Using Multi-Point Constraints with PAL2 for Detail Freebody Analysis

by
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Abstract

Typically, in the aerospace industry, freebody loads are extracted from an overall, coarsely meshed, finite element model, when a stress analysis is required on a structural detail. Since the structural detail freebody is generally a different size and has more detail than its representation in the overall model, the structural detail is not in equilibrium under these freebody loads, and the distribution of the freebody loads is not known. A finite element model, of the structural detail, employing multi-point constraints (MPC's), solves both problems. This paper discusses using MPC's to solve the above problems and derives the relations to implement MPC's in PAL2.

Introduction

A typical problem in stress analysis/design is sizing detail structure using freebody loads. Often these freebody loads are internal loads of a much larger and coarsely meshed structure. This is typical of aerospace structural analysis. Figure 1 illustrates this case.

In Figure 1, a wing torque box is shown loaded by lift and fixed at the fuselage. A wing torque box is a wing's basic load carrying structure. Each structural member is in equilibrium under a set of freebody loads which are actually torque box internal loads.

Consider the middle rib from the torque box as a freebody as shown in Figure 2. Figure 2a shows the middle rib exactly as it was represented in Figure 1 with the rib freebody loads which are torque box internal loads. However, Figure 2b shows the design details of the middle rib. Notice the detail freebody is actually a different size so that the freebody loads from Figure 2a do not balance the detail freebody due to the unbalanced moment. Additionally, there is a hole.

Figure 3 shows the rib freebody loads in their exact relative locations from Figure 2a (remember these loads are in equilibrium) and the detail freebody from Figure 2b properly located with respect to these loads. These loads must now be distributed from their equilibrium locations to the detail freebody points. The MPC load distribution property does this by connecting the freebody loads to the detail freebody points using a distribution scheme as shown in Figure 3 Detail B. In the two sections that follow, MPC's are defined, the distribution property derived, and a distribution scheme is discussed.

Defining MPC's and Deriving the MPC Load Distribution Property

MPC's are defined in eqn 1. As an example, consider Figure 4.

eqn 1 $\sum Ci*ui = 0.$

Figure 4 shows two truss members of equal length (L), area (A), and Young's modulus (E) joined at point 2. Since the members are linear displacement elements (i.e. constant load/strain), the displacement at point 2 is the average of points 1 and 3. Using eqn 1, this is expressed as eqn 2a.

eqn 2a $-1.*u2 + .5*u1 + .5*u3 = 0.$

MPC's are, therefore, linear combinations of displacements. Generally, one displacement is dependent on the remaining independent displacements. In this case, u2 is dependent on independent displacements u1 and u3.

A special case of eqn 1 involves only one term. This is called a single point constraint or SPC, and is often used to prevent rigid body motion. For instance, in Figure 4, point 1 could be fixed. This is expressed by eqn 2b.

eqn 2b $-1.*u1 = 0.$

To derive the MPC load distribution property, eqn 1 is multiplied by a constant QM and grouped with Ci to give eqn 3 [1].

eqn 3 $QM*\sum Ci*ui = \sum(QM*Ci)*ui = 0.$

One law of structural mechanics states that the external work done by a constraint on a structure in equilibrium is zero. Since ui is a displacement, and Ci is dimensionless, eqn 3 is a statement of this law if the product QM*Ci is a load. In fact, QM is a generalized load associated with a constraint equation (more generally called a Lagrange multiplier) and QM*Ci is a load at the displacement ui. This is the load distribution property of MPC's. SPC loads are determined similarly.

However, it still remains to find the value of QM so that QM*Ci can be calculated. To do this, consider eqn 4.

eqn 4 [PEXT] = [P] + [RM] + [RS] = [K][u]

where :

- [PEXT] is the set of the external loads on a structure summed at each point
- [P] is the set of applied loads on a structure
- [RM] is the set of constraint loads on a structure due to the MPC's
- [RS] is the set of constraint loads on a structure due to the SPC's
- [K] is the stiffness matrix
- [u] is the displacement matrix

Since QM*Ci are these constraint loads, [RM] and [RS] may be replaced by [QM][CM] and [QS][CS], respectively. Using this relationship and moving [RM] and [RS] to the right side of eqn 4, eqn 4 may be rewritten as eqn 5.

eqn 5 $$[P] = [K][u] - [CM][QM] - [CS][QS] = [K \mid -CM \mid -CS]\begin{bmatrix} u \\ QM \\ QS \end{bmatrix}$$

Finally, eqn 1 must be included for both the MPC'S and the SPC'S giving a complete set of equations as eqn 6.

