

# EMPLOYMENT OF MSC/STI-VAMP FOR DYNAMIC RESPONSE POST-PROCESSING

Robert N. Coppolino

David F. Bella

The MacNeal-Schwendler Corporation

## Abstract

Employment of MSC/STI-VAMP as an interactive dynamic response post-processor for MSC/NASTRAN is outlined. A new modification to SOL 3 (Normal Modes Analysis) provides for determination of quasi-static residual vectors, base motion excitation matrices and other modal quantities required by the post-processor. The general operation of a MSC/STI-VAMP dynamic response post-processor is demonstrated for frequency, transient and random response calculation.

## Introduction

Two years ago, The MacNeal-Schwendler Corporation and Synergistic Technology Incorporated initiated a joint venture aimed at integration of dynamic analysis and testing<sup>(1)</sup>. This objective has been achieved, to a great extent, through utilization of MSC/NASTRAN solution sequences with newly developed DMAP modifications as well as enhanced signal analysis and modal parameter identification capabilities in MSC/STI-VAMP. In order to accommodate desired communications between finite element analysis in MSC/NASTRAN and measured data analysis in MSC/STI-VAMP, the ITAP (Integrated Test/Analysis Processor) data base utility was developed. This utility facilitates

(a) pre-test planning analysis, (b) MSC/STI-VAMP command file generation, (c) data file translation from MSC/STI-VAMP to MSC/NASTRAN (and vice versa), and (d) post-test evaluation of mathematical models and test data.

The system development cycle as summarized in Figure 1 includes a task entitled "Performance Evaluation". This task involves subjecting the mathematical description of a system to dynamic design environments for integrity evaluation. To be sure, MSC/NASTRAN has always provided this function with capabilities to perform frequency response and transient and random analysis of finite element models. MSC/STI-VAMP, which is an interactive processor, offers highly desirable versatility for analyzing the dynamic response of finite element, empirical and hybrid analytical/experimental models.

The present paper outlines a new feature of the ITAP database utility which provides the engineer with the capability to perform interactive frequency response, transient response and random response analysis of systems described in terms of normal modes. Special attention has been given to features which minimize undesirable effects of modal truncation, permit user-friendly specification of applied force and enforced motion excitation, and address the issue of initial conditions.

#### Description of Modal Behavior

Employing standard MSC/NASTRAN nomenclature, the response of a linear structural dynamic system is described by the equation set

$$\begin{aligned}
[M_{gg}]\{\ddot{U}_g\} + [B_{gg}]\{\dot{U}_g\} + [K_{gg}]\{U_g\} \\
= [P_{gs}]\{P(t)\}
\end{aligned}
\tag{1}$$

where  $\{U_g\}$  are absolute grid-set displacements and  $\{P(t)\}$  are temporarily independent applied force excitations.  $[M_{gg}]$ ,  $[B_{gg}]$  and  $[K_{gg}]$  correspond to the mass, viscous damping and elastic stiffness matrices, respectively. The matrix  $[P_{gs}]$  describes distribution of applied force excitations; it is explicitly formed in MSC/NASTRAN when the "LSEQ" option is used to specify applied loads<sup>(2)</sup>. This method of applied force specification permits the user to efficiently describe highly distributed loadings.

When enforced motions are applied at a foundation, the system displacement,  $\{U_g\}$ , may usually be described as the sum of rigid and flexible body motions as:

$$\{U_g\} = \{\hat{U}_g\} + [R_g]\{U_{base}\}
\tag{2}$$

where  $\{\hat{U}_g\}$  is the relative flexible body displacement array and  $\{U_{base}\}$  is the absolute foundation displacement array. The matrix,  $[R_g]$ , represents rigid body vectors referenced to foundation degrees-of-freedom. This matrix is generated in MSC/NASTRAN by the "VECPLOT" module.

Upon substitution of the above transformation into equation (1) the dynamic equations for relative motion are

$$\begin{aligned}
[M_{gg}]\{\ddot{U}_g\} + [B_{gg}]\{\dot{U}_g\} + [K_{gg}]\{U_g\} \\
= [P_{gs_2}] \begin{Bmatrix} P(t) \\ \ddot{U}_{base} \end{Bmatrix}
\end{aligned} \tag{3}$$

where the augmented load allocation matrix is

$$[P_{gs_2}] = [P_{gs} \quad -M_{gg}R_g] \tag{4}$$

This form of dynamic equations, while restricted to statically determinate foundations, provides an extremely convenient means of describing enforced motion excitation. It should be noted that in most cases where base excitation is specified (e.g. seismic, shipboard shock loading) no provision is made for indeterminate foundation inputs.

After subjecting the structural mathematical model to all necessary constraints, a set of normal vibration modes are calculated. The number of modes required to adequately describe modal behavior are determined by the frequency content ( $0 < f < f^*$ ) of the excitation environment<sup>(3)</sup>. Moreover, for minimization of modal truncation errors, a set of quasi-static residual vectors (equal in number to the total number of applied forces,  $\{P(t)\}$  and enforced motion inputs  $\{\ddot{U}_{base}\}$  are required. The quasi-static residual vectors, which complete the static description of structural response, may be calculated as an orthogonalized set of pseudo-modes<sup>(4)</sup> which are appended to the truncated modal set. Since the residuals in this form have the properties of normal modes with natural frequency above the truncation frequency,  $f^*$ , the augmented modal transformation is written as,

$$\{\hat{U}_g\} = [\phi_g]\{\xi\} \quad (5)$$

where  $[\phi_g]$  includes the orthogonalized residual vectors.

Application of the modal transformation to equation (3) results in a set of uncoupled modal equations (if  $\phi_g^T B_{gg} \phi_g$  is assumed diagonal and modes are unit mass normalized).

$$\begin{aligned} \ddot{\{\xi\}} + [2 \xi_n \omega_n] \dot{\{\xi_n\}} + [\omega_n^2] \{\xi_n\} \\ = [\phi_g^T P_{gs_2}] \left\{ \begin{array}{l} P(t) \\ \ddot{U}_{base} \end{array} \right\} \end{aligned} \quad (6)$$

where  $\xi_n$  and  $\omega_n$  represent the respective modal critical damping ratios and natural frequencies (in units of rad/sec).

On the basis of modal response, the following physical responses are recovered:

(a) relative flexible body displacement

$$\{\hat{U}_g\} = [\phi_g]\{\xi\} \quad (7)$$

(b) absolute acceleration

$$\{\ddot{U}_g\} = [\phi_g]\{\ddot{\xi}\} + [R_g]\{\ddot{U}_{base}\} \quad (8)$$

(c) internal member loads and stresses

$$\begin{aligned}\{F\} &= [\phi_F]\{\xi\} \\ \{\sigma\} &= [\phi_\sigma]\{\xi\}\end{aligned}\tag{9}$$

(d) foundation reaction loads

$$\{F_{\text{base}}\} = [R_g^T M_{gg} \phi_g]\{\ddot{\xi}\} + [R_g^T M_{gg} R_g]\{\ddot{U}_{\text{base}}\}\tag{10}$$

The modal member load  $[\phi_F]$  and stress recovery  $[\phi_\sigma]$  matrices are explicitly obtained in MSC/NASTRAN by use of the "DRMS1" module. It should also be noted that the coefficient matrices for equation (10) are the modal participation and rigid body mass matrices, respectively.

#### MSC/NASTRAN Implementation

The above outlined formulation has been implemented as a DMAP-alter for Solution 3 (Normal Modes Analysis). This formulation is useful, in general, for purely MSC/NASTRAN response analyses since (1) the user need not specify a big foundation mass (BFM) to apply base motion excitations and (2) mode acceleration-type accuracy is achieved with a mode displacement approach.

In order to specifically accommodate interactive response analysis in MSC/STI-VAMP, all necessary modal matrices are placed on an OUTPUT4 file to be read by the ITAP database utility. The following matrices are included in the OUTPUT4 file:

- (a) modal frequencies,  $\begin{Bmatrix} f_1 \\ \vdots \\ f_n \end{Bmatrix}$
- (b) modal displacements (selected output set),  $[\phi_g]$
- (c) rigid body displacements (selected output set),  $[R_g]$
- (d) modal excitation matrix,  $[\phi_g^T P_{gs_2}]$
- (e) modal participation matrix,  $[R_g^T M_{gg} \phi_g]$
- (f) rigid body mass,  $[R_g^T M_{gg} R_g]$
- (g) modal member loads,  $[\phi_F]$
- (h) modal member stresses,  $[\phi_\sigma]$

The user is prompted later in the ITAP database utility to provide modal damping data  $\begin{Bmatrix} \zeta_1 \\ \vdots \\ \zeta_n \end{Bmatrix}$  .

Since MSC/NASTRAN and MSC/STI-VAMP analyses are often performed on different computers, the above OUTPUT4 data may be cast in BCD rather than Binary form for computer-to-computer data transfer.

## Description of Frequency Response Functions

The generalized signal analysis capability in MSC/STI-VAMP is based to a large extent on the Fast Fourier Transform (FFT) and modern spectral analysis methods<sup>(5)</sup>. An efficient approach to computation of dynamic response resulting from harmonic, transient and random environments for linear systems is based on these methods, rather than numerical integration. The key data which is needed to proceed with the modern FFT/spectral analysis approach is system frequency response functions.

In the case of a structural dynamic system with multiple applied excitations the modal frequency response functions are

$$\xi_{KL}(f) = \frac{(2\pi f_k)^{-2}}{1-(f/f_k)^2 + i(2\zeta_k f/f_k)} [\phi_g^T P_{gs_2}]_{KL} \quad (11)$$

where "K" denotes the mode number, "L" denotes the excitation number and "i" represents  $\sqrt{-1}$ . The reader is reminded that the excitations may be applied forces, foundation input acceleration or both. In MSC/STI-VAMP, signals are typically processed over a frequency band,  $0 < f < f^*$ , where  $f^*$  corresponds to the user selected Nyquist frequency<sup>(5)</sup>. For the present application,  $f^*$ , should correspond to a frequency at or above the dynamic range of excitations<sup>(3)</sup>. Generally, all modes (including residuals) with natural frequencies above  $f^*$ , must be viewed as behaving quasi-statically, thus the denominator in equation (11) is taken as unity for such modes.



