APPLICATION OF MSC/NASTRAN AND ADS/NASOPT TO NOISE TRANSMISSION PROBLEMS

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Abstract

This paper addresses the problem of noise transmission from rotating machinery through a support structure to a foundation. The problem is attacked using dynamic finite element analysis with MSC/NASTRAN and optimization with ADS/NASOPT. First, the structure is automatically redesigned so as to drive resonant frequencies away from rotating frequencies. Then, a method is developed and illustrated for calculating the sensitivities of complex steady-state displacements to small changes in design variables. This information is passed to ADS/NASOPT so that it can redesign the structure to minimize the dynamic response directly. The methods are applied to a demonstration problem.

1.0 NOISE TRANSMISSION PROBLEMS

Noise transmission problems are important in many situations. This paper is concerned with reduction of noise or vibrations transmitted from rotating machinery through supporting structures to a foundation. For example, the foundation may be a building floor, an automobile frame, or a ship's hull. An optimization procedure is used to redesign the structure systematically so as to minimize the transmission of vibrations to the foundation due to a specified unbalance operating at a specified rotating frequency. Two approaches are discussed. In the first, the objective is to drive the supporting structure's resonant frequencies away from the operating frequency. In the second, the objective is to minimize dynamic displacements at the foundation without examining resonant frequencies directly.

In the following sections we review briefly the sensitivity analysis capability in MSC/NASTRAN, and the optimization capabilities in ADS/NASOPT. We then examine two approaches to the noise transmission problem: manipulation of modes, and direct minimization of frequency response. These ideas are then illustrated in example problems.

2.0 SENSITIVITY ANALYSIS IN MSC/NASTRAN

In version 63, MacNeal-Schwendler introduced sensitivity analysis into MSC/NASTRAN [1]. As a post-processing step, users can compute partial derivatives of certain response quantities, such as displacements and stresses with respect to "design variables". Design variables include element properties such as thicknesses or cross-sections, or material properties.

The growing recognition of the importance of sensitivity analysis on the part of users and developers of engineering software is encouraging for several reasons:

1. Broadly speaking, it is becoming increasingly important for engineers to predict not only how their designs will behave, but also the sensitivity of this behavior with respect to small changes in design parameters, operating conditions, or environmental factors. Sensitivity analysis is directly related to issues of safety, reliability, and economics.

2. It will draw structural designers and analysts closer together. With the aid of sensitivity analysis, analysts can easily predict the effects of small design changes and thus be more helpful to designers. Many designers will be quick to perceive the opportunities thus

opened. Others may sense an "invasion of their turf" on the part of analysts armed with sensitivity analysis and optimization, and thus feel compelled to cooperate more closely with analysts.

3. It opens a door to systematic tuning of finite element models via system identification. Using sensitivity data, one may systematically refine selected model parameters which are difficult to characterize a priori, by minimizing discrepancies between test data and analysis results.

4. Finally, the most widely recognized benefit is the opportunity to couple automated optimization to MSC/NASTRAN and other popular analysis codes.

The current sensitivity capability in MSC/NASTRAN offers a promising start. We hope to see future capability to support advanced techniques, such as shape optimization.

3.0 ADS/NASOPT

ADS/NASOPT is a proprietary structural optimization code that is jointly developed and distributed by Engineering Design Optimization, Inc. and CSA Engineering, Inc. [2]. ADS/NASOPT couples the design sensitivity capability of MSC/NASTRAN with the general-purpose optimizer, ADS [3]. ADS is suitable for a very wide range of optimization problems. It has been incorporated into ADS/NASOPT without modification. ADS has many control parameters which can be confusing to new users. Thus, default values have been selected in ADS/NASOPT which have proven appropriate for most structural problems.

ADS/NASOPT offers designers a complete optimization package that includes the following features.

1. Any design variables supported by MSC/NASTRAN sensitivity analysis may be used (thicknesses, areas, etc.).

2. Composite design variables may be defined. Beam cross-section properties, such as flange thicknesses, may be assigned as independent design variables using this approach.

3. Constraints may be prescribed for static displacements or stresses, natural frequencies, or buckling loads. This paper describes a recent extension to dynamic displacements and stresses.

4. Composite constraints may be prescribed as user-defined functions of simple constraints.

5. Minimum weight is the standard objective. However, users may select any quantity that may be constrained as a substitute objective. In this case, weight constraints may be given. Thus, for example, one may seek to maximize a structure's fundamental frequency within a specified weight budget.

