# DETERMINATION OF STRUCTURAL DYNAMIC RESPONSE SENSITIVITY TO MODAL TRUNCATION

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Current structural analysis methods employ large finite element models to simulate the dynamic behavior of complex systems. These models contain thousands of physical degrees of freedom and, consequently, are too cumbersome and inefficient for many analytical purposes. One technique used to produce more manageable models is the mathematical transformation of the physical degrees of freedom into a set of independent modal degrees of freedom. This modal data may then be truncated to reduce the problem size and, therefore, the cost of the analysis. The trade-off for reduced problem size is a loss in accuracy of the physical responses recovered from the model. This paper investigates the modal truncation sensitivity of transient responses which have been recovered using the mode displacement method. A new MSC/NASTRAN DMAP procedure which calculates the modal truncation sensitivity of these responses has been developed. Examples illustrate how this procedure may be used to determine if adequate modal data has been retained to produce accurate responses.

## NOMENCLATURE

| | | |
|---|---|---|
| $c$ | = | Physical Damping |
| $C$ | = | Modal Damping |
| DMAP | = | Direct Matrix Abstraction Program |
| DOF | = | Degrees of Freedom |
| $f$ | = | Physical Applied Force |
| $F$ | = | Modal Applied Force |
| $k$ | = | Physical Stiffness |
| $K$ | = | Modal Stiffness |
| MSC | = | MacNeal-Schwendler Corporation |
| $m$ | = | Physical Mass |
| $M$ | = | Modal Mass |
| $\omega$ | = | Normal Frequency (radians) |
| $\Phi$ | = | Complete Set of Normal Mode Shapes |
| $\Phi_k$ | = | Truncated Set of Normal Mode Shapes (Kept Modes) |
| $P$ | = | Modal Participation Factors |
| $S$ | = | Modal Truncation Sensitivity Factors |
| $t$ | = | Time |
| $q$ | = | Modal Displacement |
| $\dot{q}$ | = | Modal Velocity |
| $\ddot{q}$ | = | Modal Acceleration |
| $x$ | = | Physical Displacement |
| $\dot{x}$ | = | Physical Velocity |
| $\ddot{x}$ | = | Physical Acceleration |

# 1. INTRODUCTION

The use of very large finite element models has grown dramatically in recent years. This growth is a response to the demand for increasingly detailed analyses of complex structures such as the Space Shuttle and its payloads. The computational requirements for the analysis of these models are enormous, and can exceed the capabilities of all but the most powerful computer systems. A number of methods have been developed to reduce the size of these models and, thereby, the computational requirements and cost of the analysis. The most common method involves transforming the model's physical DOF into a set of independent modal DOF representing the normal modes of vibration of the model. The size of the model may be reduced by truncating these modal DOF at a specified cutoff frequency. The lower this cutoff frequency is set, the lower the cost of subsequent transient analyses will be. The trade-off for reduced cost appears when the transient physical responses of the model are recovered, as modal truncation results in a loss in accuracy of these responses. This loss in accuracy can be insignificant or be so severe as to invalidate the analysis. Therefore, it is crucial to select a cutoff frequency that will ensure sufficient accuracy without adding undue cost to the analysis.

The selection of an appropriate cutoff frequency, however, is not a straightforward task. The dynamic characteristics of the model, the frequency content of the transient forces applied to the model, and the data recovery method employed all affect response sensitivity to modal truncation. There are few general guidelines for determining cutoff frequencies and therefore these frequencies are, at best, educated guesses. There is no way to absolutely ensure the accuracy of the results short of using the entire set of modal DOF, but that defeats the whole purpose of the modal transformation technique. Therefore, a method is needed to determine the effect of modal truncation on transient physical responses. A new MSC/NASTRAN DMAP procedure has been developed which answers this need in a convenient and efficient manner. It can be employed to root out potential truncation problems and indicate when a higher cutoff frequency is needed. This reduces the uncertainties in the analysis associated with modal truncation and, thus, increases the overall confidence in the final results.

## 2. MODAL TRANSFORMATION, PARTICIPATION AND TRUNCATION SENSITIVITY

The equation of motion for the finite element model may be written

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{c}\dot{\mathbf{x}} + \mathbf{k}\mathbf{x} = \mathbf{f}(t) \qquad (2.1)$$

The normal frequencies and modes of vibration must satisfy the equation

$$(\mathbf{k} - \omega_i^2\mathbf{m})\,\boldsymbol{\Phi}_i = \mathbf{0} \qquad (2.2)$$

for $i = 1, 2, \ldots, N$ The modal matrix $\boldsymbol{\Phi}$ includes all $N$ normal modes and may be truncated to $\boldsymbol{\Phi}_k$ to represent only the kept set of modes. Truncating the modes removes the flexibility of the higher modes from the model and increases its overall stiffness. The physical DOF may be transformed into a truncated set of modal DOF using the relation

$$\mathbf{x} = \boldsymbol{\Phi}_k\mathbf{q} \qquad (2.3)$$

This transformation produces the modal equation of motion

$$\boldsymbol{\Phi}_k^T\mathbf{m}\boldsymbol{\Phi}_k\,\ddot{\mathbf{q}} + \boldsymbol{\Phi}_k^T\mathbf{c}\boldsymbol{\Phi}_k\,\dot{\mathbf{q}} + \boldsymbol{\Phi}_k^T\mathbf{k}\boldsymbol{\Phi}_k\,\mathbf{q} = \boldsymbol{\Phi}_k\mathbf{f}(t) \qquad (2.4)$$

