AN EFFICIENT STATIC CORRECTION ALGORITHM WITH APPLICATION TO THE HYDRODYNAMIC LOADING OF THIN SHELL STRUCTURES

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INTRODUCTION

An important class of design problems in the commercial nuclear power industry is concerned with the hydrodynamic loading of thin shell structures which form the boundary of a pool of water called the suppression pool. The purpose of the suppression pool is to condense high pressure steam released from the reactor pressure vessel during abnormal operating transients or postulated accident conditions. When these events occur, the steam condensation is preceded by the clearing of water and air under high pressure from within the piping connecting the pressure vessel to the suppression pool. The clearing of the water and air and the subsequent steam condensation produce transient pressure disturbances in the pool which are transmitted to the boundary. The response of the boundary structure, and its attached piping and equipment, to these loads is an important consideration in the design evaluation of the suppression pool system.

Realistic analysis of the hydrodynamic loading of suppression pool structures requires a careful treatment of the interaction between the pool water and its boundary. Considerable success has been achieved by modeling the pool as a linear acoustic fluid with pressure as the dependent variable. An efficient algorithm for modal transient solution of problems involving the coupling of a linear acoustic fluid to a linear elastic structure has been developed for use with MSC/NASTRAN [1] and implemented at the General Electric Co. by means of DMAP coding. In using this algorithm, it is necessary to select a cutoff frequency for the extraction of the natural frequencies and modes of the structure. The structure modes are subsequently combined with the fluid degrees-of-freedom to form coupled modes which are used for the calculation of the transient response of the fluid-structure system.. Typically, the cutoff frequency for the structure modes is related in some manner to the highest frequency of significance in the loading function.

Experience in the use of the fluid-structure algorithm has shown that serious inaccuracies can occur if no correction is made to account for quasi-static response of the system. Previous work [2] has dealt with the need to include a static correction to accurately represent the response of the fluid in the limit of near incompressibility. Recent experience has indicated the need to include a similar correction for accurate calculation of the structural response. In experiments performed to identify the nature of air-bubble oscillation loads, for example, it has been observed that the suppression pool structure follows the low frequency variation of the applied pressure and that its response is dominated by membrane deformation. Higher frequency bending modes are observed but they contribute significantly less than the load-following membrane response. Accurate calculation of the stress requires the inclusion of relatively long wavelength structure modes which are associated with membrane deformation of the shell. The long-wavelength modes may, however, correspond to high frequencies which would not be included from consideration of the frequency content of the loading function.

One approach to the problem of insufficient modes for load-following structural response would be to simply raise the cutoff frequency used for the modal extraction. This has the undesirable consequence of requiring the calculation of a large number of additional modes which contribute very little to the structural response. A second alternative is to utilize one of the standard static correction methods (e.g., mode acceleration). The cost of this alternative is unacceptable, however, because it requires the solution of a large system of equations, representing the static response of the structure to the instantaneous load, at each time step for which the response is required. In typical applications of the fluid-structure algorithm, the entire history of the response is required to demonstrate argreement between calculations and test data.

This paper describes a solution to the problem of omitted structure modes by the introduction of a shape function which is specifically chosen to capture the quasi-static response of the structure. The shape function is appended to the set of dry-structure modes before the coupled system modes are computed. In this way the existing DMAP coding, developed for modal solution of fluid-structure interaction problems, can be used without alteration. The method can be thought of as a first-order application of the Ritz-vector technique described in [3].