6. Fully stressed design cycles may be carried out in addition to (or as an alternative to) cycles of mathematical optimization.

7. The optimization is carried out with two iteration loops. In an inner loop, ADS carries out a cycle of optimization with limited design changes allowed. This stage uses an "approximate model", which is set up automatically. This model predicts changes in responses due to small changes in design variables, by extrapolation, and executes very rapidly. The inner loop automatically deactivates constraints that are far from being active. The outer loop involves a complete MSC/NASTRAN reanalysis. The used of approximate models allows optimization with a minimum number of MSC/NASTRAN runs, only three to eight in many cases.

8. Sophisticated users may explore the many ADS options concerning strategies, methods, etc., by setting control parameters. Other users may accept the defaults that have been selected to provide good performance on most structural optimization problems.

The major user responsibility with ADS/NASOPT is the preparation of a design model. This data file defines design variables, constraints, the objective, and control parameters. It is prepared in bulk data format and is relatively simple to prepare, as may be seen in the Appendix. However, it is necessary to take some care in making an intelligent selection of design variables. There should be enough to allow the optimizer reasonable latitude in seeking an optimum. There should not be too many, however. For example, a design with many thickness changes may not be practical to manufacture. The increased computer time due to a large number of design variables has not been significant in studies carried out to date.

4.0 MINIMIZING NOISE TRANSMISSION BY MOVING NATURAL FREQUENCIES

One approach to noise transmission problems is to attempt to manipulate resonant frequencies directly, moving them away from driving frequencies. This is an attractive approach when only one or a few modes participate significantly in the response to the input. For example, suppose one had a structural resonance at 62 Hz on a system supporting a machine running at 60 Hz. One might choose to have the optimizer attempt to maximize that particular frequency, subject to a limit on the weight increase that can be accepted. While this is a workable approach, some caution is in order.

To begin with, as the original offending mode is pushed away, one must be wary of other resonances creeping closer to the driving frequency. In the example, we might succeed in driving the 62 Hz mode up to 70 Hz, but if at the same time a 52 Hz mode were increased to 58 Hz, there may be no real gain. One way to deal with this problem is to place constraints on other modes to keep them away from the driving frequency. That is, we might specify that the 52 Hz mode not be allowed to exceed 53 Hz as the design evolves.

Second is the problem of identification of modes. The user must use mode numbers to identify modes which are to be minimized, maximized, or constrained. It is possible for modes to switch places in the ranked list. That is, one might start out identifying mode number 3 as the "first torsion mode", and mode number 4 as the "second bending mode". After a few cycles of optimization, the torsion mode might have a higher frequency than the bending mode, and there is no way for the computer to know this. The user would have to modify the design model file manually if this situation should arise. In working with natural frequencies, it is always advisable to check the modes as the optimization proceeds by looking at plots.

A related complication is that modes may change character as the design evolves. A mode that may have been characterized as "pure bending", for example, may acquire some torsional motion after the design has been modified. In general, one must assess the participation factors of each mode that is a candidate for modification. This factor is a measure of the amount of displacement in the mode vector at both the loaded points and the response points. If a mode is localized in an area away from both the load and the response points, then theoretically it will not contribute to noise transmission even though its frequency may be very close to the driving frequency. In practice, however, one should be aware of any mode that is close to the driving frequency because the structure will likely "find a way" to activate that mode. This is because the large amplification factor due to resonance, when multiplied by even a small participation factor, can easily produce significant response.

A fourth consideration is that computed natural frequencies and mode shapes are subject to all the uncertainties that accompany any finite element analysis: element limitations, model coarseness, uncertain material properties, etc. This is particularly important because dynamic responses are highly sensitive to shifts in resonant frequencies which are near the driving frequency. A good way to deal with this problem is to vary the unbalance load over a small spectrum centered on the actual operating frequency. This does not mean that the operating frequency is uncertain; it is usually quite definite. The point is that by sweeping the driving frequency over, say, plus or minus ten percent, one would pick up the worst case response that would occur if a calculated resonance with an uncertainty of ten percent had actually fallen near the driving frequency.

The user must decide whether to increase or decrease a resonance. It might be possible to drive the 62 Hz mode in the example down to, say 52 Hz, instead of up to 70. This could be done by removing material, but would only be acceptable if stress limits were not exceeded.

