STRUCTURAL OPTIMIZATION WITH MSC/NASTRAN APPLIED TO GEAR HOUSING

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PROBLEM STATEMENT

The purpose of this paper is to present a structural design optimization capability that is made possible by the sensitivity analysis option recently implemented in MSC/NASTRAN. In the past, a number of examples of relatively small scale were reported in literature using research oriented programs, but detailed description of solutions of structural design optimization problems of practical scale and complexity are scarce. In this paper, application of a structural optimization capability to a practical structural design problem is described. The example structure is a large steel bearing housing as shown in Fig. 1. This example was selected because it had most of the attributes required to demonstrate important features of the design optimization system such as: large size, displacement as well as stress constraints, load combinations, multiple boundary conditions, and reasonable number of design variables.

In reality, the structure shown in Fig. 1 is the front half of the gear housing and the entire structure is obtained by connecting a mirror image at the back of the structure shown in Fig. 1. Namely, the real depth of the housing is 26.75 x 2 = 53.5 inches. Note that the width of the pinion journal bearings is 9.188 in. and that of the bull gear is 13.0 in. For structural analysis, one half of the structure shown in Fig. 1 is modeled using 243 CBARs, 5 CELAS2s, 1375 CQUAD4s and 4 TRIA3s. (altogether 1623 elements) as shown in Fig. 2. The total number of grid points is 1467 and the number of free degrees of freedom is 7329. The maximum active column is 348 and its RMS value is equal to 244. The structure is analyzed to compute the responses of the specified side for the following loading conditions.

(a) The right half under the operating load, 1-g accessory load and self weight.

(b) The left half under the operating load, 1-g accessory load and self weight.

(c) The right half under the 100-g vertical shock load, 50-g accessory load and operational load.

(d) The left half under the 100-g vertical shock load, 50-g accessory load and operational load.

In order to obtain these responses based on the one half model, analyses of the following six subcases must be carried out and then superimposed according to the SUBCOM case control instructions.

SUBCASE 1 : Symmetric boundary condition without snubber supports
Operating load, 0.5-g vertical accessory loads

SUBCASE 2 : Symmetric boundary condition without snubber supports
Weights of endplate, bull gear and pinion gears

SUBCASE 3 : Anti-symmetric boundary condition without snubber supports
Operating load, 0.5-g vertical accessory loads

SUBCASE 4 : Symmetric boundary condition with snubber supports
50-g endplate, bull gear and pinion gears loads

SUBCASE 5 : Symmetric boundary condition with snubber supports
100-g vertical accessory loads

SUBCASE 6 : Anti-symmetric boundary conditions without snubber supports
100-g vertical accessory loads.

The operating loads applied through the journal bearings are assumed to be distributed over the ±30° sector with COS(3t) pattern about the direction of each arrow given in Fig. 3. The operating loads are assumed to be acting on the arcs located two-thirds of the bearing width from the front edge for pinion gears. For the bull gear, two-thirds of the load is applied to the inner edge and one-thirds is applied at the outer edge. Fig. 4 gives the gravity loads applied through bearings which are distributed over ±90° sectors with COS(t) pattern. Axial positions of these loads are almost the same as the operating loads. In Fig. 5, the accessory loads are presented. They are applied at specified points forward of the front edge and on the center axes, but each of them are distributed through RBE3 elements to the whole front edge circumference of the bearing support. The vertical as well as the horizontal shock loads are specified; however, it was clear that the horizontal shock loads are insignificant in the entire design process, thus only vertical shock loads are considered. The magnitude of the vertical shock loads are computed at 100-g for accessories and 50-g for others. For pinions, the vertical shock loads are applied at the innermost edges of the bearings, while two-thirds is applied on the inner edge and one-thirds on the outer edge for the bull gear.

The boundary conditions specified for this structure are summarized in Fig. 6. The snubber supports are special elastic supports that are considered to be active only for vertical shock loads, hence it is necessary to change the boundary conditions depending upon the load conditions.

The design requirements are summarized as follows. The von Mises stress computed at any of the specified 1390 stress recovery points should not exceed 51,000 psi. Displacement constraints are slightly more involved and classified into three categories as follows.

Center-Center distance change between the bull gear and any of the pinion gear supports : $\leq$ 0.0097 inch

Transverse rotation of the axes,
Bull gear axis : $\leq 1.326 \times 10^{-4}$ rad.

Pinion gear axes : $\leq 1.344 \times 10^{-4}$ rad.

Distortion of the cylindrical surfaces
$|\mathrm{Max}\ dr - \mathrm{Min}\ dr|$ for bull gear : $\leq$ 0.00236 inch

" for pinion gears : $\leq$ 0.00200 inch

where dr stands for the radius change of the originally cylindrical bearing support.

As the result of analysis of the initial design provided by preliminary design process, it was revealed that this original design was not acceptable because there were many stress points where von Mises stresses exceeded the allowable limit of 51,000 psi. The maximum stress observed was about 75,000 psi, indicating 47% violation. There are a number of violations of displacement constraints up to as much as 90% as shown in Table 1.

The problem to be solved is to modify the plate thickness distribution of this structure, so that all stress and displacement constraints are satisfied, with minimum increase in structural weight. The structure is divided into 23 plates and 7 bars, i.e. altogether 30 subsections. Each subsection is assigned to have one independent design variable that can control modification of thickness of a plate or cross sectional area of a bar. The bars are aligned along the free edge of the webs so that the combined sections form "T" sections.

SYSTEM ORGANIZATION

1. Numerical Optimization Methods

It is well recognized that one of the most important aspects of engineering design process is making good decisions within many difficult and conflicting constraints and objectives. Many of the decisions are made by engineers based on their knowledge, experience and creativity. Most often these decisions can be made only by virtue of human intelligence and flexibility. However, certain classes of quantitative decisions are found to be carried out extremely well by high speed computers. If relatively large number of quantities have to be decided simultaneously while satisfying many constraints, the amount of data to be considered in the design process increases far beyond the level of human capability. What is required in this case is not flexible intelligence, but high volume numerical data processing capabilities.

For structural design, basic layout of the structure usually requires human intelligence. On the other hand, many sizing decisions (or gauge decisions) can be formulated in various forms that are amenable to computerized data processing tasks. In the AIAA History of Key Technology paper, Ref. 1, Schmit describes his conception of the idea of applying numerical search techniques to structural design problems. In essence, he found that structural gauge decisions were mathematically equivalent to the scarce resource allocation problems studied extensively by investigators in the general field of operations research. He applied the mathematical programming techniques originally developed to solve problems in operations research to structural design problems and found that these techniques would be useful for structural design.

The key elements of mathematical programming techniques are simple and intuitively easy to understand. It is a numerical search method that finds a minimum or maximum of a scalar function $F(x_1, x_2, \ldots, x_n)$ with respect to multiple number of real variables x_1 throught x_n. For structural design problems, these variables may be thickness of plates, cross sectional area of bars or moment of inertia of beams. The objective function, F, to be minimized may be structural weight, failure probability or fabrication cost, etc. For almost all cases, the design must satisfy certain design criteria, such as stress, deformation, stability, natural frequencies, or design code specifications. These design requirements are also functions of design variables $(x_1, x_2, \ldots, x_n)$ and without losing generality, any one of them may be formulated in either one of the following forms:

$$g(x_1, x_2, \ldots, x_n) \leq 0$$

$$h(x_1, x_2, \ldots, x_n) = 0$$

For example, a condition that a stress should not exceed the given allowable stress may be expressed as:

$$s / s_a - 1.0 \leq 0$$

where s is the stress of interest and s_a is the allowable stress limit. Accordingly, the class of structural member size optimization problems can be cast into a general form as:

$$\text{Minimize } F(x_1, x_2, \ldots x_n)$$

$$\text{Subject to:}$$

$$g_j(x_1, x_2, \ldots x_n) \leq 0 \qquad j = 1, 2, \ldots J$$

$$h_k(x_1, x_2, \ldots x_n) = 0 \qquad k = 1, 2, \ldots K$$

There are a number of computer programs available to solve this class of problems. An optimization program requires an external program that evaluates all the functions involved, i.e., F, g_j and h_k for given values of $(x_1, x_2, \ldots x_n)$. In the early days of the 1960's, finite element structural analysis programs were used to evaluate these functions. The critical problem encountered in such attempts was an extraordinarily large amount of computer time required to evaluate these functions many times as requested by the optimization program. It is not unusual that the number of function evaluations required for convergence went up to several hundreds. If one has to carry out finite element analyses hundreds of times even on today's super computers, not to mention on the computers in the 1960's, the cost and time become prohibitive. One of the reasons that an optimization program requests so many function evaluations is that it needs to evaluate the gradients or sensitivities of these functions with respect to design variables via a finite difference scheme. Therefore, if the senstivity data can be computed externally and more efficiently, then there is a good chance that the number of function evaluations can be reduced by an order of magnitude. There has been no general purpose finite element analysis program (except for relatively small research oriented codes) that provides this vital information. This is the very significance of the sensitivity analysis recently implemented in MSC/NASTRAN.