Frequency response functions associated with physical responses of interest are formed as

$$\begin{aligned}
 \hat{U}_{IL}(f) &= \sum_K \phi_{IK} \xi_{KL}(f) \\
 \sigma_{JL}(f) &= \sum_K \phi_{\sigma JK} \xi_{KL}(f) \\
 F_{JL}(f) &= \sum_K \phi_{F Jk} \xi_{KL}(f)
 \end{aligned} \tag{12}$$

where I and J correspond to the indices of response quantities of interest. When acceleration response is of interest, modal accelerations are required which are simply " $-(2\pi f)^2$ " times the modal displacements,  $\xi_{KL}(f)$ . The acceleration frequency response functions are

$$\ddot{U}_{IL}(f) = \sum_K \left( \phi_{IK} \ddot{\xi}_{KL}(f) \right) + R_{IL} \tag{13}$$

where the  $R_{IL}$  term is needed only when the particular excitation is a foundation acceleration. Similarly, the foundation reaction frequency response functions are, for cases with base motion input,

$$\begin{aligned}
 F_{\text{base}JL}(f) &= \left( \sum_K [R_g^T M_{gg} \phi_g]_{JK} \ddot{\xi}_{KL}(f) \right) \\
 &+ [R_g^T M_{gg} R_g]_{JL}
 \end{aligned} \tag{14}$$

The above noted functions are computed in MSC/STI-VAMP using command sequences generated in a new ITAP database utility option. These frequency response functions generally satisfy engineering requirements for study of subject system's frequency response characteristics.

#### Calculation of Transient Response

Transient response to applied excitation is readily calculated for a linear dynamic system by exploiting the convolution property<sup>(5)</sup>. In particular, given an excitation history,  $P_L(t)$ , its Fourier transform,  $P_L(f)$ , can be used to compute the Fourier transform of dynamic response

$$\hat{U}_I(f) = \hat{U}_{IL}(f) P_L(f) \quad (15)$$

The corresponding time history is calculated as the inverse FFT in MSC/STI-VAMP. If several excitations are present, then the total response is calculated as the sum of individual responses to all input histories.

When using the above strategy to determine response transients, special attention must be given to initial conditions. In general, initial conditions for a system described by equation (1) consist of a complete specification of initial displacement and velocity. For nearly all practical situations, however, there are two generic initial condition categories, namely, (a) the system is initially at rest (zero initial conditions) or (b) the system is initially in a state of static equilibrium (flexible body deformation balances applied loads).

Since FFT operations inherently assume that time dependent functions are periodic, specialized considerations must be made to satisfy cases (a) and (b) noted above. If we define an excitation history over a time window of length  $T$ , where the duration of interest is  $T/2$ , we can tailor the excitation to satisfy requirements associated with zero initial conditions or static equilibrium initial conditions. This is best illustrated with a simple example as noted by the force histories in Figure 2. For zero initial conditions the square wave defined over the period  $T$  suffices. For static equilibrium initial conditions we define the actual square wave over  $0 < t < T/2$  and define an artificial jump to the value at  $t = 0$  somewhere within the interval  $T/2 < t < T$ . The resulting time history responses for a 1 HZ single degree-of-freedom oscillator with  $\zeta_n = 0.05$  are illustrated in Figure 3. It is clear that appropriate tailoring of excitation histories permits satisfaction of the two important initial condition cases.

### Response to Random Excitation

There are a variety of classifications of random excitation environments which may be investigated in MSC/STI-VAMP. A large subset of problems which are of interest fall within the category of ergodic, multiple random excitation<sup>(5)</sup>. These excitation environments are generally specified in terms of an input spectral density matrix

$$[G_{xx}(f)] = \begin{bmatrix} G_{11}(f) & G_{12}(f) & \dots \\ G_{21}(f) & G_{22}(f) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} \quad (16)$$

where  $G_{ij}(f)$  is the one-sided autospectral density function associated with the  $i$  th excitation and  $G_{ij}(f)$  is the one-sided cross-spectral density function associated with the  $i$  th and  $j$  th excitations. In terms of discrete Fourier transforms, the cross-spectral density function among inputs denoted by  $X_i(f)$  and  $X_j(f)$  is defined as

$$G_{ij}(f) = \frac{2}{T} E[X_i(f)X_j^*(f)] \quad (17)$$

where  $E$  denotes the "expected value" and  $T$  is a selected sampling interval<sup>(5)</sup>.

In many cases investigators arbitrarily assume that the individual environments are uncorrelated (i.e.  $G_{ij}(f) = 0$  at all  $f$  for  $i \neq j$ ). When this is not actually the case, incorrect system random response may be obtained. When evaluating the possible linear relationship among any two excitations, the ordinary coherence function

$$\gamma_{ij}^2(f) = \frac{G_{ij}(f)G_{ji}(f)}{G_{ii}(f)G_{jj}(f)} \quad (18)$$

provides a useful measure. Ordinary coherence is a real valued function bounded by  $0 < \gamma_{ij}^2(f) < 1$ . If the value is zero at all frequencies, then the two excitations are uncorrelated. Alternatively, if  $\gamma_{ij}^2(f) = 1$ , at a particular frequency, then the two excitations can be fully linearly related at that frequency.

Response to ergodic random excitation is calculated for a particular output using the appropriate frequency response function for an output "Y" due to input "L" as  $H_{YL}(f)$ . The response autospectrum is calculated for an N-input system by the equation

$$G_{YY}(f) = \sum_{L=1}^N \sum_{M=1}^N H_{YL}(f) G_{LM}(f) H_{YM}^*(f) \quad (19)$$

where the "\*" superscript denotes the complex conjugate function. Since the input spectral density matrix is Hermitian, the output autospectrum is always a real, positive valued function.

A quantity of interest in random response analysis is the mean square value of response,  $\psi_Y^2$ . This value may be determined from the autocorrelation function,  $R_{YY}(\tau)$ , which is the inverse Fourier transform of the autospectrum,  $G_{YY}(f)$ ; in particular,

$$\psi_Y^2 = R_{YY}(0) \quad (20)$$

and the RMS response is  $\psi_{YY}$ . Many other useful relationships are to be found in Reference 5. In particular,  $\psi_Y^2$  can be estimated from calculated discrete values of the  $G_{yy}(f)$  terms in equation (19).

#### MSC/STI-VAMP Implementation

The mathematical operations required to calculate frequency response functions, transient response and random response are invoked by specification of sequences of two-letter commands<sup>(6)</sup>. When the required operations follow

standard patterns with variations for specific dynamic systems, automated command sequences may be generated, and these are invoked in MSC/STI-VAMP using a "@" followed by the command sequence name.

The ITAP database utility contains a variety of generic command file generators for quick-look data analysis and multi-input/multi-output random data analysis. At present, work is being finalized on a new command file generator option to prepare sequences for frequency response, transient response and random response post-processing.

### Illustrative Example Analysis

The performance of MSC/STI-VAMP as a dynamic response post-processor, as well as the modified MSC/NASTRAN Solution 3 sequence, is demonstrated in the simple example described herein. Consider the cantilevered beam of length, 120 in., and flexural stiffness,  $EI = 10.433 \text{ lb-in}^2$  and mass distribution,  $\rho A = 2.185 \times 10^{-4} \text{ lb-sec}^2/\text{in}^2$  illustrated in its first "Y" bending mode in Figure 4. Applied excitations to this structure were specified using the "LSEQ" option at grid point 4 (36" from the base) and as lateral "Y" acceleration at the base. A frequency range of  $0 < f < 10 \text{ HZ}$  was selected and a single lateral "Y" mode at 2.67 HZ was found in this range. Quasi-static residual vectors in the frequency range above 10 HZ are illustrated in Figures 5-7. These vectors numbering "7" are associated with the "Y" force at grid point 4 and six rigid body motions associated with base excitation. The three illustrated residuals are the only ones required to complete the static response description of the beam as noted by inspection of  $[\phi_g^T P_{gs_2}]$  in Table 1.

On the basis of the MSC/NASTRAN computed modal data, frequency response functions were calculated in MSC/STI-VAMP. The functions are associated with Y-displacement (relative) at grid point 4 and bending moment at the same location due to two inputs, namely an applied force at grid point 4 and a base Y-acceleration. The four resulting functions are illustrated in Figures 8-11. It is interesting to note the effect of residuals on these functions. The functions plotted as dashed lines are those formed using only modes in the 0-10 HZ range. The significance of residuals becomes very clear when transient response is calculated.

As a first transient response case, the beam is excited by an applied gradual step input force illustrated in Figure 12. The response spectrum associated with this excitation, given in Figure 13, clearly indicates that nearly all "dynamic" activity is in a frequency band below 3 HZ<sup>(3)</sup>. Using the frequency response function estimates with and without residual effects from Figure 8, the resulting lateral displacement response given in Figure 14, clearly indicates the importance of residuals. Without the use of residuals, the appropriate static displacement is not achieved.

In a second transient response case, the appropriate calculation procedure satisfying static equilibrium initial conditions is illustrated. The step-down oscillatory force excitation illustrated in Figure 15 results in lateral displacement and bending moment histories given in Figures 16 and 17, respectively, which exhibit the appropriate static behavior prior to "step-down".

The final example transient problem exhibits the system response to base excitation as calculated in MSC/STI-VAMP. The short-duration lateral-Y base acceleration given in Figure 18 has an associated acceleration shock spectrum (Figure 19) which indicates that the majority of dynamic activity is below 8 HZ<sup>(3)</sup>. Relative displacement, absolute acceleration and bending moment histories for the beam are illustrated in Figures 20-22, respectively.

An idealized example of multi-input random response analysis illustrates a small portion of the MSC/STI-VAMP capabilities in such applications. The input spectral density matrix provided in Figure 23 describes an input environment with  $G_{11}(f)$  as the applied force input autospectrum and  $G_{22}(f)$  as the base acceleration input autospectrum. The cross-spectra  $G_{12}(f)$  and  $G_{21}(f)$  indicate the presence of some level of correlation between the two separate excitations. Coherence between the two inputs, following equation (18), is presented in Figure 24. An illustration of the components which combine to produce the output acceleration autospectrum at grid point 4 is presented in Figure 25. It is clear in this example that response due to the input cross-spectrum is non-negligible. Finally, the response autocorrelation function is presented in Figure 26 indicating the mean square acceleration response level.

### Concluding Remarks

This paper outlines the basic procedures to be included in a new MSC/STI-VAMP based post-processor which interactively calculates dynamic response of linear structural dynamic systems. Data required to perform frequency response, transient and random response consists primarily of enhanced modal output matrices from MSC/NASTRAN. Excitation environments are to be specified in MSC/STI-VAMP.



The modified Solution 3 sequence which is summarized in this paper contains several highly desirable features which are useful even in strictly MSC/NASTRAN based analyses. These features include (1) automatic inclusion of quasi-static residual effects in orthogonalized modal form, (2) elimination of the need for a big foundation mass (BFM) specification for base motion excitation analysis, and (3) explicit formulation of modal and transformation matrices.

