ACCURACY OF VIBRATION ANALYSIS FOR THIN CYLINDRICAL SHELL BY MSC/NASTRAN

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ABSTRACT

The " good modelling "for the dynamic analysis by the finite element method are of importance for getting reliable results with minimum computational cost. Accuracy of QUAD4 and QUAD8 isoparametric shell elements in MSC/NASTRAN, when applied to the free vibration analysis of thin cylindrical shells having freely supported ends, has been investigated by numerical computations. The calculated eigen frequencies, the mode shapes and the corresponding stresses were compared with the theoretical solutions given by Arnold and Warburton. As far as the present analysis results are concerned, the excellent performance of QUAD4 has been revealed for a wide range of vibration forms, especially in calculating the membrane behavior dominant modes, which are of practical importance owing to their high excitability under pressure loading.

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1. Introduction

Dynamic analysis of Structures by the finite element method requires much more computational cost than the static one. It depends highly on the number of d.o.f. (degree of freedom) in the analysis model. Therefore, how to find the finite element model with minimum number of d.o.f. to get practically acceptable results draws strong interest of many structural engineers. Though a number of general purpose programs capable of linear dynamic analysis are found today, the necessary informations for keeping the accuracy of numerical results are commonly poor. This might be partly due to the widely variated dynamic characteristics of structures, all of which can not be investigated solely by the developer of the analysis code. As a result, it is required for the practical user to have the ability of "good modelling". He should understand the basic matters of the analysis code, which cannot be treated as a " black box " any more[8], and the intimate contact with the code developer is indispensable.

In this paper, the accuracy of free vibration analysis for thin cylindrical shells with simply supported ends by using MSC/NASTRAN is investigated by numerical computations. The error origins in the free vibration analysis for this kind of problems might be for the most part

1) Spatial approximation performance of the used elements, such as stiffness and inertia properties, for representing the vibration modes
2) Reduction technique of the dynamic degree of freedoms, such as the Guyan reduction method

Most part of the paper is focused primarily on the first one, i.e. the spatial approximation performance of QUAD4 and QUAD8 shell elements for representing the vibration modes of simply supported thin cylindrical shell is investigated relating to the accuracy of eigen-frequencies, their mode shapes and the corresponding stresses, which will be compared with the theoretical values. Though many investigators have concentrated their interest to the accuracy of eigen-frequencies, the latter two are also of practical importance from the view point of dynamic transient analysis. Because of the dependence of the eigen frequencies on the total energy of the corresponding modes, their accuracy never guarantee that of the calculated mode shapes, i.e. the non-existence of locally disturbed deformation, which is critical for the resulting stress values. It is noted here that, to remove the error due to the second origin above mentioned, the direct application of inverse power method with no dynamic reduction is used for the present eigenvalue extraction.

QUAD4[1,2] and QUAD8 belong to the so called " degenerated shell element ", which is based on the isoparametric concept using the reduced integration technique. QUAD4 is known for its pioneering work in the development of this type of four node quadrilateral plate bending elements, which show extensive high performances for wide range of plate thickness problems inspite of their very simple formulations. QUAD8 is considered to be the higher order version of QUAD4[1], though its theoretical description has not been published yet. A number of recent contributions in this area can be found in the references 4-7).

From the numerical investigation performed here, though the element shapes are confined to the rectangular one with moderate aspect ratio, the surprisingly excellent accuracy of QUAD4 element for a wide range of vibration forms will be revealed, as far as the application to thin cylindrical shell problems are concerned. On the other hand, QUAD8 will be found to show some deficiencies in calculating the membrane behavior dominant modes, which are of practical importance owing to its high excitability under pressure type of loading.

All the calculation results presented in this paper are obtained by using the 61-B version of MSC/NASTRAN installed in the UNIVAC 1181 computer at IHI.

2. Preparations for the Numerical Investigations

2-1 Theoretical Solution

The theoretical solution for the thin cylindrical shell with simply supported ends as shown in Fig.1 was given by Arnold and Warburton[10]. The solution for the other end conditions can be found, for example, in Reference 11).

The vibrational modes for the present problem can be expressed in the following forms.

$$u = U \cos(m\pi x/\ell)\cos(n\theta)\cos(pt) \quad \text{(Axial Displacement)}$$
$$v = V \sin(m\pi x/\ell)\sin(n\theta)\cos(pt) \quad \text{(Tangential Displ.)}$$
$$w = W \sin(m\pi x/\ell)\cos(n\theta)\cos(pt) \quad \text{(Radial Displacement)}$$

Here, 'n' and 'm' are the number of circumferential and axial waves, respectively. For thê brevity of investigation, the analysis performed in the present paper is confined to the case ' m=1 ' .

The solution procedure by Arnold and Warburton is based on the energy method with using strain relations given by Timoshenko, which is comparable to the use of displacement equations by Flügge. Once determined the eigenfrequency by the characteristic equations, the amplitude ratios U/ W and V/ W can be calculated easily and the corresponding mode shapes are determined. Setting the maximum modal displacement to take the unit value, the corresponding stresses are easily calculated. A FORTRAN code, which follows the above solution procedures, has been developed so that the numerical solution for any dimensional cylindrical shell can be instantaneously obtained.

2-2 Finite Element Modelling

From the symmetricity consideration, the analysis by NASTRAN were performed for the shaded portion as is shown in Fig. 2,i.e.

a 90-degree sector. The shell dimensions and the material property used in the numerical calculation are also shown there.

The cylindrical coordinate is introduced and throughout the analysis all the rotational degrees of freedom with respect to the axis perpendicular to the shell surface, i.e. z-axis, are restrained.[9]

The introduction of the boundary conditions for AB, BD, DC and CA as is shown in Fig. 2 allows us to calculate the vibration mode of the even number of circumferential waves.

An example of finite element meshes by using QUAD8 is shown in Fig. 3. In the mesh patterns used in the present analysis all the nodes are equally spaced both to the axial and circumferential directions. The division number for the axial direction is fixed to be five, whereas that for the circumferential direction is varied from 6 to 16 to see the convergence characteristc of the elements. Hereafter, the mesh patterns are indicated as ' N_c X N_a ', for example ' 8 X 5 ', which means N_c division for the circumferential direction and N_a axially. Due to the above regular mesh pattern, the element shape is confined to the rectangular one, with the aspect ratios about 1.9 (' 6 X 5') to 0.7 ('16 X 5').

The superiority of the lumped mass approximation to the consistent one for the shell vibration problems has often been reported[9] from the view point of accuracy for the eigen frequencies and the efficiency of the numerical computation. In the present calculation, the lumped mass approximation is used throughout the analysis except otherwise stated.

As was stated previously, the eigen-value extraction is performed by the direct application of inverse power method with no dynamic reduction except otherwise stated.

3. Results and Discussions of the Numerical Investigations

3-1 Accuracy of Eigen Frequencies

The solid curves in Fig. 4 shows the theoretical eigen frequencies versus the number of circumferential waves for the three radius-thickness ratios $R/t = 50$, 284.7 and 600. The analysis by NASTRAN were carried out for the above three cases, especially that of $R/t = 284.7$ is primarily investigated.

(1) R/t = 284.7 by QUAD4

The ratios of the calculated frequencies to the theoretical values are shown in Table 1 , and they show surprisingly excellent accuracy even for the modes of larger 'n', the number of circumferential waves. Increasing the division number for the circumferential direction, the convergence trend to the theoretical values can be observed. It should be noticed that the mesh pattern ' 8 X 5 ' has been thought to be quite insufficient for calculating the modes of 'n' larger than 10 with using simple elements such as QUAD4.

(2) R/t = 284.7 by QUAD8

Table 2 shows the results by QUAD8 element. Compared with those by QUAD4 in Table 1, some relative inferiority is found for the modes of smaller 'n' = 2 and 4, except for the values obtained by consistent mass approximation, though they are practically acceptable. As will be shown later, the inferiority is due to the local disturbance of the calculated mode shapes.

For the modes of 'n' larger than 6, where the bending behavior becomes dominant, the results by QUAD8 using the mesh pattern ' 8 X 5 ' are almost comparable to those by QUAD4 using the mesh pattern ' 12 X 5 '. Taking into account the total d.o.f. of the analysis models, QUAD8 does not show the merit of its higher order formulation. It should be noted that for these modes the accuracy of the eigen frequencies by lumped mass approximation is superior to that by consistent one.