DESCRIPTION OF STATIC CORRECTION METHOD

The MSC/NASTRAN program can be used for the transient analysis of a system in which a linear acoustic fluid is coupled to a linear elastic structure. Mathematical details are described in [1]. The matrix formulation of the undamped coupled equations for structure displacement, u, and fluid pressure, p, can be written as

$$(\bar{M}s^2 + \bar{K}) \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} 0 \\ F \end{bmatrix} \qquad (1)$$

with

$$\bar{M} = \left[\begin{array}{c|c} M & 0 \\ \hline A & C \end{array}\right] ; \qquad \bar{K} = \left[\begin{array}{c|c} K_S & -A^T \\ \hline 0 & K_F \end{array}\right]$$

and

$s(\ )$ = d/dt ( )
M = structure mass matrix

K_S = structure stiffness matrix

C = fluid "mass" matrix

K_F = fluid "stiffness" matrix

A = fluid-structure coupling matrix

F = volumetric flow source applied to the fluid

The fluid "mass" matrix in Equation (1) is a diagonal matrix with elements equal to the reciprocal of the fluid bulk modulus. The fluid "stiffness" matrix is proportional to the reciprocal of the fluid density and is derived from the finite-element formulation of the Laplacian operator. The coupling matrix, A, contains non-zero terms only at row/column positions corresponding to matched pairs of fluid and structure grid points on the fluid-structure interface. The matrix terms of A are equal to the elements of area associated with the interface points.

The matrix formulation of Equation (1) is appropriate for numerical solution in physical coordinates by direct integration. For large problems of the type being discussed here, direct integration is not practical. A more efficient method, based on modal analysis, is used. The use of the modal method requires a set of specially coded DMAP algorithms which interact with the standard MSC/NASTRAN rigid formats.

In summary, the modal solution procedure is as follows:

1) Compute natural frequencies and mode shapes for the dry structure.
2) Compute the coupling matrix, A.
3) Reformulate the problem with fluid pressures and structure modal coordinates as the dependent variables.
4) Compute the coupling terms which appear in the reformulated coefficient matrix. These terms involve the modal properties of the dry structure and the coupling matrix, A.
5) Compute the fluid "mass" and "stiffness" matrices and enter them in the reformulated coefficient matrix.
6) Compute eigenvalues and modes for the coupled system. Each mode represents a set of pressures at the fluid grid points and a corresponding set of modal displacements for the structure.
7) Perform modal transient analysis using the coupled eigenvalues and modes. (Modal damping can be introduced at this stage of the solution process.)

In the calculation of both the dry-structure and coupled modes (steps 1 and 6 above) an upper-bound cutoff frequency must be selected. Typically, the cutoff frequency is related in some manner to the highest frequency of significance in the loading function.

A major potential deficiency of the above procedure is that it may not include enough structural modes to simulate load-following behavior in which the shell responds to the interfacial pressure in an essentially quasi-static manner. The membrane modes required to represent load-following response may easily correspond to frequencies significantly above the cutoff frequency selected on the basis of the spectral content of the loading function.

The correction procedure proposed here is based on the solution of the static form of the matrix equation (1), namely

$$K_S\, u - A^T p = 0 \qquad (2)$$

and

$$K_F\, p = F_o \qquad (3)$$

The function F_o in (3) represents an arbitiarily normalized fluid loading having the same spatial distribution as the transient loading function, F(t), in (1). (It is implicitly assumed that the spatial distribution of F does not vary significantly with time.) The first step in the static correction procedure is to solve (3) to obtain the spatial variation of the wall pressure, i.e.,

$$p = K_F^{-1} F_o \qquad (4)$$

Equation (4) is now used to substitute for p in (2), which is then solved to obtain the static shape function, u_R^*.

$$u_R^* = K_S^{-1}\, A^T\, K_F^{-1}\, F_o \qquad (5)$$

Suppose that the dry-structure eigenvalue problem has been solved to obtain a mode set ϕ_i $(i = 1, 2, \ldots, N)$, orthogonalized such that

$$\phi_j^T M_S \phi_i = \delta_{ij} \qquad (6)$$

A static shape, u_R, orthogonal to the mode set, ϕ_i, can be constructed from u^*_R by the standard Gram-Schmidt procedure

$$u_R = u^*_R - \sum_{i=1}^{N} [\phi_i^T M_S u_R^*]\phi_i \qquad (7)$$

with the result that

$$\phi_j^T M_S u_R = 0 \qquad (j = 1, 2, \ldots, N) \qquad (8)$$

The orthogonalized shape, u_R, is appended to the mode set ϕ_i. The frequency, ω_R, corresponding to the shape function, u_R, is given by the Rayleigh quotient

$$\omega_R^2 = \frac{u_R^T K_S u_R}{u_R^T M_S u_R} \qquad (9)$$

The calculation of the eigenvalues and mode shapes of the coupled fluid-structure system proceeds in the manner described above, using the augmented dry-structure mode set.