The MSC/STI-VAMP post-processing capabilities, as illustrated in the cantilevered beam example, provide an efficient means for hands-on dynamic response studies. Among the features of this capability are (1) automatic satisfaction of practical initial conditions in transient response, (2) multi-input random and transient analysis methods, and (3) extensive library of functions for study of signal histories and spectra.

Both MSC/NASTRAN and MSC/STI-VAMP are codes which are structured to permit customized analysis as required by the user. The interactive command language in MSC/STI-VAMP is a welcome complement to MSC/NASTRAN's DMAP library. While the ITAP database utility serves as a package to integrate testing and analysis disciplines, the user has the option to develop customized procedures to fit his organization's particular needs.

#### References

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- 3) Coppolino, R.N., "Practical Considerations in Modal Transient Response Analysis and Response Spectrum Superposition," SAE Technical Paper Series, Paper No. 841581, October, 1984.
- 4) Coppolino, R.N., "Employment of Residual Mode Effects in Vehicle/Payload Dynamic Loads Analysis," Payload Flight Methodology Workshop Proceedings, NASA CP-2075, November, 1978, pp 323-346.
- 5) Bendat, J.S., and Piersol, A.G., Random Data Analysis and Measurement Procedures, 2nd ed., Wiley-Interscience, New York, 1986.
- 6) "MSC/STI-VAMP Operators Manual," 1986.

# SYSTEM DEVELOPMENT CYCLE

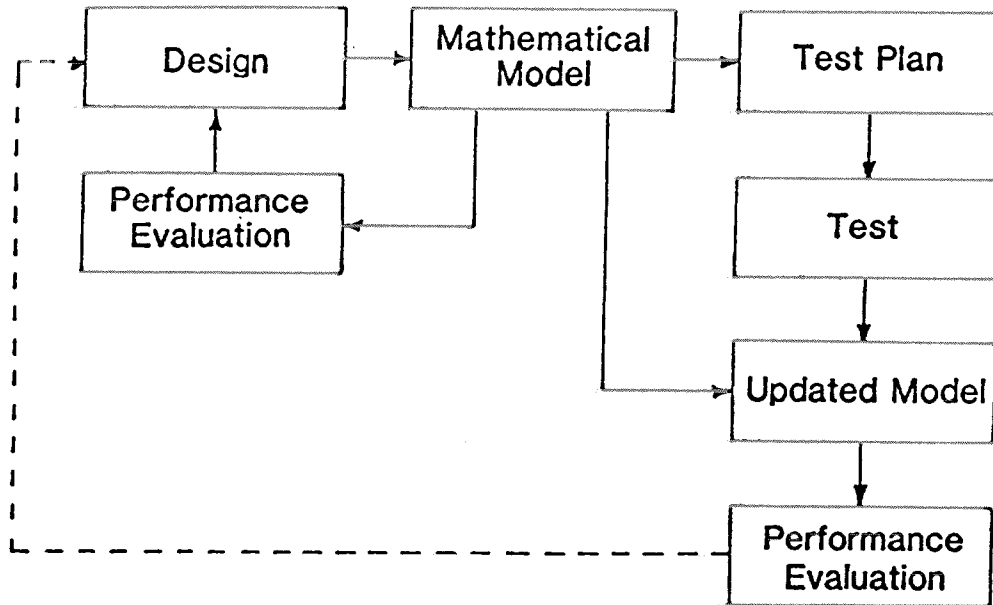


FIGURE 1

MSC/STI VAMP (VAX version)  
 FORCED RESPONSE OF A SDOF SYSTEM  
 FORCE HISTORIES

MSC/STI-VAMP  
 2-FEB-87  
 11:01:01

VAMP>

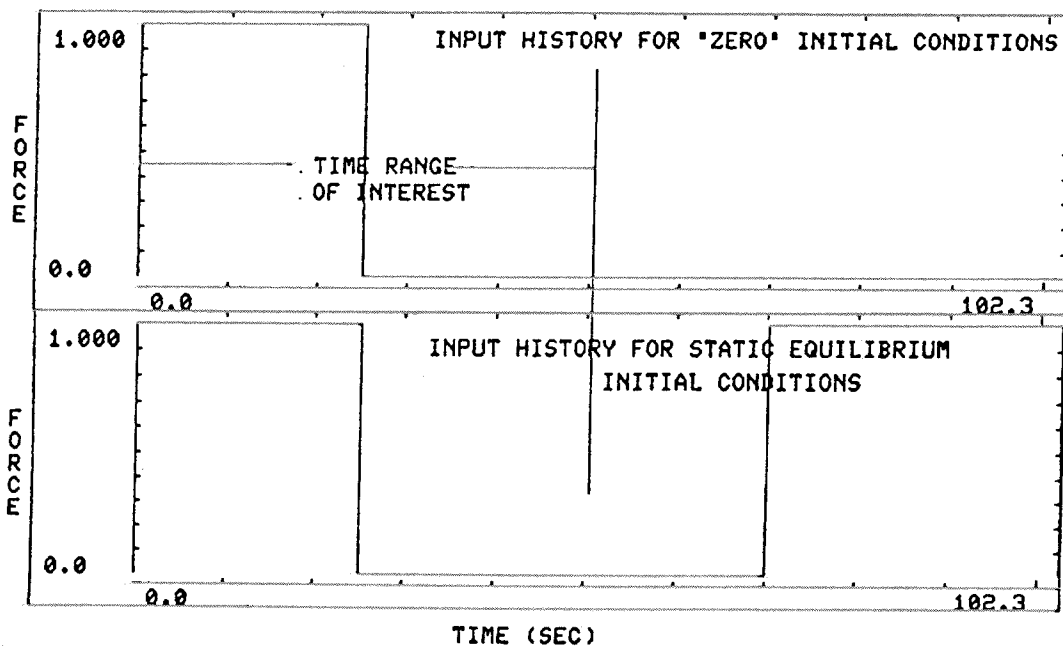


FIGURE 2

VAMP>

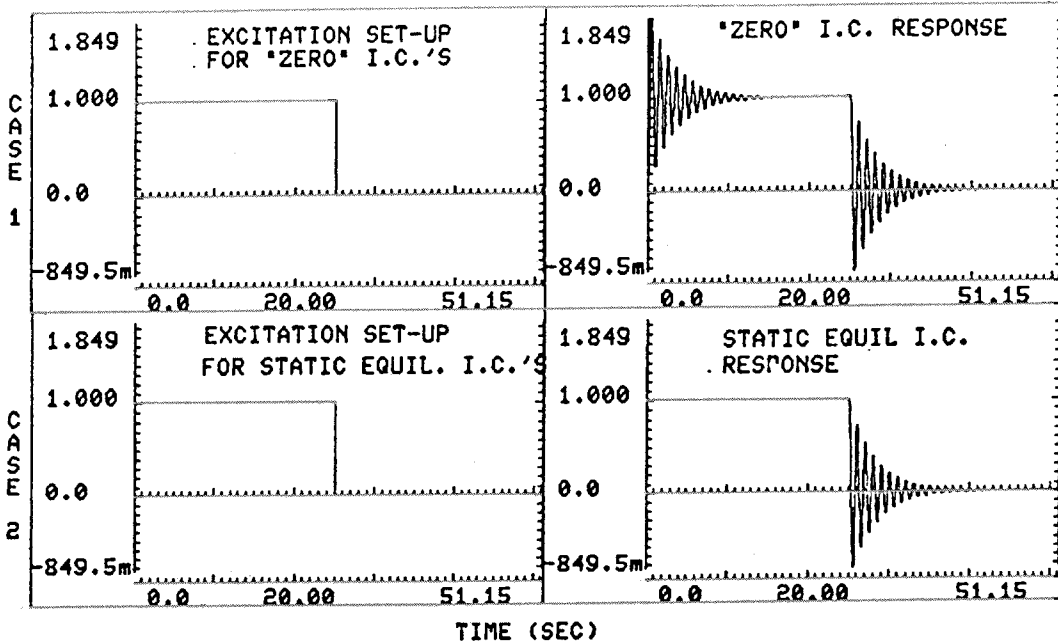


FIGURE 3

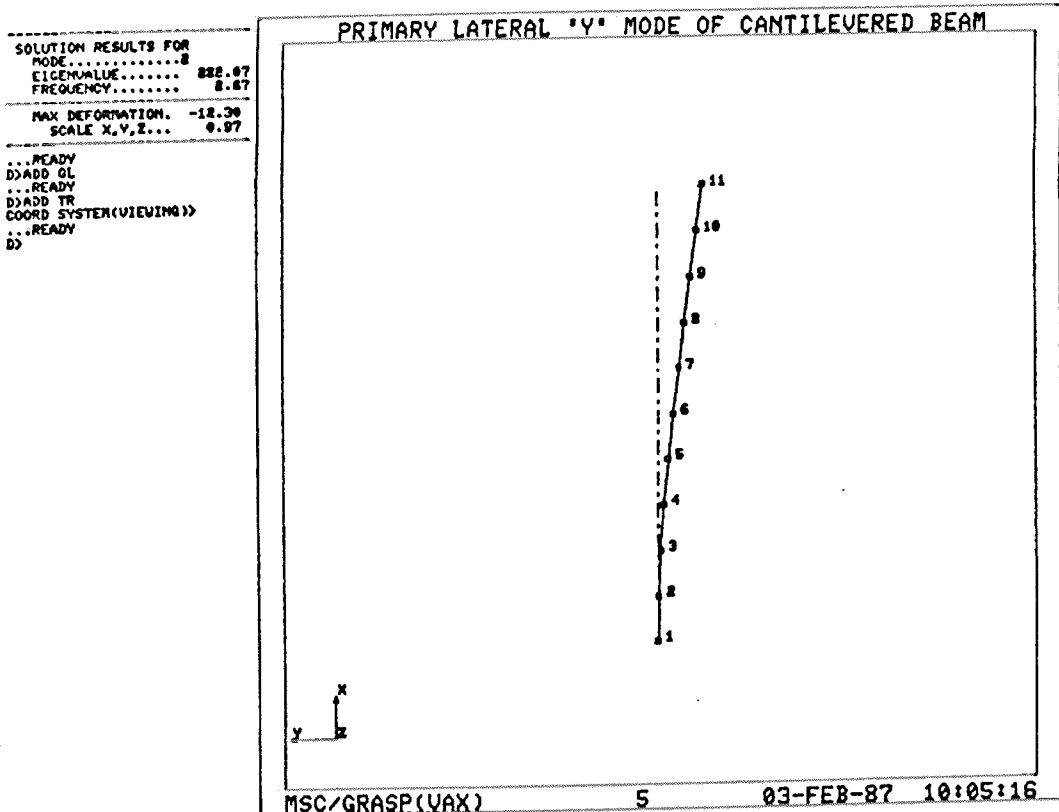


FIGURE 4

SOLUTION RESULTS FOR  
 MODE.....4  
 EIGENVALUE..... 1.08+04  
 FREQUENCY..... 18.57  
 ---  
 MAX DEFORMATION. 11.93  
 SCALE X,Y,Z... 1.00