(3) R/t = 50 and 600 by QUAD4 and QUAD8

To confirm the performance of QUAD4 and QUAD8 for the other radius-thickness ratio R/t, the analysis are carried out for the two cases, R/t = 50 and 600, using the mesh pattern ' 8 X 5 '.

Table 3 shows the results for R/t = 600, and the inferiority of QUAD8 for the modes of 'n'= 2 and 4 is again found.

Table 4 show the results for R/t = 50, the relatively thicker plate situation. Here, the inferiority of QUAD8 observed in the other two cases disappeared, and as will be shown later, the local disturbance of the mode shape was not found any more.

For the schematic comparison with the theoretical values, the the calculated freauencies by QUAD4 with using the mesh pattern ' 8 X 5 ' are plotted in Fig. 4 .

3-2 Accuracy of Mode Shape

(1) Mode Shapes by QUAD4

The calculated mode shapes for 'n'= 2, 4, 6, 8 by QUAD4 with using the mesh pattern ' 8 X 5 ' are shown in Fig. 5, which follows the satisfactory results.

It was found in the foregoing accuracy investigation of the frequencies that the mesh pattern ' 8 X 5 ' gives surprisingly good results even for the modes of 'n' larger than 10 inspite of its relative coarseness. The mode shapes for 'n' = 12 and 14 by this mesh pattern are compared in Fig. 6 with those obtained by QUAD4 using the mesh pattern ' 16 X 5 '. It is seen that the common nodal displacements, designated by circles, display good coincidence to each other, which shows the high performance of QUAD4 in spite of its very simple formulation.

(2) Mode Shapes by QUAD8

Fig. 7 shows the calculated mode shape by QUAD8 using the mesh pattern ' 8 X 5 '. For the mode of 'n'= 2 , elementwise local disturbances of the mode shape are observed and also very slight disturbance for the mode of 'n'= 4. To see the characteristic of the disturbance, the nodal displacement to the radial direction 'w' along the mid-section of the cylindrical shell are compared with the theoretical values for the mode 'n' = 2, which is shown in the table of Fig. 8. Very interesting is the fact that the nodal displacements at the corner nodes of QUAD8 show high accuracy, whereas those at the mid-side nodes follows discrepancy. The behavior indicates the elementwise local disturbance of the mode shape.

The same trend can also be found for the thinner plate thickness R/t = 600 as is shown in Fig. 9, where the disturbances are magnified.

On the other hand, the mode shapes for R/t = 50 follows no such deficiency as is shown in Fig 10.

Several solution ways for overcoming the above deficiency found in the analysis of very thin cylindrical shells were tried, and one of the succeeded results is shown in Fig. 11, where the finer mesh divisions ' 16 X 5 ' was applied. Here, the slight disturbance is still observed for the mode 'n' = 2. However, the use of the finer mesh division is unpracticable from the view point of computational cost.

All the foregoing results are obtained by using the lumped mass approximation. Remembering that the calculated frequencies by the consistent mass approximation (R/t = 284.7, ' 8 X 5 ') follows no inferiority, their mode shapes could be expected to give the better results, which are shown in Fig. 12. The much improved and seemingly satisfactory mode shapes are observed, with slight disturbance still for the mode 'n' = 2.

From the above examinations of the mode shapes it might be tentatively deduced that that the disturbance of the mode shape calculated by QUAD8 for the smaller number of 'n' would appear for the case of very large R/t together with the use of lumped mass approximation. It seems that this might be due to the poor approximation ability of the lumped mass for representing the inertia property of this kind of higher order elements when used for the analysis of very thin cylindrical shells.

3-3 Accuracy of Stresses

The comparison of the calculated mode stresses with the theoretical values for R/t = 284.7 are shown in Tables 5 and 6, where the maximum mode displacements are set to be 1.0 (mm). The location for comparison is shown in Fig. 3 (point 'Q'), and the inner and outer surface stresses are considered from the practical point of view. The mesh pattern used is ' 8 X 5 '.

(1) It is seen that the results by QUAD4 shows very good coincidence with the theoretical values for the modes up to 'n' = 8.

(2) On the other hand, those by QUAD8 exhibits the poorer coincidence for the modes of 'n' = 2 and 4, which might be due to the local disturbance of the mode shape discussed previously. The results by using the consistent mass approximation are also shown in the round brackets of the same tables, which give the improved stress values, however, poorer still for the mode 'n' = 2 than those by QUAD4. It should be emphasized here that

inspite of the slight mode shape disturbance as is observed in Fig. 2, the corresponding stress values include the unexpected amount of error. Further , it should be noted that the results by QUAD8 show no better performance than QUAD4 for the modes of larger 'n' from the view point of the total d.o.f. per circumferential wave length.

4. Summary and Conclusion

As far as the present numerical investigations are concerned, it has been revealed that QUAD4 plate bending element show the excellent high performance for the vibration analysis of thin cylindrical shells. Though the investigated element shape is confined to the rectangular one, not only the accuracy of eigen frequency but also that of mode shape and the corresponding stress are remarkably excellent inspite of its simple formulation, and the necessary information for the minimum mesh division to keep the practically acceptable accuracy has been obtained. To confirm its general applicability, however, further investigations for the distorted element shape might be necessary.

On the other hand, QUAD8 shows somewhat inferiority in calculating the membrane behavior dominant modes, which are essential for the strength assesment of shell structures, when applied to very thin cylindrical shells. Further, for the modes of larger number of circumferential waves, where the bending behavior is dominant, it does not seem to give the better performance than QUAD4 on the basis of total d.o.f., and the merit of its higher order formulation is not found.

Because of the obvious superiority of QUAD4 to QUAD8, the application of QUAD4 to the present area of problems is preferable.

Acknowledgement

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Table 1 Comparison of the Calculated Eigen Frequencies with the Theoretical Values (QUAD4, R/t = 284.7)

	Mesh Pattern	f / f^* (f : calculated Value) 6 X 5	8 X 5	12 X 5	16 X 5	f^* Theoretical Value (Hz)
Number of Circumferential Waves	2	1.011	1.004	1.000	0.998	96.53
	4	1.022	1.012	1.006	1.003	43.28
	6	1.033	1.017	1.008	1.005	23.79
	8	1.059	1.018	1.003	0.999	18.99
	10	1.065	1.022	0.998	0.994	22.43
	12	0.909	1.014	0.998	0.993	30.03
	14	- #	0.957	0.996	0.992	40.01
	16	- #	0.808	0.988	0.991	51.82

- # : Not found

Table 2 Comparison of the Calculated Eigen Frequencies with the Theoretical Values (QUAD8 , R/t = 284.7)

		f / f^* (f : Calculated Value)					f^* Theoretical Value (Hz)
Mesh Pattern		6 X 5	8 X 5	12 X 5	16 X 5	8 X 5	
Mass Approx.		LUMP	LUMP	LUMP	LUMP	CONSISTENT	
Number of Circumferential Waves	2	0.779 #	0.951 #	0.979 #	0.989	1.000	96.53
	4	0.960 #	0.990 #	0.992	0.998	1.001	43.28
	6	0.994	1.000	0.998	1.000	1.006	23.79
	8	1.009	1.005	1.000	1.000	1.014	18.99
	10	1.004	1.003	1.000	1.000	1.019	22.43
	12	0.929	1.000	0.998	0.999	1.029	30.03
	14	1.029	0.994	0.998	0.998	1.041	40.01
	16	1.034	0.938	0.997	0.998	1.209	51.82

indicate the local disturbance of the mode shape

Table 3 Comparison of the Calculated Eigen Frequencies with the Theoretical Values (R/t = 600, Mesh Pattern 8 X 5)

		f/f^* (f : Calculation)		f^* (Hz) Theoretical Value
Element Type		QUAD4	QUAD8	
Number of Circumferential Waves	2	1.005	0.985 #	96.52
	4	1.013	0.944 #	43.15
	6	1.018	0.986	22.77
	8	1.025	1.001	14.88
	10	1.034	1.008	13.20
	12	1.028	1.005	15.28
	14	0.973	0.997	19.43
	16	0.825	0.935	24.80

indicate the local disturbance of the mode shape

Table 4 Comparison of the Calculated Eigen Frequencies with the Theoretical Values (R/t = 50 , Mesh Pattern 8 X 5)

		f/f^* (f : Calculation)		f^* (Hz) Theoretical Value
Element Type		QUAD4	QUAD8	
Number of Circumferential Waves	2	1.009	0.999	96.37
	4	1.008	1.001	43.29
	6	0.991	0.999	49.91
	8	0.985	0.996	77.58
	10	0.976	0.992	117.6
	12	- #	0.987	167.4
	14	- #	0.979	225.3
	16	- #	0.923	294.4

