ENGINEERING SYSTEMS

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MSC/NASTRAN Dynamic Analysis: Modal or Direct?

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ABSTRACT

MSC/NASTRAN offers dynamic analysis capability using direct or modal formulations with or without superelements. This paper compares computer resources and results accuracy using direct and modal analysis of a linear, nonsuperelement vehicle-type structure. A novel approach to response spectra capability is used to determine forcing function frequency content. A real eigenvalue primer is also included in an appendix.

INTRODUCTION

Background

Recently the author presented a brief lecture series on MSC/NASTRAN dynamic analysis to a group of engineers in the automotive industry. The experience level within the group varied between several months and a few years. All were familiar with conventional linear static analysis (SOL 24) and real eigenvalue analysis (SOL 3) to determine eigenvalues and eigenvectors using Generalized Dynamic Reduction (GDR,1)*. A few individuals had used either modal frequency response (SOL 30) or geometric and nonlinear-material transient analysis (SOL 99). Management concluded that a basic course in dynamic analysis would be useful for future analysis assignments.

This presented the author with an opportunity to review a concisely-written introductory text on matrix representation of linear (and somewhat nonlinear) transient and modal frequency response applications (2). Also, the new MSC/NASTRAN handbook for dynamics (3) was reviewed, as well as the text accompanying an MSC video-lecture course on dynamic and nonlinear analysis (4). This rather overwhelming collection of theoretical expertise was condensed to an eight hour summary with the objective of the lectures to choose the appropriate MSC/NASTRAN solution capability, create the model, execute dynamic analysis and produce output results with emphasis on XY plotting, and interpret the results.

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* See Reference section

Paper Organization

This paper presents dynamic analysis choices available to the engineer performing conventional linear dynamic transient analyses and provides guidelines on when certain methods are appropriate. A novel application of MSC/NASTRAN Response Spectra (5) determines frequency content of a forcing function, and a transient analysis application to a vehicle-type structure emphasizes a rational choice between direct or modal solution formulation using material damping options.

Appendix A is presented to briefly review real eigenvalue and eigenvector basics. MSC/NASTRAN computer timing formulae are presented to emphasize estimation of this important parameter in Appendix B.

MSC/NASTRAN DYNAMIC TRANSIENT ANALYSIS OPTIONS

Background

Figure 1a presents the usual strategic decision dilemma facing the engineering analyst of selecting a dynamic transient analysis method (6). The figure assumes the modal analysis approach will be used only for linear analysis with damping such that the dynamic equations are uncoupled (see Appendix A). This strategy ignores the MSC/NASTRAN capability of coupled modal methods and nonlinear capability using NOLINi Bulk Data. The author proposing Figure 1a strategy assumes nonzero initial condition capability is available for a modal formulation which is not true in MSC/NASTRAN.

The direct transient analysis approach in Figure 1a is completely general in MSC/NASTRAN with optional DMIG-input of mass, stiffness and damping matrices in small displacement linear-material analysis (SOL 27). Geometric and material nonlinear analysis (SOL 99) has recently become available. Non-zero initial conditions are available for both the linear material and nonlinear material formulations (TIC cards).

Figure 1b shows an abreviated flow chart (7) for MSC/NASTRAN linear-material small displacement direct or modal transient analysis. The figure shows that the direct or modal approach is implemented by appropriate modules early in the program with common modules used in equation time integration, data recovery, and XY plotting.

Figure 2a shows the MSC recommendation for proper use of eigenvalue extraction options including GDR (8). It is the author's opinion that GDR is appropriate for all modal transient solutions regardless of the number of the DOFs in the structural model because:

- o A common group of cards (ASET1, QSET1, DYNRED, etc) insures that this method will be used on large models where any other method would have drastic, adverse economic effects during computation.

- Any penalty for using GDR over conventional Givens or Modified-Givens methods on small models is negligible.

Maximum DOF bounds are suggested by authors for choosing a modal or direct transient analysis formulation. This paper shows horrendous CPU-time required for a 1500 time step solution of a moderate-size model (about 5000 DOF), which emphasizes that the direct method be used only on up to a few hundred DOF models. Models over 10,000 DOF should use modal formulation and superelements.

Forcing Function Frequency

Transient analysis authors (2,9) emphasize proper choice of time step in linear as well as nonlinear analyses (see Figure 1a). In linear analysis a large time step will not sample tabulated input forcing functions adequately and output XY plots will not be smooth. However, MSC/NASTRAN guarantees convergence for the linear case regardless of time step size. In nonlinear analysis time step size can have a significant influence on convergence (with a smaller time step more likely to converge).

Many analysts (including the author) do not have convenient access to a Fast Fouier Transform or Harmonic Analysis program to predict forcing function frequency content. For this reason the author presents a novel application of the MSC/NASTRAN response-spectra-curve generation option (5) to estimate forcing function frequency content in the next section.

FORCING FUNCTION FREQUENCY CONTENT USING RESPONSE SPECTRA

Introduction

The author has found that a single degree of freedom (SDOF) oscillator model may be used to determine frequency content of an arbitrary forcing function using MSC/NASTRAN Response Spectrum analysis (5). The simple steps may be summarized as:

- Construct a SDOF oscillator using a large mass (CMASS2) and a unit spring (CELAS2).

- o Apply the TLOAD1 - TABLED1 time history to the large mass as an enforced velocity using SOL 27 with an appropriate number of time steps to insure proper tabular-data sampling and smooth response curve XY plotting.
- o Examine a standard MSC/NASTRAN XY plot displaying the mass relative displacement response spectra (imaginary part in response spectra printout).
  - o Forcing function frequency peaks will be evident, with the absolute magnitude of the harmonic coefficients displayed.

Example One Illustration

Figure 3a shows time history of a 0.1 second period square wave (10 Hz) for time period of 0.25 seconds. This function is applied to the SDOF oscillator described above as an enforced velocity. Figure 3b shows the relative-displacement response spectra output by the program. The figure shows frequency peaks at one, three, five, etc. times fundamental frequency of 10 Hz. Absolute amplitude of the frequency peaks decrease monotonically with frequency as predicted by the square wave Fourier series shown in Fig 3a.

Complete MSC/NASTRAN run deck details are shown in Figure 4. The figure shows 500 0.5 millisecond time steps used with output at each time step.

Example Two Illustration

Figure 5a shows an enforced triangular wave velocity function (10 Hz) applied for 0.25 seconds to the SDOF oscillator described above. As before, Figure 5b shows the relative-displacement response spectra. Frequency peaks occur at one, three, five, etc. times fundamental frequency, with the absolute amplitude of the peaks attenuated much more than the Example One result. These amplitude peak magnitudes agree quite well with the Fourier series coefficients shown in Figure 5a.

Practical Application

A typical input forcing function recently encountered in an acceleration analysis of a vehicle-type structure is shown for a 0.25 second period in Figure 6a. TABLED1 cards described this function using 128 time-amplitude pairs with smallest time step of about one to two milliseconds.

Figure 6b shows relative-displacement response spectra for the Figure 6a function applied as an enforced velocity excitation to the SDOF oscillator described above. A total of 2500 0.1 millisecond time steps were used with output at 0.5 millisecond intervals. The resulting absolute relative-displacement response spectra shows a peak at 4 - 6 Hz with amplitude attenuated at the higher frequencies (200 Hz range in Figure 6b).

Conclusion

A SDOF oscillator response spectra analysis with an enforced velocity input has been shown to yield relative displacement response spectra output that identifies frequency peaks with absolute displacement amplitude proportional to Fourier series coefficient magnitude. It has been further observed that sufficiently small time steps are required to adequately sample the function (as shown in Figure 6a). A frequent TSTEP-output interval is required to insure that resulting relative-displacement response spectra contains high frequency components.

MSC/NASTRAN DYNAMIC TRANSIENT APPLICATION

Background

This section illustrates application of MSC/NASTRAN modal and direct transient formulation to a vehicle-type structure. Computer CPU timing formulae given in Appendix B are used to estimate computer time resource requirements for eigenvalue extraction and transient excitation of a moderate-size model. Also a small model is used to compare natural frequencies to the moderate-size model using both lumped and coupled-mass options on the small model.

Finite Element Models

Figure 7a shows frame members (CBAR) of a vehicle-type model*, while Figure 7b shows skin elements (CQUAD4). MSC/NASTRAN MSGMESH (10) was used to generate a small, coarse model and a moderate-size finer mesh model. Physical attributes of the two models are shown on the figure. The MSGMESH preprocessor was convenient for this application, especially the automatic EQUIV feature. A few EGRIDS, CGEN, and CBARG cards were easily changed to produce the two models during each analysis run. The EQUIV feature insured welded attachment between frame and skin at all points. However, the discerning reader will note that the X-braces are attached to the skin at fewer locations for the small model.

The vehicle-type structure was supported at each of three corners with a grounded-spring - viscous-damper arrangement. The left front corner had a road-induced vertical enforced displacement applied at the lower end of the suspension as shown in the figure.

Structure Natural Frequencies

Table 1 compares natural frequencies for the small and moderate-size models obtained using GDR to 200 Hz. The moderate-size model used lumped mass (default) analysis, while the small model was analyzed using lumped and coupled mass (PARAM,COUPMASS,1). The table shows good correlation to about 100 Hz with the small lumped mass result closer than the small coupled mass model in predicting moderate-size model results.

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*Any similarity between this model and any existing or conceptual vehicle is purely coincidental.

Table 2 compares IBM 3084 computer CPU-time requirements for real eigenvalue extraction (using GDR) as a function of FMAX for the moderate-size model. The table emphasizes solution economy that will result if the analyst has knowledge of forcing function frequency content as shown above. Higher frequency structure modes will not be required if only low frequency excitation is present. Hence, FMAX may be reduced. Table 2 also shows that the DYNREDU module dominates computer CPU-time requirements when using GDR. Total CPU-time estimates (quite accurate for FMAX = 100) are based on Appendix B formulae.

Modal and Direct Transient Analysis

The Figure 6a time function was applied as an enforced displacement to the moderate-size model (Figure 7) at the front left corner to 0.25 seconds and constrained to zero motion for the balance of a 1.5 second interval. The TABLED1 tabulation of the Figure 6 function was multiplied by five to yield peak enforced displacement of about 13 centimeters. A total of 1500 one millisecond time steps were used with output at 15 time step intervals to insure clear XY plots. Modes to 100 Hz was used in the modal transient analysis based on the forcing function frequency content (see Figure 6b).

The aluminum frame structure was given a 2 percent of critical material damping (GE entry on MAT1), while the composite skin was at 15 percent of critical. No modal (frequency dependant) damping was used during the modal formulation.

Figures 8 - 10 show modal and direct transient displacement results at the enforced suspension, the other three suspensions, and at three locations on the main structure. As expected the modal and direct transient analyses yielded identical results.

Table 3 compares IBM 3084 CPU-time requirements for the direct and modal analyses. It is quite obvious from the table that the analyst could recompute (noncheckpointed) structural modes to 100 Hz using GDR and still use only one-sixth of the computer CPU time when compared to the direct transient result for 1500-time-step analysis. The table also shows direct-formulation CPU time for 500 and 1000 time steps to demonstrate that TRD1-module time is proportional to the number of time steps (see Appendix B).

CONCLUSIONS

This paper has compared direct and modal formulation transient analysis solutions using MSC/NASTRAN conventional analyses (SOL 27 and SOL 31). It is evident from the results presented here that:

- A simple application of MSC/NASTRAN response-spectra-curve generation capability yields frequency content and relative absolute magnitude of the Fourier coefficients.

- A knowledge of forcing function frequency content is useful in setting upper limits for FMAX when extracting eigenvalues using GDR.

- Direct transient analysis is not cost-effective for moderate-size structures requiring several hundred time steps.

- Modal transient is preferred because natural frequencies are usually desired, a checkpointed run may be restarted, and solution accuracy is comparable to direct transient analysis.

Figures 11 show in cartoon form a summary of direct and modal transient analysis features as well as application of the response spectra technique to determine frequency content of proposed forcing functions.