...READY  
 D>ADD OI  
 ...READY  
 D>ADD TR  
 COORD SYSTEM(VIEWING)  
 ...READY  
 D>

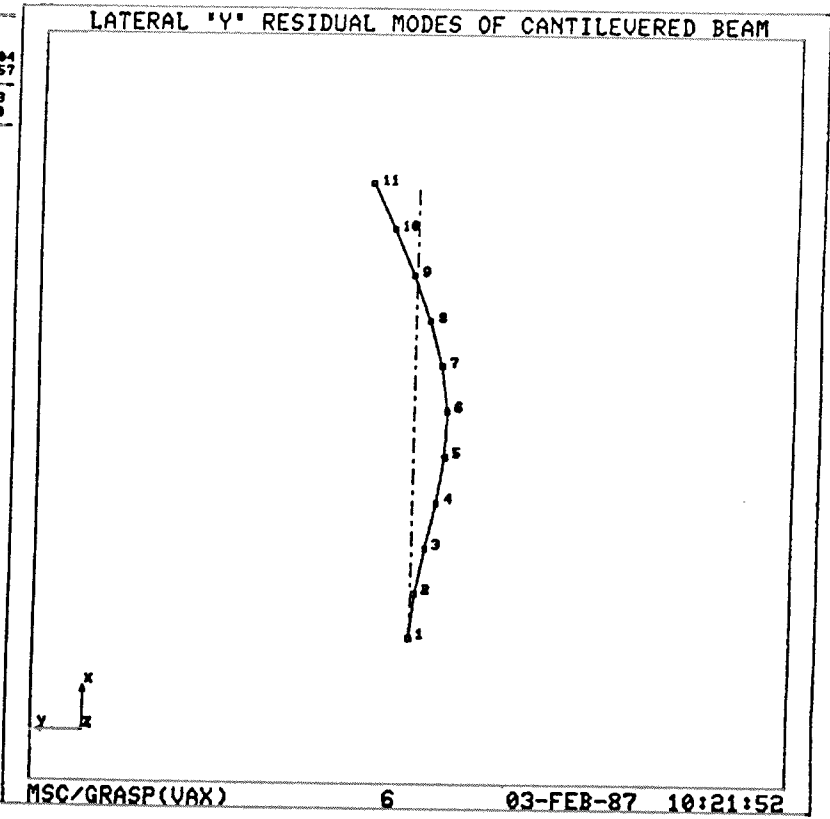


FIGURE 5

SOLUTION RESULTS FOR  
 MODE.....7  
 EIGENVALUE..... 9.49+04  
 FREQUENCY..... 49.63  
 ---  
 MAX DEFORMATION. -10.49  
 SCALE X,Y,Z... 1.14