In some cases, start-up and shut-down conditions are of interest, in addition to normal operating conditions. If in such cases there is a resonance below the operating frequency, then unacceptable response may be produced when the rotating frequency passes through that resonance. This problem could probably not be solved by manipulating modes.

In spite of these caveats, direct manipulation of natural frequencies is a good method of attack for certain problems. The method is illustrated in the example problem explained below.

5.0 MINIMIZING NOISE TRANSMISSION BY MINIMIZING FREQUENCY RESPONSES

A second approach to the noise transmission problem is to attempt to minimize the response directly. In this approach, the optimizer is given much more latitude than when manipulating modes directly. In essence, it can either try to shift frequencies, or try to simply reduce the response by beefing up members where needed (and shedding unnecessary weight elsewhere), as in a static problem. Of course, the optimizer has no "knowledge" of statics and dynamics at all. It simply seeks a constrained minimum as usual. The physics of the problem are implicit in the behavior values and sensitivities that are passed to ADS by ADS/NASOPT. Frequency constraints may be included in this approach, in cases where natural frequencies fall very close to the driving frequency.

Sensitivity analysis of frequency response outputs is believed to be a new development. Hence the algebra that was worked out, and its implementation in DMAP code is explained in the following section. As most users know, there are two approaches to dynamic response problems in MSC/NASTRAN: direct and modal. The choice of one method over the other is based mostly on economics. In modal coordinates, the equations of motion are greatly simplified, but one must pay the cost of computing sufficient eigenvalues and eigenvectors. This method is preferable when more than a very few driving frequencies are to be analyzed. In the direct method, the steady-state equations of motion are set up and solved directly using complex arithmetic. See Reference 4 for more discussion of these points. The development that follows considers both the direct and modal methods.

In this development, the object is to minimize the average magnitude of the complex responses at two points where the support structure mounts to the foundation. Translation magnitudes in each of the three coordinate directions at each point, a total of six quantities, are averaged. Depending on the application, responses in one particular direction may be more important than others.

6.0 FREQUENCY RESPONSE SENSITIVITY CALCULATION

MSC/NASTRAN can currently calculate sensitivities of static displacements, static stresses, natural frequencies, and buckling loads. In order to handle frequency response sensitivity calculations, a complete frequency response sensitivity DMAP program was written. In addition, some DMAP modifications to the standard superelement frequency response solution sequence (SOL 68 and SOL 71) were required.*

We first consider the direct approach using SOLution 68. The effort here involved minor modifications to SOL 68, plus a complete DMAP code for computing complex response sensitivity.

SOL 68 was modified as follows: The FRRD1 module, which normally solves the complex equations of motion, was removed. It was replaced by a loop, executed once for each driving frequency, in which the following steps were carried out:

* Rigid formats 26 or 30 can be used when no superelements are required. Alters would be needed to create a data base and store the data blocks needed for design sensitivity. An advantage of this approach is that less disk storage is required. There is generally no reason to save any data blocks besides those required for sensitivity analysis, since every MSC/NASTRAN analysis must start from scratch and thus cannot recover any significant amount of data from the database.

1. Assemble the impedance matrix:

$$[H] = [K] + i\omega[B] - \omega^2[M] \quad (1)$$

2. Decompose the complex impedance matrix using the DECOMP module.

3. Save the decomposed matrix in the data base (if it were an inverse we would call it an admittance matrix, but inverses are never calculated in MSC/NASTRAN).

4. Solve for complex displacements using the FBS module.

5. Compute the magnitudes of the complex displacements and save them in the data base along with the real and imaginary parts.

These steps do the same work as FRRD1 does, but in separate steps so that the intermediate results can be saved. The standard procedure for assembling local vectors is not modified.

The sensitivity program does the equivalent of SOLution 51 (statics) or SOLution 53 (normal modes). It solves for the dynamic displacement sensitivities which are obtained by differentiating the equations of motion

$$[H]\{U\} = \{P\} \quad (2)$$

yielding

$$[\frac{\partial H}{\partial X}]\{U\} + [H]\{\frac{\partial U}{\partial X}\} = 0 \quad (3)$$

or

$$[H]\{\frac{\partial U}{\partial X}\} = -[\frac{\partial H}{\partial X}]\{U\} = \{R\} \quad (4)$$

The DMAP code handles these calculations in the following steps:

1. Fetch the real and imaginary parts of the displacement vectors, U_r and U_i. These vectors are actually matrices having one column for each driving frequency. If multiple load cases had been specified in SOL 68, they would have had a set of NDR columns for each load case (NDR = number of driving frequencies).