The numerical optimization code selected for this study is a relatively new code, ADS, developed by G. N. Vanderplaats (Ref. 2). ADS is a program that implements collection of algorithms within one package and users are given a variety of strategies and algorithms to choose from. It will be necessary to have a different class of algorithms in the future, for example, to handle mixed discrete and continuous design variables. For practical purposes, it may be useful to have different algorithms to experiment with different design iteration trajectories to avoid being trapped in relative minima. Furthermore, the one-dimensional search option of the ADS program to improve overall performance of the structural design system may be used. Also, availability of source code for possible modifications, establishment of upgrading & maintenance policy and availability of various consulting services through the users group are attractive features of the ADS program. For ordinary users of this system, these advanced features of ADS are transparent, but users who are interested in taking advantage of various techniques are provided with mechanisms to attempt their experiments. For the users that have been using CONMIN and want to remain CONMIN users, the interface to CONMIN is also available.

2. Approximation Concepts

Even though the sensitivity data may be obtained externally to the optimizer, simple combination of optimization with MSC/NASTRAN still requires an excessively large number of finite element analyses, typically more than 50. It is necessary to further trim down the numerical data processing effort by eliminating all unnecessary computation. The purpose is not to compute structural responses accurately all along the design modification path, but simply guide the design to a practical, near optimal design efficiently. To achieve this objective, approximation concepts described in Ref. 3 have been incorporated and will be extended to take advantage of all available capabilities in MSC/NASTRAN.

The two most important approximations are the temporary deletion of non-critical constraints after each analysis but prior to sensitivity analysis, and optimization with respect to the approximate model. This process reduces data processing effort in the NASTRAN sensitivity analysis as well as reducing CPU and memory requirements in the optimization stage. Theoretically, the number of stress constraints can be very large, i.e. number of stress points times number of load cases. Stress constraints that are not critical compared to specified threshold levels are deleted from further consideration after the NASTRAN analysis. Also, only a small, specified number of stresses are considered from a group of elements. For example, all plate elements whose thicknesses are identical and controlled by one design variable are considered to belong to one group. This is based on the assumption that stress redistribution within such a group may be small, even if the magnitude of stress varies when the thicknesses are changed. For this case, the most critical stress points for the original thicknesses will remain most critical after thicknesses start changing; in other words one has to monitor only a small number of stresses from this group. In a similar manner, displacement, frequency and buckling constraints are screened and deleted if any of them are judged to be non-critical at the particular optimization iteration. Furthermore, as a result of this screening, any of the load cases that are recognized to be non-critical will be deleted temporarily from consideration. This screening process is complicated when load combinations and multiple boundary conditions are involved. Currently, this process is implemented via user input through TABEDIT module, but in the near future, this capability will be automated such that no user input is required.

Another important approximation is the use of Taylor series expansion of constraint functions with respect to intermediate variables that are explicit functions of the original design variables. For the example presented in this paper, stress and displacement constraints are expressed as the linear function of reciprocals of the thicknesses of the plate elements. The reasoning behind the use of reciprocals is explained in detail in Ref. 3. Let t_i be the thickness of elements that belong to the i-th group, s_{ik} is the k-th membrane stress in the i-th group and N_{ik} is the stress resultant, i.e.

$$N_{ik} = t_i \times s_{ik} \quad \text{or} \quad s_{ik} = N_{ik} / t_i$$

For statically-determinate structures, the internal distribution of stress resultant remains unchanged regardless of the changes in the thickness distribution, hence the relation given above is maintained exactly. For statically indeterminate structures, the change of stress resultant is assumed to be moderate, then the relation above may be approximately valid. In this case it is more reasonable to expand stress in terms of a reciprocal of the thickness to obtain better approximation. Once the sensitivity is computed with respect to the variable x_i, which is related to t_i as:

$$x_i = 1.0 / t_i$$

and the approximate expression for the stress s_{ik} becomes:

$$\underline{s}_{ik} = s^0_{ik} + \sum_{j=1}^{n} (x_j - x^0_j) \frac{\partial s_{ik}}{\partial x_j} \, .$$

Insignificant terms in the summation in the equation given above may be deleted for further reduction of sensitivity analysis effort, but the results given in this paper were obtained using all the terms.

3. Sensitivity Analysis Restructuring

A new DMAP program has been developed to expand the available scope of the response sensitivity analyses and also to reduce the non-essential data processing effort. If a load condition is given as a combination of several subcases, displacement sensitivity vectors for all the subcases are computed first and the displacement sensitivity vector for the combined load case is assembled. Then response sensitivity data for stress, deformation, etc., are generated. The sequence of this process is found to be important to accommodate constraints imposed on nonlinear functions of displacement components, such as von-Mises stresses. Also this DMAP program contains an option to use multiple boundary conditions such that one can use a half or a quarter model of a symmetric structure and assemble sensitivity of responses to unsymmetric loadings by means of superposition of displacement sensitivity vectors computed previously for the different boundary and loading conditions. A summary of the capabilities of this program is given in Table 2.

4. Program Structure

One iteration of structural resizing operation consists of execution of four or five modules, as shown in Fig. 7. It is possible to carry out multiple numbers of iterations in a single batch job by a chaining operation using the job control language. However, for large scale problems like the one presented in this paper, the user usually would like to see the intermediate results to make sure the process is progressing well without wasting computational effort for no pay-offs. Three new modules are written in FORTRAN; they are Initial Sort, Constraint Sort and Optimization modules. The functions of these modules, shown in Fig. 7, are briefly outlined as follows.

Initial Sort Module : This module is processed only once at the outset, unless structural model configuration is modified. This module creates tables and vectors that are not affected by the changes of design variables.

Constraint Sort Module : The NASTRAN response analysis provides FORTRAN readable output files. Based on the results contained in these files, the objective and constraint functions are computed. Then, the constraint screening procedure described in the previous section is carried out. One of the outputs of this module is an updated NASTRAN bulk data file which contains DSCONS cards generated based on the screened constraint data. Automated load case deletion option was not exercised in the present example. One of the options of this module is to generate resized models using one iteration of fully stressed design, and if necessary, update the NASTRAN bulk data of the analysis model, as shown in Fig. 7.

Optimization Module : This module reads the response sensitivity files and creates the Taylor series expansions of constraint functions with respect to the specified intermediate variables. The user has to specify the bounds of each design variable, because the following optimization will be carried out against the Taylor series approximation models and validity of such models should be limited to the neighborhood of the current starting design point. At the end of the optimization, the NASTRAN bulk data files are updated to reflect the member size changes. The user is then ready to go back to analysis of the newly sized structure with NASTRAN.