- # Not Calculated

Table 5 Comparison of the Calculated Circumferential Normal Stresses σ_y with the Theoretical Values (R/t = 284.7 , Mesh Pattern : 8 X 5)

	Inner Surface σ_{iy} (Kg/mm^2)			Outer Surface σ_{oy} (Kg/mm^2)		
Element Type	QUAD4	QUAD8	Theoretical Value	QUAD4	QUAD8	Theoretical Value
Number of Circumferential Waves 2	-1.143	-0.875 (-1.074)	-1.157	-1.222	-0.886 (-1.180)	-1.238
4	-0.084	-0.100 (-0.088)	-0.089	-0.344	-0.311 (-0.345)	-0.360
6	0.205	0.187 (0.200)	0.215	-0.301	-0.287 (-0.301)	-0.324
8	0.350	0.336 (0.349)	0.389	-0.375	-0.364 (-0.378)	-0.422
10	0.398	0.397 (0.422)	0.491	-0.405	-0.406 (-0.431)	-0.501
12	0.314	0.355 (0.408)	0.490	-0.315	-0.357 (-0.411)	-0.493
14	0.119	0.198 (0.252)	0.341	-0.119	-0.199 (-0.253)	-0.251
16	7.1E-5	3.8E-5 (3.9E-5)	0.0	2.0E-4	4.6E-6 (1.1E-6)	0.0

Note ; (1) Comparisons are performed at the element center 'Q' as is shown in Fig. 3

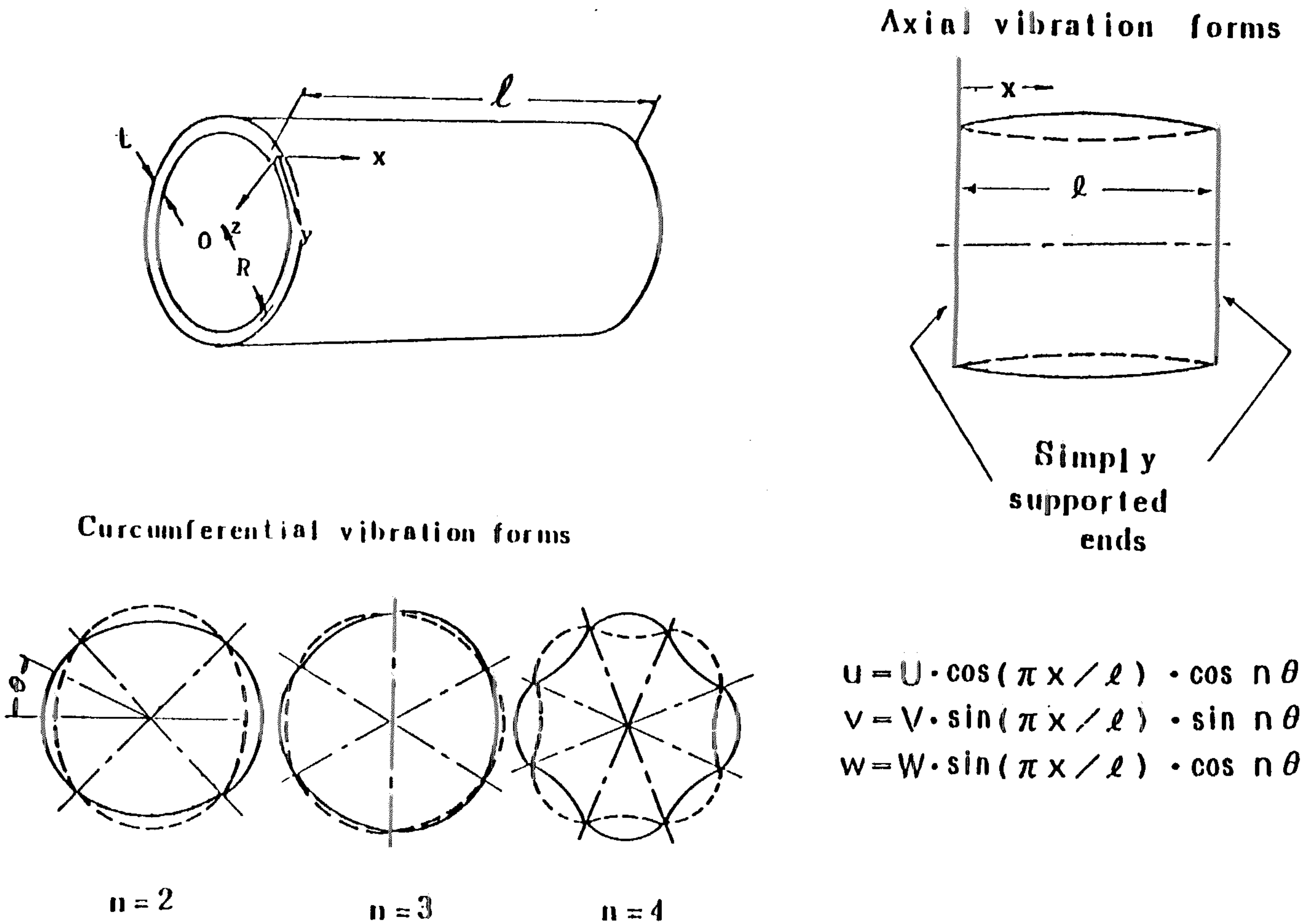
(2) The values in the round brackets are calculated by consistent mass approximation

Table 6 Comparison of the Calculated Axial Normal Stresses σ_x with the Theoretical Values (R/t = 284.7 , Mesh Pattern : 8 X 5)

	Inner Surface σ_{ix} (Kg/mm^2)			Outer Surface σ_{ox} (Kg/mm^2)		
Element Type	QUAD4	QUAD8	Theoretical Value	QUAD4	QUAD8	Theoretical Value
Number of Circumferential Waves 2	-1.155	-0.832 (-1.078)	-1.149	-1.253	-0.898 (-1.184)	-1.253
4	-0.696	-0.638 (-0.672)	-0.697	-0.840	-0.767 (-0.821)	-0.853
6	-0.288	-0.276 (-0.278)	-0.295	-0.493	-0.478 (-0.491)	-0.524
8	-0.066	-0.067 (-0.066)	-0.068	0.350	0.336 (0.334)	0.389
10	0.041	0.040 (0.042)	0.056	-0.234	-0.239 (-0.251)	-0.287
12	0.064	0.069 (0.081)	0.106	-0.136	-0.167 (-0.188)	-0.220
14	0.029	0.047 (0.061)	0.088	-0.046	-0.081 (-0.101)	-0.132
16	1.7E-4	9.1E-6 (6.8E-6)	0.0	2.8E-4	1.9E-6 (1.9E-6)	0.0

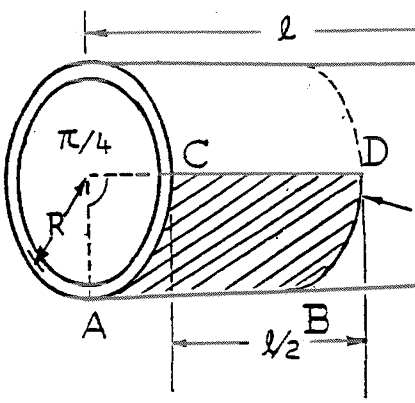
Note : (1) Comparisons are performed at the element center 'Q' as is shown in Fig. 3