REFERENCES

1. Joseph, J. A., Editor, 'MSC/NASTRAN Application Manual', The MacNeal-Schwendler Corporation, Los Angeles, CA, 1985, Section 2.4

2. Craig, R. R., 'Structural Dynamics', John Wiley and Sons, New York, 1981, Chapters 11 - 13

3. Gockel, M. A., Editor, 'Handbook for Dynamic Analysis', The MacNeal-Schwendler Corporation, Los Angeles, CA, June, 1983

4. Anderson, W. J., 'Study Guide - Dynamics and Nonlinear Analysis', Video Lecture Series 1002, The MacNeal-Schwendler Corporation, Los Angeles, CA, May, 1985, Chapters 1 - 15

5. Joseph, Ibid, Section 2.15

6. Craig, Ibid, Chapters 15 and 18

7. Gockel, Ibid, Section 13

8. Gockel, Ibid, Section 5

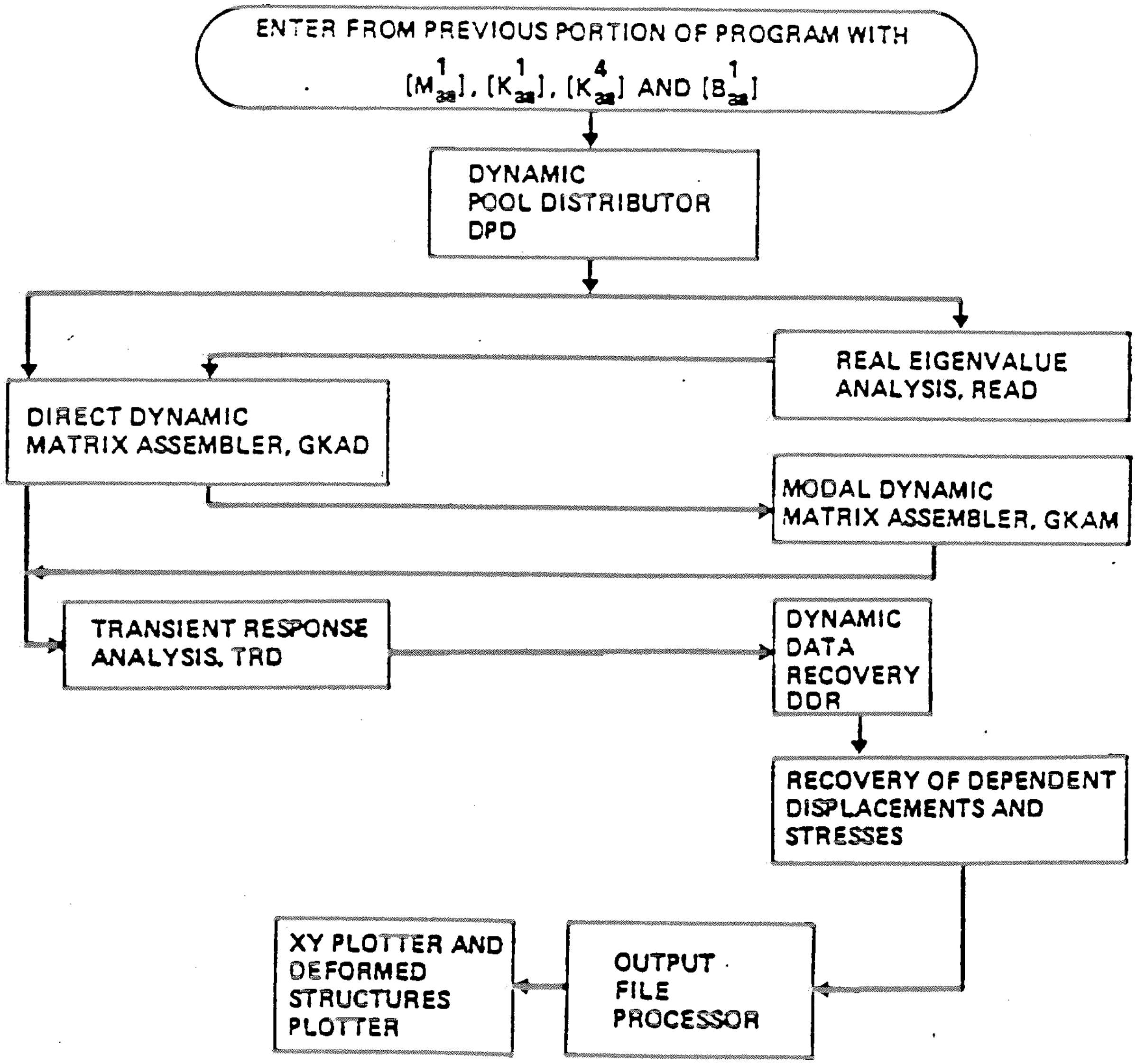
9. Joseph, Ibid, Section 2.14.7

10. Peterson, L., Editor, 'MSGMESH Analyst's Guide', The MacNeal-Schwendler Corporation, Los Angeles, CA, December, 1980

11. Thompson, W. T., 'Theory of Vibration with Applications', Prentice-Hall, Englewood Cliffs, NJ, 1981, Appendix C

```
            MODAL      +---------+  DIRECT
        +----------| SELECT |----------+
        |          |STRATEGY|          |
        |          +---------+          |
        V                               V
● UNCOUPLED EQUATIONS          ● LINEAR, NONLINEAR
● LINEAR                       ● ARBITRARY DAMPING
● UNCOUPLED DAMPING            ● RESPONSE ACCURACY
● RESPONSE FROM MODES            IMPROVES WITH SMALL
  SUBSET                         TIME STEPS
```

a. Transient analysis strategy.



b. MSC/NASTRAN transient analysis flow chart, simplified.

FIGURE 1. Transient Analysis Strategy and MSC/NASTRAN solutions

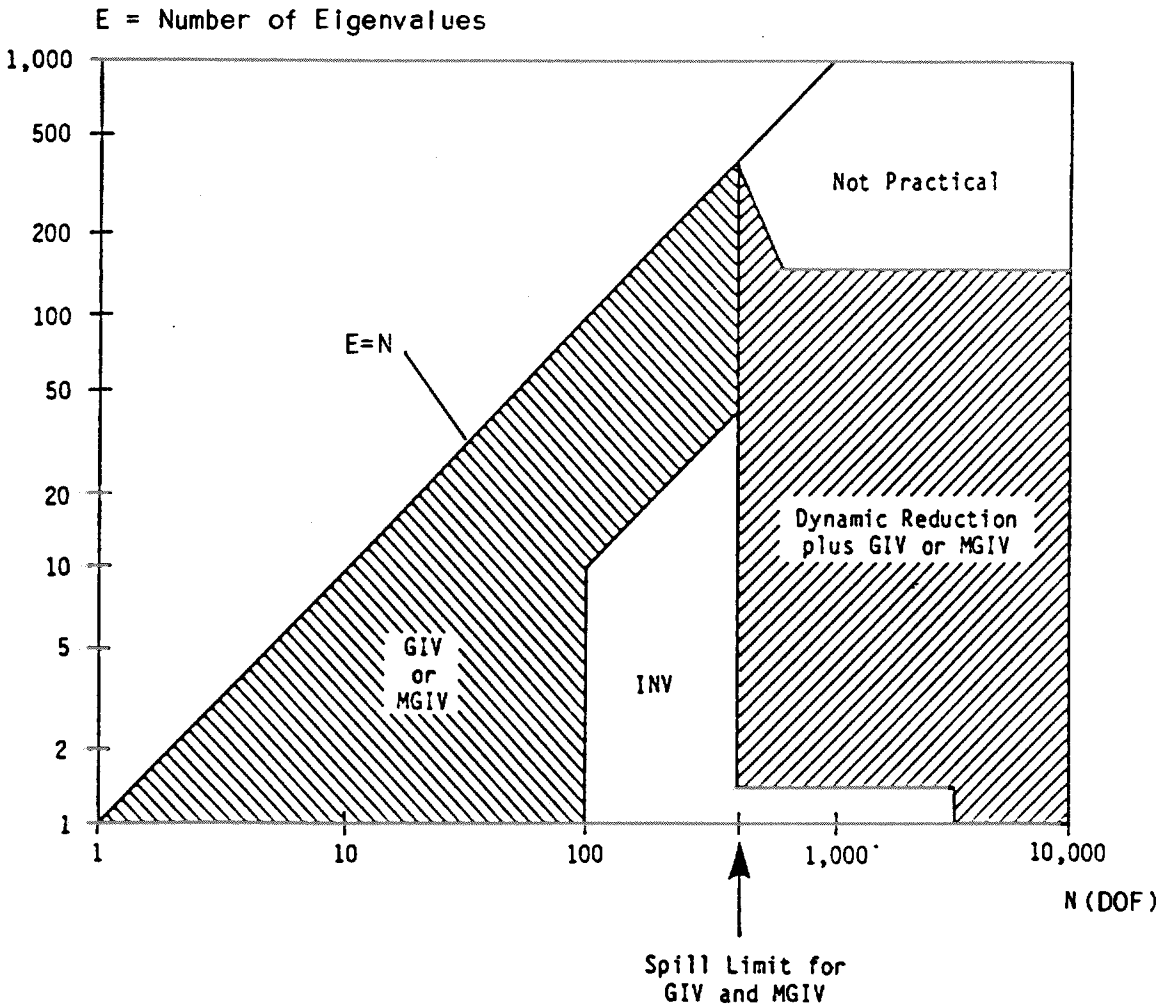
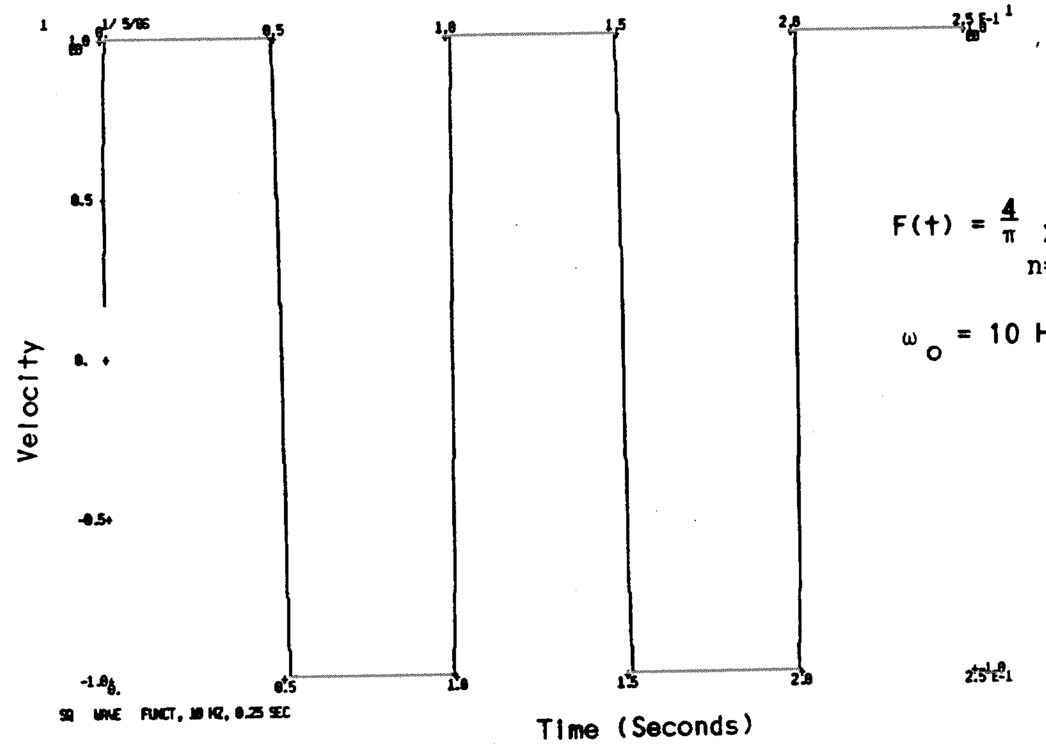