or

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}(t) \qquad (2.5)$$

The time history responses of the modal DOF are then solved using numerical integration. The time history responses of the physical DOF are recovered as follows:

$$\mathbf{x}(t) = \boldsymbol{\Phi}_k\mathbf{q}(t) \qquad (2.6)$$

$$\dot{\mathbf{x}}(t) = \boldsymbol{\Phi}_k\dot{\mathbf{q}}(t) \qquad (2.7)$$

$$\ddot{\mathbf{x}}(t) = \boldsymbol{\Phi}_k\ddot{\mathbf{q}}(t) \qquad (2.8)$$

This is known as the mode displacement method of data recovery.

A concept closely related to modal truncation sensitivity is commonly known as modal participation. The equation for displacement modal participation (velocity and acceleration modal participation are similar and excluded for brevity) is written

$$P_{xi} = \Phi_i q_i(t_o) \tag{2.9}$$

and represents the individual contribution of the $i$th normal mode to the total displacement, i.e., the mode's participation in the response. Modal participation and truncation sensitivity are both calculated at a single time point, commonly the time at which the maximum response occurs. Modal truncation sensitivity is now defined as

$$S_{xi} = \sum_{h=1}^{i} \Phi_h q_h(t_o) \tag{2.10}$$

and is simply the contribution of the $i$th normal mode and all proceeding modes to the total response, i.e., the response that would be recovered if only the first $i$ modes were kept.

## 3. EXAMPLES

Two examples of modal truncation sensitivity analysis are presented: a simple beam problem and a Space Shuttle landing event simulation.

The first problem consisted of a cantilevered beam. It was represented by an MSC/NASTRAN finite element model comprised of four free physical DOF, four BAR elements, and four CONM2 concentrated masses. This model is shown in Figure 3.1. Four normal modes were generated for the model using Solution 63. The mode shapes and their corresponding frequencies are shown in Figure 3.2. A normal force was then applied to the free end of the beam as follows

$$f(t) = 1000 \, \mathrm{SIN}(2\pi \, 8 \, t) \tag{3.1}$$

and the displacement time histories of each physical DOF were recovered using Solution 72. The modal truncation sensitivity of each response maxima was then calculated. The load cases, times and GRID points were selected for truncation analysis through the NASTRAN Case Control Deck shown in Figure 3.3; no DMAP changes were required. The resultant modal truncation sensitivity plots are shown in Figure 3.4.

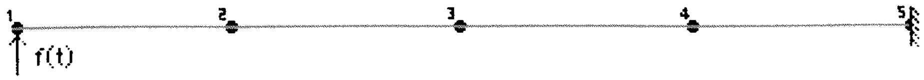
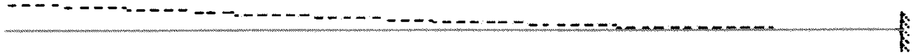
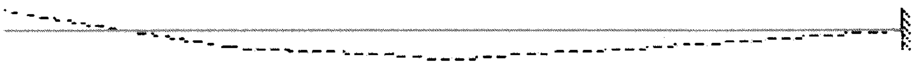


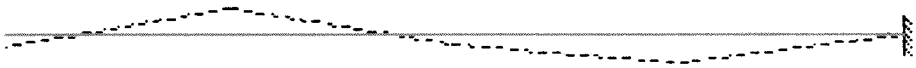

Fig. 3.1 Cantilevered Beam Model



Mode 1 Freq. 0.50 Hz
(a)



Mode 2 Freq. 3.23 Hz
(b)



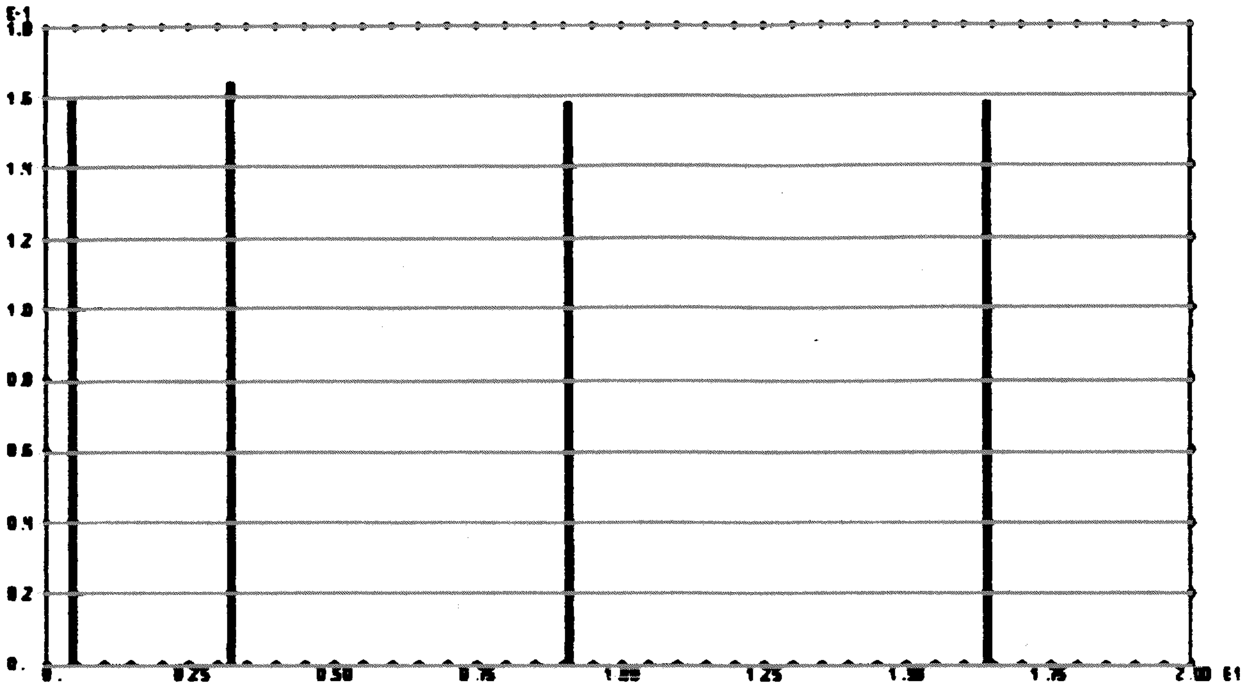
Mode 3 Freq. 9.12 Hz
(c)