EXAMPLE OF APPLICATION

The potential importance of the static correction can be illustrated by the calculation of the response of a boiling water reactor (BWR) suppression pool structure to the pressure disturbance caused by air-bubble oscillation following actuation of a safety relief valve (SRV). Figure 1 shows a section view of the toroidal shell structure which contains the suppression pool. The toroidal shell is made up of sixteen 22.5° mitred cylindrical sections. Its radius to thickness ratio is about 280. Also shown is the SRV piping which is used to relieve overpressure transients in the reactor pressure vessel. When an SRV is opened, steam enters the line and compresses the air ahead of it. The compressed air is discharged into the pool where it forms bubbles which expand and contract as they rise to the surface. Figure 2 shows a typical example of the torus shell pressure produced by bubble oscillation following SRV actuation. Figure 3 shows the shell hoop stress measured at bottom-center. The measured stress clearly indicates that the structure follows the applied load quite closely.

The finite-element model of the toroidal shell structure used to calculate the response to the air bubble load is shown in Figure 4. It represents a 22.5° sector of the toroidal shell, the ring stiffener at the mitre joint and one pair of support columns. QUAD4 elements were used to model the shell and BAR elements the ring stiffener and the support columns. The loading function shown in Figure 5, representing the air-bubble oscillation pressure, was applied to the fluid-structure interface. It has maximum amplitude at the bottom of the pool and attenuates to zero as the free surface is approached.

Two calculations were performed to illustrate the importance of the static correction. In the first, the response was calculated by modal superposition using all the structure modes up to 30 Hz (23 modes). From examination of the applied load (Figure 5), which is essentially a damped sinusoid at about 7 Hz, one might judge that inclusion of all modes below 30 Hz would be adequate. In the second calculation, the static correction shape was appended to the same 23 modes (as described above) before the transient calculation was made. The static shape corresponded (by the Rayleigh quotient) to a frequency of 46 Hz. Figures 6 and 7 show the hoop stress at bottom center of the shell with and without the static correction. The effect of the static correction on both frequency and amplitude is seen very clearly. For the case without static correction, the hoop stress is dominated by response at about 26 Hz and the peak amplitude is about one-third of what was measured (see Figure 3). With the static correction the hoop stress follows the load at 7 Hz and the peak amplitude agrees with the measured value.

CONCLUSION

The above example illustrates the potential importance of the static correction to problems involving the hydrodynamic loading of thin shell structures. The use of the modal method without static correction can lead to substantial errors in the frequency, amplitude, and character (e.g., bending vs. membrane) of the structural response. The algorithm proposed in this paper has significant advantages in cost and convenience over the alternatives of raising the cutoff frequency or using the standard mode acceleration approach. It is especially well suited for use with the symmetric modal method for the solution of fluid-structure interaction problems with MSC/NASTRAN.

