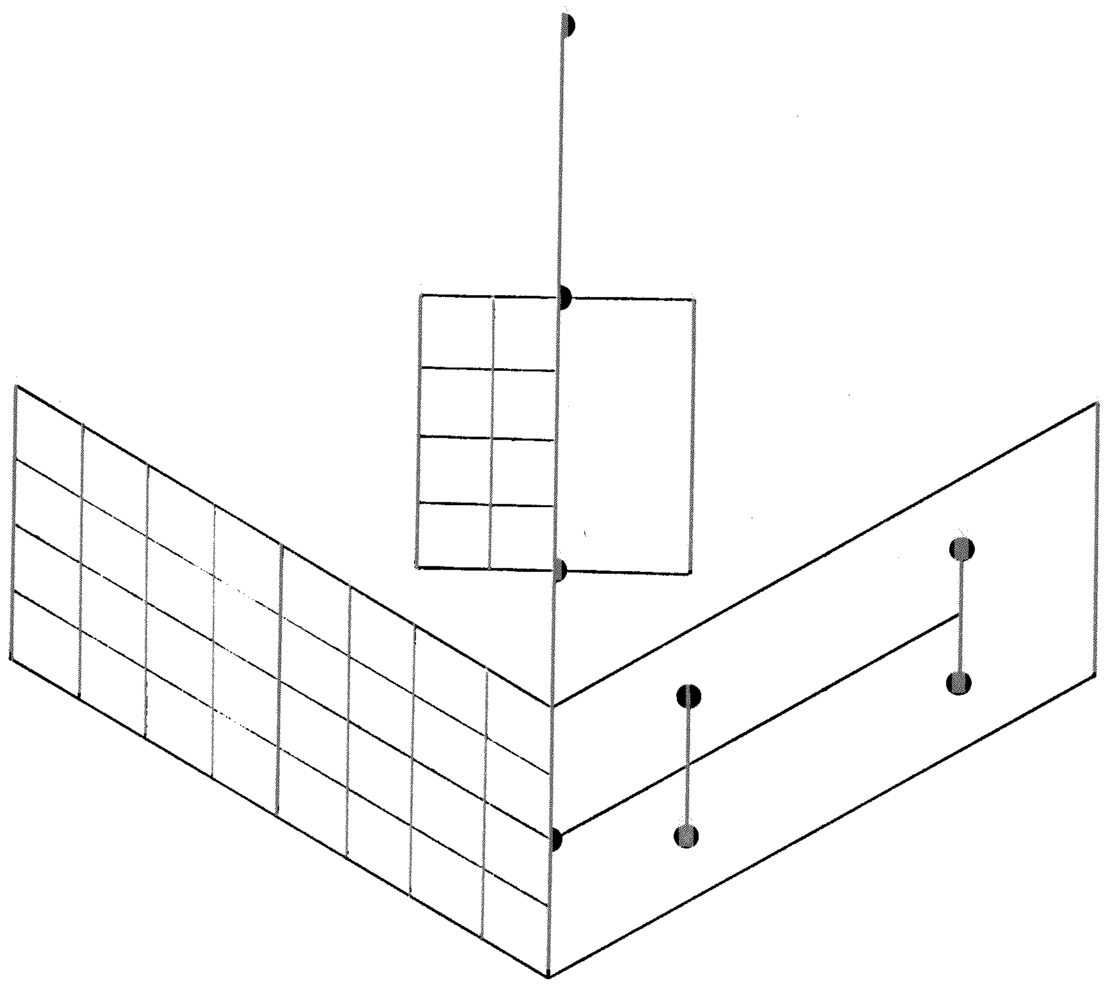

FIGURE 1. Idealization of FSW Configuration