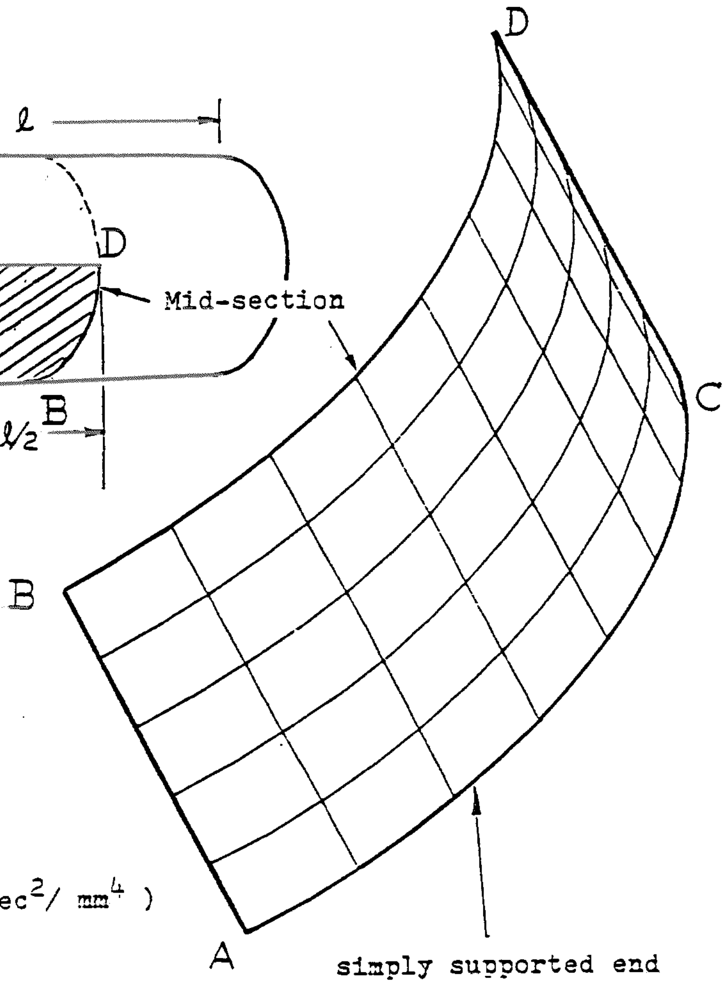
(2) The values in the round brackets are calculated by consistent mass matrix approximation



$$u = U \cdot \cos(\pi x / \ell) \cdot \cos n\theta \cdot \cos pt$$
$$v = V \cdot \sin(\pi x / \ell) \cdot \sin n\theta \cdot \cos pt$$
$$w = W \cdot \sin(\pi x / \ell) \cdot \cos n\theta \cdot \cos pt$$

Fig. 1 Coordinate Systems and Vibration Forms with Simply Supported End Condition





Dimensions

ℓ = 5941.6 (mm)

R = 4223.8 (mm)

Material Properties

ρ = 7.959E-10 (Kg sec^2/ mm^4)

E = 19700 (Kg/mm^2)

ν = 0.3

Restraint Conditions

Along $\overline{AB}$ and $\overline{CD}$

$v = \theta_x = \theta_z = 0$

Along $\widehat{AC}$ (Supported End)

$v = w = \theta_x = \theta_z = 0$

Along $\widehat{BD}$ (Midsection)

$u = \theta_y = \theta_z = 0$

Fig. 2 Finite Element Model and Boundary Conditions

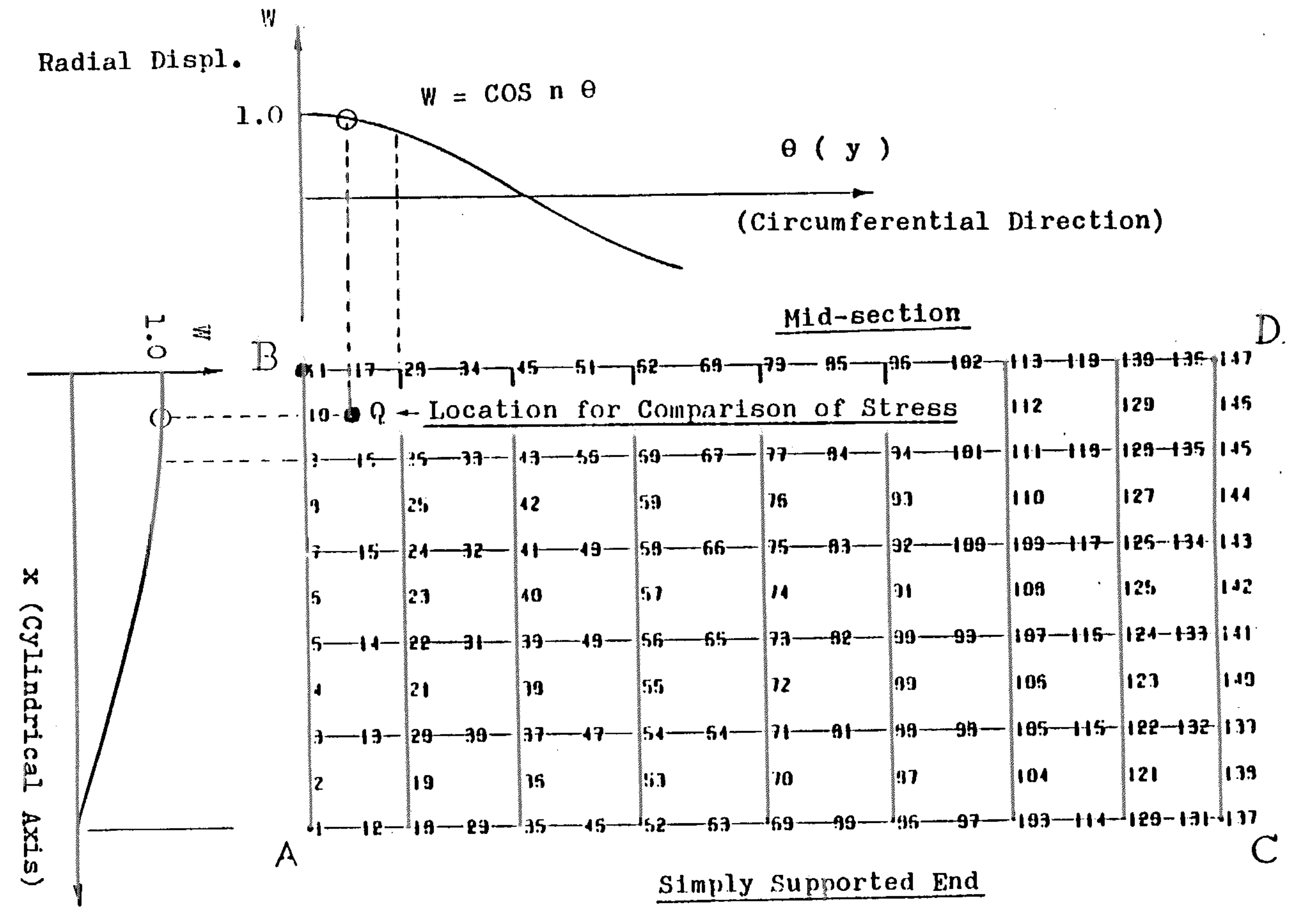


Fig. 3 An Example of Finite Element Meshes (QUAD8, 8 X 5)
and the Location for Comparison of Stress Values