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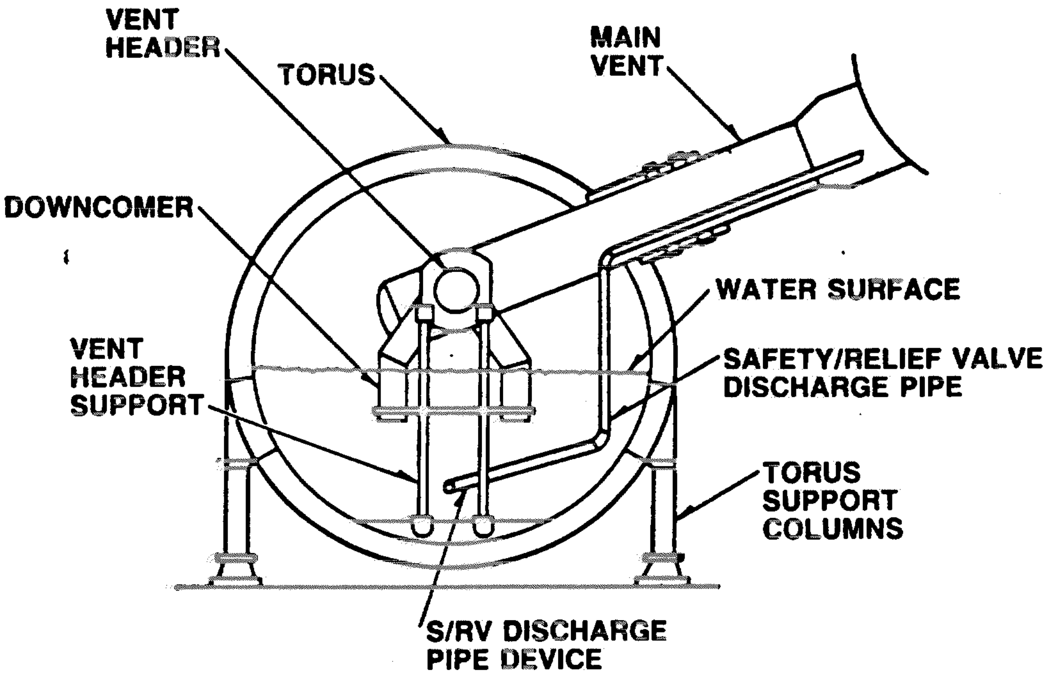