RESULTS

As stated previously, the original design violates both stress and displacement constraints. Uniform scaling is the standard technique to meet strength and stiffness requirements. However, for this example, uniform scale up cannot be performed because 391 out of 1623 elements are of fixed sizes, not affected by design variables. Also, one of the loads is the inertia load that is a function of design variables, thus the displacement and stress may not be reduced by scaling the design variables. In this example, an engineer was provided with the analysis results and he arbitrarily redesigned the structure so that all constraints were satisfied, without paying attention to weight increase. The weight increased to 11,808 lbs. from the original 6,330 lbs., but this design did satisfy all the stress and displacement constraints. This redesign process, to obtain a feasible design, is generally not necessary since ADS has capabilities to overcome constraint violations, but it is convenient to have feasible designs through the design iteration process. In this case, the redesigned structure with its weight equal to 11,808 lbs. was chosen as the starting design and the method of feasible directions was selected as the optimization algorithm (ISTRAT=0, IOPT=1, IONED=4).

The design variable and weight iteration history is given in Table 3 and the weight history is plotted in Fig. 8. Within one iteration, each of the 30 independent design variables are allowed to change ± 100%; for example, a plate with its thickness of 1.2 inches can be modified to any thickness between 0.6 inches and 2.4 inches. No more than one iteration stage was processed in one batch job, since iteration results were examined manually at the end of each iteration. The process was terminated after 5 iterations since less than 1% of weight reduction was achieved for the last 2 consecutive iterations and the design variables were not changing appreciably. The final structural weight, 6,851 lbs. is 8.2%

heavier than the original design, but satisfies all stress and displacement constraints. At this final design, 15 stress, bull gear axis rotation, the right most pinion roundness and 7 side constraints are active.

Figs. 9 and 10 present von Mises stress contours for the original and the final designs, respectively. The identical stress contour labels were used for these two plots, so that it is readily observed that high stress concentrations and many areas with high stress gradients in the original design were eliminated in the final design by the optimization process. Displacement constraint violations, especially transverse rotation of axes, are more difficult to satisfy, but the material distribution of the final design is reasonable and intuitively acceptable. For example, all the bearing supports whose distortion exceeded the upper limit were modified so as to increase their stiffnesses significantly. Transverse rotation of the bull gear axis was reduced by increasing the bearing support thickness as well as the stiffener thicknesses.

Run times experienced on CRAY-1S are summarized in Table 4, where the CPU times for one typical iteration are given. The numbers in parenthesis are the times required before the improvements listed in Table 2 were implemented. As can be seen clearly in the right-most column, CPU time expended outside of NASTRAN is less than 3.3% of the total time, therefore it is more important to implement schemes that can reduce data processing effort in NASTRAN. Sensitivity analysis is still expensive and this is more so when the numbers of design variables and load cases are increased. Approximations used in this study do not go beyond the level described in Ref. 2, but it will be possible to introduce further improvements in the sensitivity analysis.

CONCLUSIONS

Based on the sensitivity analysis capability of MSC/NASTRAN, an efficient and general structural optimization system has been developed. This system is capable of solving problems of practical scale and complexity as demonstrated in this paper. The total computational cost required to obtain the optimal design was approximately 17 times that of the original displacement and stress analysis. This cost factor may be reduced somewhat in the future. Currently, a practical estimated cost for a typical design optimization problem involving 20-30 design variables and less than 10 load conditions may be 10 to 20 times the cost of a static analysis.

REFERENCES

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2. Vanderplaats, G. N., Sugimoto, H. and Sprague, C. M. "ADS-1 : A New General Purpose Optimization Program", AIAA Paper No. 83-0831. Proc. AIAA/ASME/ASCE/AHS 24th SDM Conference, Lake Tahoe, Nevada May 1983

