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FINITE ELEMENT SIMULATION OF COUPLED AUTOMOBILE ENGINE DYNAMICS

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by

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Abstract

A procedure has been developed to simulate engine dynamics and determine displacements and forces in the reference frames of the moving components. The method is based on a harmonic domain representation of displacements and forces and the coupling between harmonics at component interfaces. A pre-processor generates the required coupling matrices and inertial loading vectors for each harmonic. A DMAP Alter assembles the matrices, solves for the harmonic displacements and provides plots of the displacements and forces over the two revolution period of the engine. A comparison is made between an analytic solution for a rigid component engine and the simulation procedure with both rigid and flexible components.

INTRODUCTION

Computer solutions to dynamics of rotating structures have tended to concentrate on cases with simple interactions between the rotating structure and its non-rotating supports. Fortunately many structures may be analyzed in a rotating coordinate system. If the non-rotating supports may be modeled with simple isotropic springs they may be transformed to the rotating system. Examples are turbines and centrifugal pumps where the support stiffness may be approximated.

For complex rotating structures the solutions are more difficult to obtain. If the rotating structure is neither symmetric nor connected to simple components, the structural vibrations may couple to the cyclic rotations. An example is the helicopter blade system where motions are internationally generated as harmonics of the rotational speed in order to provide control. The resulting differential equations of motion contain time-varying coefficients which are not easily solved by conventional finite element methods.

The methods described in this paper provide for time varying coupling of conventional finite element structure models. MSC/NASTRAN may be used to model engine blocks, crankshafts, bearings connecting rods, pistons, and attached mechanisms. Each component is assumed to have small deformations and displacements in its own moving coordinate system. The components are connected together by constraining the displacements and forces at the interface points. The method is described below.

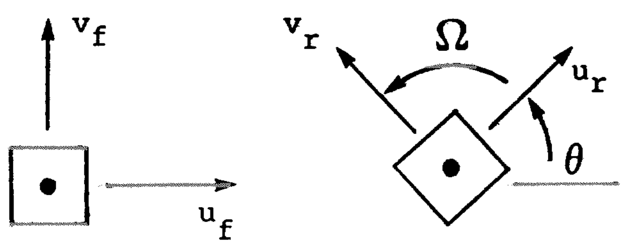
HARMONIC SPACE

The methods shown below are not unique to the solution of rotating structures. They are basically an extension of the rigid body engine balancing process and are related to proprietary unpublished methods used in the helicopter industry.

The basic assumptions are that each structural component undergoes small deformations in its own moving coordinate system. Furthermore, the motion is assumed to be a periodic function represented by a set of the harmonics of coupled rotating systems. Each connection between different components will be represented by complex coefficients connecting the harmonics of the fundamental motions.

Connections

The fundamentals of the method are shown below by connecting two points:



Equation 2 becomes:

$$\begin{Bmatrix} u_r \\ v_r \end{Bmatrix} = \frac{1}{2}\left[\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} e^{i\theta} + \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix} e^{-i\theta}\right] \begin{Bmatrix} u_f \\ v_f \end{Bmatrix} \tag{3}$$

By substituting Eq. (1) into both sides of Eq. (3) we will have an equation of all harmonics. However, we may separate the different time functions, $e^{ik\theta}$, for unique values of k and it can be shown that

$$\begin{Bmatrix} u_r{}^k \\ v_r{}^k \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} \begin{Bmatrix} u_f{}^{k-1} \\ v_f{}^{k-1} \end{Bmatrix} + \frac{1}{2} \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix} \begin{Bmatrix} u_f{}^{k+1} \\ v_f{}^{k+1} \end{Bmatrix} \tag{4}$$

for $k = -\infty \ldots -1, 0, 1 \ldots +\infty$.

The significance of Eq. (4) is that the connections are constant coefficients between the different harmonic displacements. In actual practice the series k may be truncated to a finite set, -N to N.

Finite Elements

Each structural component is assumed to have linear stiffness, mass, and damping matrices (K, M, B) in each moving coordinate system. Furthermore, by minimizing the energy variation over the fundamental time period we may separate the harmonic displacements. For example the potential energy δU is

$$\delta U = \frac{1}{2} \lfloor u \rfloor^T [k]\{\delta u\} \qquad (5)$$

where u is defined by Eq. (1).