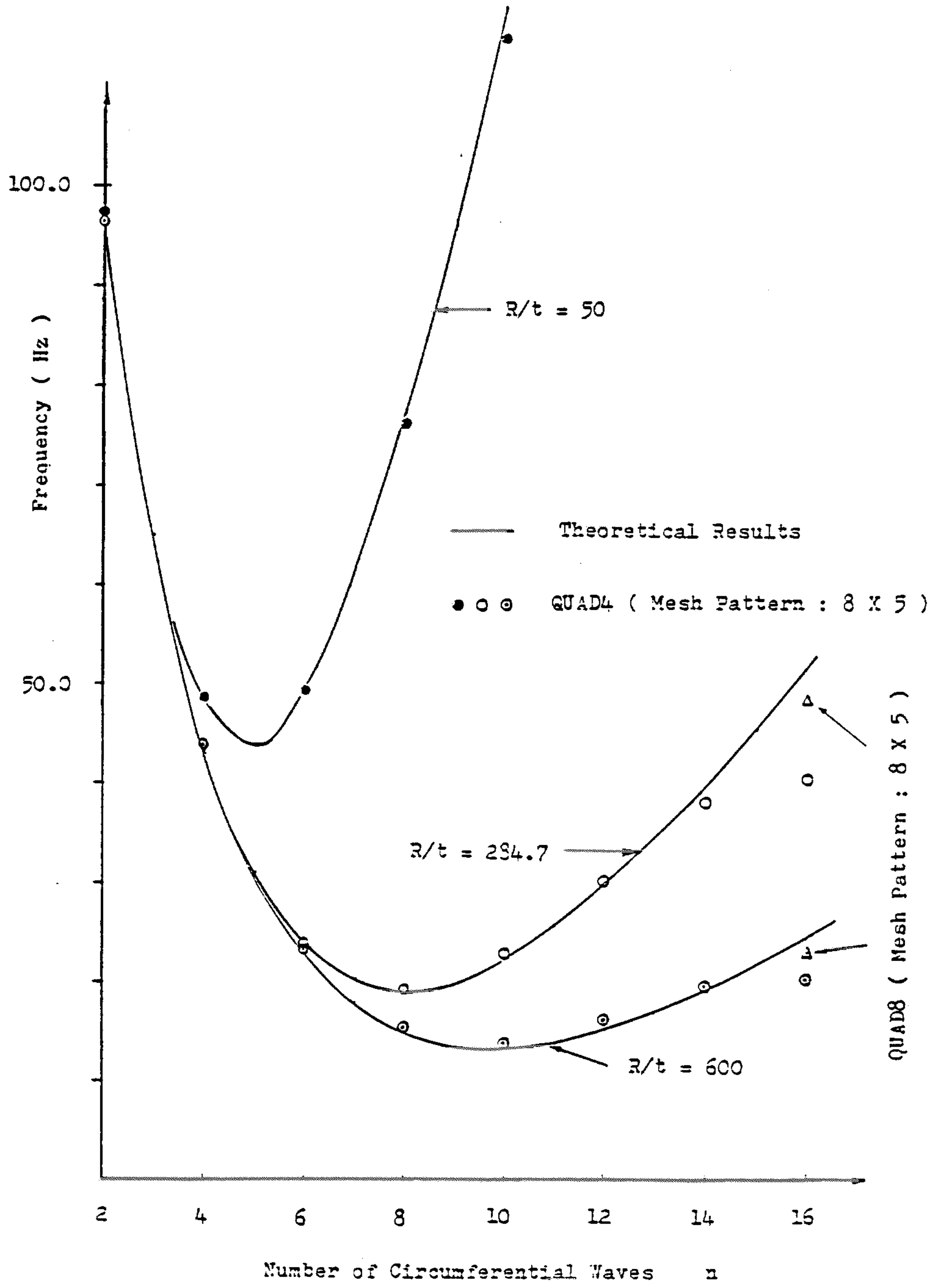


Fig. 4 Schematic Comparison of the Calculated Frequencies with the Theoretical Value (Mesh Pattern : 8 X 5)

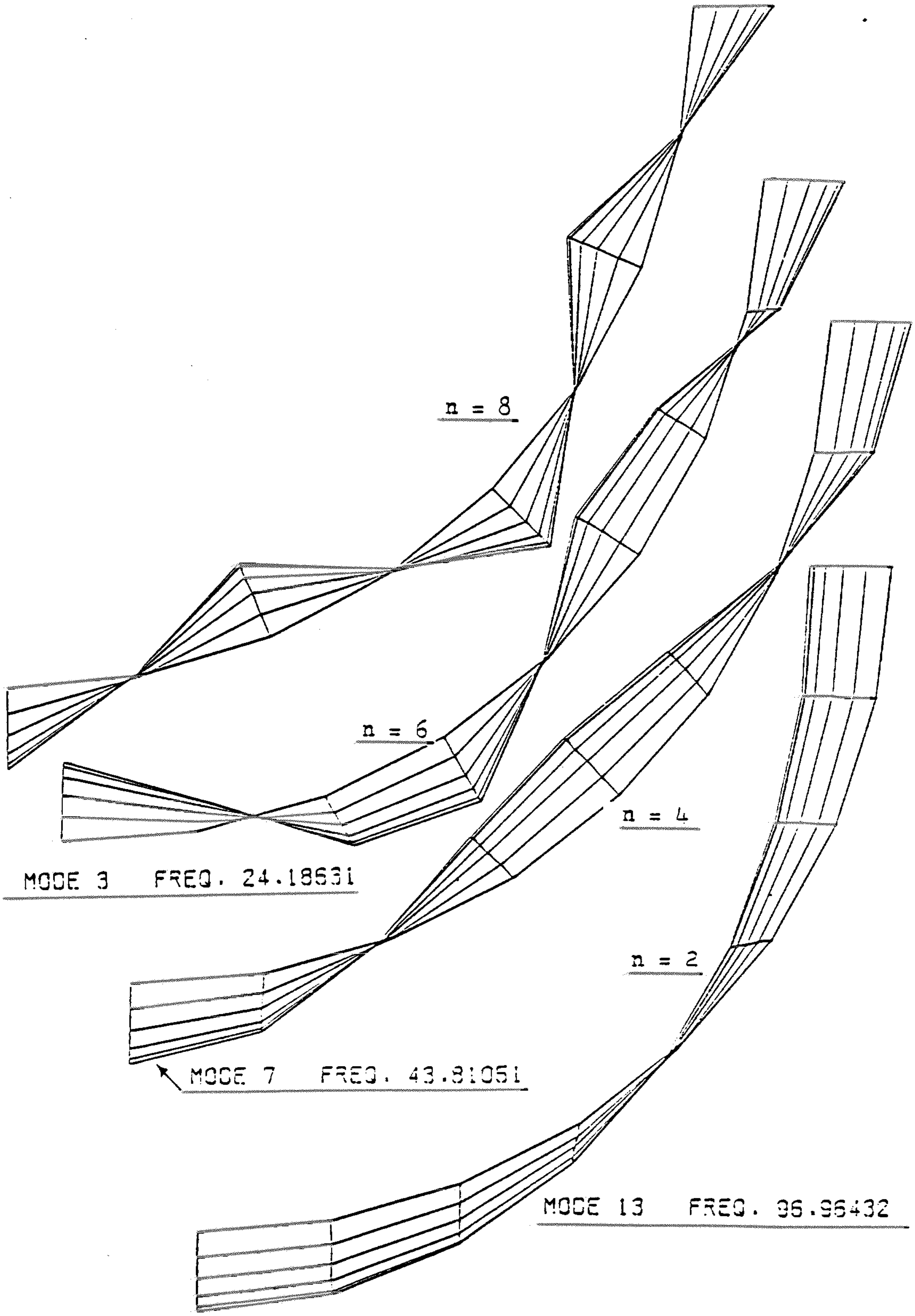


Fig. 5 Calculated Mode Shapes by QUAD4 and the Mesh Pattern 8 X 5 (R/t = 284.7)