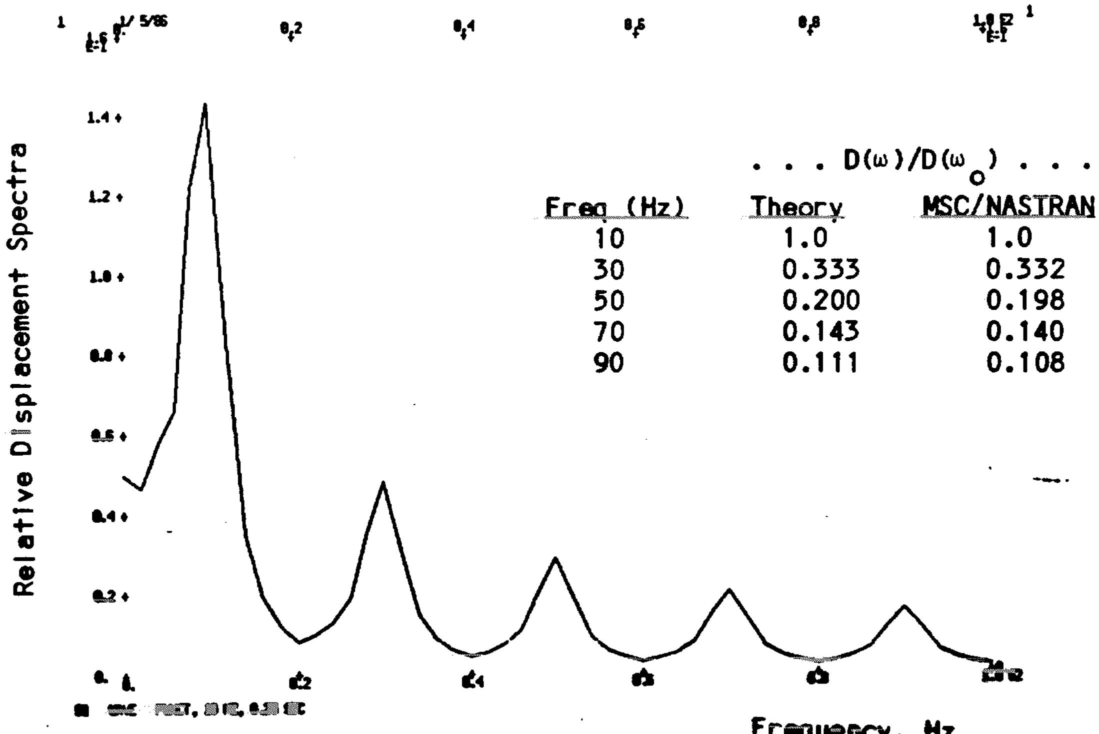
FIGURE 2. MSC Eigenvalue Extraction Guidelines



$$F(t) = \frac{4}{\pi} \sum_{n=1,3,5...} \frac{1}{n} \sin n \omega_o t$$

$$\omega_o = 10 \text{ Hz}$$

a. Time History



. . . $D(\omega)/D(\omega_o)$ . . .

Freq (Hz)	Theory	MSC/NASTRAN
10	1.0	1.0
30	0.333	0.332
50	0.200	0.198
70	0.143	0.140
90	0.111	0.108

b. Displacement Response Spectra

FIGURE 3. Example Problem One, Square Wave

```
N A S T R A N   E X E C U T I V E   C O N T R O L


ID SEIS RESP
SOL 27
TIME 1ØØØ
ALTER 462,462
CEND
```

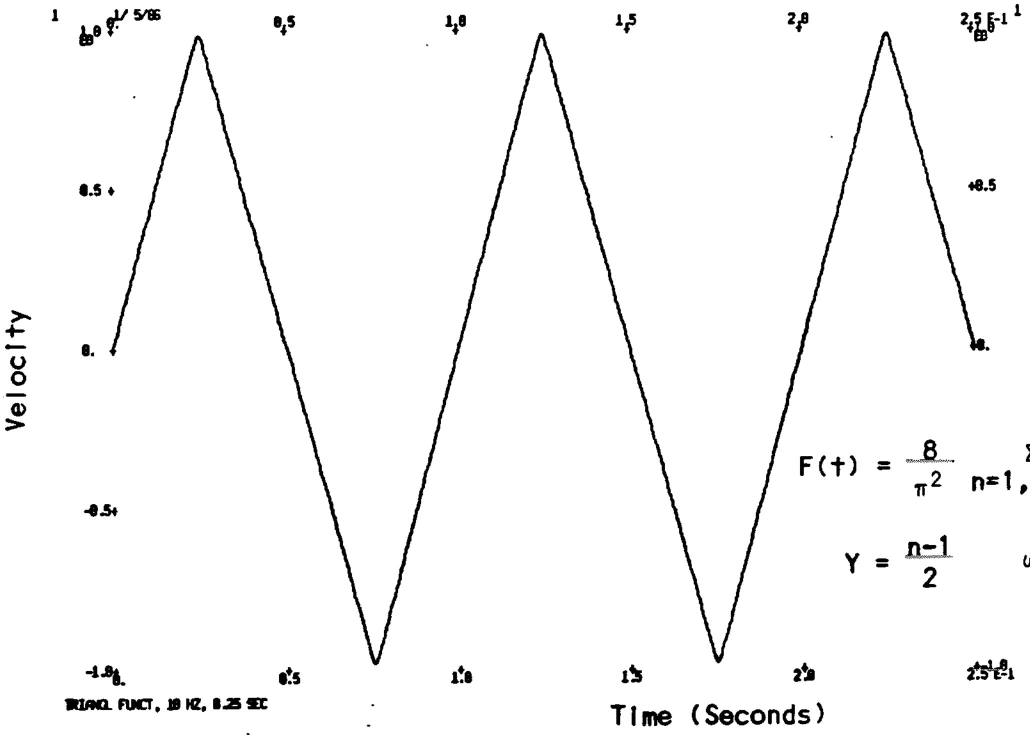
```
C A S E   C O N T R O L


TITLE= SQ   WAVE   FUNCT, 1Ø HZ, Ø.25 SEC
SET 1=7
DISP(PLOT)=1
VELO=1
DLOAD=12
TSTEP=1
OUTPUT (XYPLOT)
PLOTTER NAST
XPAPER 26.
YPAPER 2Ø.
XDIVISIONS 5
YDIVISIONS 5
XTITLE FREQUENCY  HZ
YTITLE RESPONSE DISPL
XYPLOT,DISP,SPECTRAL,1/7(T1IP)
BEGIN BULK
```

```
S O R T E D   B U L K   D A T A   E C H O

.   1  ..   2  ..   3  ..   4  ..   5  ..   6  ..   7  ..   8  ..   9  ..  1Ø  .
CELAS2  71      1.      7       1
CMASS2  3Ø      1.+3    7       1
DAREA   1       7       1       1.+3
DTI     SPSEL   Ø
DTI     SPSEL   1       2       3       7       ENDREC
FREQ    2       Ø.
FREQ1   3       Ø.      2.      5Ø
GRID    7                                               23456
PARAM   AUTOSPC YES
PARAM   RSPECTRAØ
TABLED1 12                                                              +T12
+T12    Ø.      Ø.      .ØØØ5   1.      .Ø5     1.      .Ø5ØØ1  -1.     +T12A
+T12A   .1      -1.     .1ØØ1   1.      .15     1.      .15ØØ1  -1.     +T12B
+T12B   .2      -1.     .2ØØ1   1.      .25     1.      ENDT
TLOAD1  12      1               2       12
TSTEP   1       5ØØ     .ØØØ5   1
ENDDATA
```

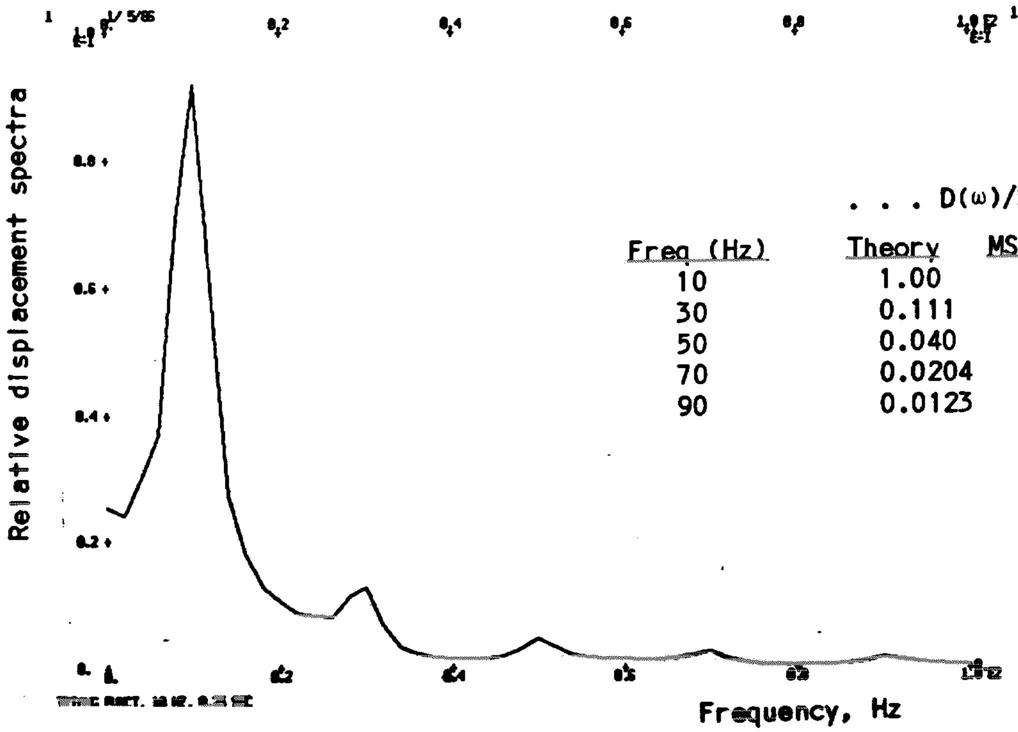
FIGURE 4. MSC/NASTRAN Response Spectra Run Deck, Example Problem One