...READY  
 D>ADD OI  
 ...READY  
 D>ADD TR  
 COORD SYSTEM(VIEWING)  
 ...READY  
 D>

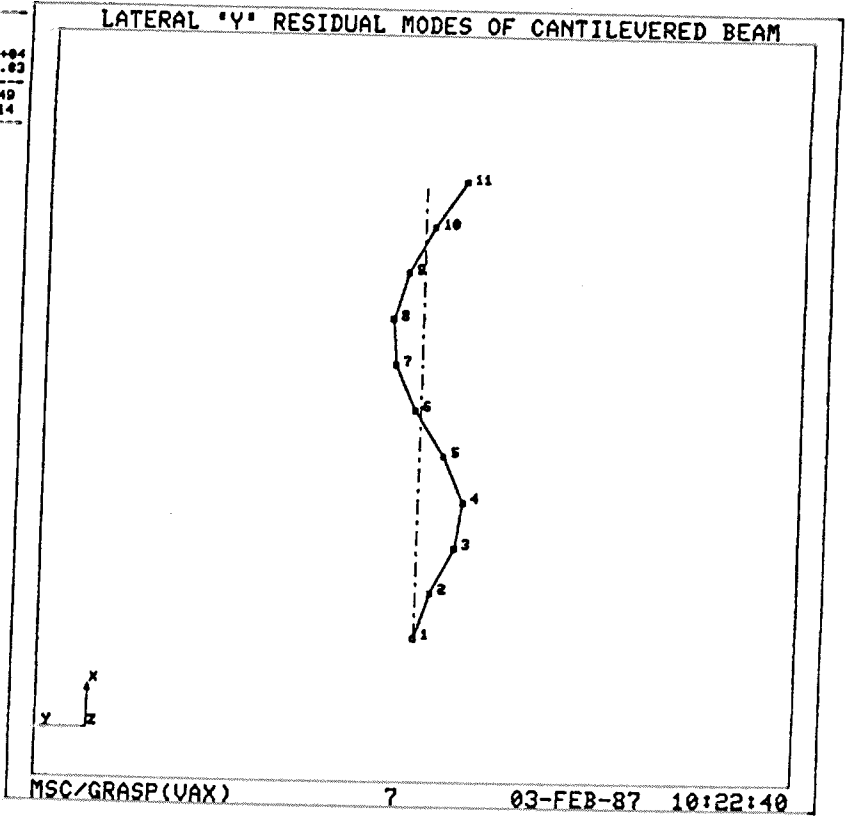


FIGURE 6

SOLUTION RESULTS FOR  
 MODE.....9  
 EIGENVALUE..... 1.00+06  
 FREQUENCY..... 166.25

MAX DEFORMATION. -18.32  
 SCALE X,Y,Z... 0.97

...READY  
 D>>ADD CL  
 ...READY  
 D>>ADD TR  
 COORD SYSTEM(VIEWING)>  
 ...READY  
 D>

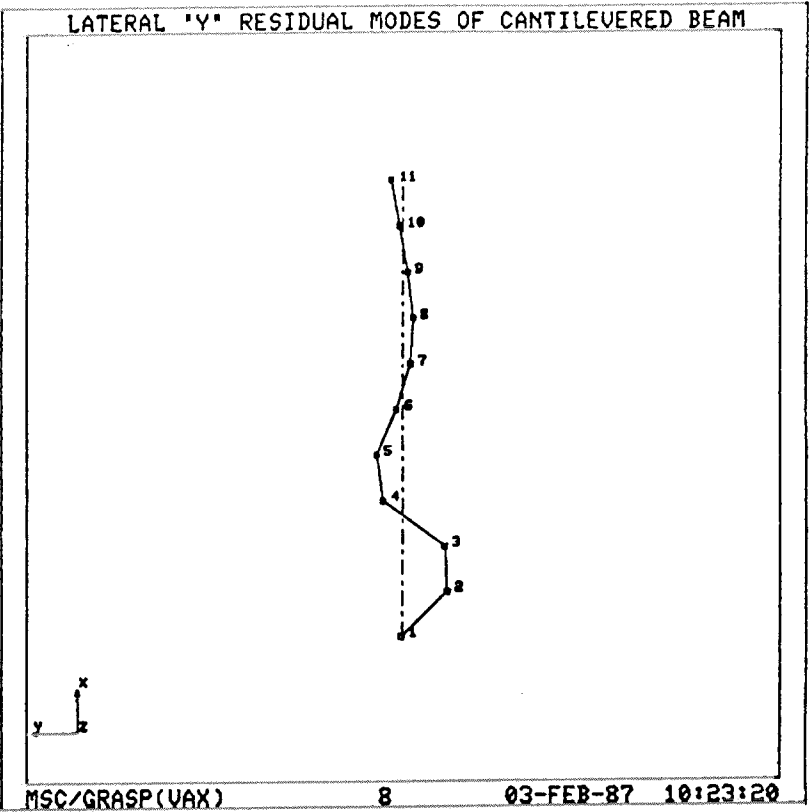


FIGURE 7

MSC/STI UAMP (VAX version)  
 FORCED RESPONSE OF A CANTILEVERED BEAM  
 FREQUENCY RESPONSE FUNCTIONS

MSC/STI-UAMP  
 29-JAN-87  
 16:57:43

VAMP>

Y-RESPONSE DUE TO APPLIED LOAD

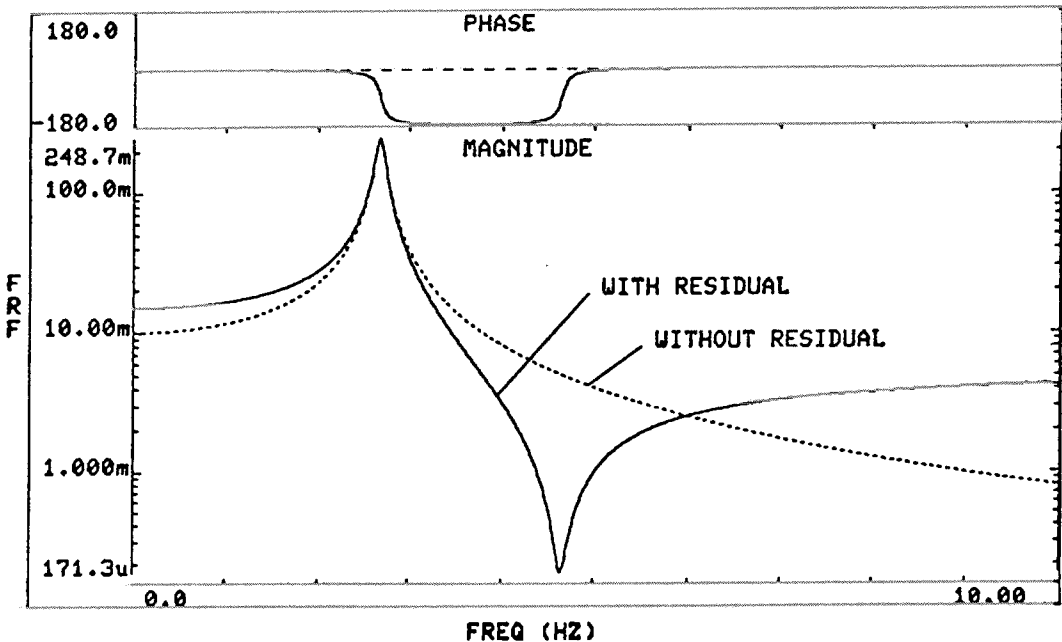


FIGURE 8

UAMP>

BENDING MOMENT DUE TO APPLIED LOAD

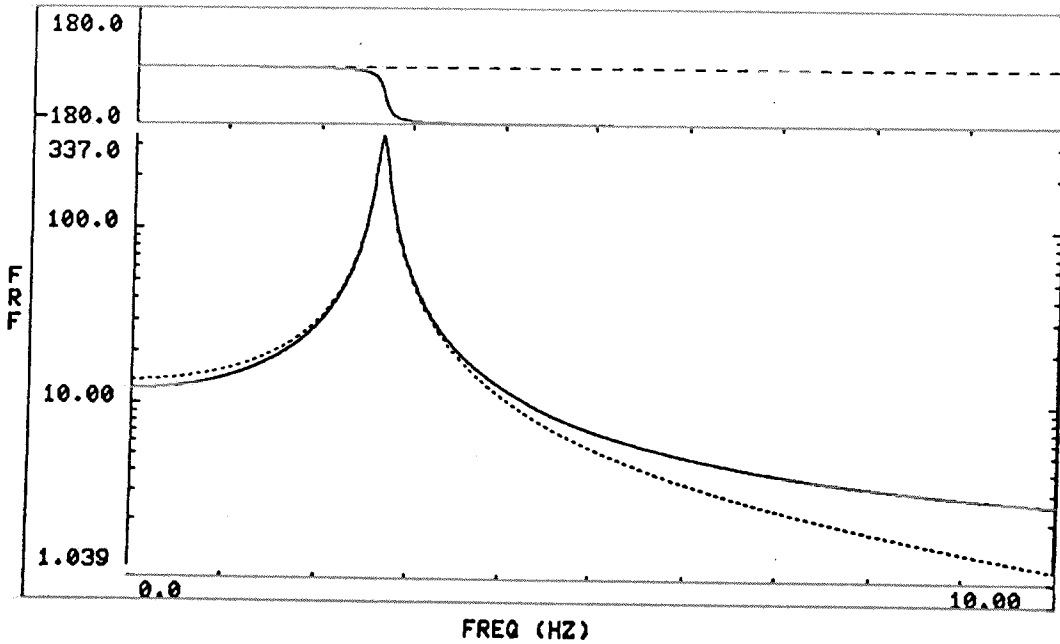


FIGURE 9

UAMP>

Y-RESPONSE DUE TO BASE ACCEL.

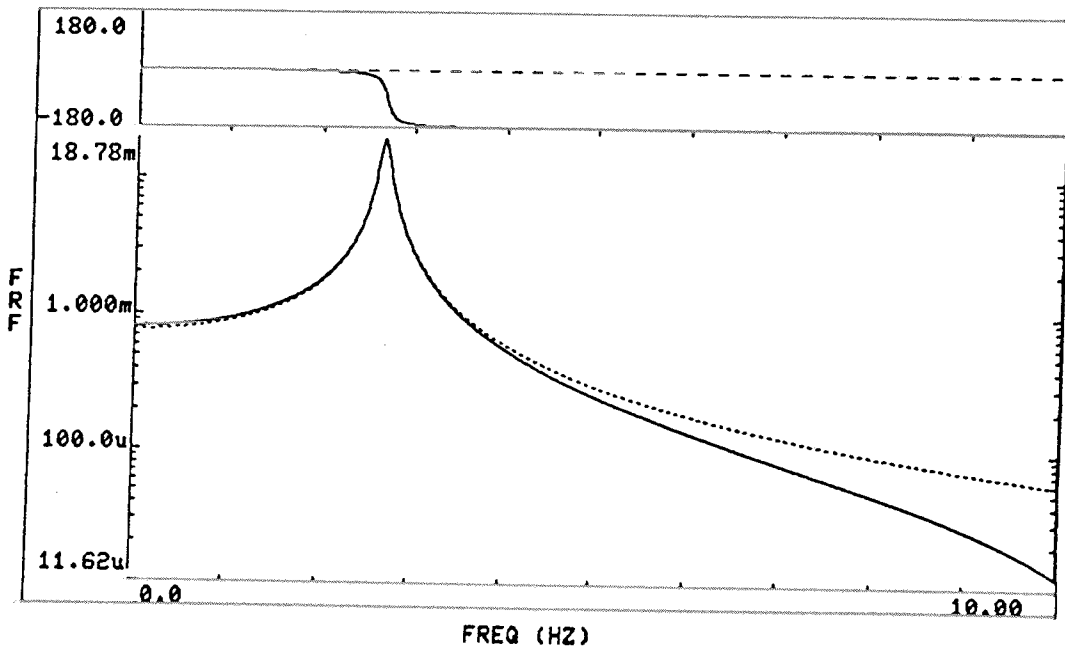


FIGURE 10

BENDING MOMENT DUE TO BASE ACCEL.