If we evaluate the integral of δU over the time period, $\Omega(t-t_o) = 4\pi$ (for a 4-cycle engine), we obtain:

$$\delta U = \sum \delta U_n \quad , \qquad n = 0, 1, \ldots n$$

where

$$\delta U_n = \frac{1}{2} \lfloor u^{-n} \rfloor^T [K]\{\Delta u^n\} + \frac{1}{2} \lfloor u^n \rfloor^T [K][\Delta u^{-n}] \qquad (6)$$

where δU_n are the individual harmonic energy variations.

Therefore the system stiffness matrix consists of disjoint stiffness matrices which will be coupled by the equations connecting the angular motions.

System Formulation for Simple Rotations

The displacement vector for each harmonic consists of both rotating and nonrotating terms. Eq. (4) generates constraint equations of the form:

$$\{ u_r^k \} = [H^-] \{ u_f^{k-1} \} + [H^+] \{ u_f^{k+1} \} \qquad (7)$$

where all of the connections defined by Eq. (4) are collected into the H matrix. The actual connection process is performed using the Lagrange multiplier technique. Let

$$\lfloor u^k \rfloor = \lfloor u_f{}^k, u_r{}^k, \lambda^k \rfloor \qquad (8)$$

The assembled stiffness matrix has the form

$$[K] = \begin{bmatrix} K_o & G^+ & & & & & & \\ G^- & K_o & G^+ & & & & & \\ & & \ddots & & & & & \\ & & G^- & K_o & G^+ & & & \\ & & & G^- & K_o & G^+ & & \\ & & & & G^- & K_o & G^+ & \\ & & & & & & \ddots & \\ & & & & & G^- & K_o & G^+ \\ & & & & & & G^- & K_o \end{bmatrix} \tag{9}$$

where

$$\lfloor u \rfloor = \lfloor u^{-N}, \ldots u^{-1}, u^{0}, u^{1} \ldots u^{N} \rfloor \tag{10}$$

$$[G^+] = \begin{bmatrix} 0 & 0 & -H^+ \\ 0 & 0 & 0 \\ -H^+ & 0 & 0 \end{bmatrix}^T \tag{11}$$

$$[K_o] = \begin{bmatrix} K_{ff} & 0 & 0 \\ 0 & K_{rr} & I \\ 0 & I & 0 \end{bmatrix} \tag{12}$$

$$[G^-] = [\ G^+\ ]^\dagger \tag{13}$$

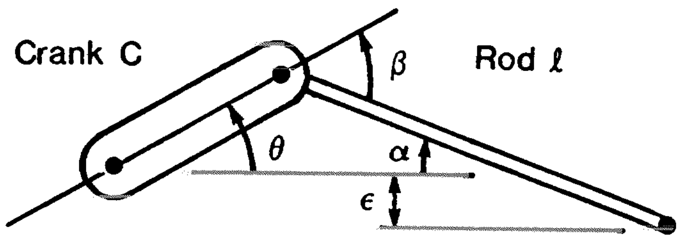
The assembled mass and damping matrix contain disjoint partitions for each harmonic of the form

$$[M_k] = -k^2\Omega^2 \begin{bmatrix} M_{ff} & 0 & 0 \\ 0 & M_{rr} & 0 \\ 0 & & 0 \end{bmatrix} \tag{14}$$

$$[B_k] = i k\Omega \begin{bmatrix} B_{ff} & 0 & 0 \\ 0 & B_{rr} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{15}$$

Crankshaft-Rod Connection

The angle between the connecting rod and the crankshaft or piston is not a single trigonometric constraint. This is shown by the sketch below.



Here the connections are expressed by the angles α, β, and θ where $\theta = \Omega t$.