3. Schmit, L. A. and Miura, H. "Approximation Concepts for Efficient Structural Synthesis", NASA CR-2552 March 1976

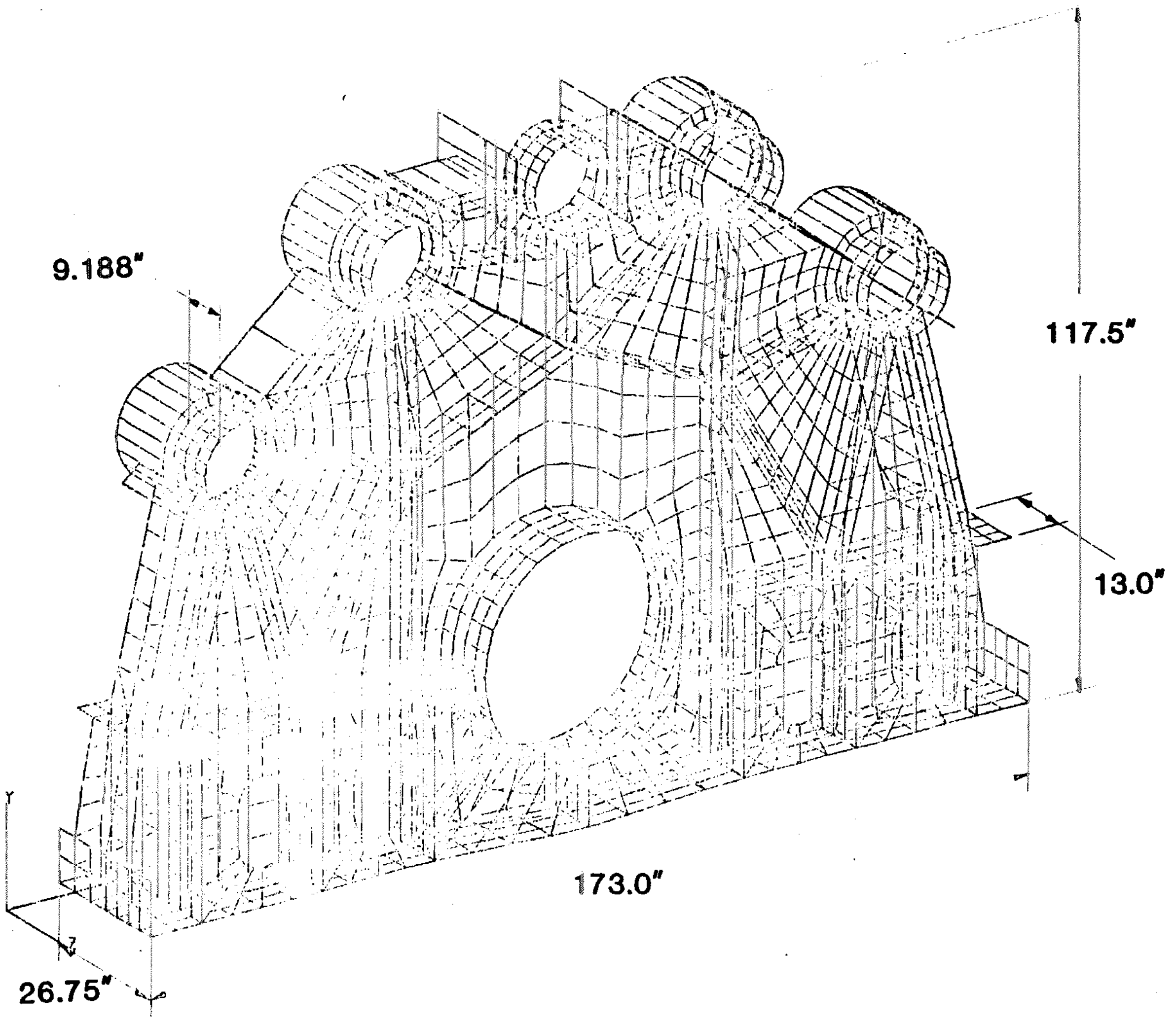