2. Call module DSVG1 twice to compute the sensitivities vectors

$$\{EGK_r\} = [\frac{\partial K}{\partial X}]\{U_r\} \quad ; \qquad \{EGK_i\} = [\frac{\partial K}{\partial X}]\{U_i\} \tag{5}$$

and then set

$$\{EGK\} = \{EGK_r\} + i\{EGK_i\} \tag{6}$$

These sensitivity vectors have one set of columns for every design variable. That set includes one subset of columns for every load case (the present DMAP was set up for only a single load case), and each load case subset consists of one column for every driving frequency.

3. Compute the real and imaginary parts of the acceleration vectors:

$$\{\ddot{U}_r\} = -\omega^2\{U_r\} \quad ; \qquad \{\ddot{U}_i\} = -\omega^2\{U_i\} \tag{7}$$

4. Call DSGV1 twice to compute the inertia sensitivity vectors

$$\{EGM_r\} = [\frac{\partial M}{\partial X}]\{\ddot{U}_r\} \quad ; \qquad \{EGM_i\} = [\frac{\partial M}{\partial X}]\{\ddot{U}_i\} \tag{8}$$

and then set

$$\{EGM\} = \{EGM_r\} + i\{EGM_i\} \tag{9}$$

5. Compute the combined complex sensitivity vector {EG} = {EGK} + {EGM}.

6. In a loop, executed once for each driving frequency, do the following:

   a. Fetch the decomposed impedance matrix [HDDL].

   b. Partition out of {EG} the columns which apply to the particular driving frequency associated with this pass through the loop. Put them in matrix [RHS].

c. Use the FBS module to compute the real and imaginary displacement sensitivity vectors:

$$\{\frac{\partial U}{\partial X}\} = -[H]^{-1}\{EG\} \qquad (10)$$

d. Split the complex displacement sensitivity into real and imaginary parts:

$$\{\frac{\partial U}{\partial X}\} = \{\frac{\partial U_r}{\partial X}\} + i\{\frac{\partial U_i}{\partial X}\} \qquad (11)$$

e. Compute the sensitivity of the magnitude of the complex displacement vector in terms of the real and imaginary displacement vectors, and their respective sensitivity vectors:

$$\{U_m\} = \sqrt{\{U_r\}^2 + \{U_i\}^2} \qquad (12)$$

$$\{\frac{\partial U_m}{\partial X}\} = \frac{\{U_r\}\{\frac{\partial U_r}{\partial X}\} + \{U_i\}\{\frac{\partial U_i}{\partial X}\}}{\{U_m\}} \qquad (13)$$

7. Run the DSMA module to set up the matrix of constraint values and sensitivities based on the DSCONS card appearing in the bulk data deck. Print them out and/or write them out on a binary file for ADS/NASOPT to use.

8. Run the DSVG1 and other modules to compute the weight and weight sensitivities.

The DMAP code does not support multiple load conditions, but could be extended to handle these.

The above procedure is complicated further when a symmetric structure is represented by a half model analyzed with separate symmetric and antisymmetric runs. It is important to remember that the magnitude of a complex number is a nonlinear function of its real and imaginary parts. Thus the magnitude of the summed (symmetric + antisymmetric) response is not equal to the sum of the magnitudes of the symmetric and antisymmetric parts. The procedure outlined above has to be modified, and the following steps taken:

1. Calculate the symmetric frequency response.

2. Calculate the antisymmetric frequency response.

3. Calculate the sensitivities of the real and imaginary parts of the symmetric response.

4. Calculate the magnitude of the symmetric response. NOTE: this step and the next step are carried out only to print out intermediate results to be sure they are plausible before proceeding. They do not contribute directly to the final result.

5. Calculate the sensitivities of the symmetric magnitude in terms of the real and imaginary parts, and their respective sensitivities, and print them out.

6. Calculate the sensitivities of the real and imaginary parts of the antisymmetric response.

7. Add the symmetric and antisymmetric real and imaginary displacements. Add the symmetric and antisymmetric real and imaginary displacement sensitivities. Calculate the magnitude of the summed response, and its sensitivities.

Once the final sensitivity values are ready, they can be passed to ADS/NASOPT which handles them just like a static problem.