$$F(t) = \frac{8}{\pi^2} \sum_{n=1,3,5..} \frac{(-1)^y \sin n \omega_o t}{n^2}$$

$$Y = \frac{n-1}{2} \qquad \omega_o = 10 \text{ Hz}$$

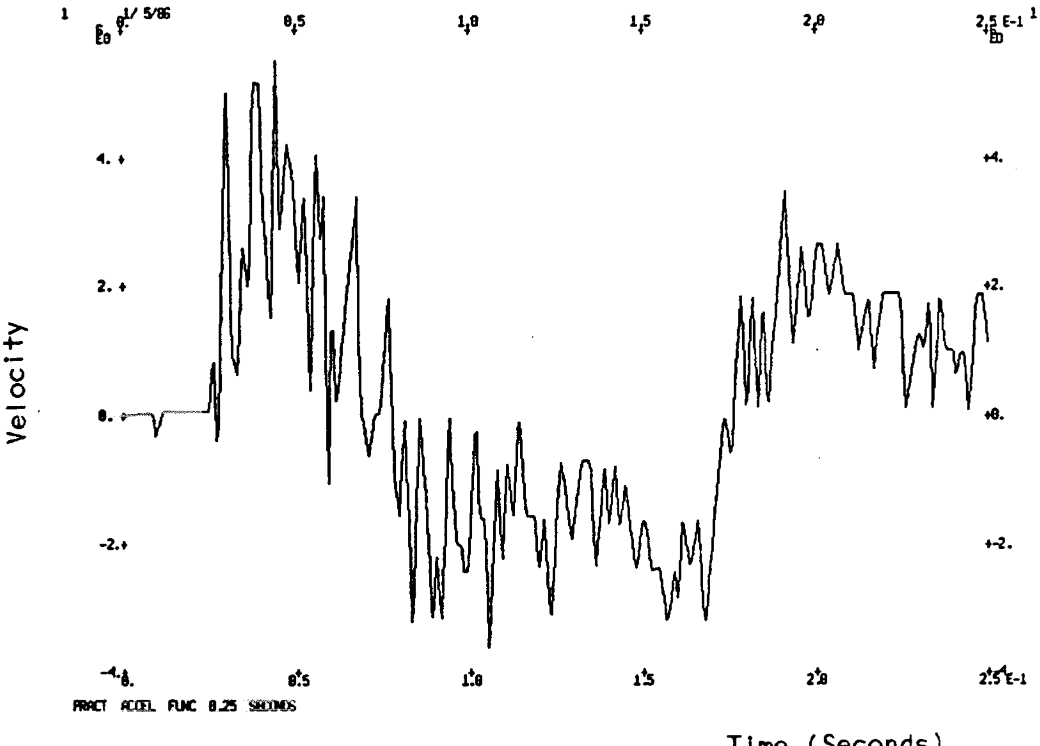
a. Time history



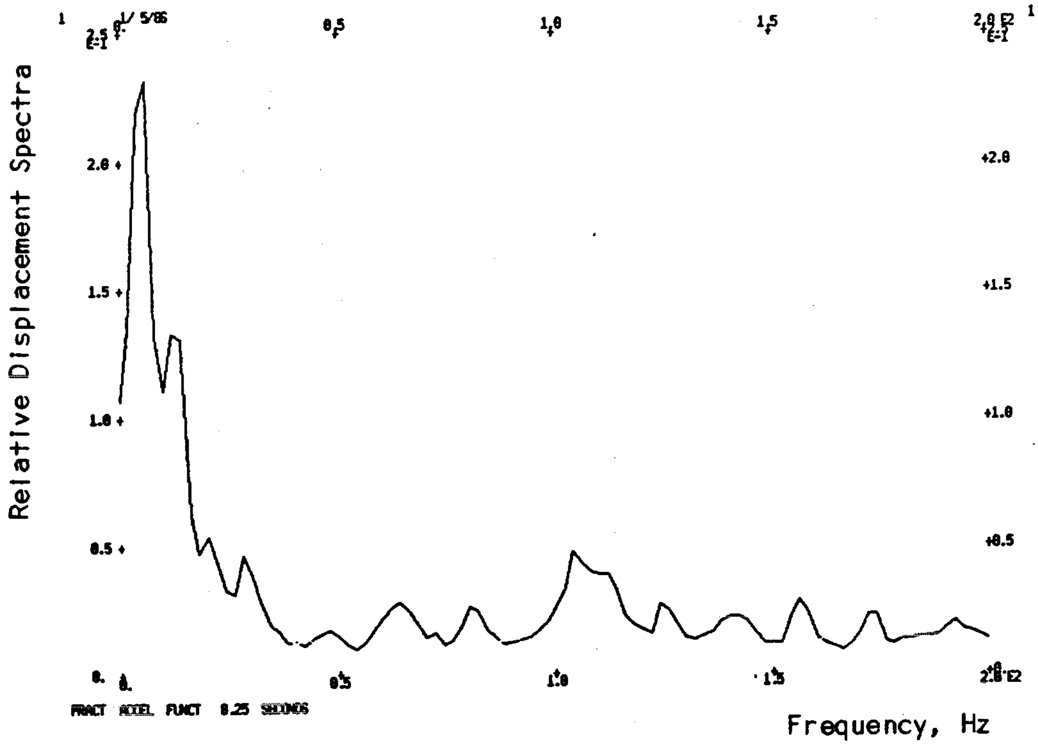
Freq (Hz)	Theory $D(\omega)/D(\omega_o)$	MSC/NASTRAN $D(\omega)/D(\omega_o)$
10	1.00	1.0
30	0.111	0.135
50	0.040	0.0488
70	0.0204	0.0223
90	0.0123	0.0148

b. Displacement response spectra

FIGURE 5. Example Problem Two, Triangular Wave



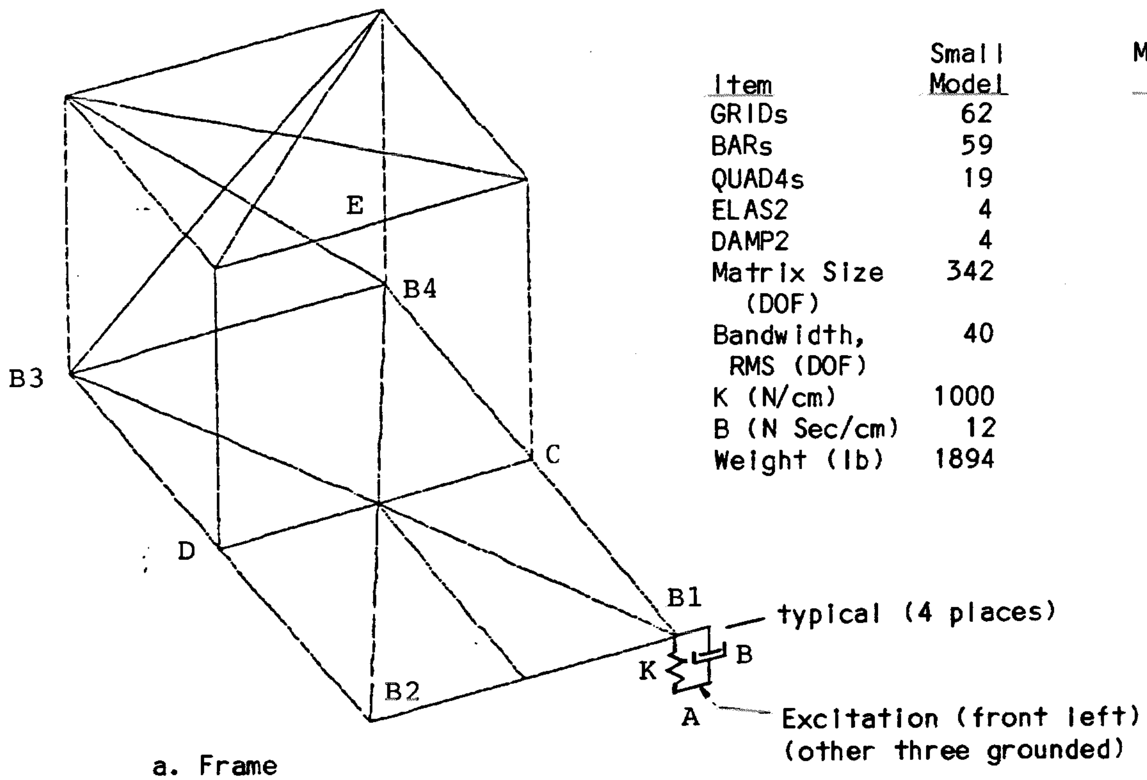
a. Time history



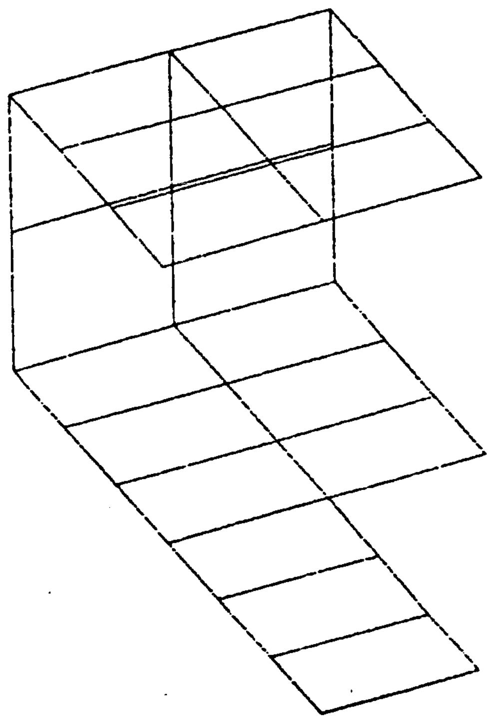
b. Displacement Response Spectra

FIGURE 6. Practical Input Function

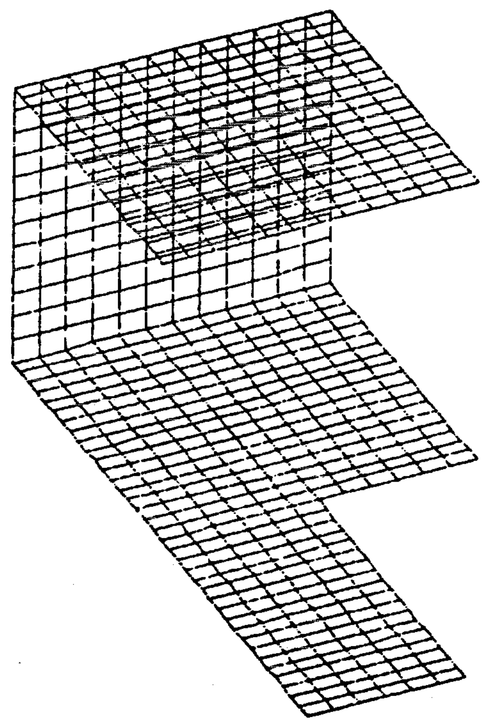
Item	Small Model	Moderate-Size Model
GRIDs	62	987
BARs	59	354
QUAD4s	19	684
ELAS2	4	4
DAMP2	4	4
Matrix Size (DOF)	342	5266
Bandwidth, RMS (DOF)	40	82
K (N/cm)	1000	1000
B (N Sec/cm)	12	12
Weight (lb)	1894	1894