FIG. 1 **KEY PHYSICAL DIMENSIONS**

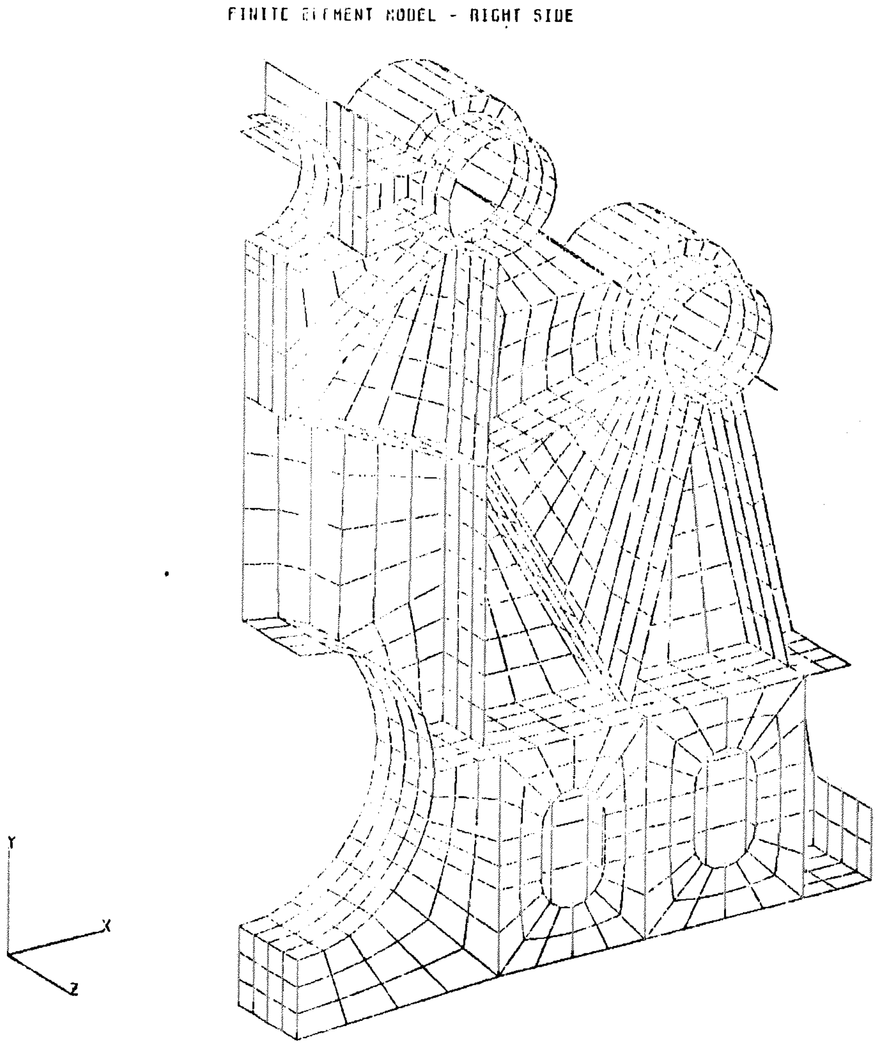


FIG. 2 FINITE ELEMENT ANALYSIS MODEL

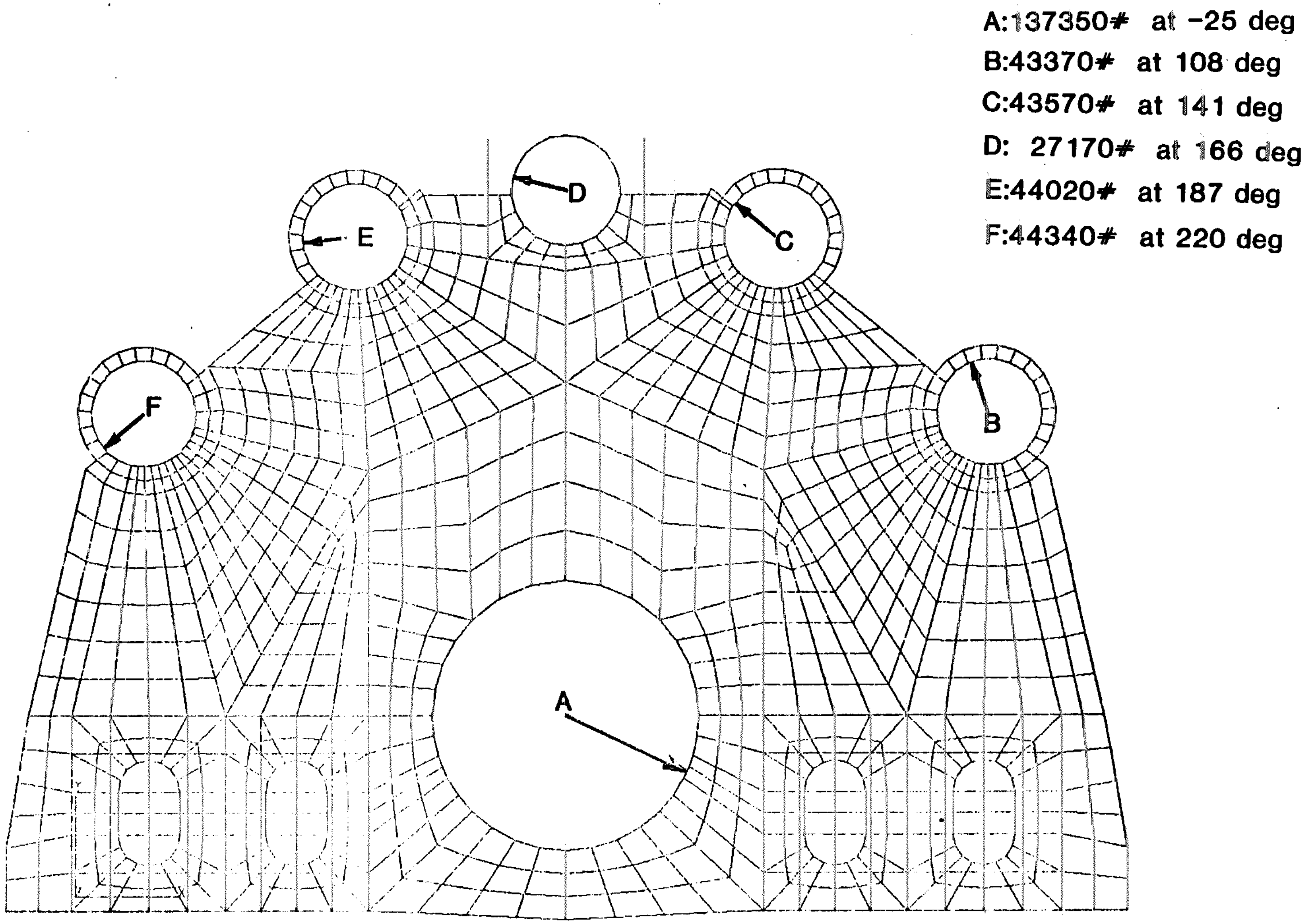


FIG. 3 **OPERATING LOADS**