The approach required for SOLution 71 is slightly different. In this case the equations are solved in modal coordinates, and the eigenvectors are involved in the solution. The alter required for SOL 71 is essentially the same as that used in SOL 68. The FRRD2 module is replaced by equivalent DMAP code that stores intermediate data needed for later sensitivity calculations. It is necessary to recover all displacements in physical coordinates, a potentially time-consuming step.

Straightforward calculation of sensitivities would involve eigenvector derivatives, since eigenvectors are involved in transforming the equations of motion to modal coordinates, and in recovering physical displacements. Nelson's method for eigenvector sensitivities has now been coded in DMAP [5]. However, this calculation is known to be time-consuming, and the premise adopted here is that eigenvector sensitivities can be bypassed. The idea is that although the responses have been computed using modal coordinates, the sensitivities can be obtained by differentiating the original equations in physical coordinates, then solving the sensitivity equations by converting them to modal coordinates first. We return to the equations of motion (1), and the derivative expression (4). Just as we assume we can obtain a reasonable approximate solution to the equations of motion using a truncated set of modes, we assume that we can solve the sensitivity equations (12) in a similar manner. We let

$$\{\xi\} = [\Phi]^T\{\frac{\partial U}{\partial X}\} \tag{14}$$

so that (12) becomes

$$[\Phi]^T[H][\Phi] = [\Phi]^T\{R\} \tag{15}$$

After (15) has been solved, the sensitivities in physical coordinates are recovered from

$$\{\frac{\partial U}{\partial X}\} = [\Phi]^T\{\xi\} \tag{16}$$

In many cases it may be wise to compute an average or RMS response over a certain frequency spectrum and use that value as the objective. One reason for this is that the "choppy" nature of a frequency response curve may easily lead to relative minima which can stall the optimizer, as shown in Figure 1. A single value is much less likely to lead to relative minima.

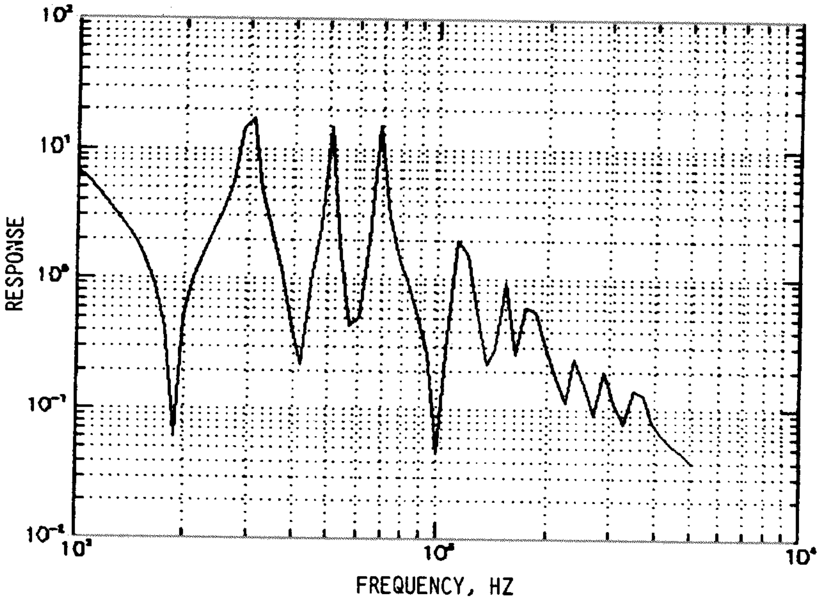

Figure 1 Typical frequency response function

7.0 EXAMPLE PROBLEMS

Figure 2 shows an academic cantilever beam test problem. The objective is to minimize the weight subject to a constraint on the fundamental frequency. The design model for this problem is shown in the Appendix. This problem verifies the ADS/NASOPT frequency constraint by comparison with two other researchers' work [6 and 7]. Figure 3 and Table 1 show the results, and the comparison with others' results. Of course, our intent in this work is to switch the roles of weight and frequency. That is, we wish to minimize or maximize a particular frequency subject to a constraint on weight, and, in most cases, constraints on other frequencies. However, this problem still provides a good validation of the capability since switching objectives is a fairly simple operation. The objective and constraints may be changed manually at any design stage by ADS/NASOPT users.