FIGURE 1. SUPPRESSION POOL STRUCTURE

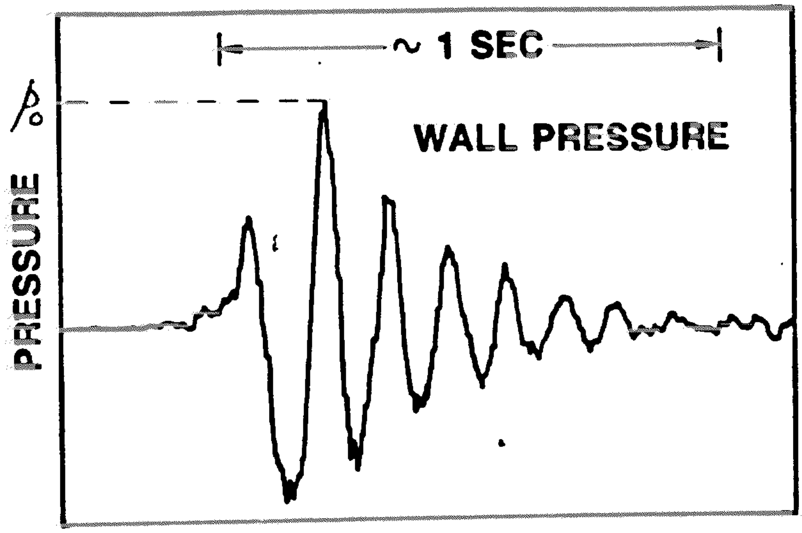


FIGURE 2. MEASURED TORUS SHELL PRESSURE

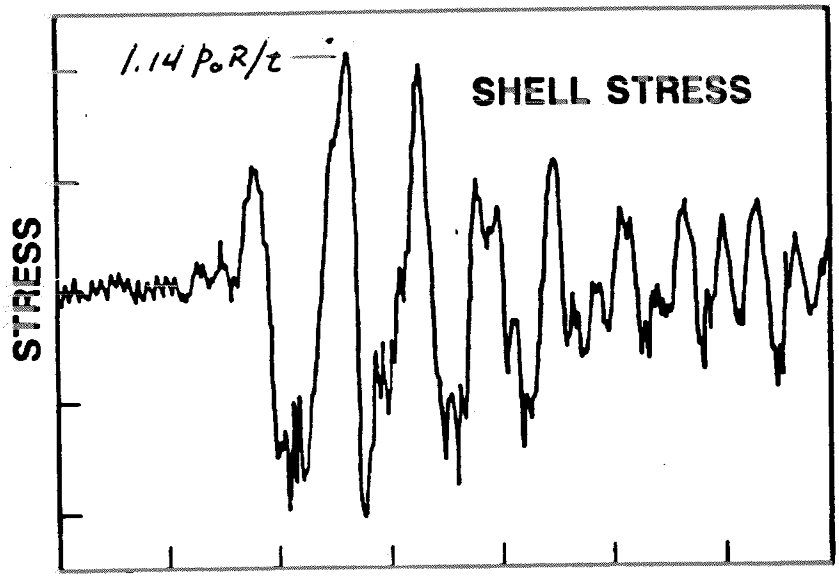