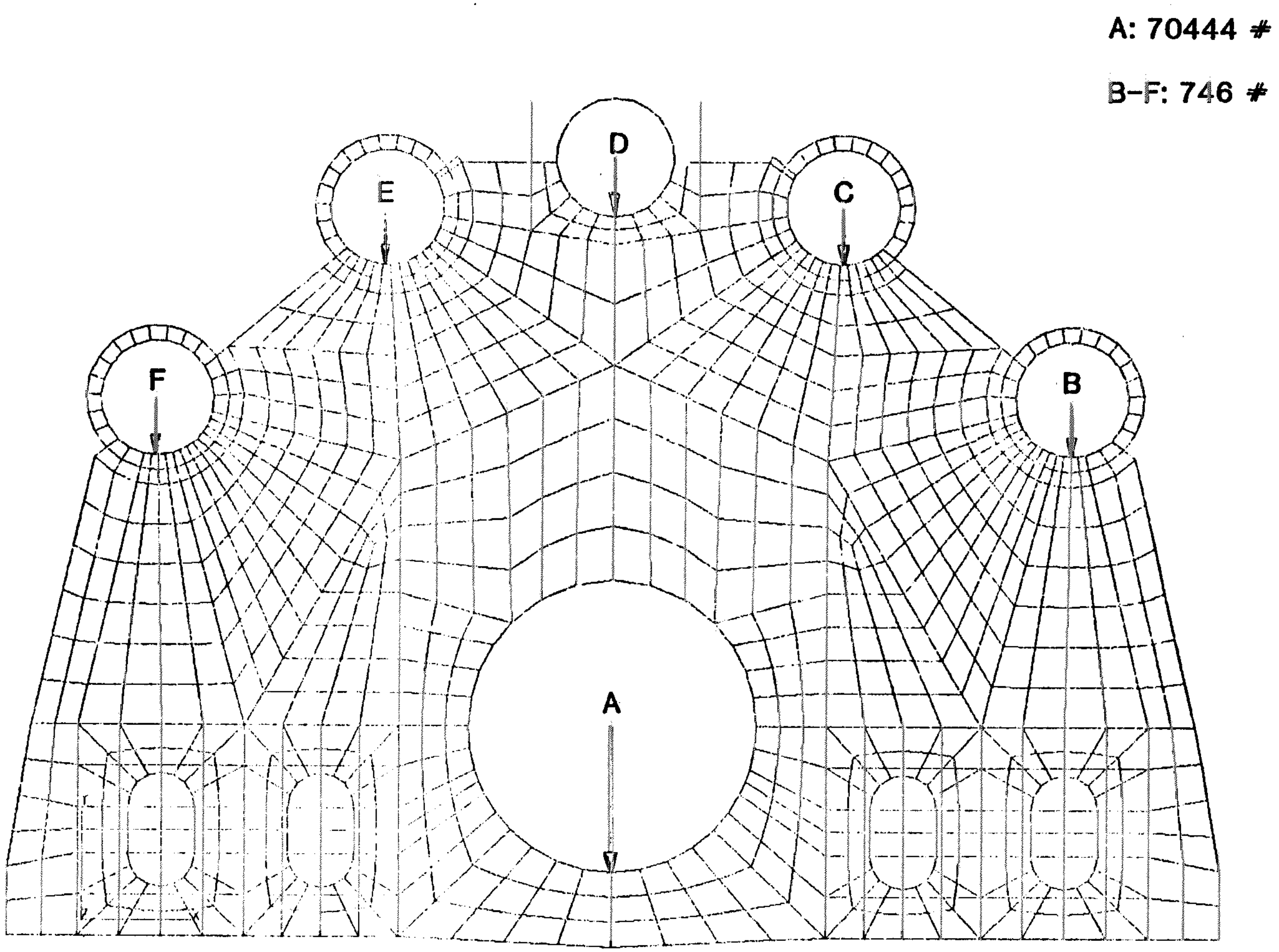


FIG. 4 **PINION AND BULL GEAR**
1G GRAVITY LOADS

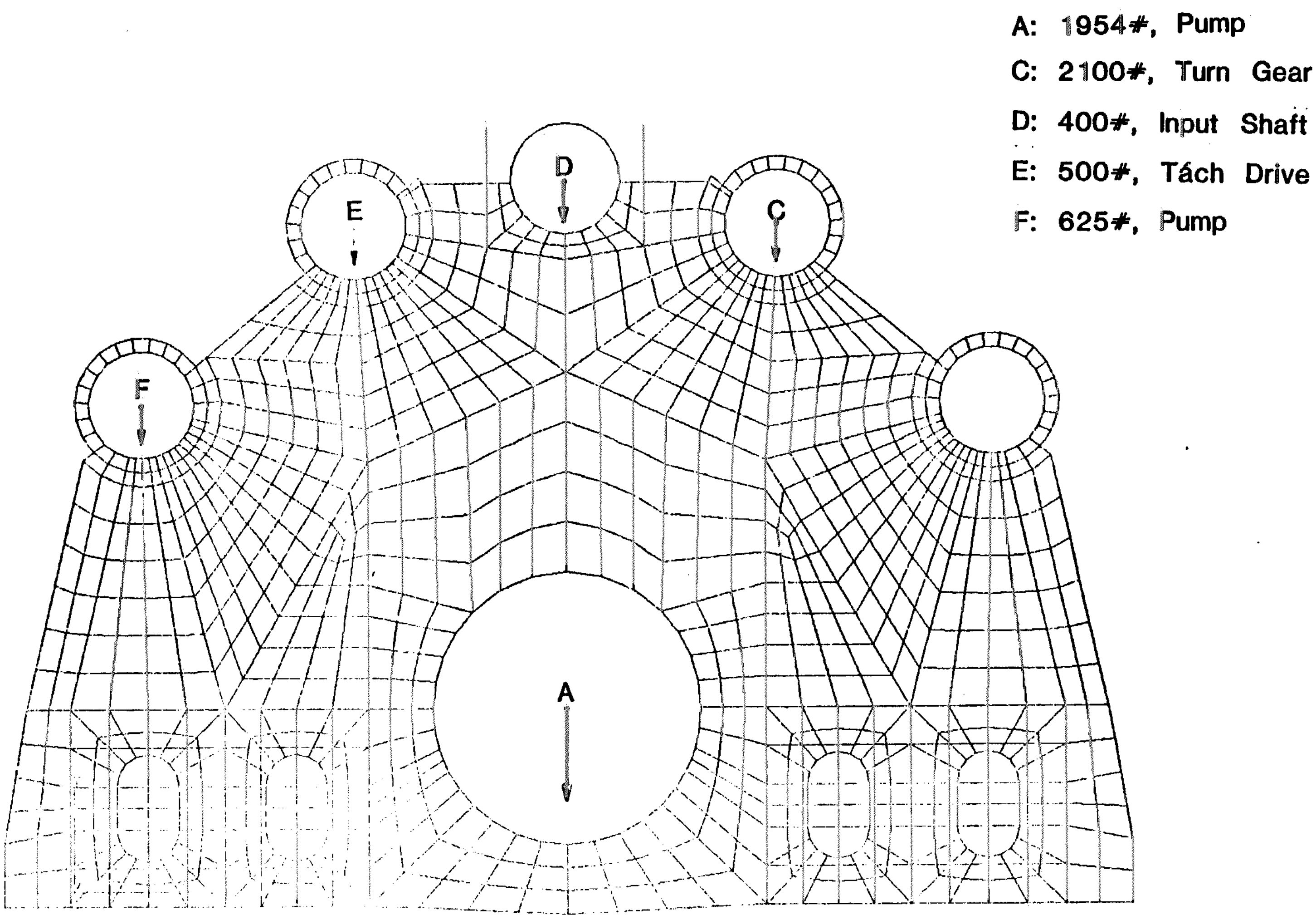


FIG.5 **ACCESSORIES**