a. Frame



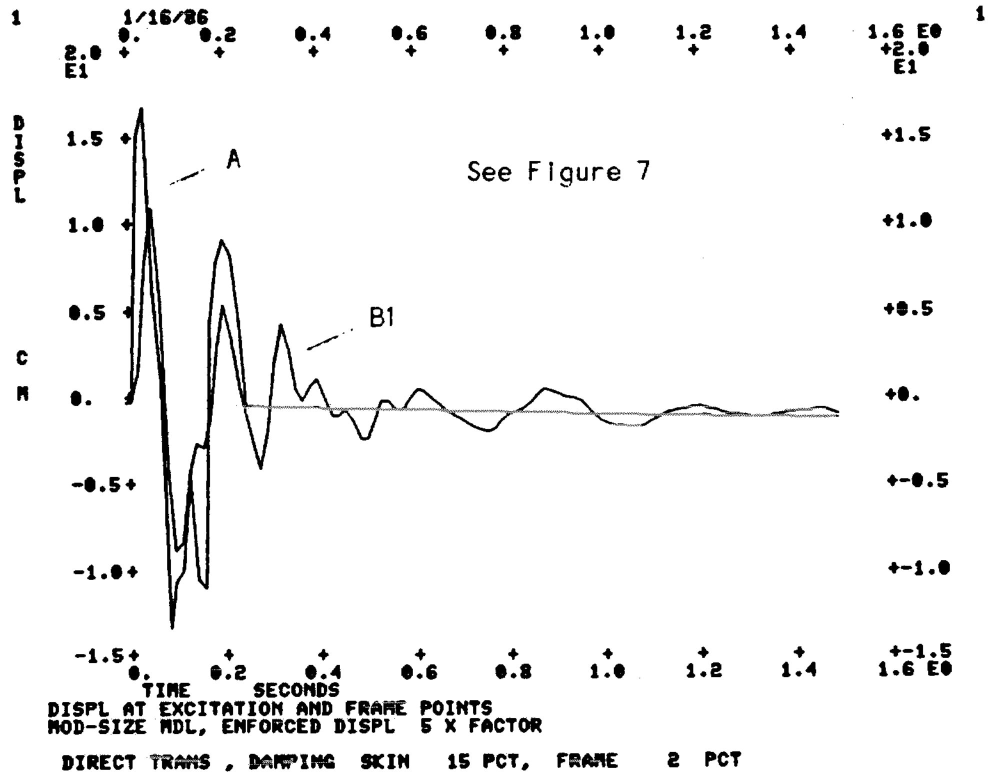
t = 1.4cm



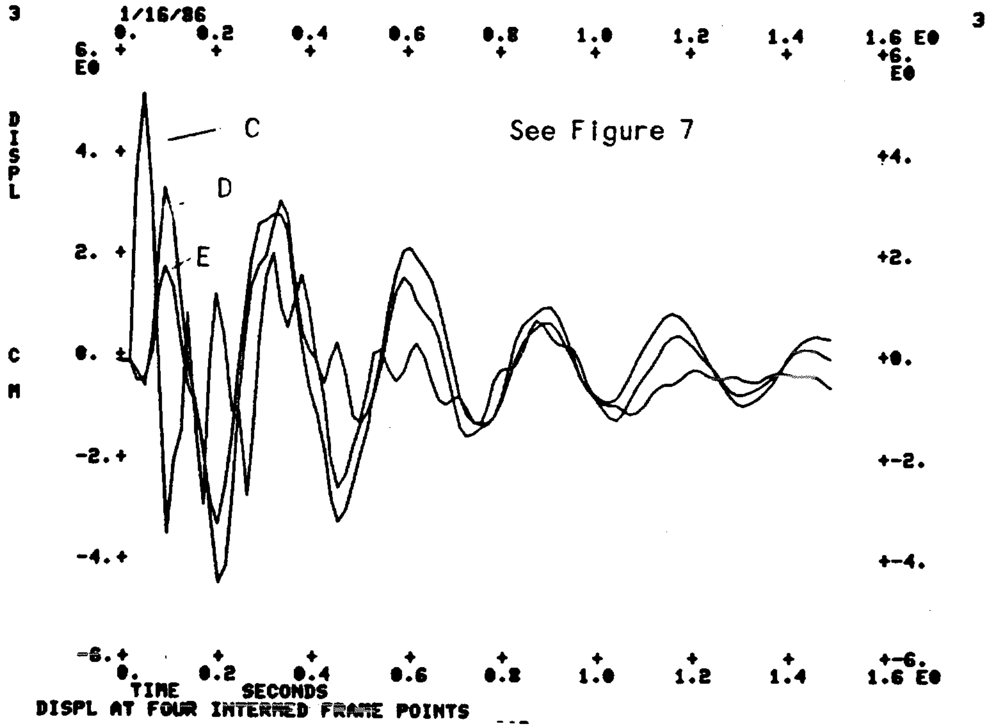
b. Small-model, skin

c. Moderate-size model, skin

FIGURE 7. Vehicle-type Structure Models

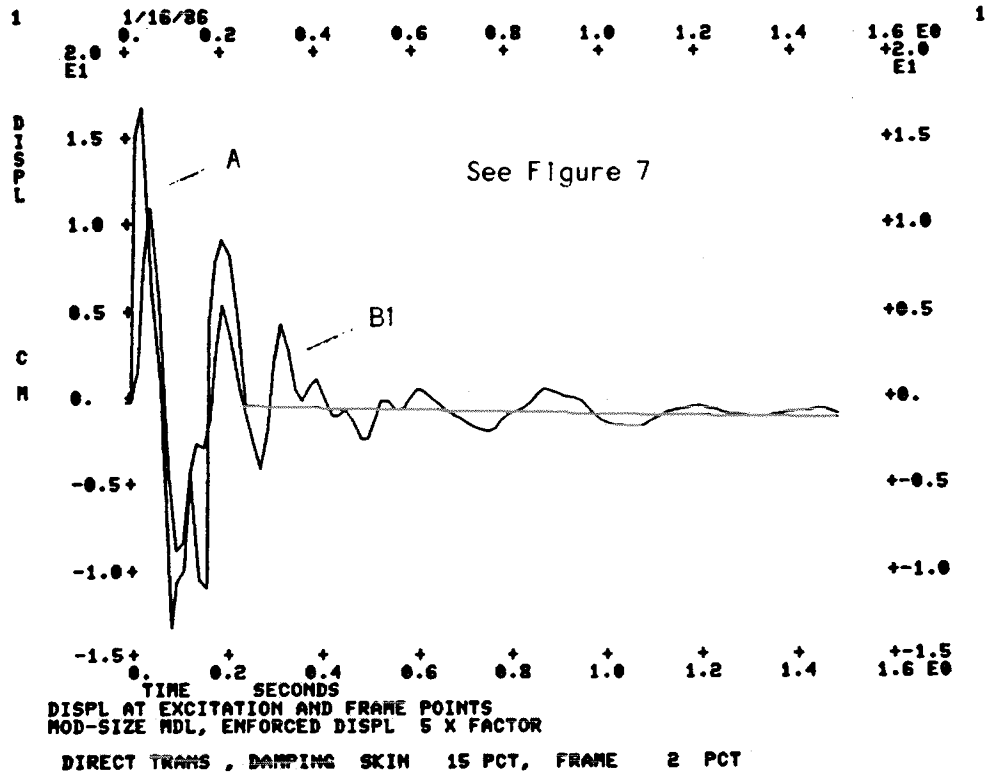


a. Excited Suspension displacements

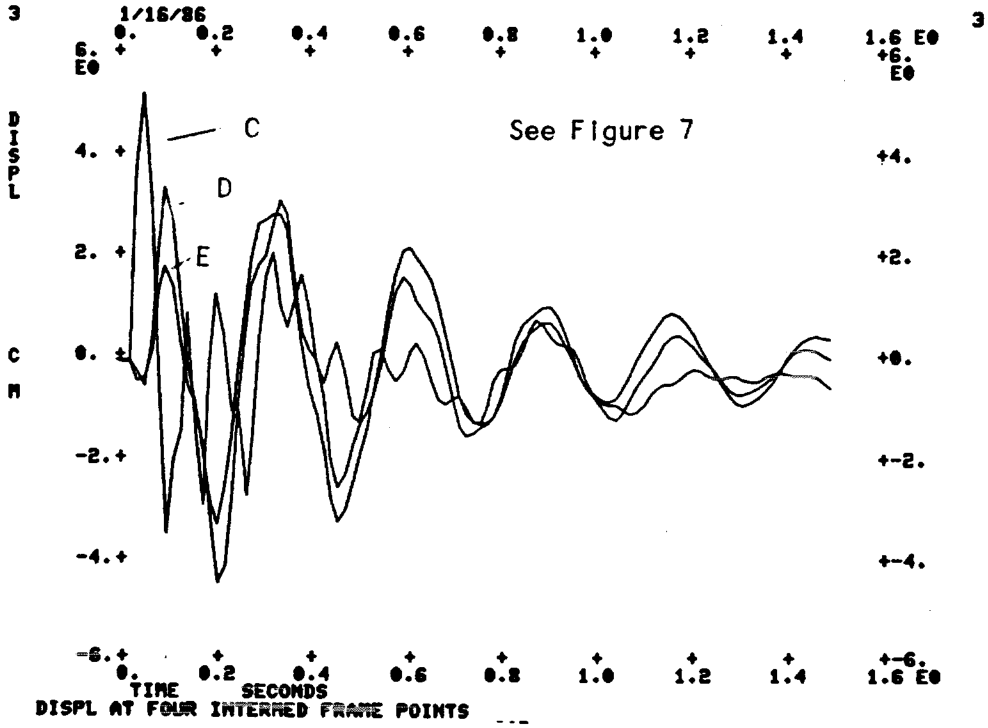


b. Frame displacements

FIGURE 8. Vehicle-type Structure Transient Displacements, Direct Formulation



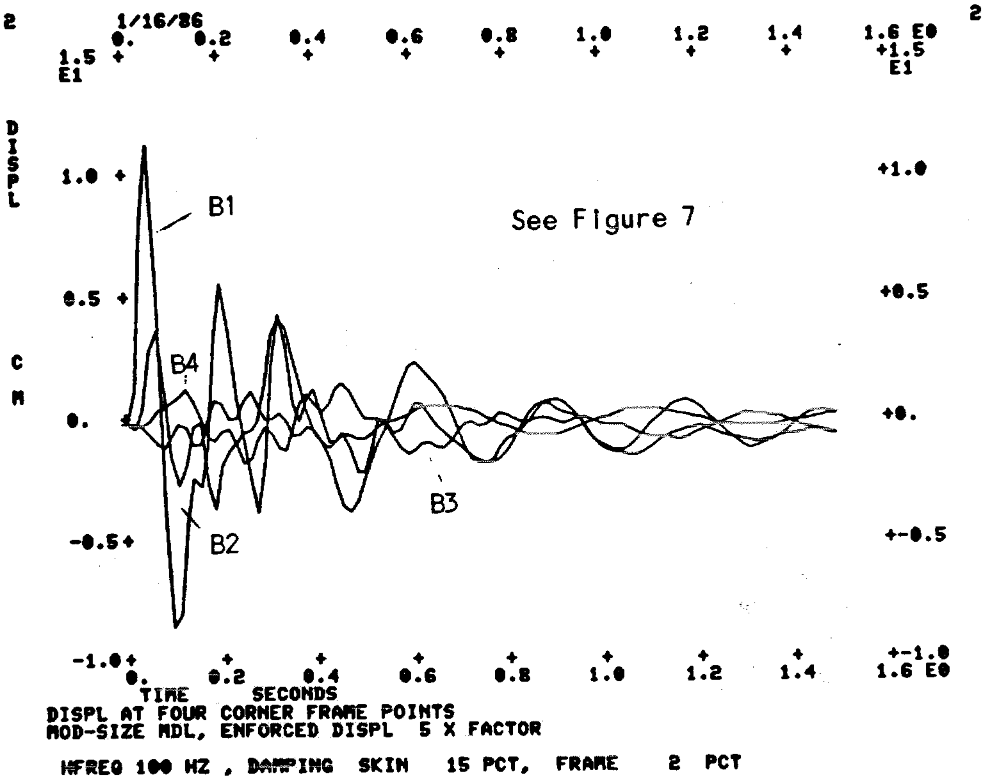
a. Excited Suspension displacements