From geometry

$$\ell \sin \alpha = C \sin \theta + \varepsilon \tag{16}$$

and

$$\beta = \theta + \alpha \tag{17}$$

Since both α and β are periodic functions of θ their trigonometric functions may also be expressed as a Fourier series in $e^{i\theta}$, e.g.:

$$\cos \alpha = \sum_m (A_m e^{im\theta} + B_{me}{}^{-im\theta}) \tag{18}$$

The connections between harmonic displacements are in the form

$$u_c^k = \sum_m \left(C_m u_\ell^{k-m} + D_m u_\ell^{k+m}\right) \qquad (19)$$

Note that the series m may be truncated to a few terms for small angles α. The result is that the constraints will couple several harmonic displacements together.

OVERALL SYSTEM DESCRIPTION

Capabilities

The capabilities of the resulting system are:

1) An unlimited number of cylinders, pistons, and crank journals are allowed.

2) Coupling between crank to block, rod to crank, piston to rod, and piston to block is provided.

3) Kinematic inertial loads are automatically calculated.

4) Combustion pressures may be specified as time functions or directly as harmonic loads.

5) The fundamental period is two revolutions to account for the alternate firing.

6) Full force and stress output is provided for all finite elements.

Assumptions and Limitations

The system is primarily limited by the assumption of linear motions relative to the kinematic solution.

1) All connections between moving components are local connections between two grid points. Overall forces will be calculated but detailed stress analysis (for instance: between the rod journal and the crank throw) should be performed by a separate load analysis.

2) Variations in engine speed should be reasonably small for high accuracy. For instance one cylinder engine with a light flywheel may vary as much as 5-10 degrees before accuracy is affected.

3) Nonlinear effects such as large bearing gaps and piston slap must be treated with equivalent linear stiffness elements.

Implementation into MSC/NASTRAN

The procedure for performing the analysis contains the following parts

1) A pre-processor program for MSC/NASTRAN generates the coupling matrices H and G along with the partition information for the matrices. The inputs consist of geometric parameters, component interfaces and loading information. The output consists of MSC/NASTRAN input data and a verification file.

2) A DMAP Alter deck is used to formulate and solve the problem for a selected engine RPM. The major steps are to assemble the complete matrices and decompose a single matrix.

3) A DMAP Alter is used to combine the resulting complex vector results into the MSC/NASTRAN transient analysis output format for complete output display over the fundamental period.

Discussion of the Solution

The form of the matrix solution is conveniently implemented with the MSC/NASTRAN frequency response method. However, unless small reduced matrices are used, the potential cost could become very high since complex matrix decomposition costs more than four times that of real symmetric solutions. Furthermore, the sizes of the solution matrices are multiplied by (2N + 1) which could require large memory and running time.

Fortunately the matrices for each component are linear and lend themselves to matrix reduction methods. The engine block, crankshaft, and drivetrain may each be reduced to 2-20 grid points apiece. A final matrix in the range of 1000 - 2000 degrees of freedom could be solved on a typical mainframe computer.

The overall capabilities allow analysis of virtually any type of cyclic mechanisms. Multiple cylinders, crankshafts, and attached mechanisms could be included in the model. Both fixed and moving components may be flexible as long as small deformations are assumed. Balance mechanisms may be attached either as explicit moving components or as part of the finite element model.

EXAMPLE PROBLEM

A simple one cylinder engine is used for a comparison with the analytic solution. Reference 1 developed theoretical values of the inertial torque at the flywheel for a one cylinder engine in terms of harmonics of one engine revolution. The method does not take into account component flexibility and is based strictly on engine geometry and a uniform rotational speed. Also included in the comparison is an engine model with 'very' flexible components to demonstrate the effect flexibility may have on the resulting torque.

The model, shown in Figure 1, has the following geometric properties.

Crank radius = 3.0 inches
Rod length = 30.0 inches
Engine speed = 1000 RPM

Table 1 presents the harmonic magnitude for the three models. Comparison of the analytic solution and the rigid component MSC/NASTRAN analysis reveals less than a 1% difference in the results. Inspection of the flexible component results clearly demonstrates the effect which flexibility can have on the nature of the problem. Figure 2 plots the rigid component results for the analytic solution. The MSC/NASTRAN results overlay the analytic results so closely that the differences could not be observed on the scale of the graph. Figure 3 compares the rigid and flexible component results to graphically demonstrate the significance of the differences.