Mode 4 Freq. 16.43
(d)

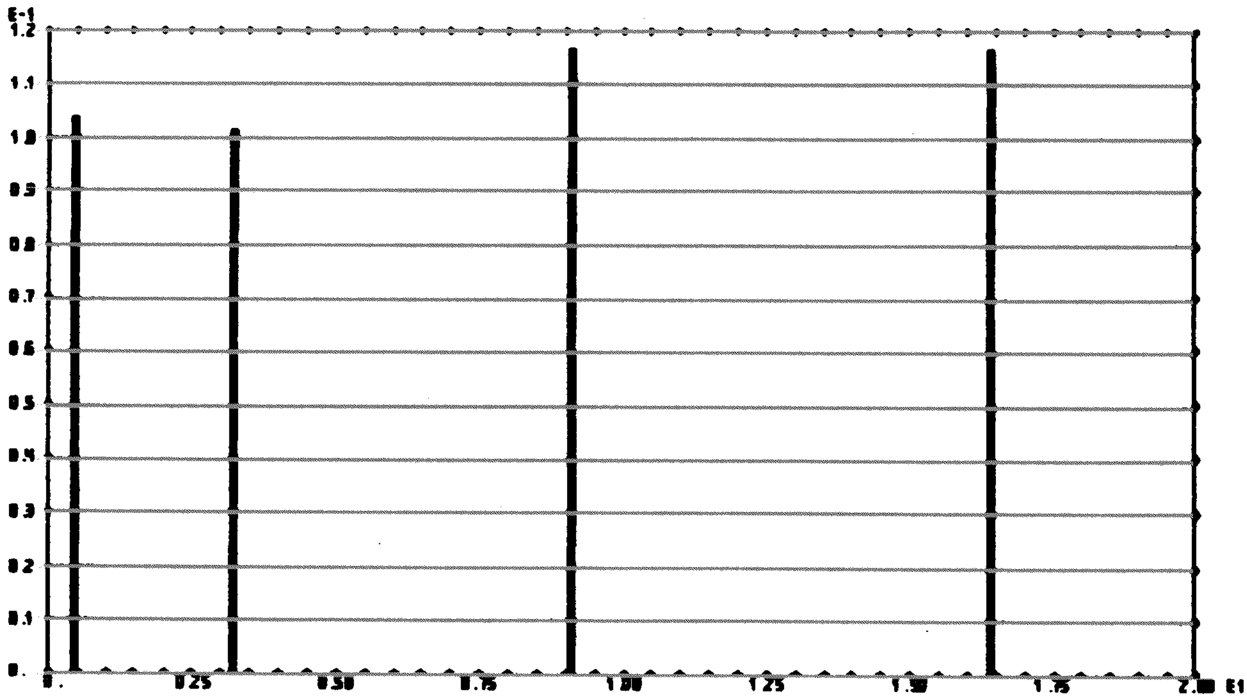
Fig. 3.2 Cantilevered Beam Normal Modes of Vibration

```
TITLE = CANTILEVERED BEAM MODEL - 8 HZ MODAL TRUNCATION SENSITIVITY
OLOAD(PLOT) = ALL
SUBCASE 100                  $ GRID 1 SUBCASE
    DLOAD = 0                $ DYNAMIC LOAD CASE
    SET 1 = 0.46             $ TIME AT MAXIMUM RESPONSE
    OTIME = 1                $ OUTPUT TIME SET
    SUBCASE 101              $ GRID 1 PLOTTING SUBCASE
    LABEL = 0.46 SECONDS
SUBCASE 200                  $ GRID 2 SUBCASE
    DLOAD = 0                $ DYNAMIC LOAD CASE
    SET 2 = 0.47             $ TIME AT MAXIMUM RESPONSE
    OTIME = 2                $ OUTPUT TIME SET
    SUBCASE 201              $ GRID 2 PLOTTING SUBCASE
    LABEL = 0.47 SECONDS
SUBCASE 300                  $ GRIDS 3 & 4 SUBCASE
    DLOAD = 0                $ DYNAMIC LOAD CASE
    SET 3 = 0.53             $ TIME AT MAXIMUM RESPONSE
    OTIME = 3                $ OUTPUT TIME SET
    SUBCASE 301              $ GRIDS 3 & 4 PLOTTING SUBCASE
    LABEL = 0.53 SECONDS
OUTPUT(XYPLOT)               $ PLOTTING PARAMETERS
PLOTTER NAST
XPAPER = 26.0
YPAPER = 20.0
XGRIDLINES = YES
XDIVISIONS = 50
YDIVISIONS = 10
XVALUE PRINT SKIP = 4
YAXIS = YES
XMAX = 20.0
XMIN = 0.0
XTITLE =                          FREQUENCY (HZ)
YTITLE =                          DISPLACEMENT GRID 1 T2
XYPLOT OLOAD 101/1(T2)       $ PLOT GRID 1(T2) IN SUBCASE 101
YTITLE =                          DISPLACEMENT GRID 2 T2
XYPLOT OLOAD 201/2(T2)       $ PLOT GRID 2(T2) IN SUBCASE 201
YTITLE =                          DISPLACEMENT GRID 3 T2
XYPLOT OLOAD 301/3(T2)       $ PLOT GRID 3(T2) IN SUBCASE 301
YTITLE =                          DISPLACEMENT GRID 2 T2
XYPLOT OLOAD 301/4(T2)       $ PLOT GRID 4(T2) IN SUBCASE 301
```