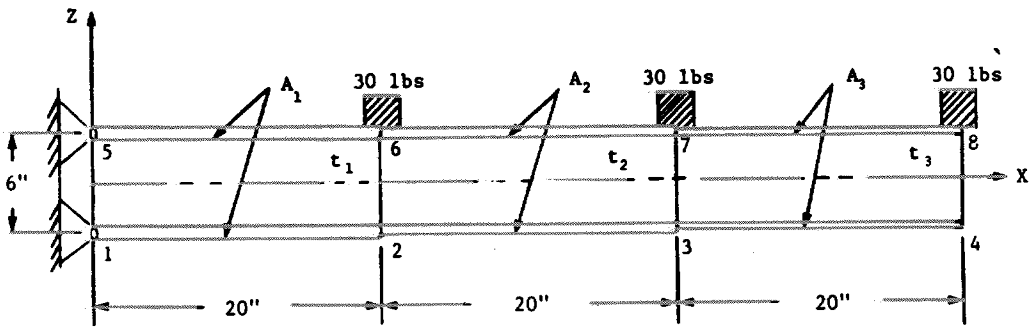

Figure 2 Cantilever Beam Test

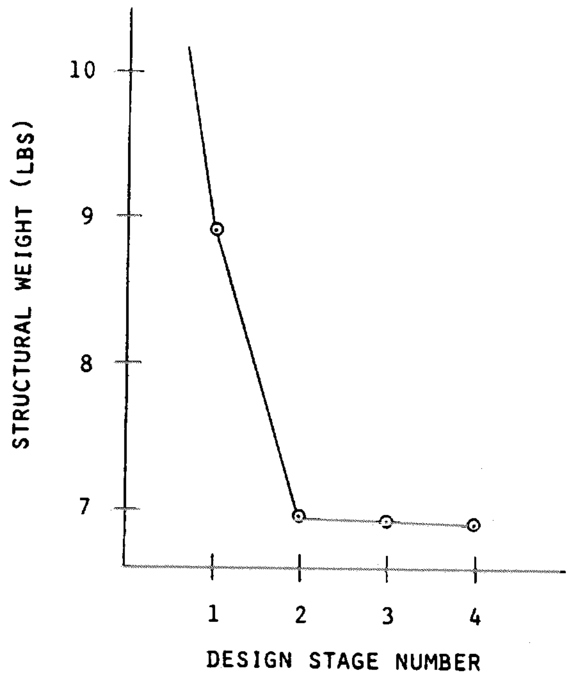

Figure 3 Cantilever beam weight history

TABLE 1 CANTILEVER BEAM DESIGN HISTORY

DESIGN STAGE				INITIAL	1	2	3	4	ACCESS-1 OPTIMUM	TURNER OPTIMUM
WEIGHT (LBS)				19.200	8.903	6.964	6.944	6.907	7.000	7.000
FUNDAMENTAL FREQUENCY (HZ)				26.097	21.022	19.939	20.027	20.017	20.	20.
BARS	1	AREA	SQ IN	1.000	0.802	0.778	0.827	0.852	0.871	1.13 - 0.69
	2			1.000	0.397	0.409	0.423	0.428	0.441	0.69 - 0.28
	3			1.000	0.333	0.121	0.108	0.104	0.108	0.28 - 0.00
WEB	4	THICKESS	INCHES	0.200	0.105	0.069	0.058	0.051	0.044	0.037
	5			0.200	0.067	0.051	0.046	0.043	0.040	0.034
	6			0.200	0.067	0.031	0.028	0.027	0.026	0.023

The frame structure shown in Figure 4 is representative of a realistic structure that was optimized recently by CSA Engineering. It is a symmetric structure, and only half is shown. The half model includes 1762 grid points, and 1762 elements. The design model includes 22 design variables. There are stress constraints associated with static loads (three cases), along with response constraints due to the dynamic load (a single case). Rotating machines are mounted on each half of the frame, and two legs are shown, mounting to a foundation of some sort. The machine has parts rotating at 60 Hz. Since the structure is symmetric, only half is modelled, and the total response is calculated by summing symmetric and antisymmetric responses. The loads are calculated by assuming a certain unbalance "r", and then applying vertical and horizontal acceleration loads to the axis of rotation, equal to ω^2/r. A 90-degree phase lag is specified between the vertical and horizontal components.

First, the natural frequencies were attacked directly. The objective was to remove all modes from the range 50-75 Hz. Initially there were two offending modes, at about 66 and 69 Hz, plus one at 49 that threatened to enter the forbidden range. Above this was a fourth mode at about 88 Hz. Static stress constraints were also specified, although it happened that none of these were critical during the dynamic design process.