eqn 6

$$\begin{bmatrix} P \\ 0. \\ 0. \end{bmatrix} = \left[\begin{array}{c|c|c} K & -CM & -CS \\ -CM & 0. & 0. \\ -CS & 0. & 0. \end{array}\right] \begin{bmatrix} u \\ QM \\ QS \end{bmatrix}$$

As an example of eqn 6, consider Figure 4 again. Eqn 7 is the appropriate matrix using eqns 2, and the truss element stiffness matrix.

eqn 7

$$\begin{bmatrix} P1 \\ P2 \\ P3 \\ \hline 0. \\ \hline 0. \end{bmatrix} = \left[\begin{array}{ccc|c|c} -1. & -1. & 0. & -.5 & -1. \\ -1. & 2. & -1. & 1. & 0. \\ 0. & -1. & 1. & -.5 & 0. \\ \hline -.5 & 1. & -.5 & 0. & 0. \\ \hline -1. & 0. & 0. & 0. & 0. \end{array}\right] \begin{bmatrix} u1 \\ u2 \\ u3 \\ \hline QM \\ \hline QS \end{bmatrix}$$

where EA/L = 1.

Applying P1 = 0., P2 = 0., P3 = 1. , the answers are given in Table 1a.

Referring to Table 1a, the displacement at point 2 is the average of points 1 and 3 as in eqn 2a. The RMi are zero since eqn 2a is consistent with the element displacement function. The RSi simply balances the applied load at point 3. The [PEXT] are the sum at each point of the previous loads.

Now, consider Figure 4 again except that eqn 8 is used for the MPC giving eqn 9.

eqn 8 $\quad -1.*u2 + 1.*u3 = 0.$

eqn 9

$$\begin{bmatrix} P1 \\ P2 \\ P3 \\ \hline 0. \\ \hline 0. \end{bmatrix} = \left[\begin{array}{ccc|c|c} 1. & -1. & 0. & 0. & -1. \\ -1. & 2. & -1. & 1. & 0. \\ 0. & -1. & 1. & -1. & 0. \\ \hline 0. & 1. & -1. & 0. & 0. \\ \hline -1. & 0. & 0. & 0. & 0. \end{array}\right] \begin{bmatrix} u1 \\ u2 \\ u3 \\ \hline QM \\ \hline QS \end{bmatrix}$$

Applying P1 = 0., P2 = 0., P3 = 1., the answers are given in Table 1b.

Referring to Table 1b, the displacements at points 2 and 3 are equal as required by eqn 8. The RMi remove the applied load at point 3 and move it to point 2, while the RSi at point 1 are unchanged. This is shown by the itemized external loads that sum to [PEXT]. This illustrates the fact that MPC's create external loads necessary to enforce the MPC displacement relation. Then, these loads add and subtract from the applied loads, causing the applied loads to distribute from the independent points to the dependent points.

Choosing CMi to Distribute Applied Loads

Various distribution schemes can be chosen which then determine the CMi in eqn 1. However, this author prefers the CMi which are consistent with the elements that originally created the freebody loads. The element shape functions are used to do this. For instance, eqn 2a is an example of the truss member shape function. Figure 5 shows the recommended MPC equations for the PAL2 commonly used elements. A detailed example using Figures 2 illustrates the procedure later in this paper.

Implementing MPC's in PAL2

Many main frame programs such as NASTRAN have MPC's implemented. Although PAL2 does not have MPC's, the PAL2 facility ADCAP2 provides the global resequenced [K] and a DOF/row table which states in what [K] row a degree of freedom (DOF) is located. Using a series of user written programs running outside of PAL2, [K] is manipulated using transformation techniques, and solved. Then, through further manipulations, a set of displacements for every DOF in [K] is produced. These `displacements are inserted into PAL2 for post processing (permitting restraint of every DOF is a unique capability of PAL2). The required matrix manipulations are derived below.