The results demonstrate the accuracy of the procedure and also the differences between rigid and flexible component assumptions. The method presented in this paper allows the analysis of flexible complex geometric components without the necessity of making simplifying assumptions or solving complicated close form solutions.

ACKNOWLEDGMENT

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REFERENCE

1) DenHartog, J.P., Mechanical Vibrations, 4th Ed., McGraw-Hill Book Co., New York.

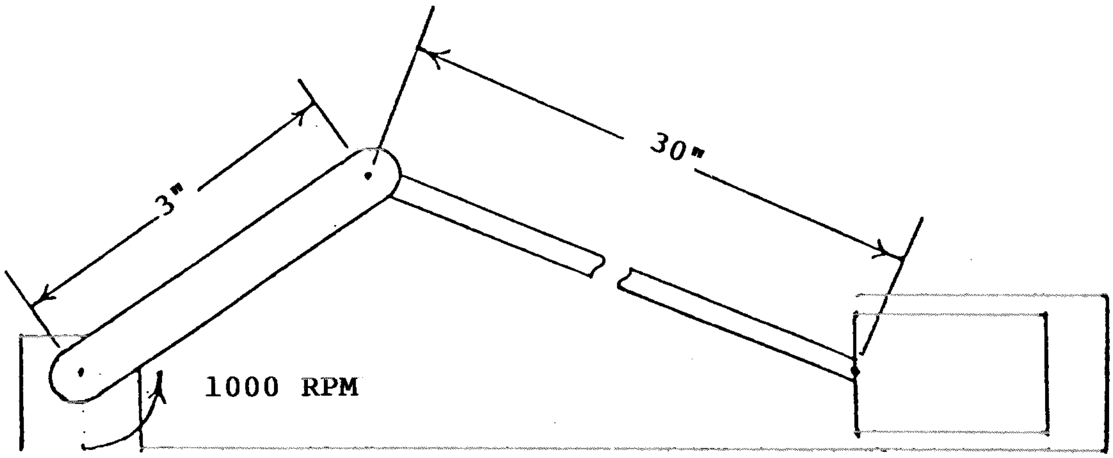