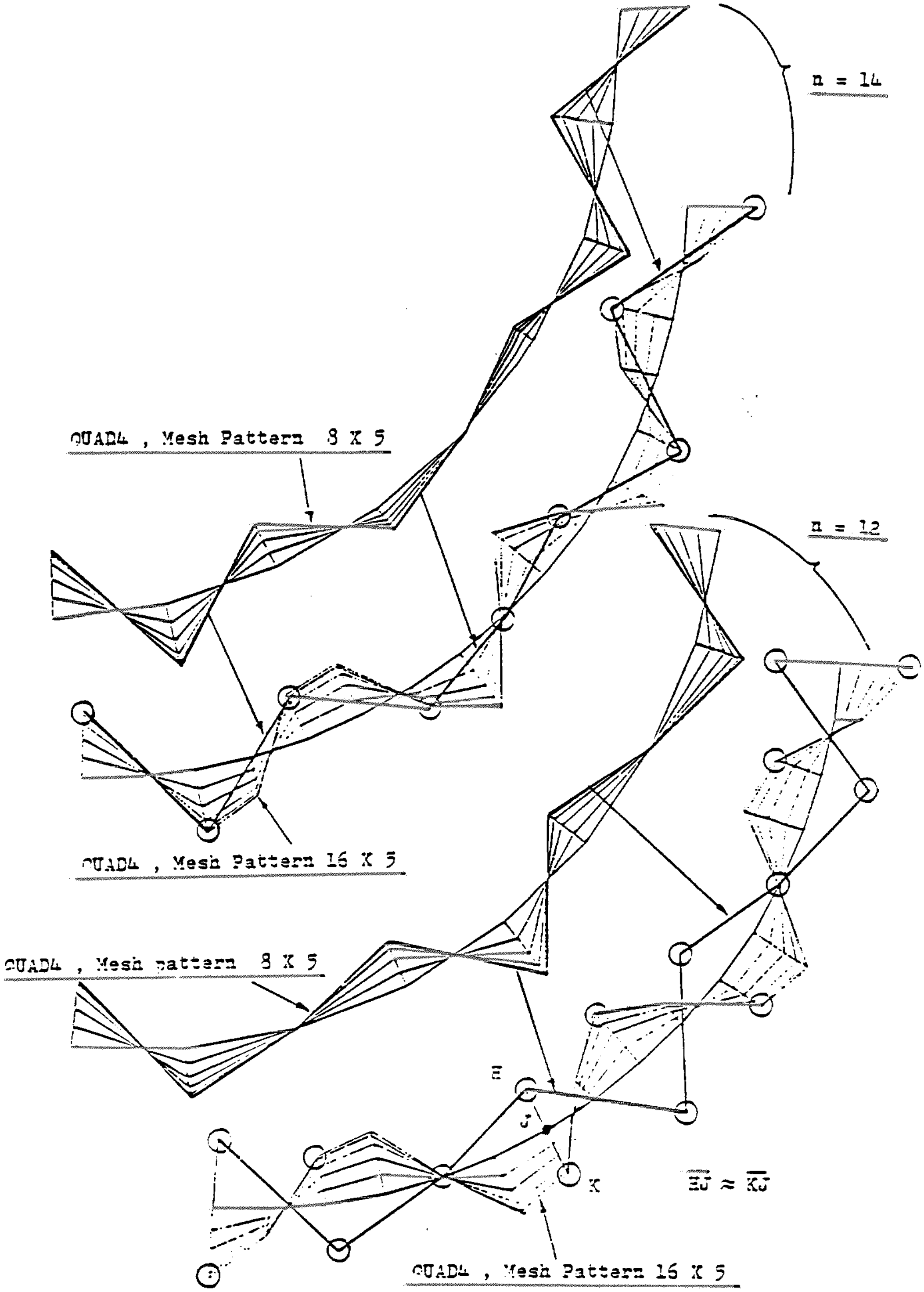


Fig. 6 Comparison of the Calculated Mode Shapes by the Different Mesh Patterns for Larger Number of Circumferential Waves (QUAD4,'8 X 5' vs '16 X 5')

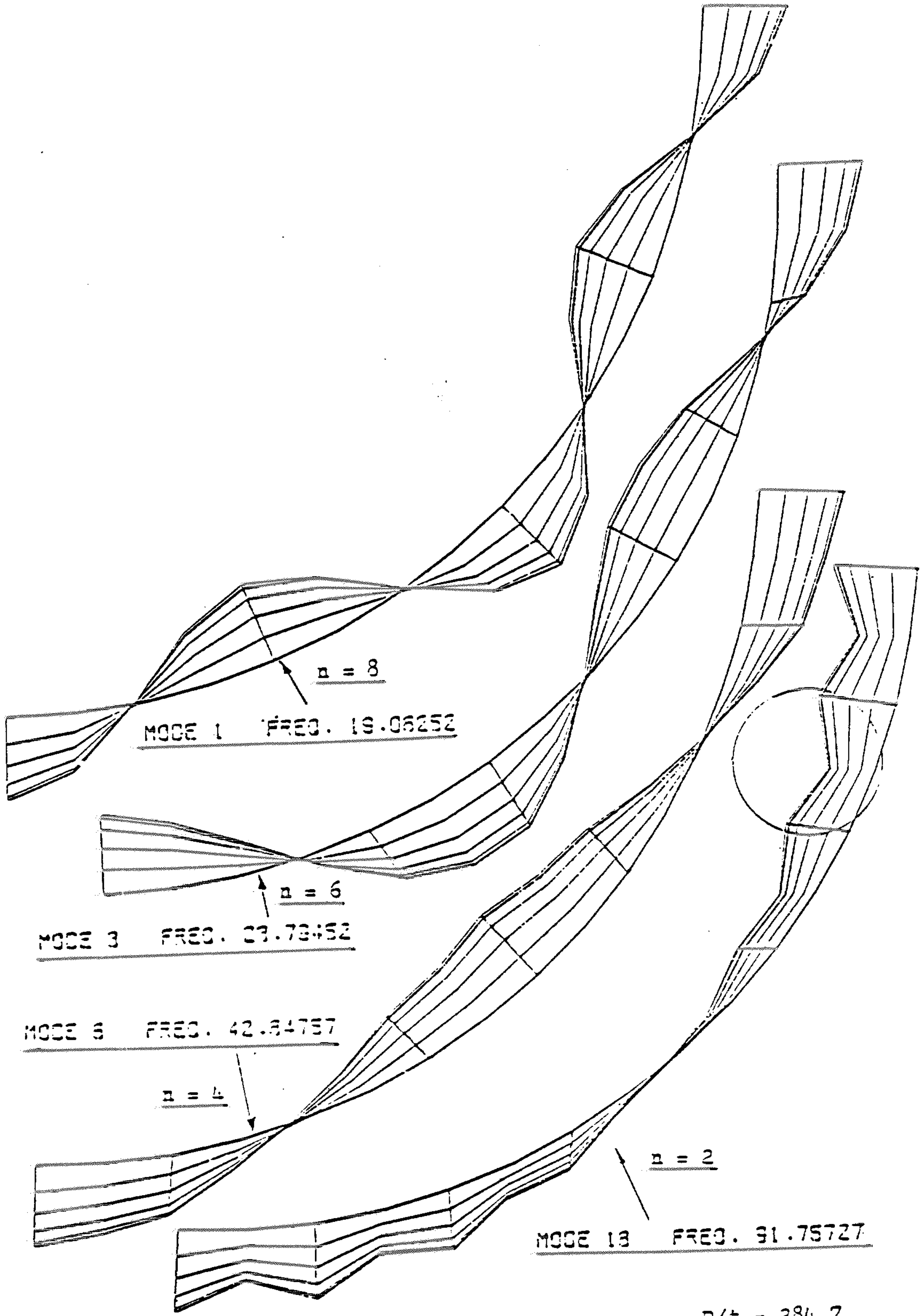


Fig. 7 Calculated Mode Shapes (QUAD8, Mesh Pattern 8 X 5)

Nodal Points Along Midsection	W : Nodal Displacement to the Radial Direction	
	QUAD8	Theoretical Value
P_1	1.000	1.000
P_2 *	0.6225	0.9808
P_3	0.9240	0.9239
P_4 *	0.5274	0.8315
P_5	0.7077	0.7071
P_6 *	0.3522	0.5556
P_7	0.3826	0.3827
P_8 *	0.1245	0.1951
P_9	3.0e-5	0.0