FIGURE 3. MEASURED SHELL STRESS

FIGURE 4. FINITE-ELEMENT MODEL OF TORUS SHELL

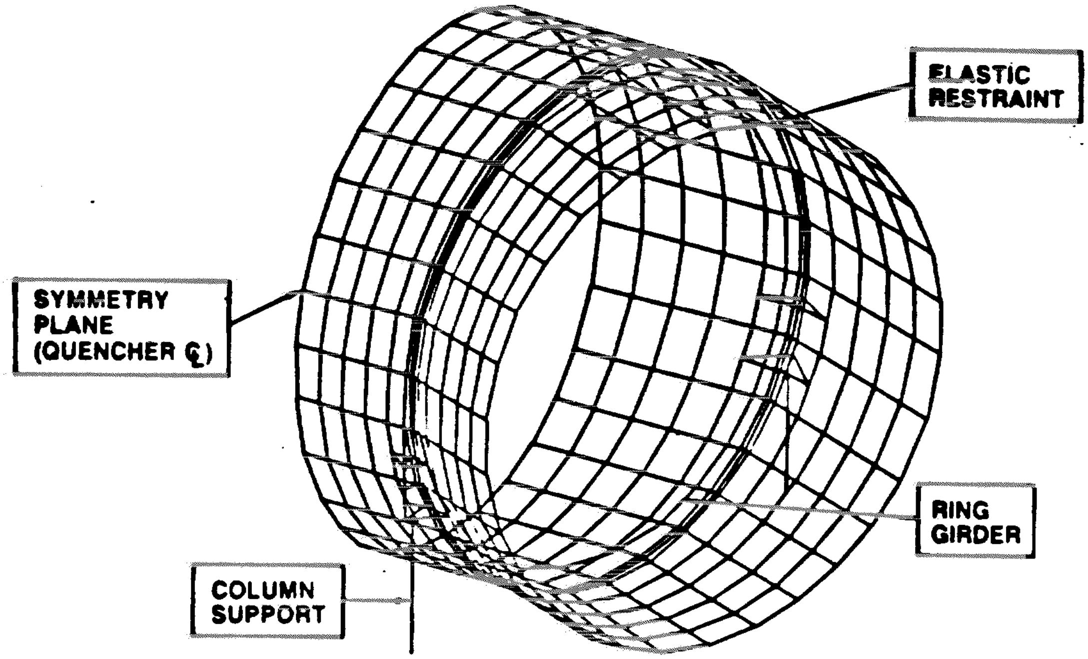
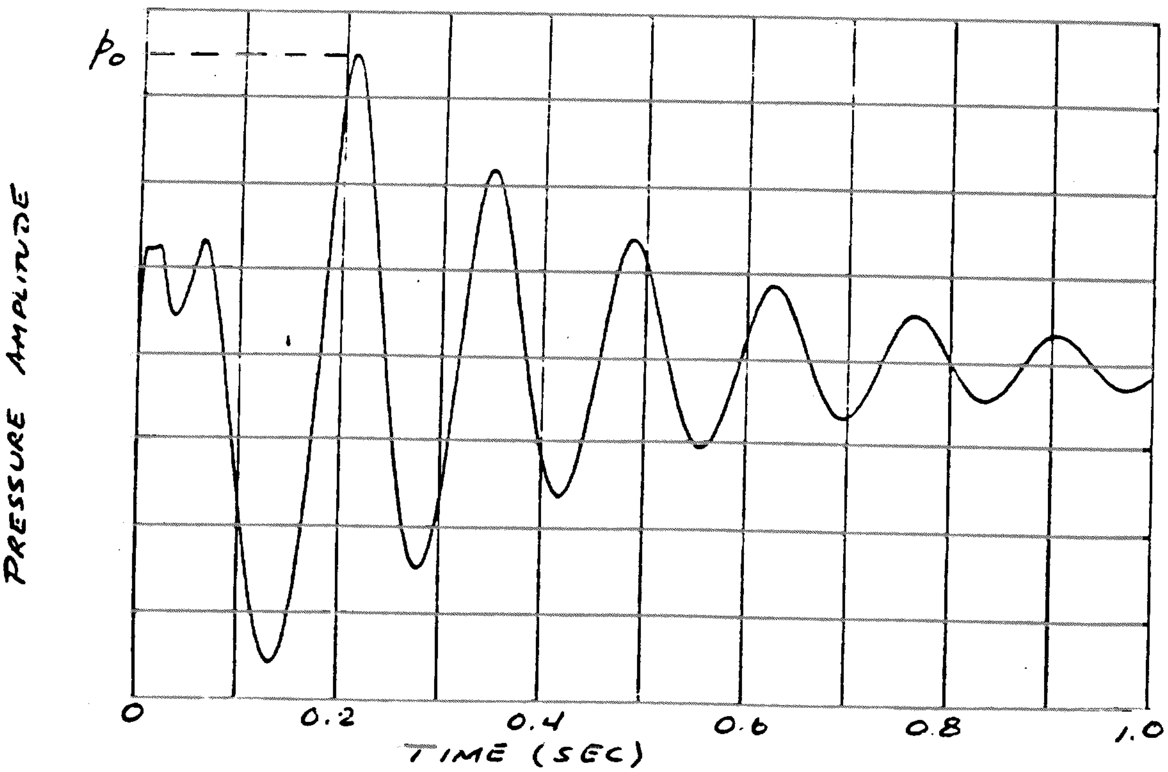