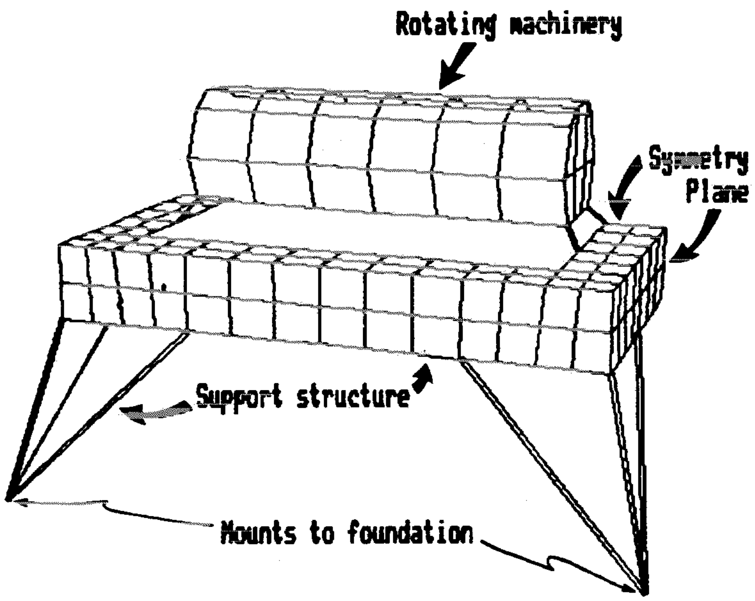

Figure 4 Frame structure supporting rotating machinery

Figure 5 shows the progress of the redesign cycles. The "design cycles" shown in the plot represent the "outer loops" discussed in section 3.0. That is, each of these cycles involves a complete MSC/NASTRAN re-analysis. The upper plot shows the evolution of the mode shapes as the design progresses. Mode 2 ("second bending in horizontal plane") is driven up to 70 Hz fairly quickly, while the mode 3 ("bending in vertical plane") progresses more slowly. Mode 4 hovers around the lower end of the range while mode 1 drifts away. The lower plot shows the accompanying weight increase.