**Fig. 3.3 Case Control Deck for Beam Modal Truncation Sensitivity**

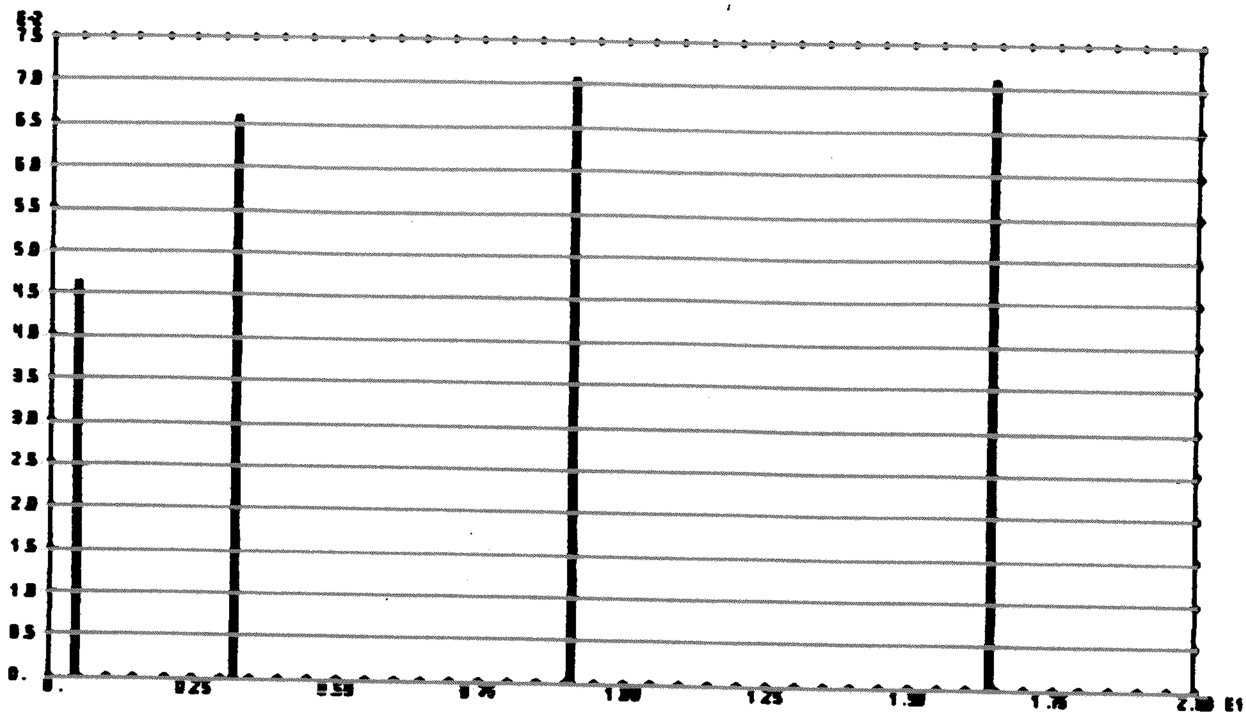


GRID 1 Displacement Vs. Cutoff Frequency
(a)