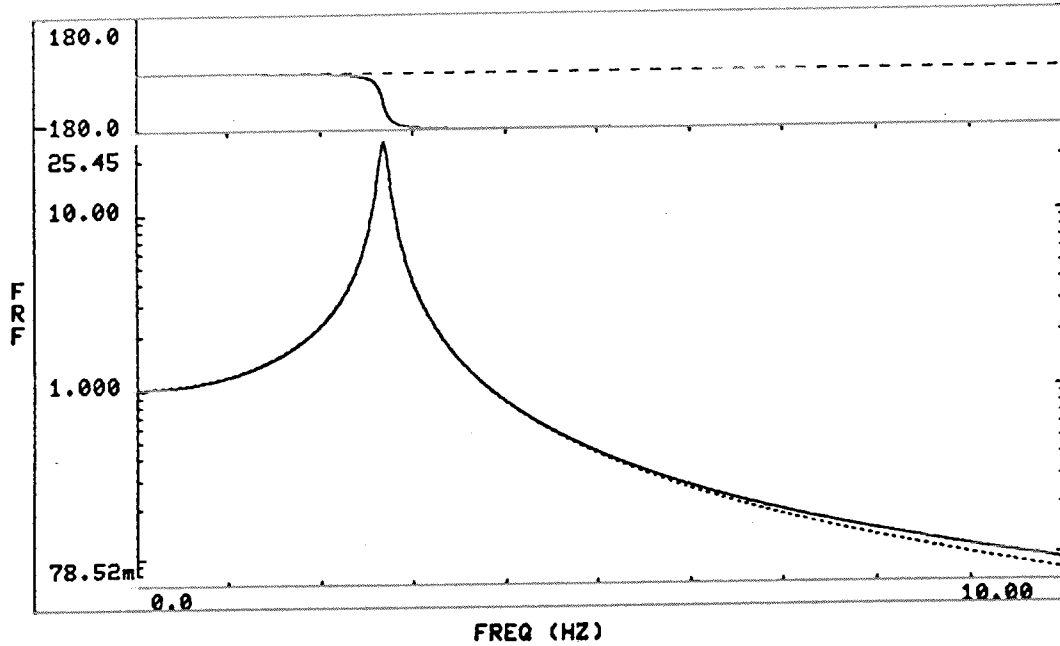


FIGURE 11

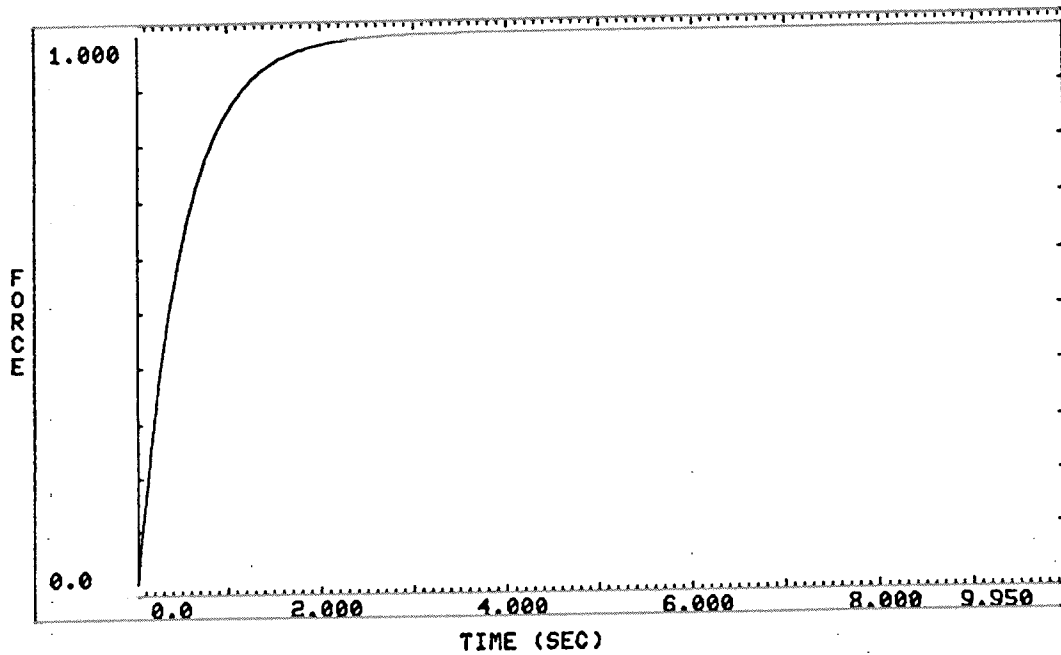


FIGURE 12



VAMP>

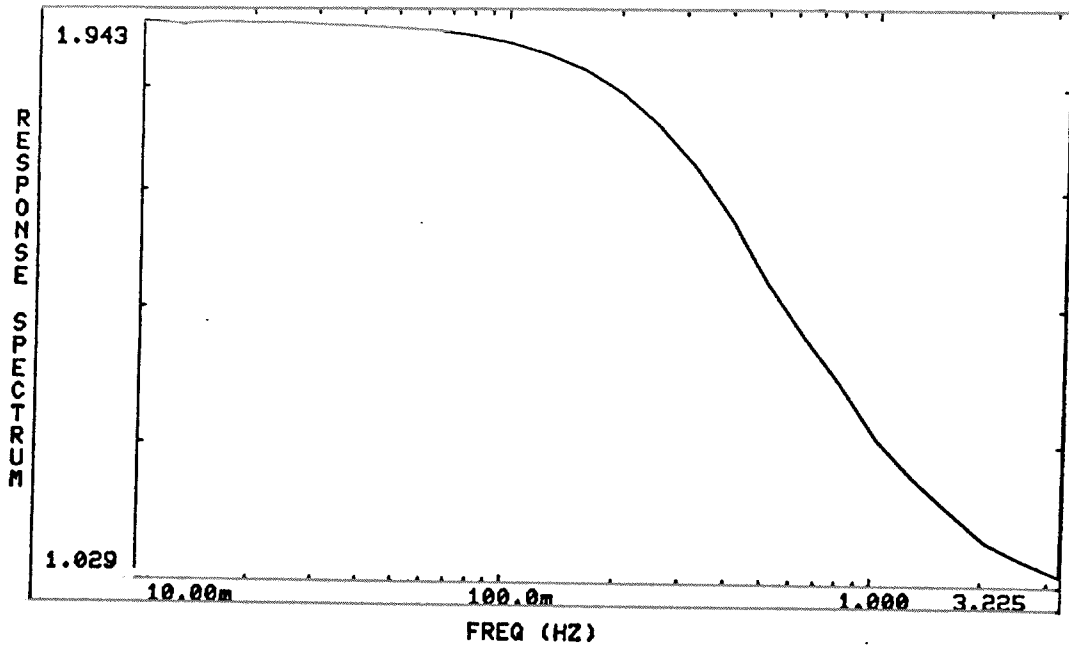


FIGURE 13

VAMP>

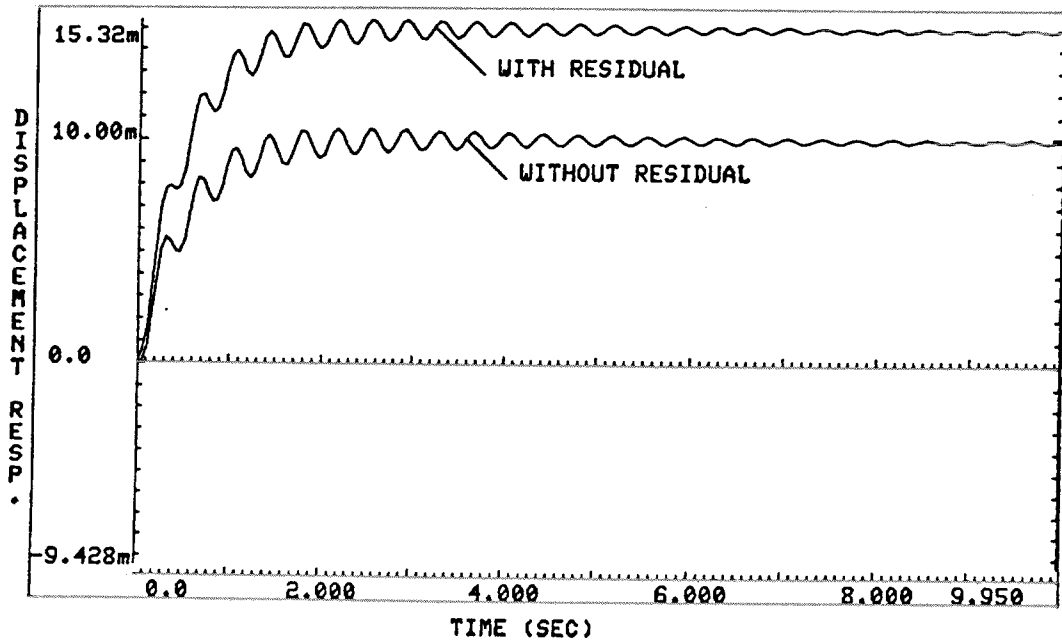


FIGURE 14

UAMP>

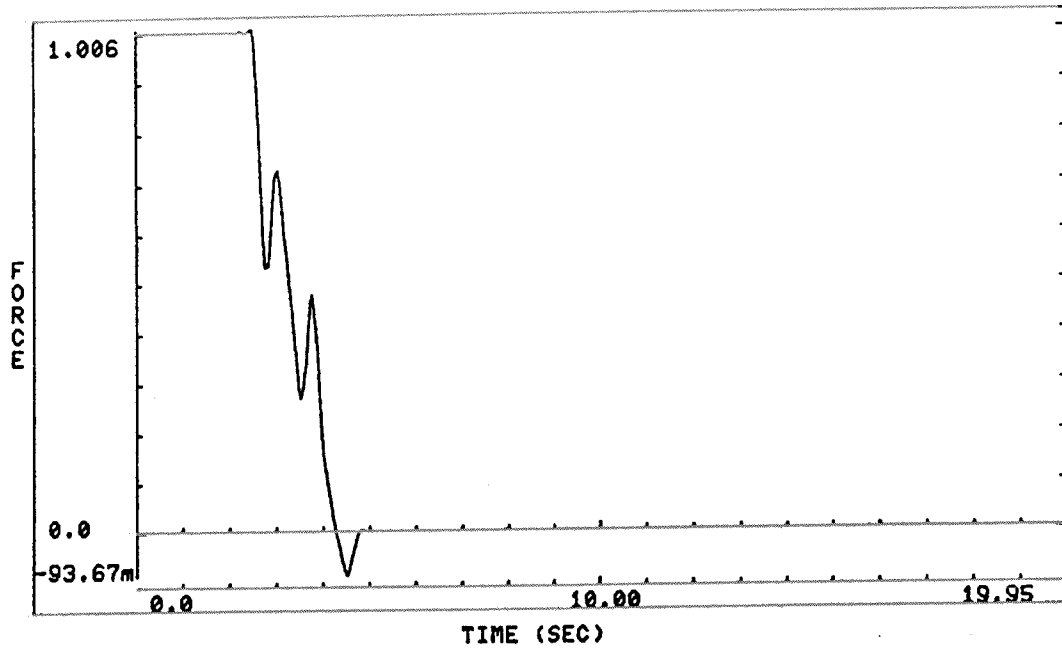


FIGURE 15

UAMP>

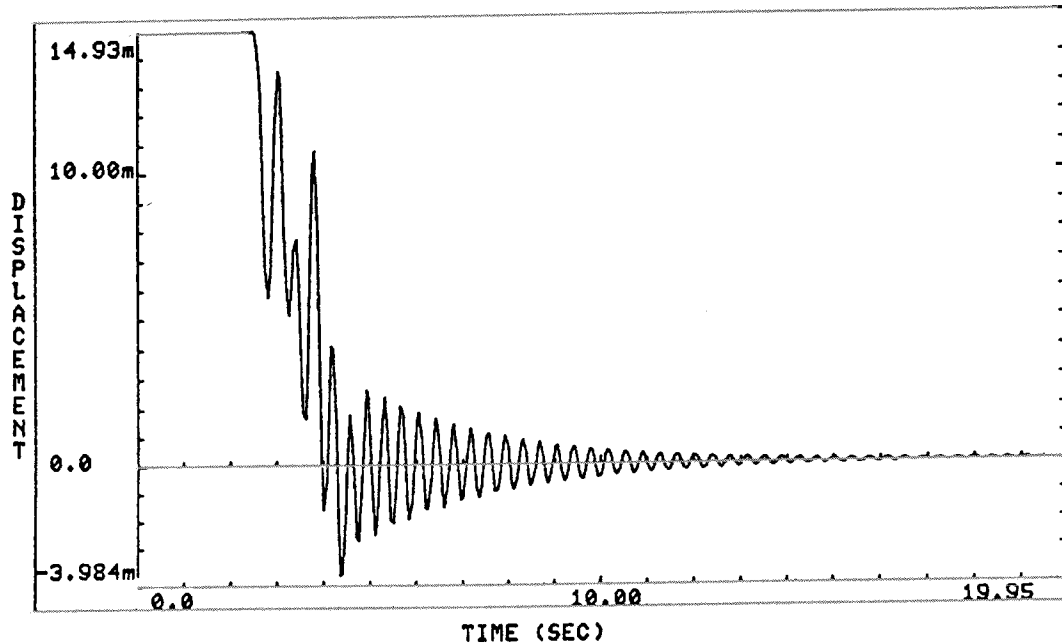


FIGURE 16

MSC/STI VAMP (UAX version)  
FORCED RESPONSE OF A CANTILEVERED BEAM  
RESPONSE TO STEP DOWN FORCE

VAMP>

MSC/STI-UAMP  
30-JAN-87  
13:21:43

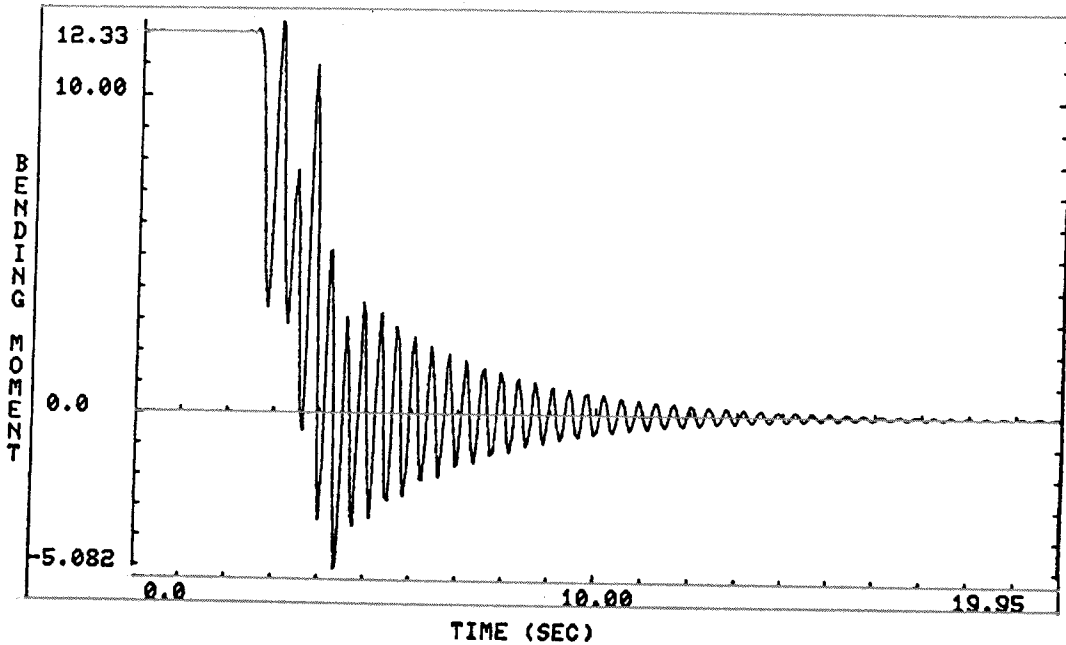


FIGURE 17

MSC/STI VAMP (UAX version)  