1G GRAVITY LOADS

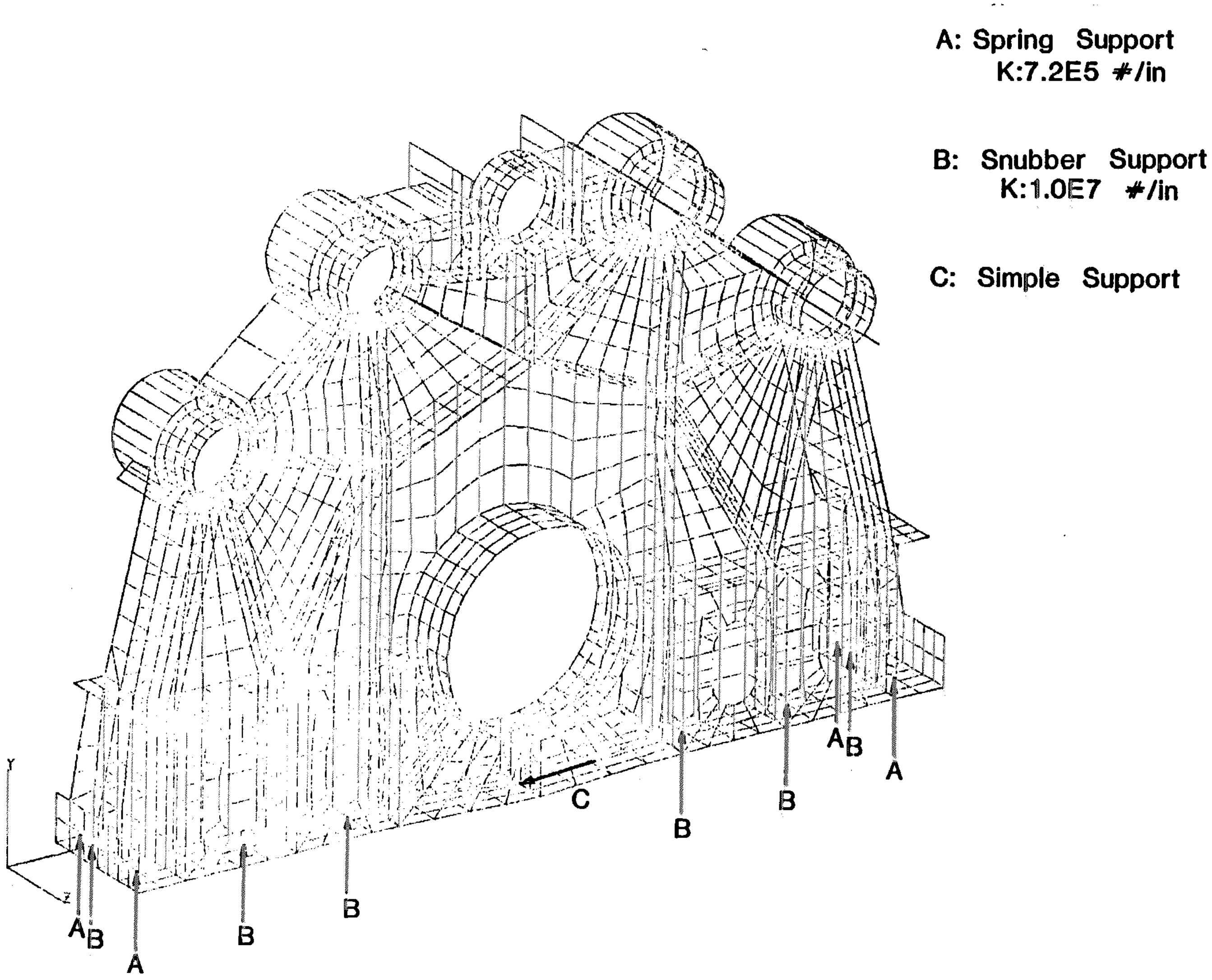


FIG.6 **BOUNDARY CONDITIONS**

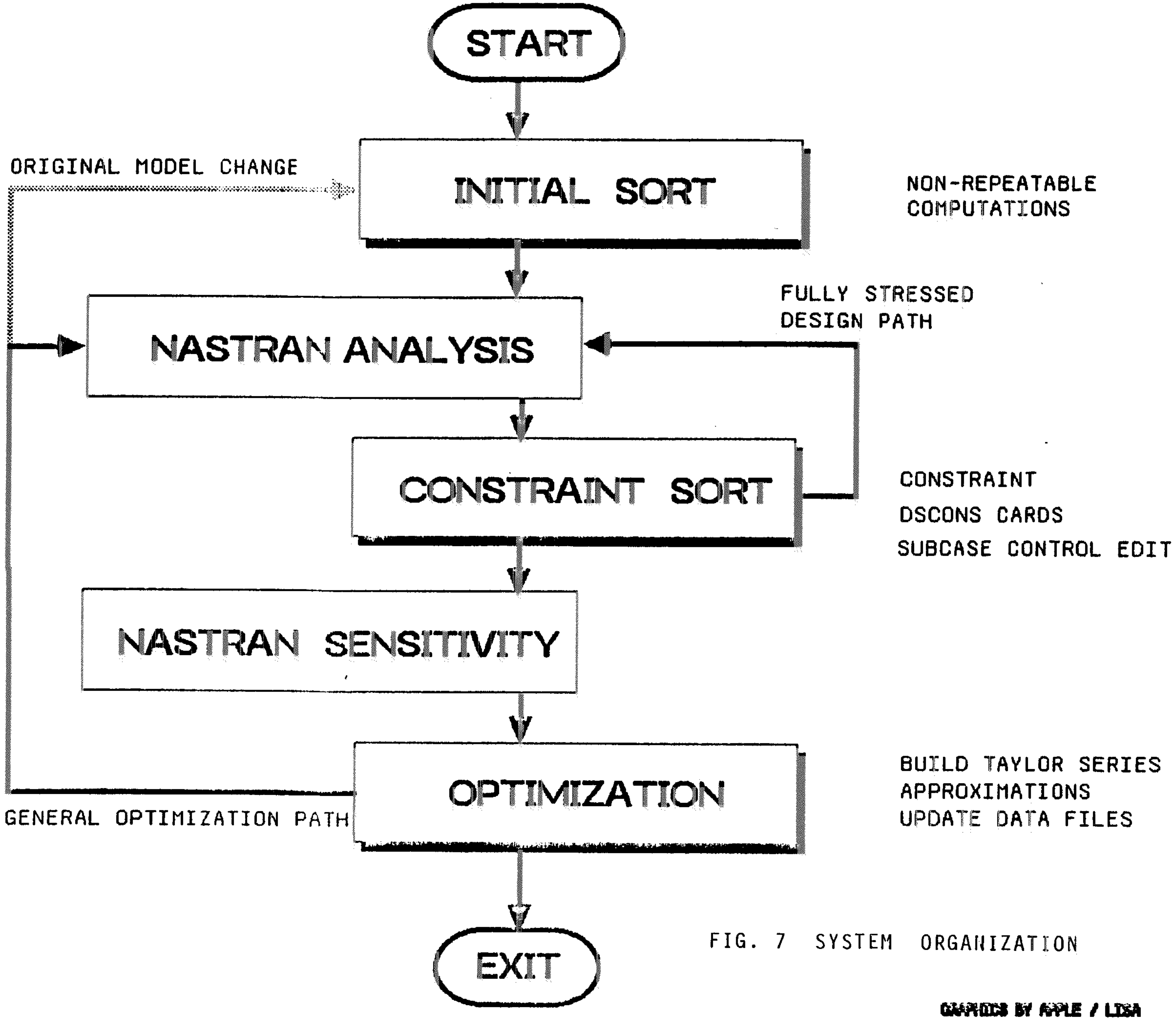
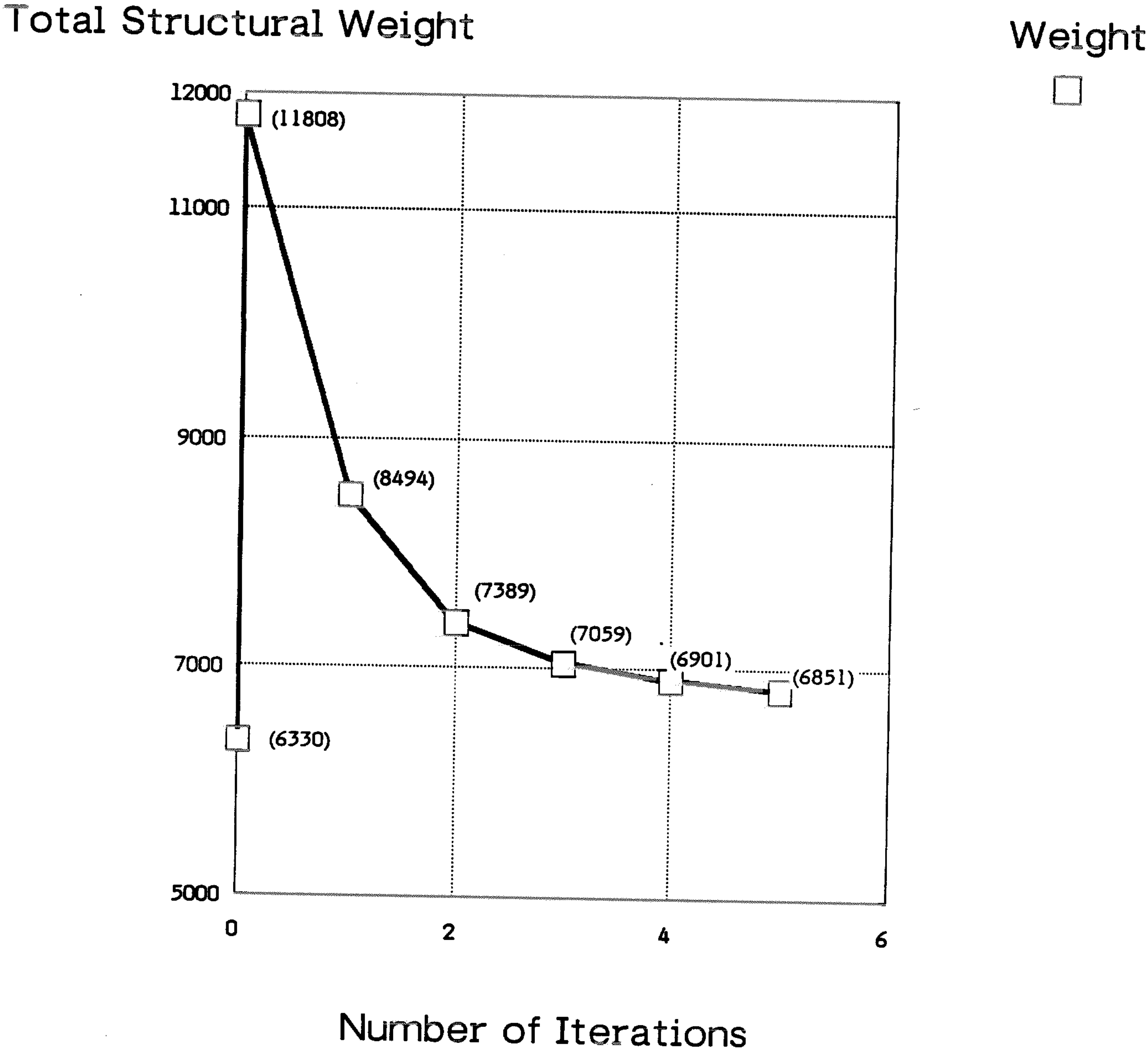