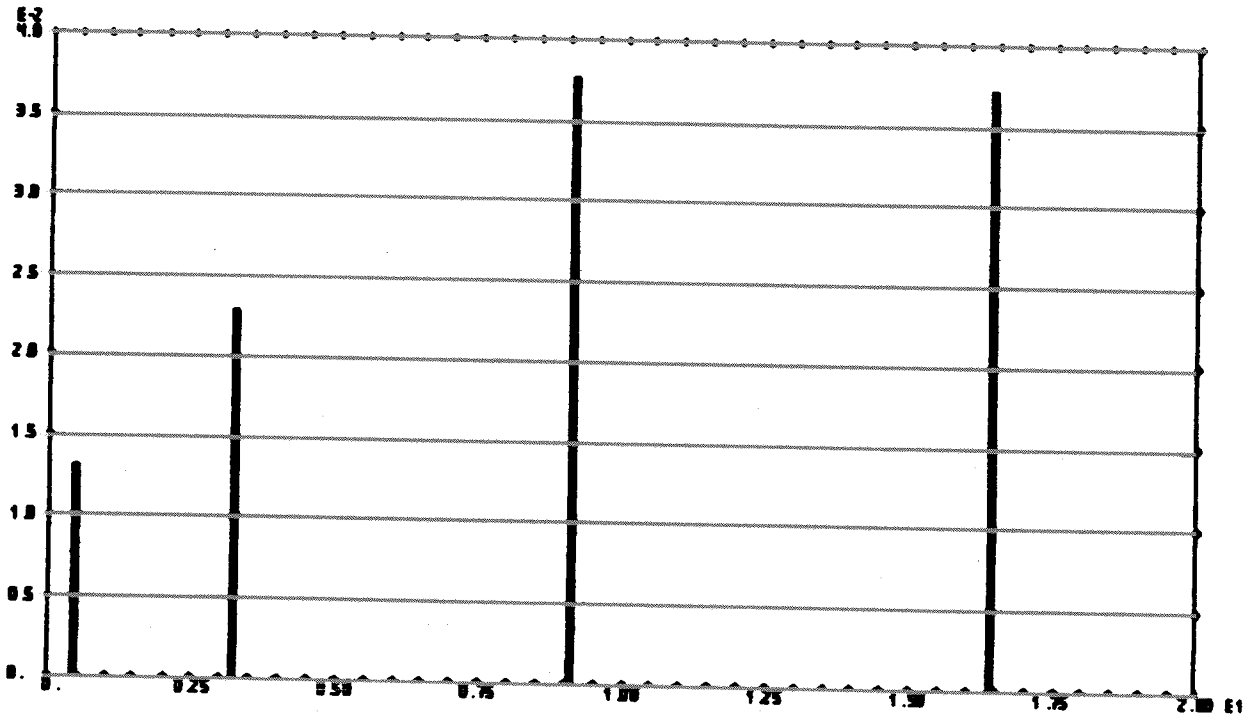


GRID 2 Displacement Vs. Cutoff Frequency
(b)

Figure 3.4 Beam Modal Truncation Sensitivity




GRID 3 Displacement Vs. Cutoff Frequency
(c)




GRID 4 Displacement Vs. Cutoff Frequency
(d)

Figure 3.4 Beam Modal Truncation Sensitivity (Continued)

What are these plots saying about the model and its responses? First, look at the plot for the maximum displacement of GRID 1 in Figure 3.4a. Note that virtually all of the response was from the first mode and that adding in the second, third, and fourth modes had very little effect. This is to be expected since the displacement of the free end of a cantilevered beam is primarily dependent on the first bending mode. Moving towards the fixed end of the beam, the stiffness of the beam increases and the responses there would be expected to depend more on the high frequency modes. This expectation was clearly fulfilled as shown in Figures 3.4b through 3.4d. The extreme case was the maximum displacement at GRID 4, where fully sixty-five percent of the response was from modes two and three. The thirty-five percent contribution of mode one represents the displacement that would be recovered if the solution were truncated to only one mode. As noted in the previous section, truncation removes the flexibility contributed by the higher modes and, therefore, reduces the overall flexibility of the model. Displacement is proportional to flexibility and, consequently, the reduction in flexibility resulted in a reduction in the displacement recovered from the truncated model. Note that mode four did not contribute significantly to any of the responses since the fourth normal frequency of 16.4 Hz was over twice the forcing frequency of 8.0 Hz. This illustrates the value of one of the few generally accepted frequency cutoff rules, namely, that modes should be calculated to at least double the highest forcing frequency.

The beam model is simple enough that a modal truncation sensitivity analysis is not needed to understand the model and its responses. The next model is much more difficult to fully understand.

There are several events during the course of a Space Shuttle mission that produce significant dynamic responses, and the one that will be examined here is the normal landing event. The finite element model employed actually consisted of two models: an Orbiter model produced by Rockwell International (Ref. 3.1) and a Centaur Integrated Support Structure model produced by General Dynamics Space Systems Division (Ref. 3.2). These structures are shown in Figure 3.5. The total size of the two models was over six thousand physical DOF. The models were merged using the Solution 63 Superelement component mode synthesis capability to produce an overall system modal model. The conventional system cutoff frequency for landing events is 42 Hz, and this resulted in the calculation of 205 system modes. The landing event modal time histories were calculated using Solution 72 with a modal initial conditions alter (Ref. 3.3). The highest forcing frequency did not exceed 20 Hz. To further complicate this problem, the responses of interest were not displacements, but rather reaction forces between the two models. These forces were recovered at the Superelement boundary using the mode displacement method.