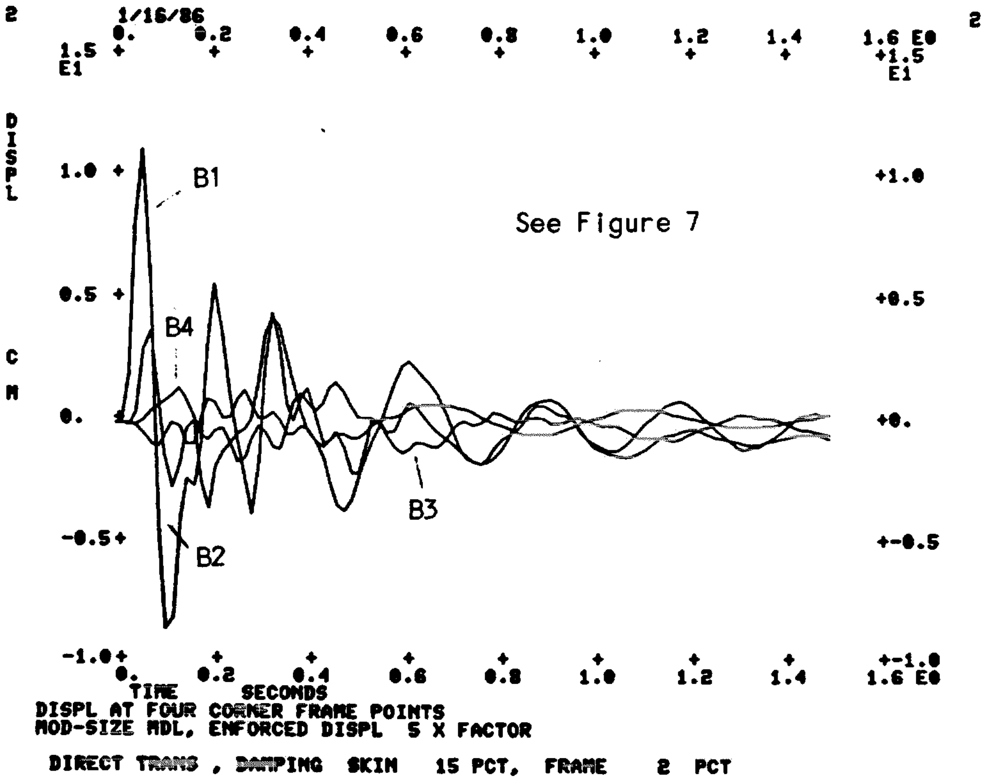
b. Frame displacements

FIGURE 8. Vehicle-type Structure Transient Displacements, Direct Formulation

FIGURE 11. Cartoons

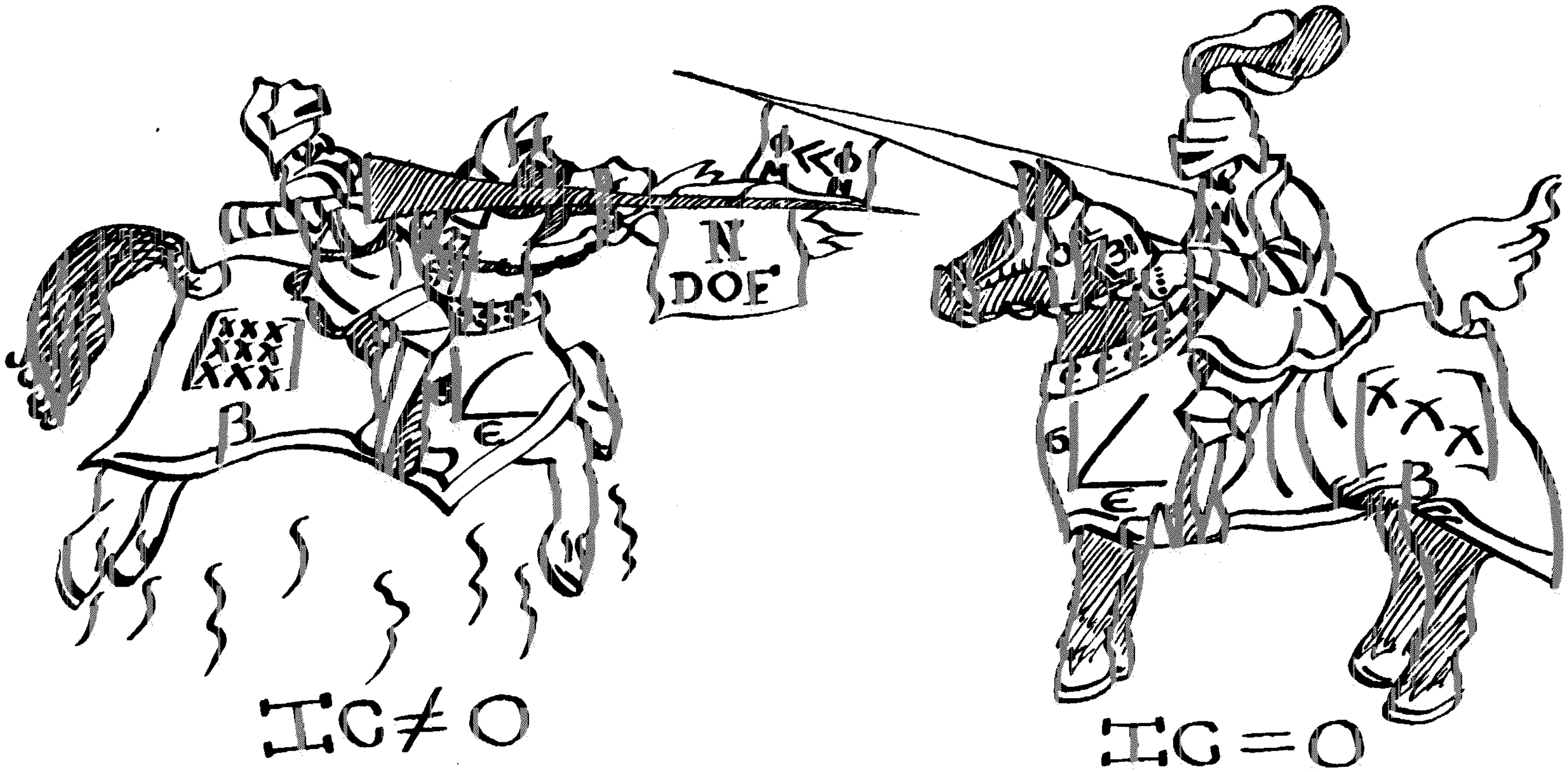


a. Modal formulation

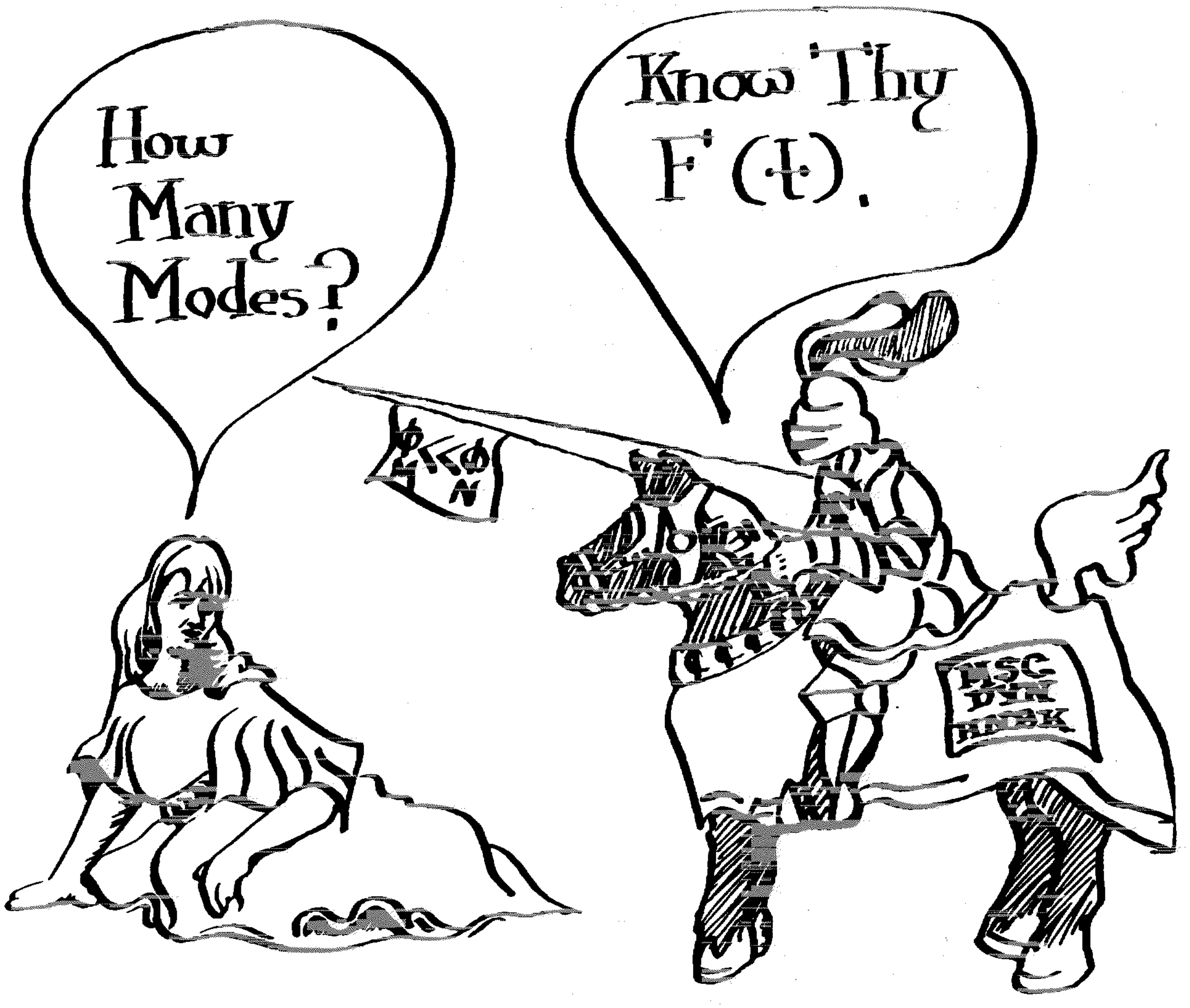


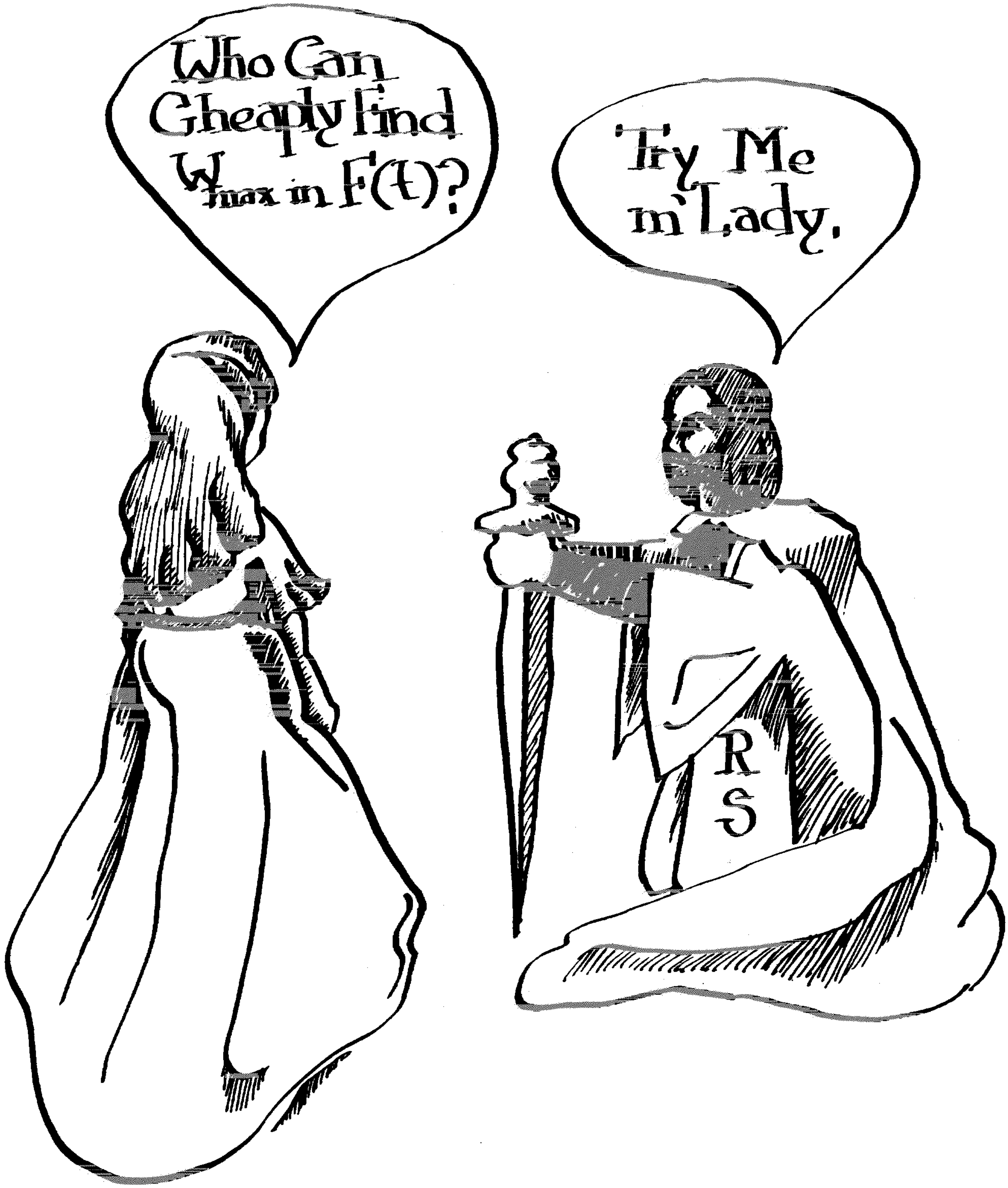
b. Direct formulation

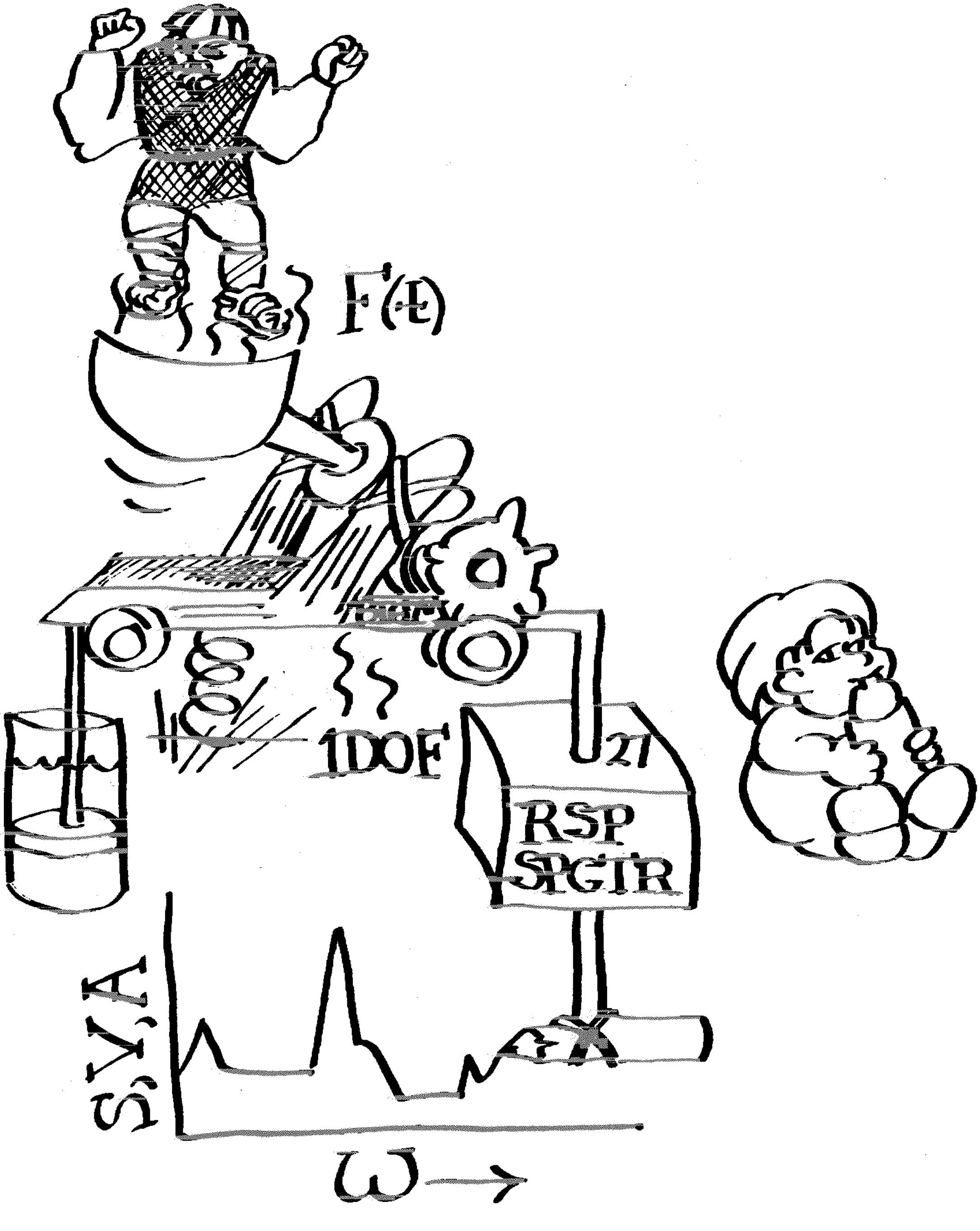
FIGURE 10. Frame Displacement at Suspensions, Modal and Direct Transient Comparison



N
DOF
IC≠0
IG=0


How Many Modes?
Know Thy F'(t).