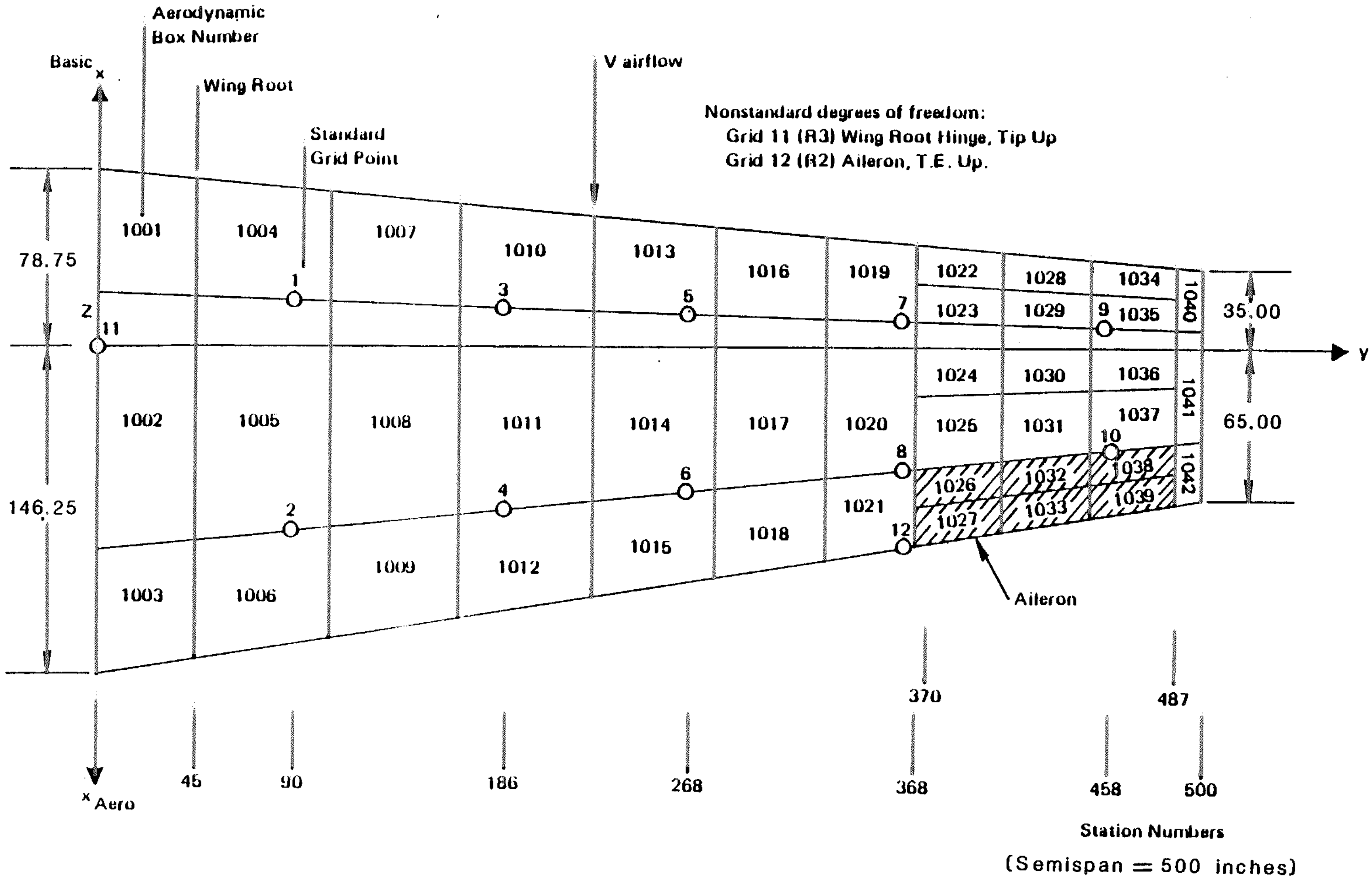

FIGURE 2. Idealization of BAH Wing for Rolling Analysis