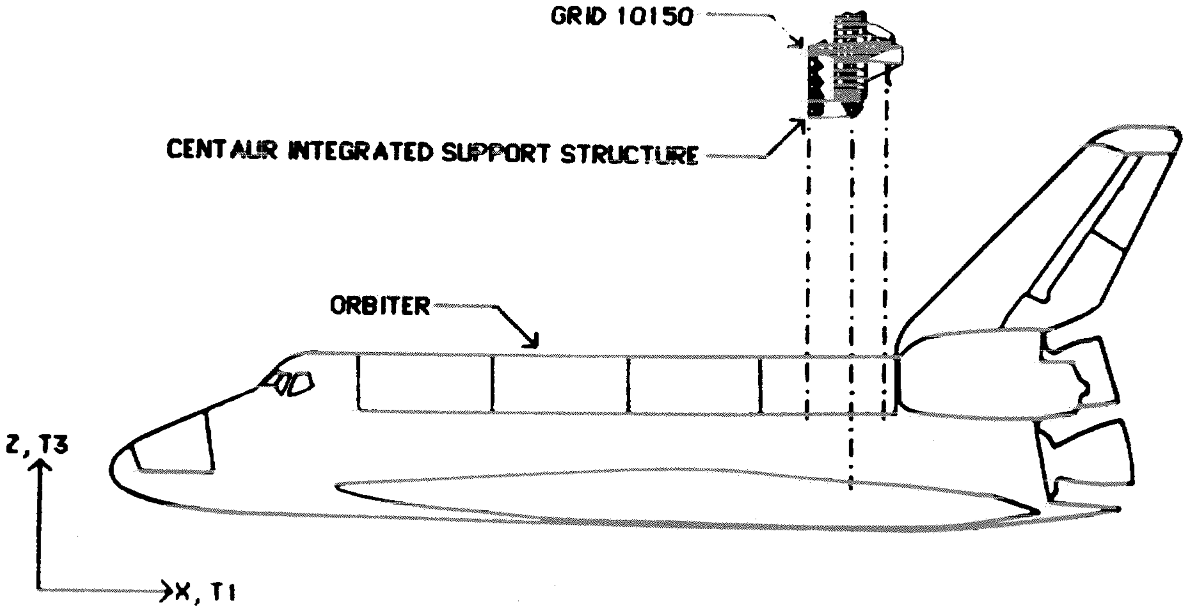
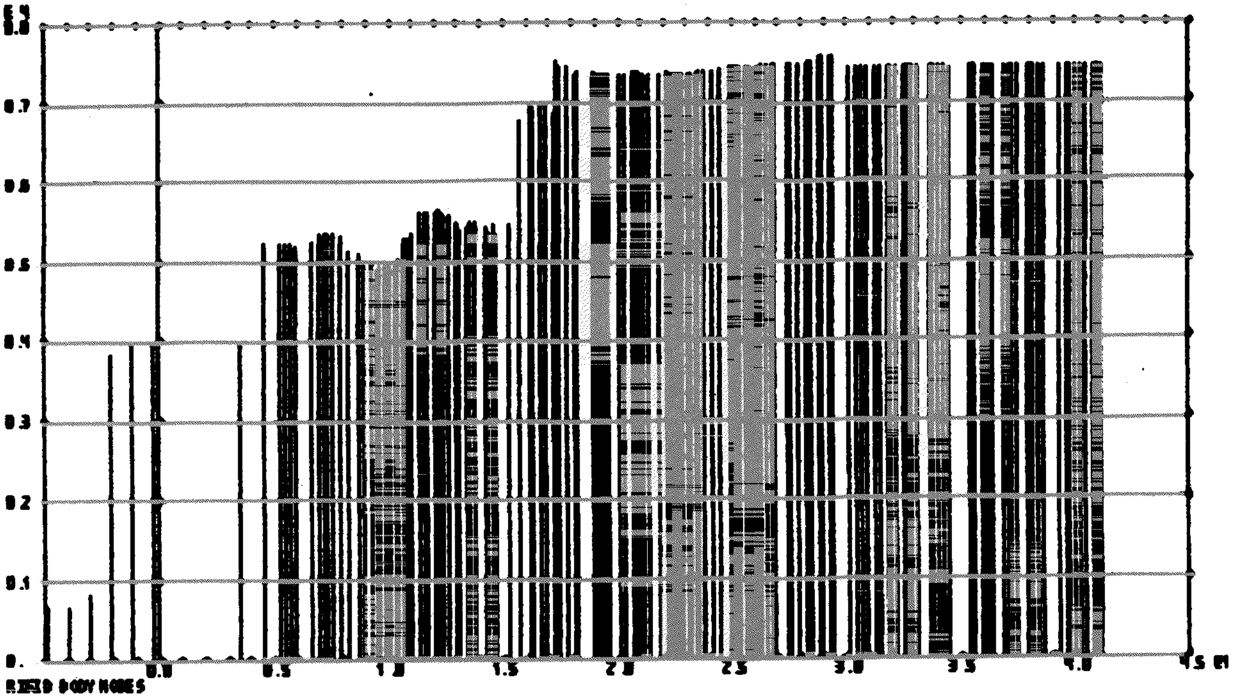
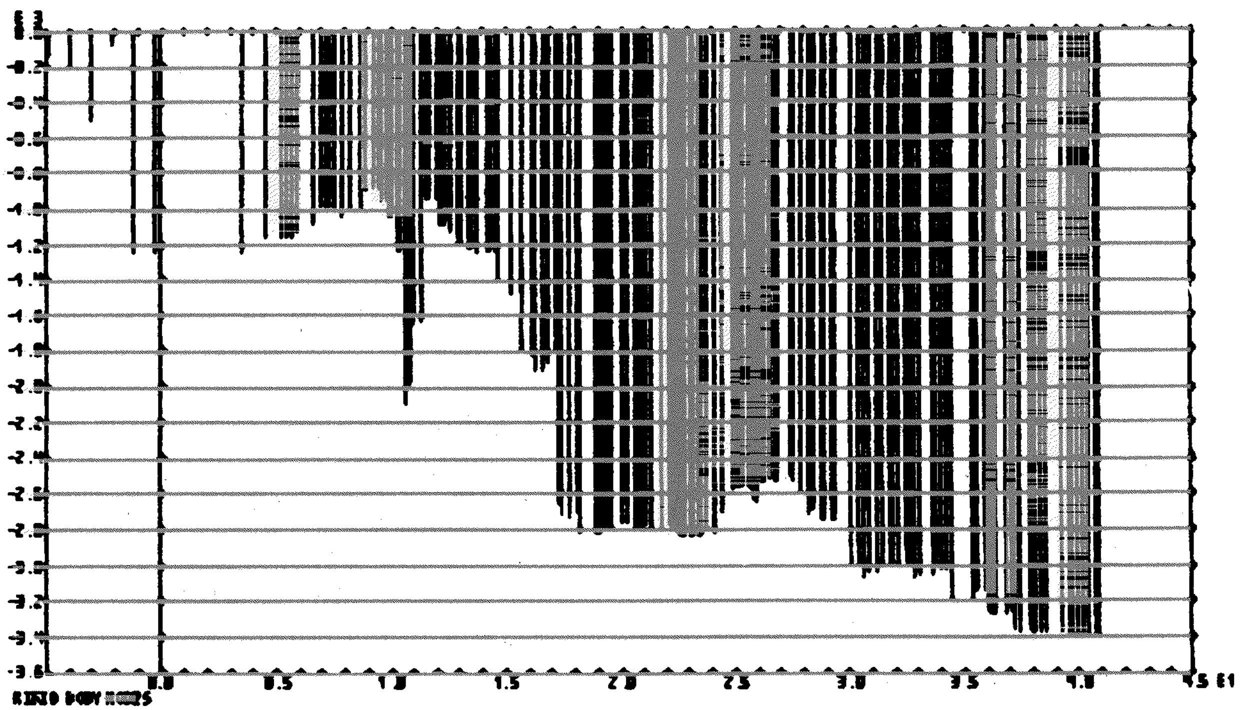