* P_2, P_4, P_6, P_8 are the Mid-side Nodes of QUAD8 Element

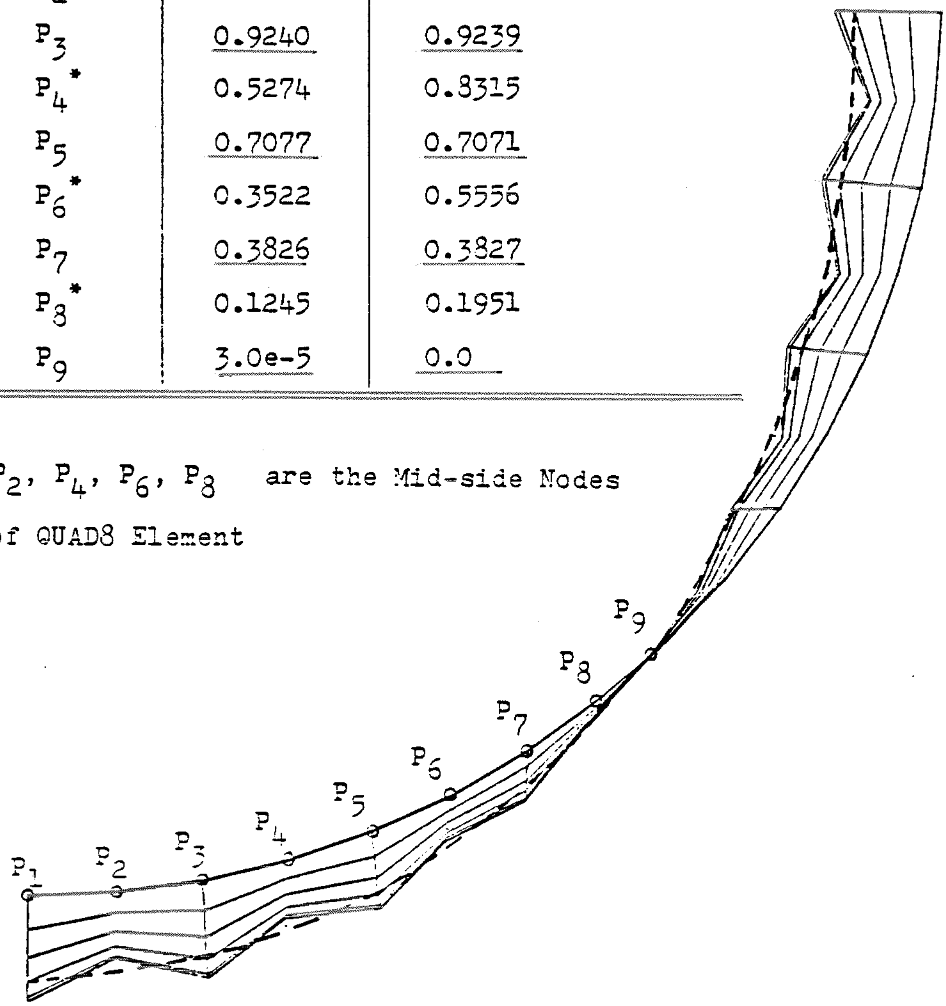
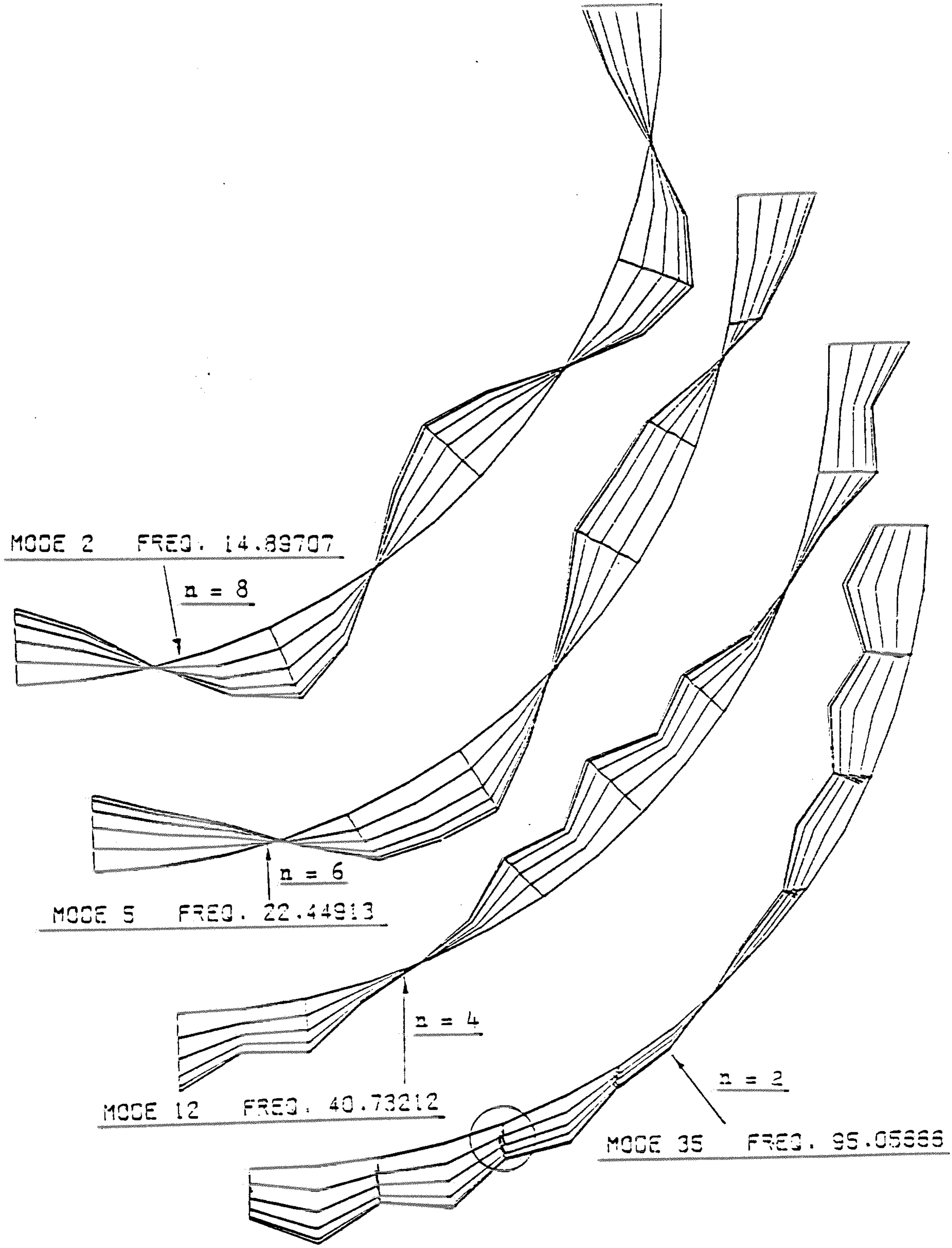


Fig. 8 Accuracy of Nodal Displacements along the Mid-section for the Vibration Mode ' n = 2 ' (QUAD8 , Mesh Pattern 8 X 5)



R/t = 600

Fig. 9 Calculated Mode Shapes (QUAD8 , Mesh Pattern 8 X 5)