FIG. 7 SYSTEM ORGANIZATION

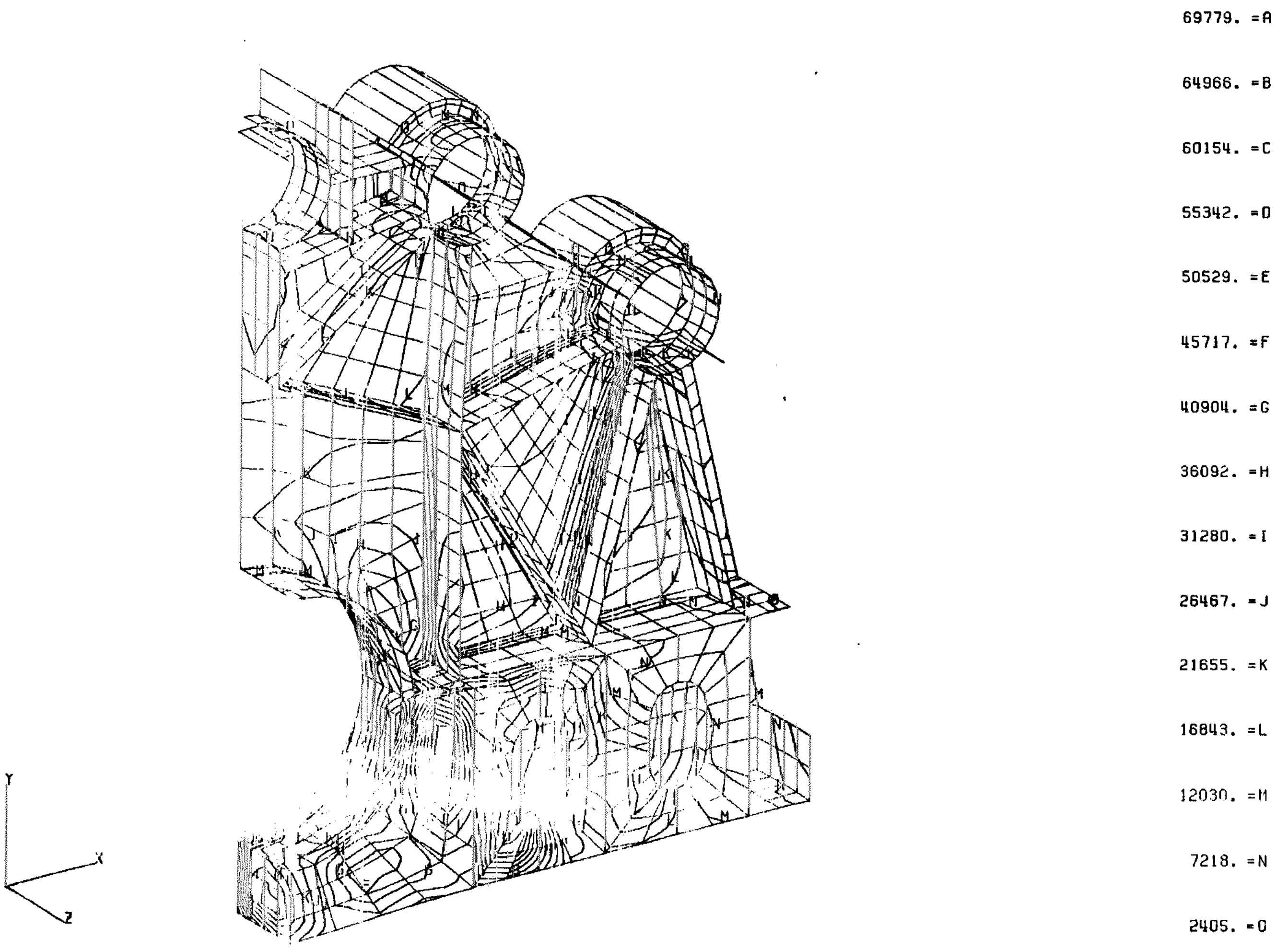
GRAPHICS BY APPLE / LISA

WEIGHT AND DESIGN
VARIABLE ITERATION HISTORY



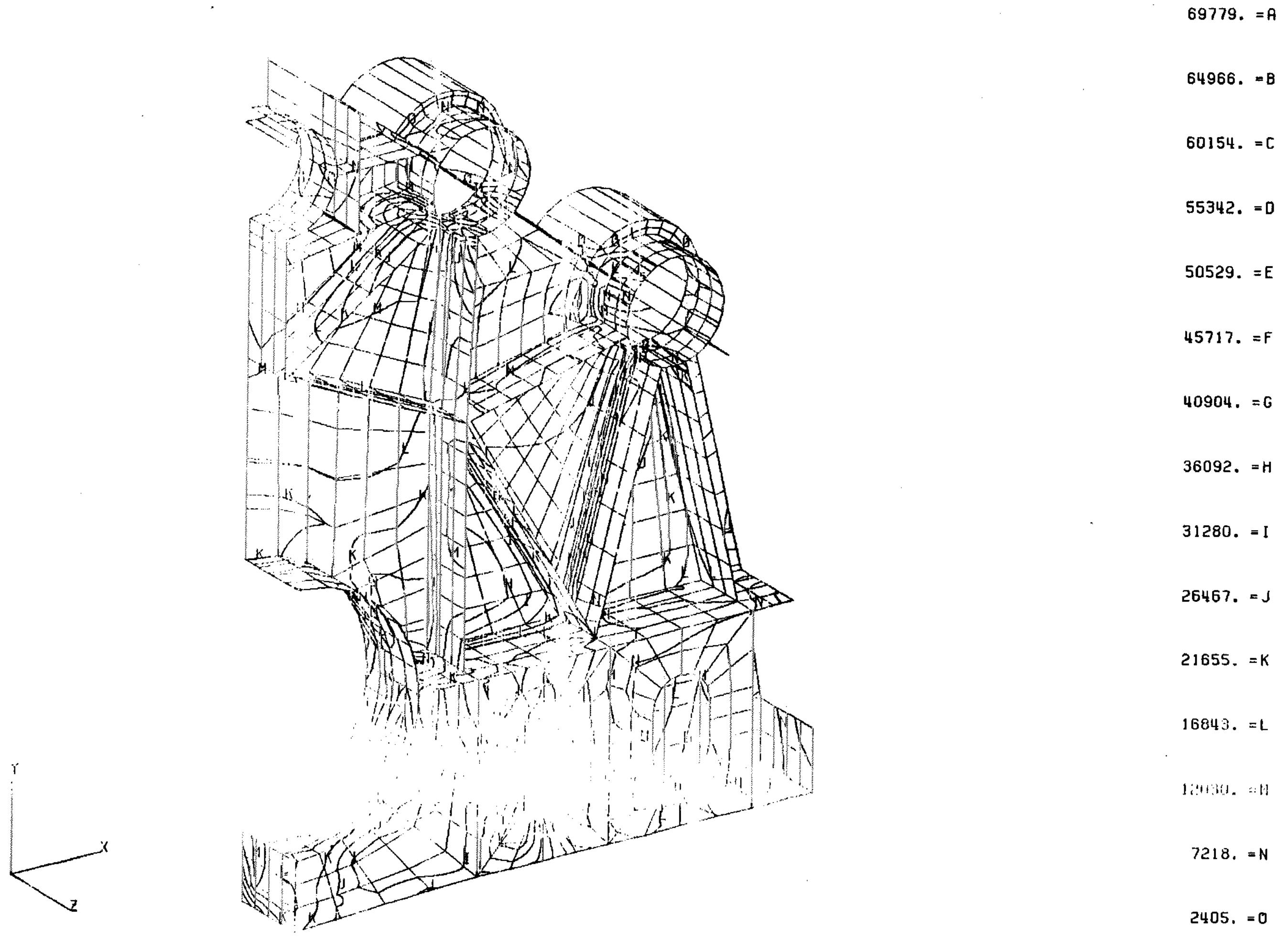
GRAPHICS BY APPLE/LISA

FIG. 8 ITERATION HISTORY



RESPONSE OF RHS FOR VERTICAL SHOCK

FIG. 9 STRESS CONTOUR - ORIGINAL DESIGN



[illegible]ONSE OF BB FOR VERTICAL [illegible]

FIG. 10 STRESS CONTOUR - OPTIMAL DESIGN

DISPLACEMENT CONSTRAINTS FOR ORIGINAL AND OPTIMAL DESIGNS

	ALLOWABLE	ORIGINAL DESIGN		OPTIMAL DESIGN	
		MAXIMUM	VIOLATION	MAXIMUM	VIOLATION
CENTER TO CENTER DISTANCE CHANGE	0.0097 in.	0.00719 in.	——	——	——
TRANSVERSE ROTATION OF AXIS					
BULL GEAR	1.326×10^{-4} rad	1.962×10^{-4} rad	48 %	1.346×10^{-4} in.	0 - 15 %
PINION	1.344×10^{-4} rad	0.774×10^{-4} rad	——	1.310×10^{-4} in.	——
DISTIRATION OF CYLINDER SURFACE					
BULL GEAR	0.00286 in	0.00409 in.	43 %	0.00188 in	——
PINION	0.00200 in	0.00380 in.	90 %	0.00179 in	——

TABLE 1 DISPLACEMENT CONSTRAINTS

SYSTEM FEATURES

I. GENERAL CAPABILITIES

1. Constraints may be specified on:

    stress components, von Mises stress
    displacements
    natural frequencies
    (frequency responses)
    (buckling loads)

2. Multiple boundary conditions

3. Load combinations

4. ADS : Most versatile robust optimizer

    8 Optimization strategies
    5 direction search algorithms
    8 uni-directional search algorithms

II. OVERALL EFFICIENCY IMPROVEMENTS

1. Dynamic deletion of inactive constraints

Example:	Totoal constraints		Retained constraints
	5620	→	153 (2.7%)

2. Taylor series expansions of constraint functions

3. DMAP program to upgrade sensitivity analysis capability

    Elimination of inactive load cases

    Support of multiple boundary conditions

    Load combinations

    Weight sensitivity calculation

Tanble 2 FEATURES OF ADS/NASOPT SYSTEM

WEIGHT & DESIGN
VARIABLE ITERATION HISTORY

DESIGN VARIABLE	ORIGINAL DESIGN	SCALED INITIAL	ITERATION				OPTIMAL DESIGN
			2	3	4	5	
1 in	0.625	1.500	0.962	1.085	1.008	1.032	1.015
2 in	0.750	2.000	3.165	4.000	4.000	4.000	4.000
3 in	0.500	2.000	1.000	0.711	0.829	0.819	0.813
4 in	0.625	1.500	0.750	0.675	0.759	0.745	0.760
5 in	0.500	2.000	1.041	1.215	1.287	1.153	1.233
6 in	1.250	1.500	1.094	0.650	0.474	0.463	0.375
7 in	0.750	3.000	1.500	0.750	0.441	0.414	0.375
8 in	2.000	3.000	1.500	0.782	0.676	0.656	0.706
9 in	2.000	3.000	2.483	2.742	2.601	2.619	2.619
10 in	1.250	2.000	1.553	0.895	0.568	0.547	0.375
11 in	2.250	3.500	4.000	3.454	3.842	3.852	4.000
12 in	3.500	4.000	3.750	3.099	2.490	2.394	2.337
13 in	1.000	2.000	1.421	1.075	0.987	0.958	0.860
14 in	0.750	2.000	1.443	1.008	0.705	0.689	0.519
15 in	0.750	2.000	1.164	0.582	0.375	0.375	0.375
16 in	0.750	1.500	1.022	0.511	0.375	0.375	0.375
17 in	0.750	2.000	1.504	0.752	0.376	0.375	0.375
18 in	0.625	1.500	1.025	0.513	0.375	0.375	0.375
19 in	0.375	0.750	0.689	0.513	0.376	0.369	0.205
20 in	0.625	1.000	0.922	0.705	0.539	0.529	0.375
21 in	1.250	1.500	0.817	0.409	0.386	0.384	0.375
22 in	2.000	3.000	4.000	4.000	4.000	4.000	4.000
23 in^2	1.000	2.000	1.898	1.821	1.923	1.902	1.789
24 in^2	0.750	2.000	1.924	1.782	1.580	1.578	1.509
25 in^2	0.750	2.000	1.817	1.406	1.124	1.106	0.772
26 in^2	0.750	1.500	1.420	1.184	1.004	0.993	0.690
27 in^2	0.750	2.000	1.922	1.562	1.060	1.056	1.019
28 in^2	0.625	1.500	1.417	1.171	0.961	0.944	0.560
29 in^2	0.375	0.750	0.745	0.729	0.703	0.703	0.701
30 in^2	0.625	1.000	0.995	0.991	1.024	1.028	1.277
Weight (lbs)	6330	11808	8494	7389	7059	6901	6851

GRAPHICS BY APPLE/LISA

TABLE 3 ITERATION HISTORY SUMMARY

RUN TIME SUMMARY (Cray 1S)

				SEC
INITIAL SORT				5.0
NASTRAN ANALYSIS				243.4
CONSTRAINT SORT				13.0
NASTRAN SENSITIVITY	(431.7)			337.0
DSTA			6.814	
Objective Sensitivity			5.853	
Displacement Sensitivity	(296.300)		195.323	
SSG3	(186.617)			
FBS		*94.609*		
SDR1	(60.505)	*51.574*		
Permutation of Displacements			13.526	
DSVG3			67.263	
Multiply Subcases			6.566	
DSTA			4.827	
SDR2			19.781	
OPTIMIZATION				1.5
			Total Per Iteration	599.9

TABLE 4 RUN TIME SUMMARY

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