Figure 3.5 Orbiter and Centaur Integrated Support Structure

The modal truncation sensitivity was calculated for two interface force maxima: one force acting on GRID 10150 in the -X (-T1) direction and the other acting on the same point in the +Z (+T3) direction. The modal truncation sensitivity plots for these forces are shown in Figure 3.6. Figure 3.6a shows that virtually all of the modal contribution to the +Z direction force was below 18 Hz. The flat sensitivity between 18 Hz and the 42 Hz cutoff indicated that enough modes were probably kept to accurately calculate this force. On the other hand, Figure 3.6b shows that the sensitivity calculated for the -X direction did not end in a plateau, but climbed steadily right up to the 42 Hz cutoff. This led to the prediction that the load would continue to climb if modes were added above 42 Hz.

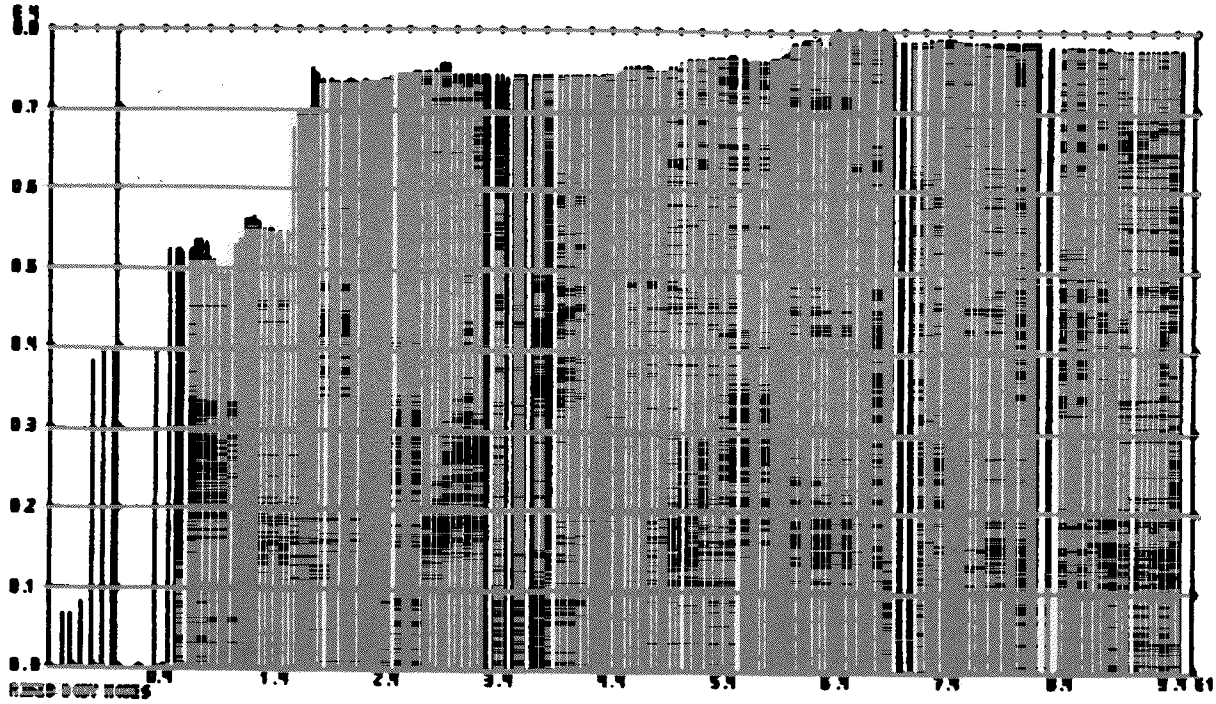


GRID 10150 T3 Orbiter Interface Load Vs. Cutoff Frequency
(a)