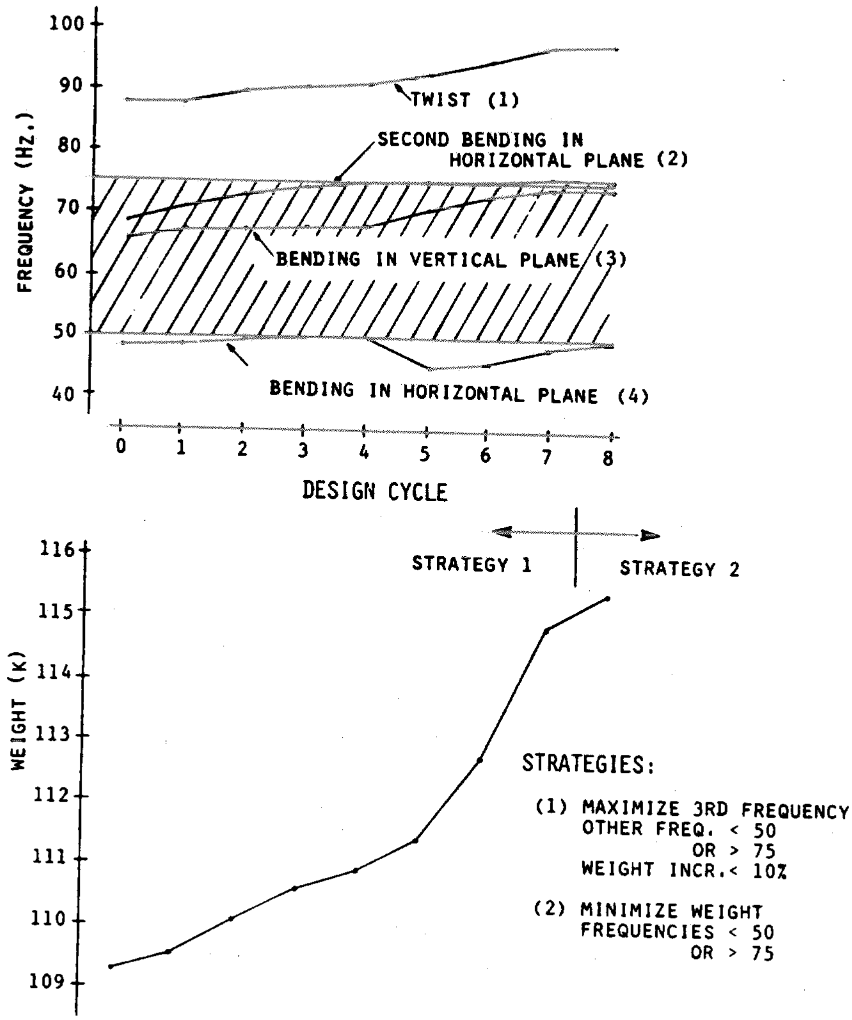

Figure 5 Frequency manipulation optimization history

Two strategies were employed for this problem. At first, the objective was to maximize mode 3 while limiting the weight increase to 10%. After this strategy had succeeded in driving all the frequencies out of the forbidden range (or nearly so), the strategy was switched in an attempt to see whether further gains could be made. The objective was switched to weight, and the frequencies were constrained, but this attempt failed. However, it is a good illustration of the choices that are available to the ADS/NASOPT user in switching objectives and constraints.

Second, a direct minimization of the response at the supporting structure's mount points was attempted. The model was again similar to the sketch shown in Figure 5, but was much more complex. The objective was to minimize the average of the magnitudes of the three complex displacement components at each mount. That is, six magnitudes were averaged, altogether.

Due to time constraints and the size of the model that was used, only limited results were obtained. However, the procedure was able to cut the combined displacement resultant by almost 50% with no increase in weight.

8.0 CONCLUSIONS

An application for automated optimization has been developed and demonstrated. The procedure makes it possible to compute a structural design that minimizes noise transmission from rotating machinery. This same procedure could be applied to other excitation sources as well, include random environmental loading. Two approaches were applied: one in which the natural frequencies were moved away from the rotating frequency, and a second in which the steady-state dynamic response at the mounts was directly minimized.

While early results are encouraging, more research is needed, especially to study the potential for relative minima due to the "choppy" nature of frequency response functions. As mentioned above, one possible approach to this problem is a smoothing or averaging procedure. Also, multiple starting points are the classical solution to relative minima.

Also, the question of how to combine complex displacement components in different directions or at different points is not simple. For example, suppose one wanted to minimize or constrain the vector resultant displacement at a point. Since in general (with damping) the three translation components would all have different phase lags, the resultant would not be a simple sinusoidal function like the individual components.

REFERENCES

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3. ADS/NASOPT User's Manual, Version 1.00, available from CSA Engineering, Inc., 560 San Antonio Rd., Palo Alto, CA 94306.

4. MSC/NASTRAN Handbook for Dynamic Analysis, Sections 4.5 and 5.3.

5. MSC/NASTRAN Application Manual, Section 5. Application Note - November 1985: "Calculation of Eigenvector Derivatives in Design Sensitivity Analysis"

6. Turner, M.J., "Design of Minimum Mass Structures with Specified Natural Frequencies", AIAA Journal, Vol. 5, pp. 406-412, March, 1967.

7. Schmit, L.A., and Miura, H., "Second Order Approximations of Natural Frequency Constraints in Structural Synthesis", International Journal of Numerical Methods in Engineering, Vol. 13, pp. 337-351, 1978.

Appendix

Sample Design Model Data File

```
$
$ Design variables
DESVAR   1       BAR1    1.0     0.1     2.0     0.05    1
DESVAR   2       BAR2    1.0     0.1     2.0     0.05    2
DESVAR   3       BAR3    1.0     0.1     2.0     0.05    3
DESVAR   4       WEB1    0.2     0.02    1.0     0.05    4
DESVAR   5       WEB2    0.2     0.02    1.0     0.05    5
DESVAR   6       WEB3    0.2     0.02    1.0     0.05    6
DVPROP   1       PROD    4       0       201
DVPROP   2       PROD    4       0       202
DVPROP   3       PROD    4       0       203
DVPROP   4       PSHELL  4       0       204
DVPROP   5       PSHELL  4       0       205
DVPROP   6       PSHELL  4       0       206
$
$ Frequency constraint
CONSTR   1       FREQ    1       30.0    3000.0
$
$ Optimization control
MOVLIM   1       6       0.5     3.0
OPTCON   1       2       0       5       7       2230    20      0       +A
+A       0       -0.5    -0.5    -0.5    -0.5    -0.5
```