FIGURE 5. AIR BUBBLE OSCILLATION PRESSURE



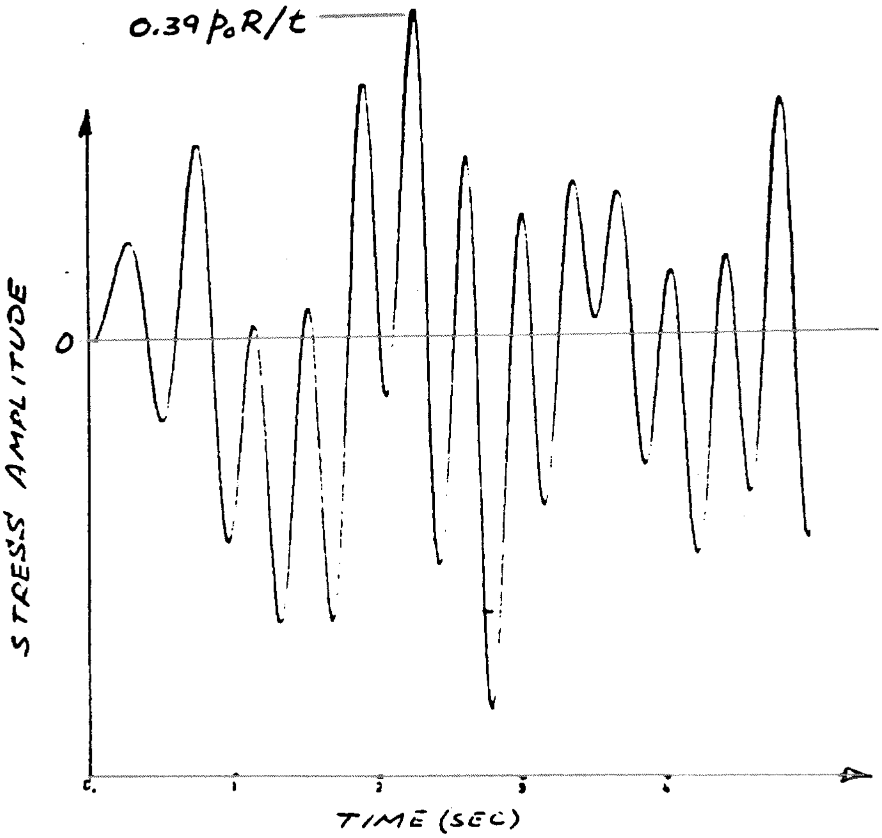


FIGURE 6. BOTTOM-CENTER HOOP STRESS (NO CORRECTION)

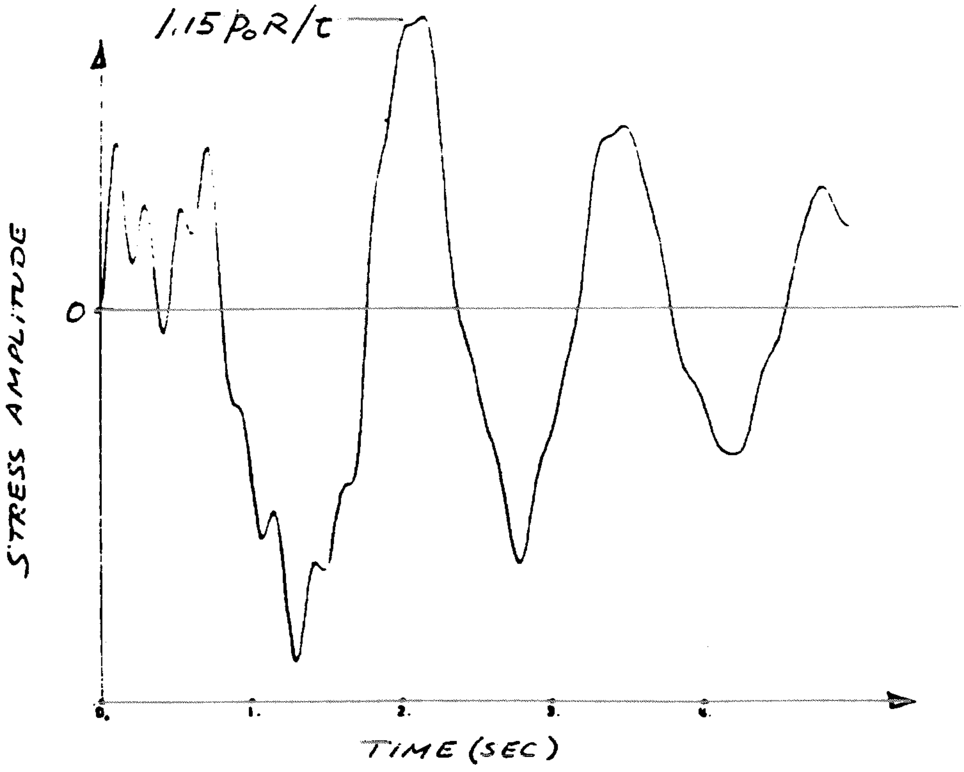


FIGURE 7. BOTTOM-CENTER HOOP STRESS (CORRECTED)