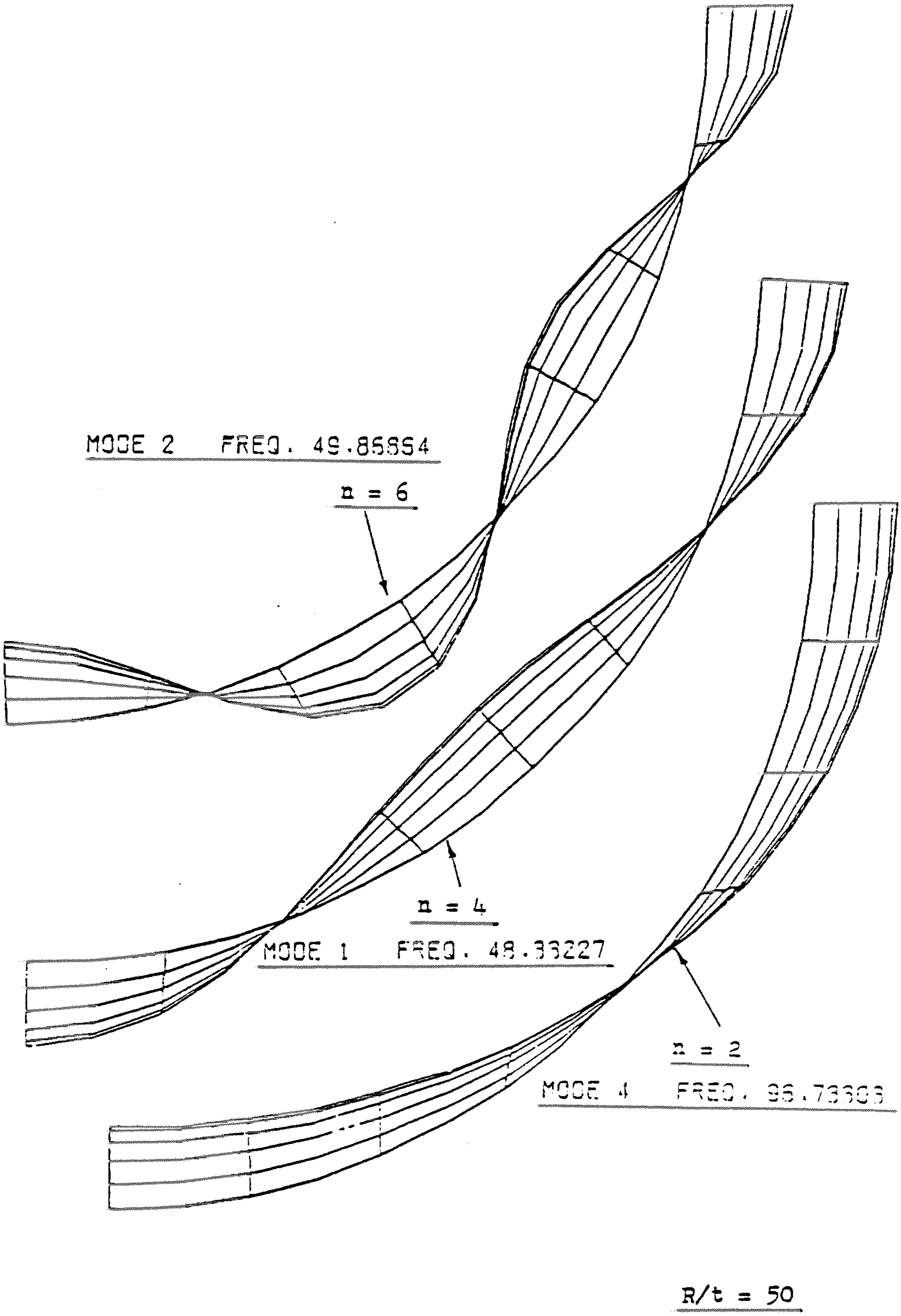
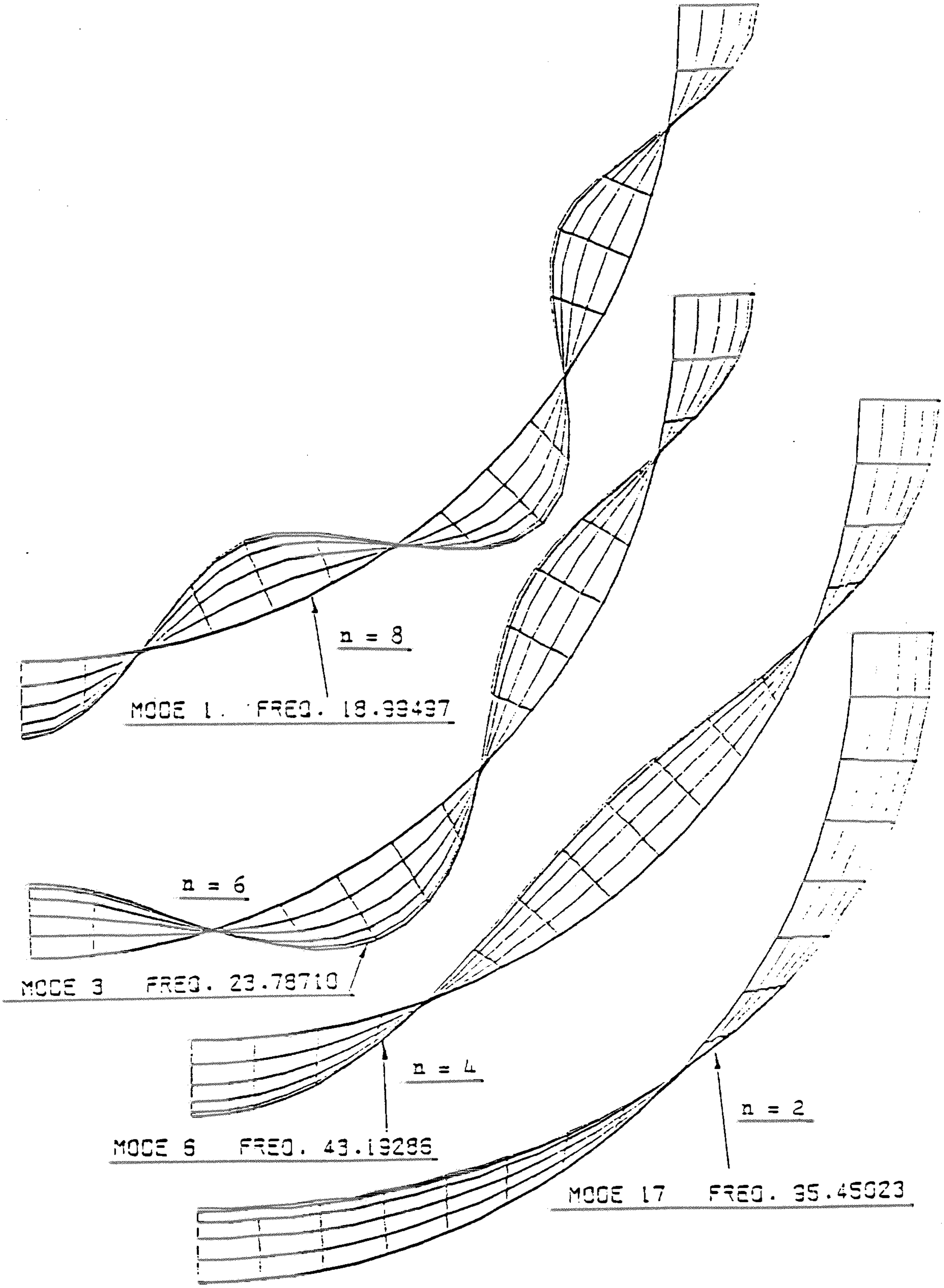


Fig. 10 Calculated Mode Shapes (QUAD8 , Mesh Pattern 8 X 5)



R/t = 284.7

Fig. 11 Calculated Mode Shapes (QUAD8 , Mesh Pattern 16 X 5)

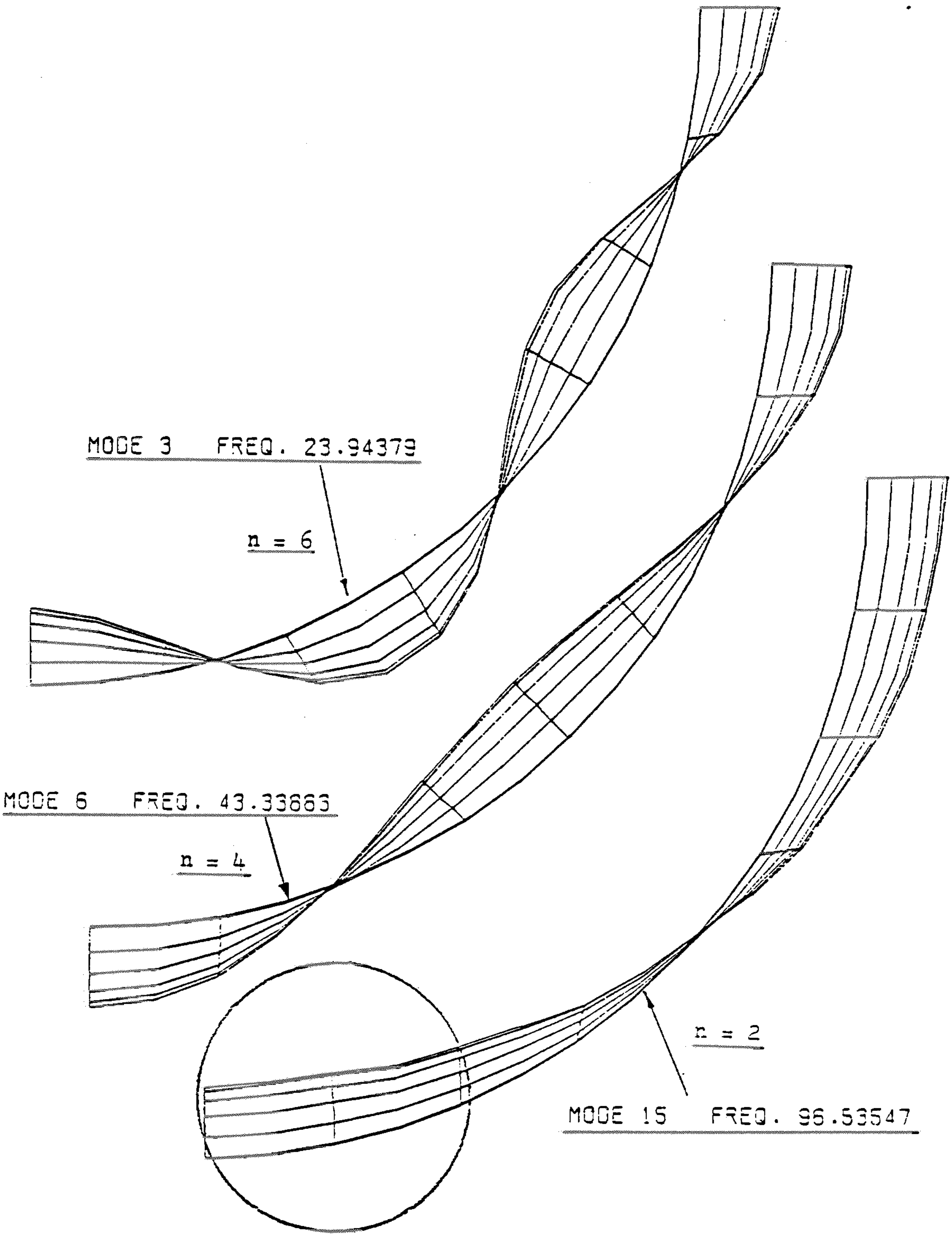


Fig. 12 Calculated Mode Shapes by Consistent Mass Approximation
(QUAD8, Mesh Pattern ' 8 X 5 ', R/t = 284.7)