FIGURE 1. Example Problem Model

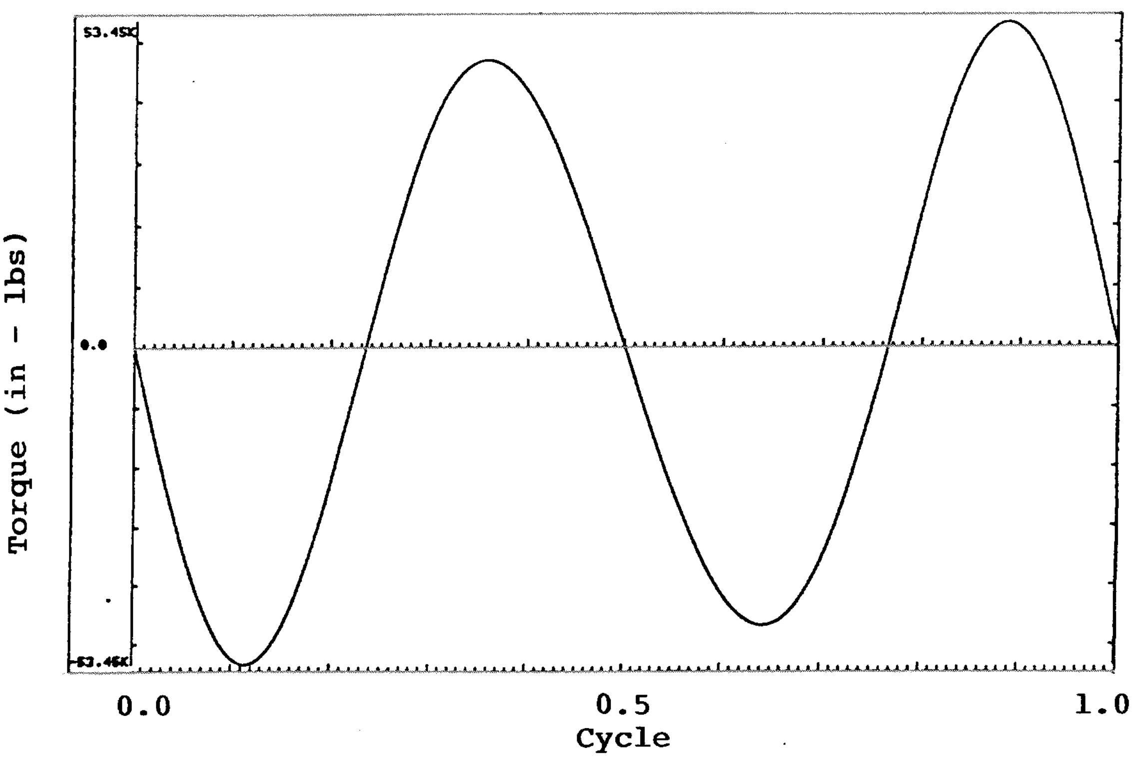

FIGURE 2. Rigid Element Model Inertial Torque

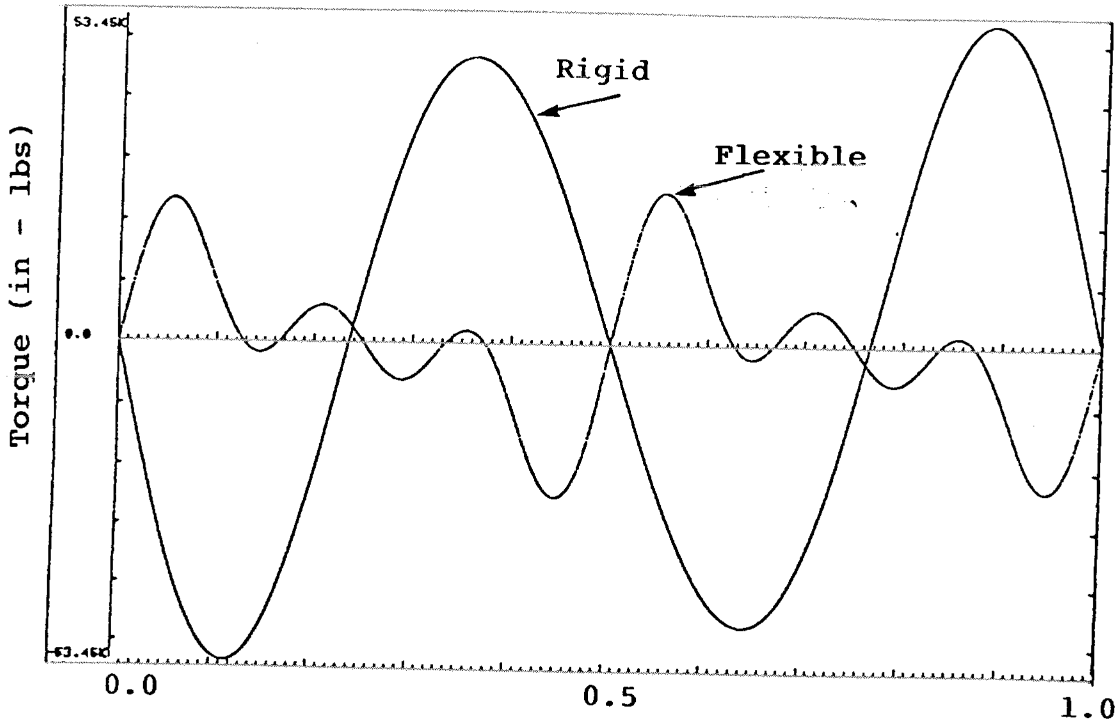

FIGURE 3. Comparison of Rigid and Flexible Element Model Inertial Torque

TABLE 1

Inertial Torque Harmonic Components

Harmonic	Analytic Solution	MSC/NASTRAN Rigid	MSC/NASTRAN Flexible
0	0.0	0.0	0.0
1	2.467E+3	2.462E+3	8.804E+1
2	4.935E+4	4.938E+4	9.982E+3
3	7.402E+3	7.453E+3	3.980E+2
4	--	2.249E+2	9.698E+3
5	--	2.194E+1	4.152E+2
6	--	1.476E0	9.454E+3