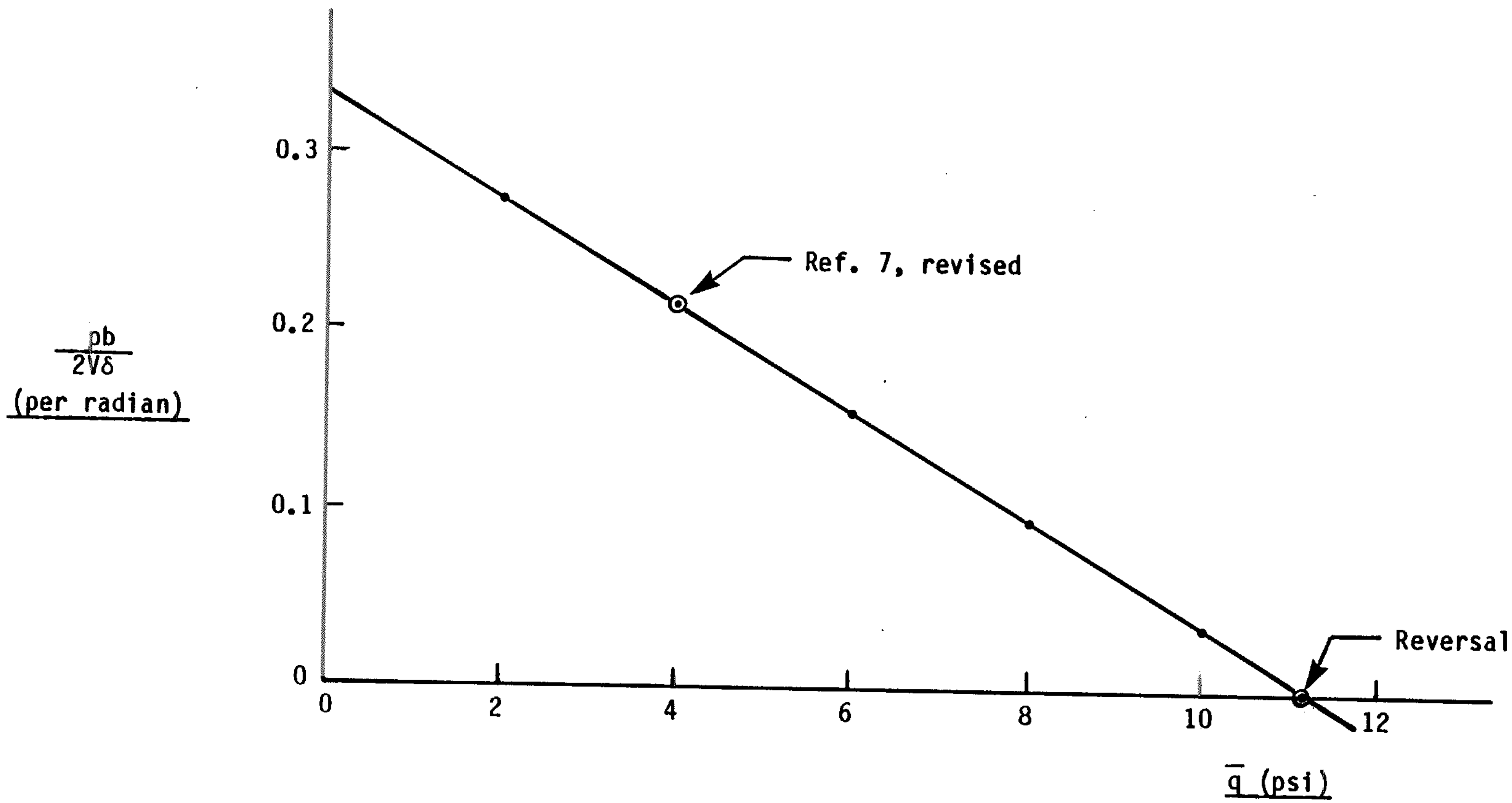

FIGURE 3. Aileron Effectiveness of BAH Wing