FORCED RESPONSE OF A CANTILEVERED BEAM  
INPUT LATERAL ACCELERATION

VAMP>

MSC/STI-UAMP  
30-JAN-87  
13:45:11

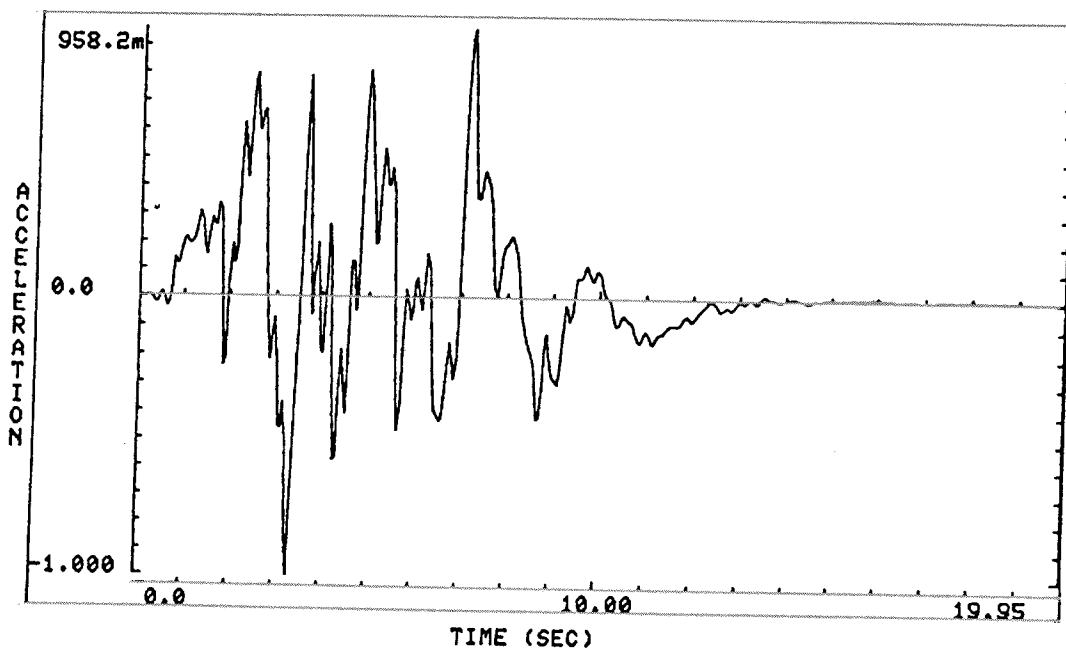


FIGURE 18

UAMP>

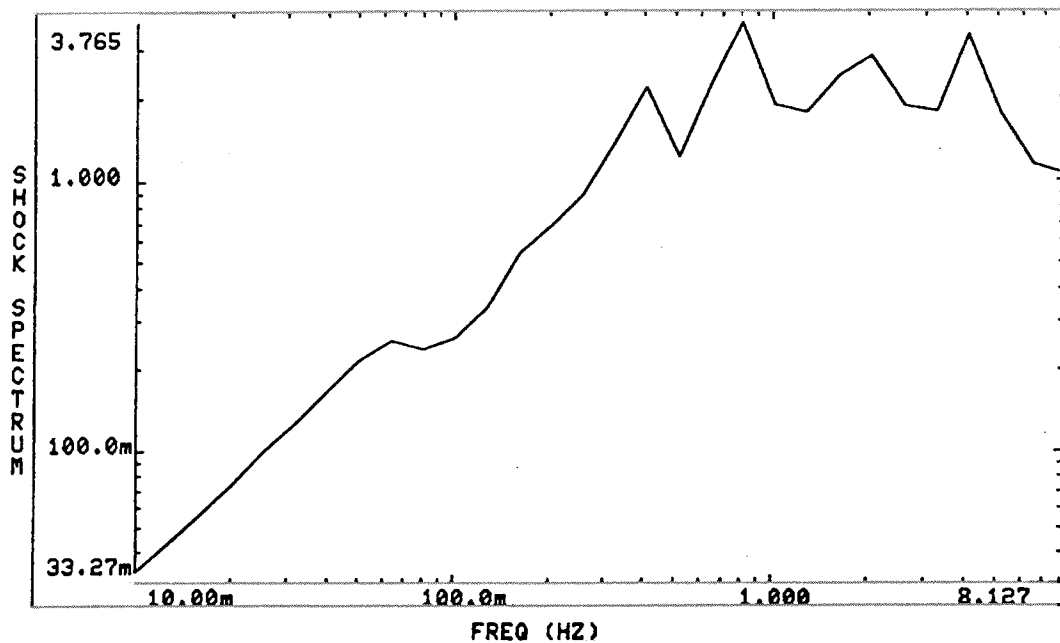


FIGURE 19

UAMP>

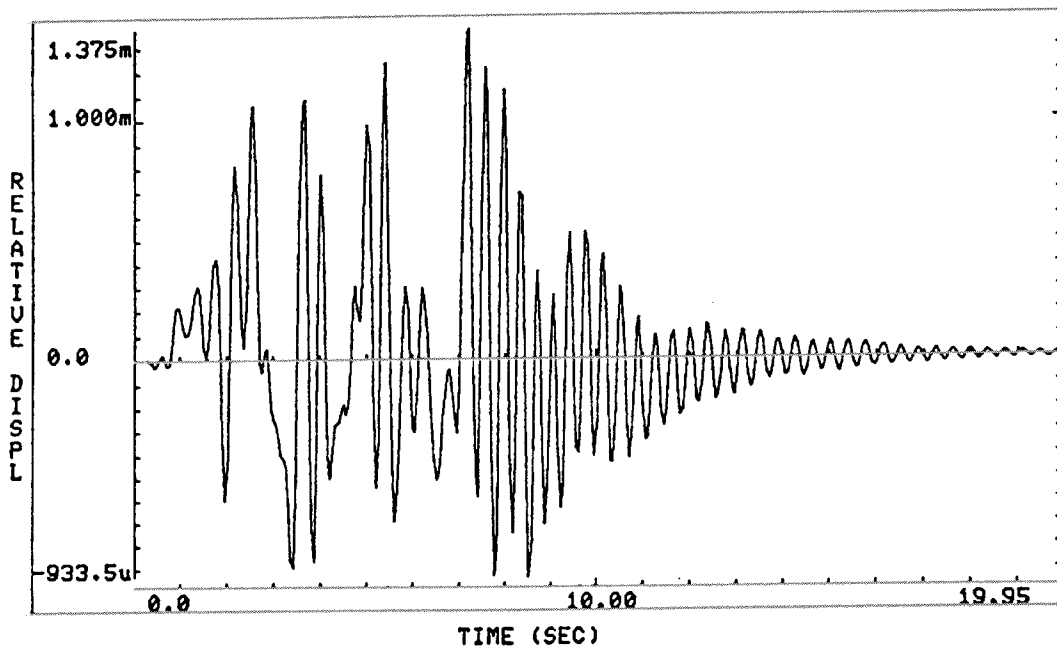


FIGURE 20

MSC/STI VAMP (UAX version)  
FORCED RESPONSE OF A CANTILEVERED BEAM  
RESPONSE TO BASE ACCEL INPUT

VAMP>

MSC/STI-UAMP  
30-JAN-87  
14:05:29

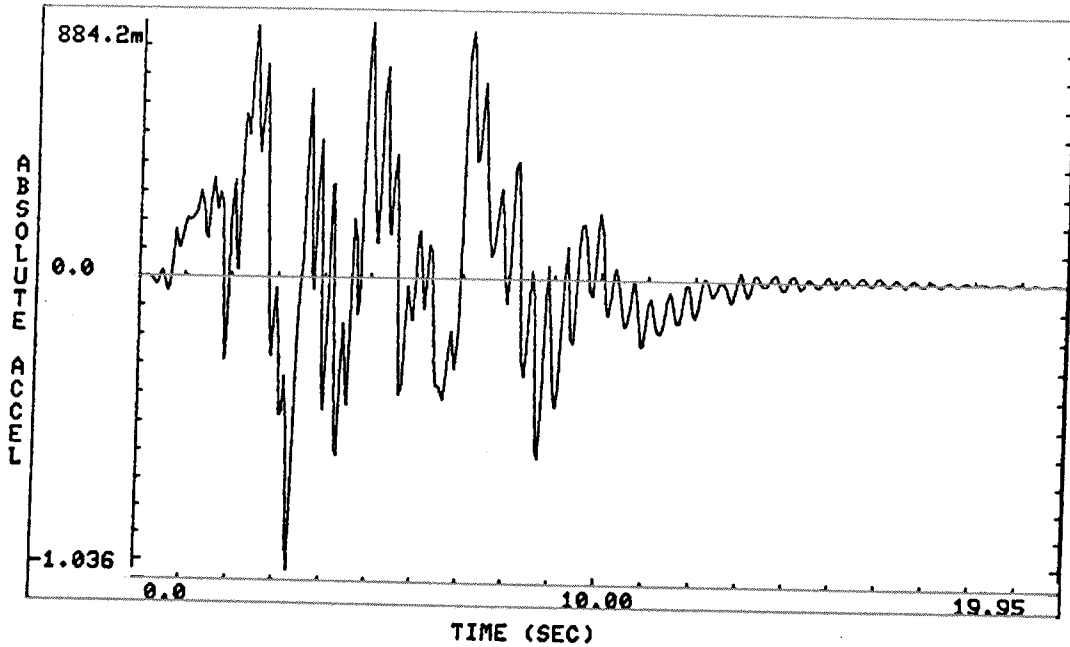


FIGURE 21

MSC/STI VAMP (UAX version)  
FORCED RESPONSE OF A CANTILEVERED BEAM  
RESPONSE TO BASE ACCEL INPUT

VAMP>

MSC/STI-UAMP  