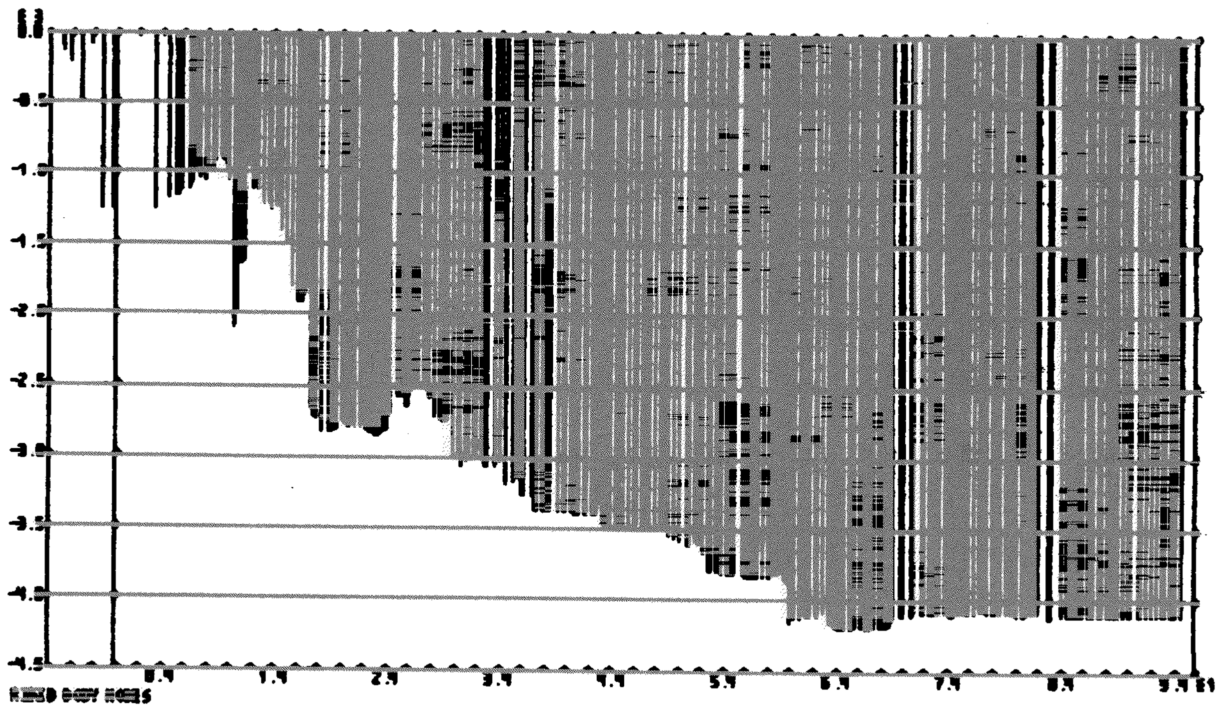


GRID 10150 T1 Orbiter Interface Load Vs. Cutoff Frequency
(b)

**Figure 3.6 Modal Truncation Sensitivity - 42 Hz Model**



GRID 10150 T3 Orbiter Interface Load Vs. Cutoff Frequency
(a)



GRID 10150 T1 Orbiter Interface Load Vs. Cutoff Frequency
(b)

Figure 3.7 Modal Truncation Sensitivity - 100 Hz Model

In order to check this prediction, 448 system modes were calculated for the model up to 100 Hz. Modal truncation sensitivity was again calculated and the resulting plots are shown in Figure 3.7. Figure 3.7a shows that the +Z direction force did not rise more than five percent over the 42 Hz to 100 Hz range, indicating that 42 Hz was an acceptable cutoff frequency for that response. Figure 3.7b shows that the force in the -X direction did not plateau until 65 Hz, indicating that 65 Hz would be the minimum cutoff frequency required to accurately recover the force. The modal truncation sensitivity analysis positively identified the 42 Hz truncation problem and the need for a higher cutoff frequency.

## 4. CONCLUSION

An MSC/NASTRAN DMAP procedure has been developed which calculates the modal truncation sensitivity of structural dynamic responses. This procedure successfully predicted that a Space Shuttle payload analysis required additional system modal data in order to accurately calculate system responses.

## REFERENCES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3.2 J. P. Creaser, *Centaur G' 5.0B7 Normal Landing Model*, GDC Memo No. D-85-002-SC, 2 January 1985.

3.3 C. C. Flanigan, *Methods for Calculating and Using Modal Initial Conditions in MSC/NASTRAN*, MSC/NASTRAN Conference on Finite Element Methods and Technology, March 1980.