Who Can Cheaply Find W_{max} in F(t)?
Try Me m'Lady.
RS


F(t)
1DOF
RSP
SPCTR
SV,A
ω →

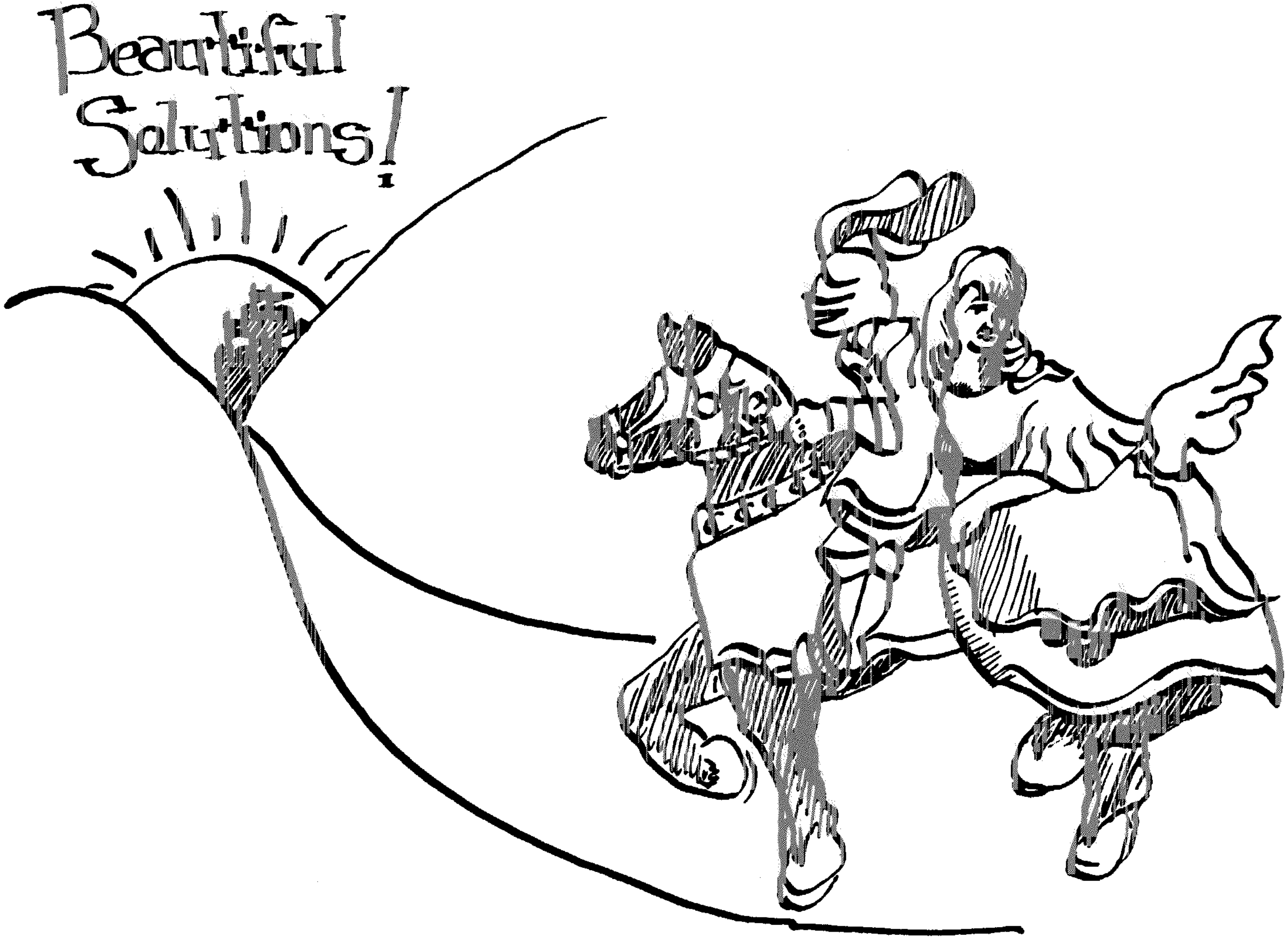

Beautiful Solutions!

TABLE 1.

Vehicle-Type Structure Models, Eigenvalues Comparison

	Frequency of Elastic Modes in Hertz		
Mode	Small, Lumped Mass	Small, Coupled Mass	Moderate-Size, Lumped Mass
1	1.5	1.6	2.2
2	2.8	2.8	2.9
3	2.9	3.0	3.5
4	9.9	10.2	10.0
19	40.6	49.6	34.3
20	41.0	51.4	37.0
21	44.9	53.6	41.6
22	50.4	56.2	45.4
23	55.3	58.5	54.5
37	84.8	95.5	73.7
38	85.6	100.6	75.6
44	97.3	119.1	87.8
45	100.7	120.1	91.4
50	112.1	145.6	99.4
51	112.9	150.1	100.8
59	157.3	193.0	116.4
60	167.0	203.1	117.5
62	185.1	214.4	123.9
63	202.1	217.8	125.4
92	1120.0	908.0	198.5
93	1150.0	928.0	200.6

TABLE 2.

Computer Time in CPU-Seconds for GDR, Moderate-Size Model

SOL 3 DMAP NO.	Modules	FMAX on DYNRED, Hertz 50	100	200
1 - 152	Misc	39.5	40.0	40.2
152	DECOMP	21.3	21.8	22.0
280	DYCNTL	23.5	25.8	24.2
302	DYNREDU	93.0	202.5	650.4
416	READ	1.3	6.3	46.0
	TOTAL*	288(144)	499(438)	1172(1527)

* Total calculated time in parenthesis.
See Figure 7 and Appendix B.
Generalized coordinates used: 35 -- 50 Hz
78 - 100 Hz
161 - 200 Hz

TABLE 3.

Transient Analysis CPU Time, IBM 3084, Moderate-Size Model

	CPU-Time in Seconds* Direct - Formulation			Modal**
Module	500	1000	1500	1500
To TRD1	95	98	99	636
TRD1***	1364(1192)	2673(2384)	4023(3575)	14(32)
Total time	1526	2838	4190	653

* Coupled equations; column headings are time steps

** GDR, FMAX= 100 Hz, H-set: 47

*** Calculated CPU-time in parenthesis (see Appendix B)

Appendix A

REAL EIGENVALUES AND MODAL TRANSIENT ANALYSIS

Introduction

Multidegree-of-freedom (MDOF) finite element models result in one independent second order differential equation for each DOF having mass (inertia) properties. Hence, direct transient analysis results in solving N equations for displacement time behavior at each GRID. These displacements may then be used to determine spatial and temporal velocity and acceleration, element stress, etc. Rather general damping is permissible with linear or nonlinear applied loads (forces and enforced motion). MSC/NASTRAN has extensive direct transient solution capability (3). This topic is not included in this appendix.

An alternative to direct transient analysis is modal transient formulation. This technique requires that the analyst be familiar with natural frequencies (eigenvalues), eigenvectors (or normal modes), generalized coordinates, decoupled equations, and modal superposition This topic has been treated thoroughly in a recent textbook (2) suitable for senior engineering students and practicing engineers. This text is the primary resource used by the author (in addition to the MSC/NASTRAN Dynamic Handbook) in presenting an introduction to modal transient analysis.

This appendix introduces real (undamped) eigenvalues, rigid body motion, eigenvectors, generalized coordinates, generalized mass, stiffness, damping, and force. Also, the well known eigenvector orthogonality principle is proven with an example. Nonlinear effects are not considered. MSC/NASTRAN terminology will be used if possible. Some matrix arithmetic knowledge is expected of the reader (11).

Three DOF, Lumped Parameter Model: Eigenvalues and Eigenvectors

Figure A1a shows an undamped three DOF structure that is free to translate as a rigid body along a supporting surface. Displacement of each mass with respect to a reference plane is shown in the figure. The system is expected to have one rigid body mode and two elastic modes (since there are three masses and two springs).

Newton's law is used to derive the three equations of motion in differential form (Figure A1b) and compacted into matrix form (Figure A1c). More complicated systems would certainly use the 'principle of virtual displacements' or Lagrange's equations to derive these equilibrium relationships.

The assumption that displacements vary sinusoidually with time for each mass results in an algebraic eigenvalue relationship shown in Figure A1d. Expanding the determinant of the Figure A1d relationship about the first row results in the characteristic polynomial in terms of lambda (which is circular frequency squared). The polynomial may be easily factored into the three roots (0, 3/4, and 2) shown in Figure A1e. These roots are called the eigenvalues of the Figure A1a system.

The roots of the eigenvalue equation are next substituted in the first and third equations of the relationship shown in Figure A1d. This result is shown in Figure A1f, where the magnitude of U1 is arbitrarily set to unity. The three DOF model eigenvectors (relative vibratory displacements at each GRID for each natural frequency) are shown in transposed form in Figure A1f. Finally the eigenvectors are shown graphically in Figure A1g.

The zero frequency eigenvalue (and uniform translation eigenvector) is called a rigid body mode and results from a singular stiffness matrix (as shown below). MSC/NASTRAN uses the SUPORT card for efficient calculation of these rigid body modes. Also MSC/NASTRAN normally expects the eigenvectors to be normalized to MASS rather than MAX as was done in Figure A1.

An MSC/NASTRAN analysis of the Figure A1 model was executed using SOL 3. The eigenvalues and eigenvectors agreed precisely with hand derived results shown in Figure A1.

Modal Matrix, Generalized Coordinates, and Generalized Properties

A matrix of the three eigenvectors derived in Figure A1f may be arranged in a modal matrix as shown in Figure A2a. This modal matrix will have N rows (where N is model DOF) and have M columns (where M is the number of eigenvectors being used). The Figure A1 model will be used to show that this modal matrix may be used to introduce generalized coordinates, derive generalized mass and stiffness, and show orthogonality of eigenvectors with respect to mass.

Figure A2b shows the usual technique used in modal-formulation finite element transient analysis, where the motion at each GRID is expressed in terms of the eigenvector and a generalized coordinate for each frequency. Hence, if the temporal behavior of each generalized coordinate is calculated, the Figure A2b relationship may be used to determine displacement components (along with velocity and acceleration) at each GRID.

Figure A2c shows that the general equation of motion (with N representing nonlinear loads set to zero) may be multiplied by the transpose of the modal matrix to get the useful result of a diagonal generalized mass and generalized stiffness matrix as shown in Figure A2c. Thus the equations of motion have been decoupled with respect to mass and stiffness.

The MSC/NASTRAN analysis for the Figure A1 model (using MAX normalization) verified the results shown in Figure A2c. As expected the printed eigenvalue table showed generalized mass and stiffness as here derived. Each of the three values of lambda are simply the ratio of generalized stiffness to generalized mass (diagonal terms in Figure A2c.)