30-JAN-87  
14:06:54

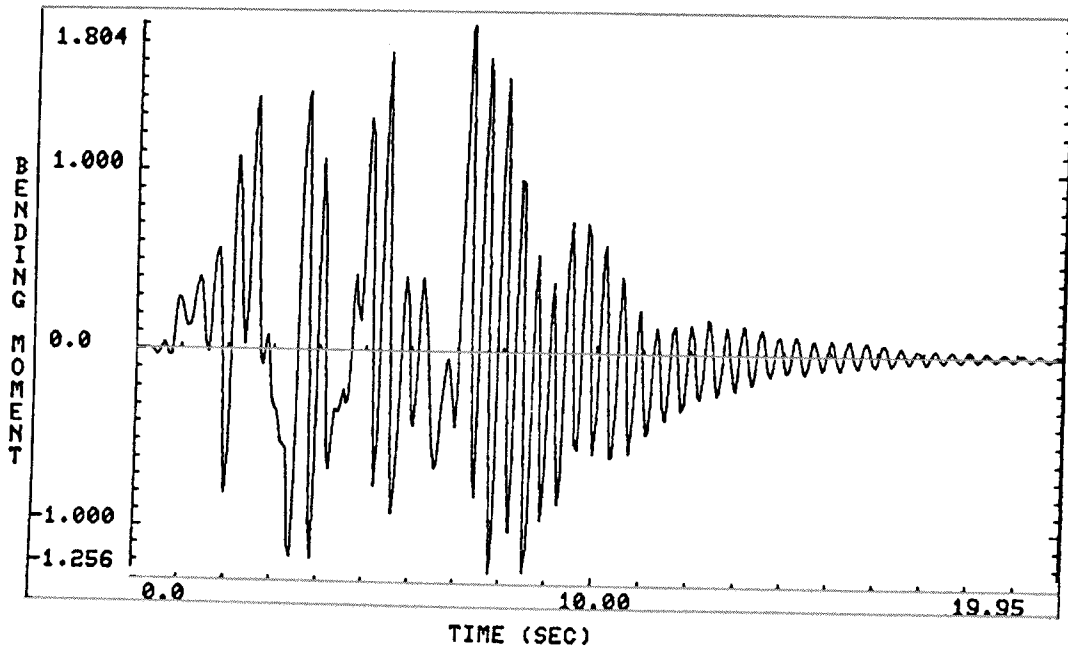


FIGURE 22

UAMP>

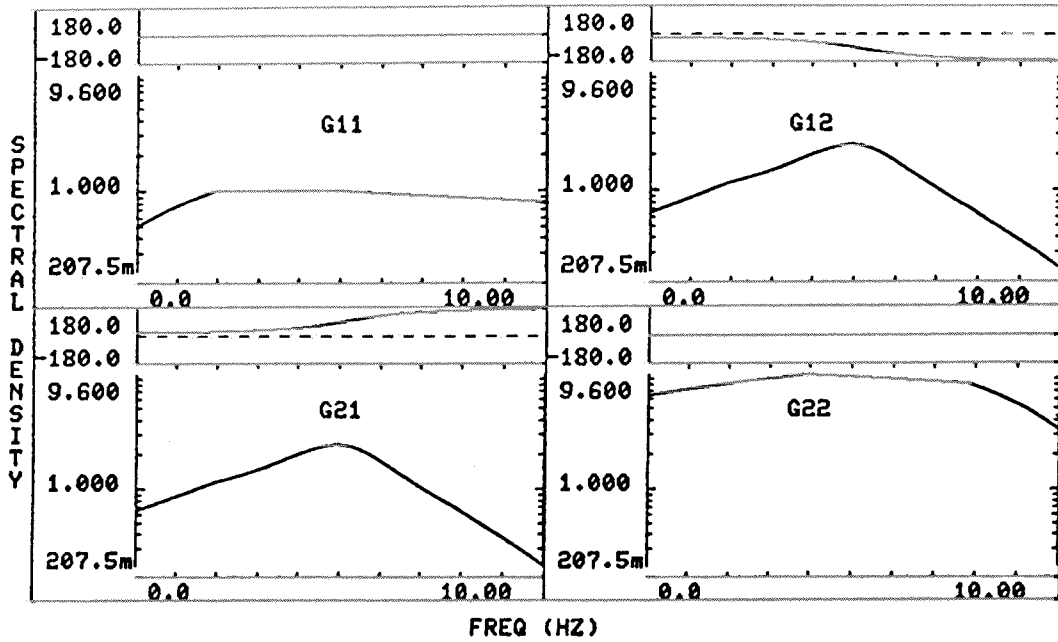


FIGURE 23

UAMP>

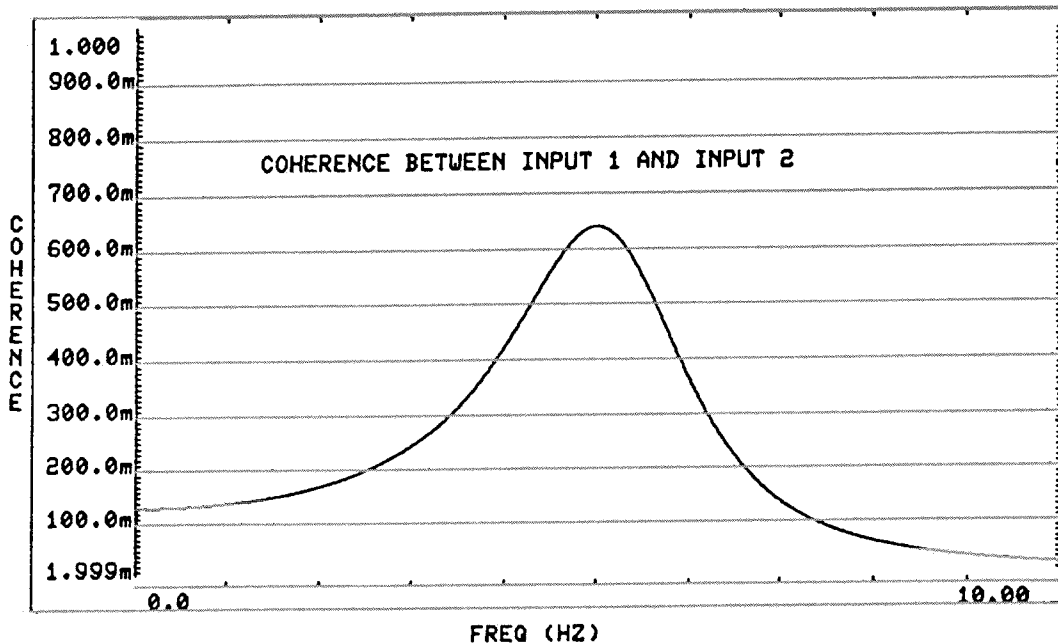


FIGURE 24

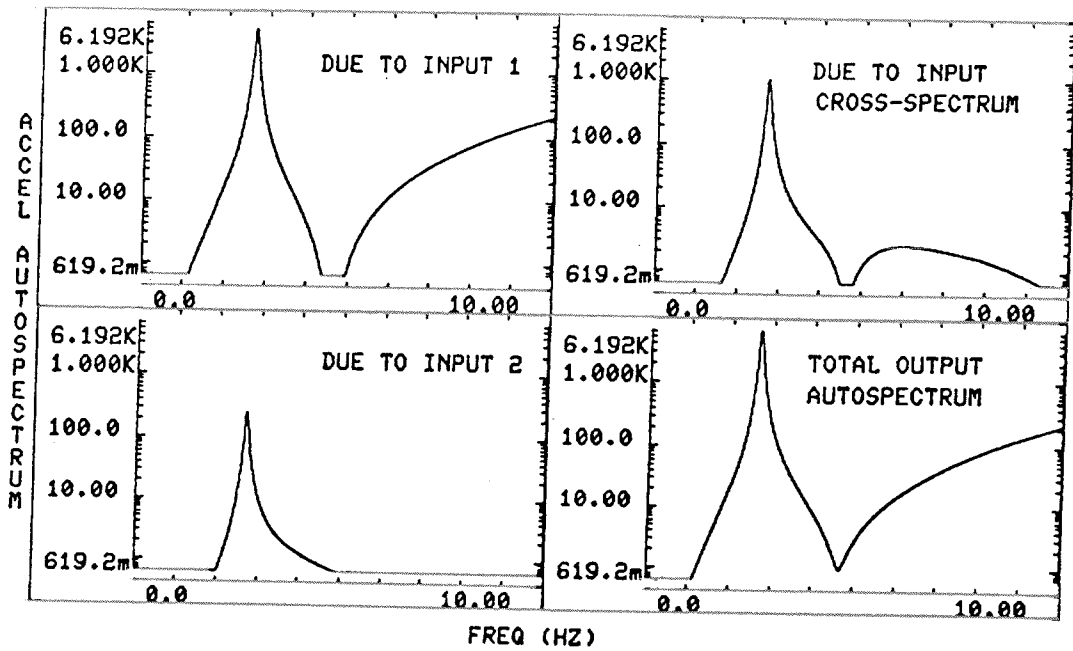


FIGURE 25

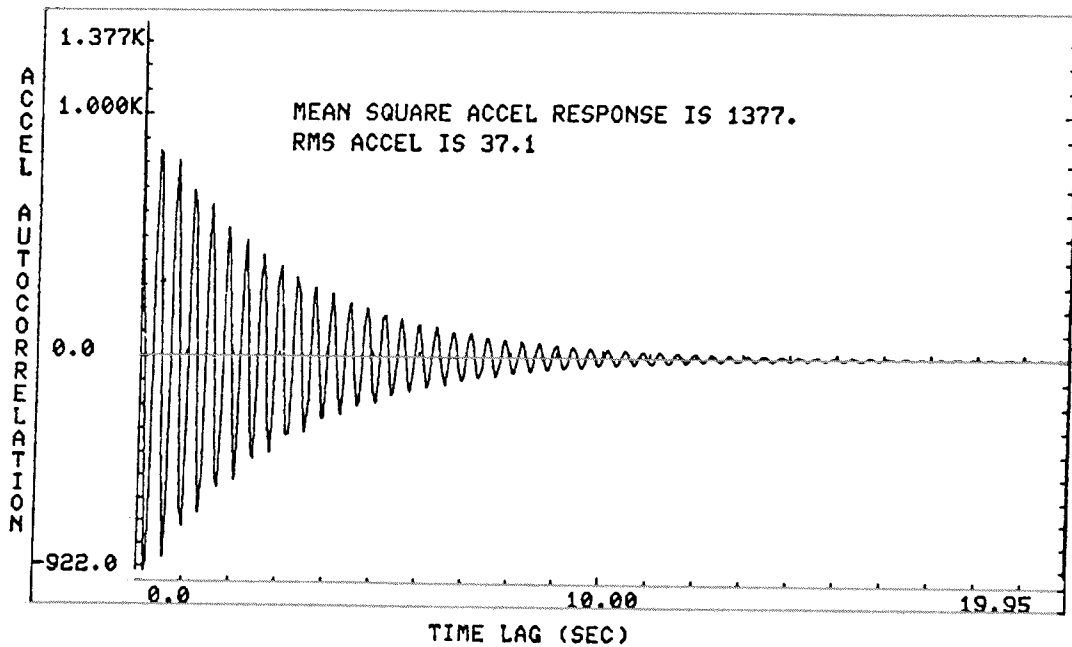


FIGURE 26

TABLE 1. MODAL AND RESIDUAL VECTOR SUMMARY

MODE	FREQ	F4Y-GAIN	ACCY-GAIN	PHI-4Y	MOM-Z
2	2.673	-1.675	-.01265	-1.675	-2270.
4R	16.570	-6.393	-.00704	-6.393	-1236.
7R	49.031	-10.498	-.00459	-10.498	27505.
9R	166.250	5.171	-.03902	5.171	175220.

NOTE: ALL OTHER MODES HAVE ZERO EXCITATION GAINS