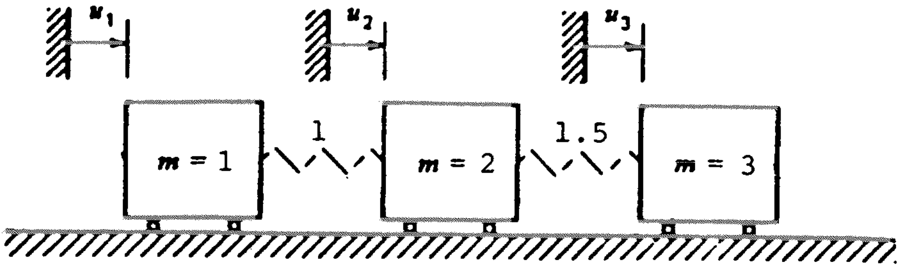
Figure A2d shows the resulting generalized damping and load from the Figure A2c operation. The editor of Reference 3 cautions that any material damping or damping elements (such as CVIS or CDAMPi) destroys any possibility of having decoupled equations in MSC/NASTRAN. Only uniform modal damping (TABDMP1) insures that the generalized coordinate equations are decoupled.

Eigenvector Orthogonality

Any two unique frequency eigenvectors are said to be orthogonal with respect to mass if the Figure A3a relationship is valid (2). Figure A3b verifies this relationship for the Figure A1 model. The eigenvectors are also orthogonal with respect to stiffness.

Conclusion

This appendix has shown a simple example problem illustrating eigenvalues, eigenvectors, generalized coordinates, generalized properties, and orthogonality of eigenvectors as used in modal formulations of MSC/NASTRAN. A model of about four or five DOF very quickly shows the benefit of using the several eigenvalue extraction techniques used in MSC/NASTRAN. References 3 and 4 should be consulted by the reader interested in the mechanics of these eigenvalue extraction techniques.



a. Physical Arrangement

$$\ddot{U}_1 + U_1 - U_2 + 0\,U_3 = 0$$

$$2\,\ddot{U}_2 - U_1 + 2.5\,U_2 - 1.5\,U_3 = 0$$

$$3\,\ddot{U}_3 + 0\,U_1 - 1.5\,U_2 + 1.5\,U_3 = 0$$

b. Equations of motion, differential form

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \\ \ddot{U}_3 \end{Bmatrix} + \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2.5 & -1.5 \\ 0 & -1.5 & 1.5 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

c. Equations of motion, matrix form

Let $U_i = Ui \cos \omega t \qquad \omega^2 = \lambda$

Then $[K - \omega^2 M]\,\{U\} = 0$

or $$\begin{bmatrix} (1-\lambda) & -1 & 0 \\ -1 & (2.5 - 2\lambda) & -1.5 \\ 0 & -1.5 & (1.5 - 3\lambda) \end{bmatrix} \begin{Bmatrix} U1 \\ U2 \\ U3 \end{Bmatrix} = 0$$

d. Eigenvalue Equation

FIGURE A1. Example Problem, 3 DOF System

$$(1-\lambda)\,[(5/2-2\lambda)(3/2-3\lambda)-9/4]-(3/2-3\lambda)=0$$

or

$$\lambda\,(3/4-\lambda)(2-\lambda)=0\;;\;\lambda_1=0,\;\lambda_2=3/4,\;\lambda_3=2$$

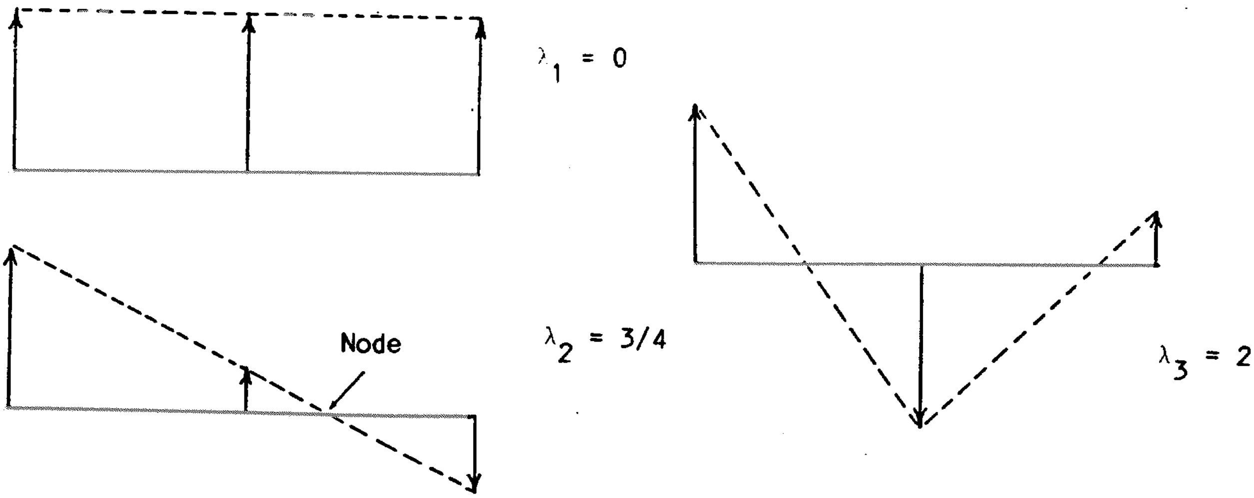
where determinant is expanded about first row

e. Eigenvalue equation roots

Mode	λ	First Eq*	Third Eq*	ϕ^T
1	0	U1 - U2 = 0	-1.5 U2 + 1.5 U3 = 0	[1 1 1]
2	3/4	1/4 U1-U2= 0	-1.5 U2 - .75 U3 = 0	[1 1/4 -1/2]
3	2	- U1 -U2 = 0	-1.5 U2 - 4.5 U3 = 0	[1 -1 1/3]

*Eigenvalue equation; U1 = 1

f. Compute eigenvectors



g. Eigenvector graphic display

FIGURE A1. Example Problem, 3 DOF System (continued)

Definition:

$$[\Phi] = \begin{bmatrix}\phi_1 & \phi_2 & \phi_3\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & 1/4 & -1\\ 1 & -1/2 & 1/3\end{bmatrix}$$

$$[M] = \begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix} \quad K = \begin{bmatrix}1 & -1 & 0\\ -1 & 2.5 & -1.5\\ 0 & -1.5 & 1.5\end{bmatrix}$$

a. Example (Fig. A1a) Modal, Mass, Stiffness Matrices

$$\{U\} = \begin{Bmatrix}U_1\\ U_2\\ U_3\end{Bmatrix} = [\phi]\ \{\xi\} = \begin{bmatrix}1 & 1 & 1\\ 1 & 1/4 & -1\\ 1 & -1/2 & 1/3\end{bmatrix}\begin{Bmatrix}\xi_1\\ \xi_2\\ \xi_3\end{Bmatrix} = \begin{Bmatrix}\xi_1 + \xi_2 + \xi_3\\ \xi_1 + 1/4\ \xi_2 - \xi_3\\ \xi_1 - 1/2\ \xi_2 + 1/3\ \xi_3\end{Bmatrix}$$

b. Structure displacement and generalized coordinates

$$[\phi]^T[M][\phi]\{\ddot{\xi}\} + [\phi]^T[B][\phi][\dot{\xi}] + [\phi]^T[K][\phi][\xi] = [\phi]^T\{P(t) + \underbrace{N}_{\text{null}}\}$$

$$[\phi]^T[M][\phi] = m = \begin{bmatrix}1 & 1 & 1\\ 1 & 1/4 & -1/2\\ 1 & -1 & 1/3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & 1/4 & -1\\ 1 & -1/2 & 1/3\end{bmatrix} = \underbrace{\begin{bmatrix}5 & 0 & 0\\ 0 & 15/8 & 0\\ 0 & 0 & 10/3\end{bmatrix}}_{\text{diagonal}}$$

$$[\phi]^T[K][\phi] = [m] = \begin{bmatrix}1 & 1 & 1\\ 1 & 1/4 & -1/2\\ 1 & -1 & 1/3\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ -1 & 2.5 & -1.5\\ 0 & -1.5 & 1.5\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & 1/4 & -1\\ 1 & -1/2 & 1/3\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 0 & 0\\ 0 & 45/32 & 0\\ 0 & 0 & 20/3\end{bmatrix}}_{\text{diagonal}}$$

NOTE: $\lambda 1 = 0/5 = 0$ $\lambda 2 = (45/32)/(15/8) = 3/4$ $\lambda 3 = (20/3)/(10/3) = 2$

c. Example generalized mass and stiffness

$[\phi]^T[B][\phi]$ (damping) $[\phi]^T\{P(t) + N\}$ (force) $\{N\}$ - nonlinear

d. Generalized damping and force

FIGURE A2. Generalized Properties for Example Problem

$$\{\phi_J\}^T[m]\{\phi_K\} = 0$$

a. Eigenvector Orthogonality definition

$$\phi_1 = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \qquad \phi_2 = \begin{Bmatrix} 1 \\ 1/4 \\ -1/2 \end{Bmatrix} \qquad \phi_3 = \begin{Bmatrix} 1 \\ -1 \\ 1/3 \end{Bmatrix}$$

$$\{\phi_1\}^T [M] \{\phi_2\} = [1 \quad 1 \quad 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ 1/4 \\ -1/2 \end{Bmatrix} = [1 \quad 2 \quad 3] \begin{Bmatrix} 1 \\ 1/4 \\ -1/2 \end{Bmatrix} = 0$$

$$\{\phi_2\}^T [M] \{\phi_3\} = [1 \quad 1/4 \quad -1/2] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \\ 1/3 \end{Bmatrix} = \left[1 \ \tfrac{1}{2} \ -\tfrac{3}{2}\right] \begin{Bmatrix} 1 \\ -1 \\ 1/3 \end{Bmatrix} = 0$$

$$\{\phi_3\}^T [M] \{\phi_1\} = [1 \ -1 \ 1/3] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = [1 \quad -2 \quad 1] \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = 0$$

NOTE: Orthogonality with respect to stiffness also true.

b. Example problem verification

FIGURE A3. Eigenvector Orthogonality

Appendix B

MSC/NASTRAN COMPUTER TIMING FORMULAE

Introduction

MSC/NASTRAN documentation (8) provides extensive computer timing formulae required by the dynamics analyst. A simplified formula for GDR eigenvalue extraction computer time (1) has been found to be quite accurate. However, the equation solution formulae are quite vague. This appendix will present some actual data for transient analysis equation solution of the moderate-size model described in the text.

GDR Timing

Computer CPU time for GDR eigenvalue extraction is given as:

TGDR=M*O*C**2 [1 + (16PS/MC) + (3Q/C) + (4 + 7P/M)(Q/C)**2] Seconds

where

- M = machine constant (0.92 microseconds for IBM 3084)
- O = o-set (model DOF after single and multipoint constraints)
- C = RMS bandwidth in DOF
- PS/M = machine factor ratio (1.25 for IBM 3084)
- P/M = machine factor ratio (0.9 for IBM 3084)
- Q = generalized coordinates for GDR (DOF). (Set to 1.5 times number of eigenvalues below FMAX)

The first term gives CPU time for two static decompositions of the o-set stiffness matrix. The other bracketed terms sum to about 0.5 for the usual case of Q/C about equal to 0.1 to 0.15. Hence, TGDR is about equal to three static decompositions of the o-set stiffness matrix.

Equation Solution

Reference 3 (page 5.5-7) states: 'The CPU time for direct transient response can be estimated by combining the times for stiffness matrix formation, mass matrix formation, matrix decomposition of the dynamic matrix (see Section 4.6.2), and equation solution where the number of right-hand sides is equal to the number of time steps.'

The present author agrees that the direct transient solution time depends on number of time steps (see Table 3, text). This table gives an average time of 2.688 CPU seconds per time step for each of the three direct transient solutions. (There are T+2 equations, where T equals the number of time steps.) However, the equation solution formula (in above defined notation) gives:

CPU time = 2*T*O*C*M.

Let T=1 (one time step). The moderate-size model (Figure 7,text) has O= 5266 and C=82. M=0.92 microseconds for an IBM 3084 computer. This gives a CPU time of 0.795, or about one third of the actual 2.688 CPU seconds required. Hence, a rough estimate of direct transient solution (TRD1) CPU time is given as:

CPU time = 6*T*O*C*M.

This formula is used in calculating estimated times shown in Table 3 (text).