[u] may be rewritten as eqn 10a by means of a transformation.

eqn 10a [u] = [MPC][uI]

where: [MPC] relates the DOF's in [K] to the independent DOF's [uI] by means of eqn 1

For example, eqn 10b relates the DOF's from Figure 4 to the independent DOF's u1 and u3 (as in eqn 2a).

eqn 10b
$$\begin{bmatrix} u1 \\ u2 \\ u3 \end{bmatrix} = \begin{bmatrix} 1. & .0 \\ .5 & .5 \\ 0. & 1. \end{bmatrix} \begin{bmatrix} u1 \\ u3 \end{bmatrix}$$

Then, [uI] is rewritten as eqn 11a also by means of a transformation.

eqn 11a [uI] = [SPC][uR]

where: [SPC] relates the independent DOF's in [uI] to the active (reduced set) DOF's in [uR]

For example, eqn 11b relates [uI] from Figure 4 to the active [uR] assuming eqn 2b.

eqn 11b
$$\begin{bmatrix} u1 \\ u3 \end{bmatrix} = \begin{bmatrix} 0. \\ 1. \end{bmatrix} [u3]$$

Combining eqns 10a and 11a gives eqn 12.

eqn 12 [u] = [MPC][uI] = [MPC][SPC][uR]

Further, substituting eqn 12 into [P] = [K][u] gives eqns 13.

eqn 13a [P] = [K][u] = [K][MPC][SPC][uR] = [K][CST][uR]

eqn 13b [CST] = [MPC][SPC]

For instance, assuming P3 = 1. in Figure 4 gives eqns 13c, 13d, and 13e.

eqn 13c
$$\begin{bmatrix} 0. \\ 0. \\ 1. \end{bmatrix} = \begin{bmatrix} 1. & -1. & 0. \\ -1. & 2. & -1. \\ 0. & -1. & -1. \end{bmatrix} \begin{bmatrix} 1. & 0. \\ .5 & .5 \\ 0. & 1. \end{bmatrix} \begin{bmatrix} 0. \\ 1. \end{bmatrix} [u3] =$$

eqn 13d
$$\begin{bmatrix} 0. \\ 0. \\ 1. \end{bmatrix} = \begin{bmatrix} 1. & -1. & 0. \\ -1. & 2. & -1. \\ 0. & -1. & -1. \end{bmatrix} \begin{bmatrix} 0. \\ .5 \\ 1. \end{bmatrix} [u3]$$

eqn 13e [CST] = [MPC][SPC] =
$$\begin{bmatrix} 0. \\ .5 \\ 1. \end{bmatrix}$$

[K][CST] in eqn 13a is generally unsymmetrical which does not permit using efficient equation solving programs. However, [K][CST] is made symmetrical by premultiplying eqn 13a by [CST] giving eqns 14.

eqn 14a [CSTT][P] = [PR] = [CSTT][K][CST][uR] =[KR][uR]

eqn 14b [KR] = [CSTT][K][[CST]

where: [CSTT] is the transpose of [CST]
[PR] is the reduced applied load set
[KR] is the reduced stiffness matrix

For instance, applying eqns 14 to eqn 13d gives eqns 15.

eqn 15a
$$[\ 0. \quad .5 \quad 1.\ ] \begin{bmatrix} 0. \\ 0. \\ 1. \end{bmatrix} = [\ 0. \quad .5 \quad 1.\ ] \begin{bmatrix} 1. & -1. & 0. \\ -1. & 2. & -1. \\ 0. & -1. & -1. \end{bmatrix} \begin{bmatrix} 0. \\ .5 \\ 1. \end{bmatrix} [u3]$$

eqn 15b $[\ 1.\ ] = [\ .5\ ][u3]$

Once eqn 14a is solved for [uR], [u] is obtained by eqn 12, and then [u] is input to PAL2 for post processing. For instance, u3 = 2 from eqn 15b. This result is expanded using eqn 12, giving eqn 16.

$$\text{eqn 16} \quad \begin{bmatrix} u1 \\ u2 \\ u3 \end{bmatrix} = \begin{bmatrix} 0. \\ .5 \\ 1. \end{bmatrix} [\ 2.\ ] = \begin{bmatrix} 0. \\ 1. \\ 2. \end{bmatrix}$$

Running a finite element model employing MPC's in PAL2 consists of the following steps:

1) Create a model in PAL2.
2) Define the MPC's, SPC's, and freebody loads and place this information into a file such as MSLIN.
   a) MPC's: define the independent and dependent point numbers and global displacement numbers (e.g. global translation x is 1), and the appropriate CMi.
   b) SPC's: define the constrained point numbers and global displacement numbers.
   c) Freebody loads: define the load values, the point numbers and the displacement numbers.
3) Run PAL2.
4) Run ADCAP2.
   a) Select "Status" to create the DOF/row table.
   b) Select "Equations" to create [K]; select NASTRAN Output4 format to obtain nine significant figure stiffness values.
5) Read the DOF/row table and MSLIN to create [MPC], [SPC], and [P].
6) Read [K] and convert to a full symmetric matrix from a triangular matrix.
7) Multiply [MPC][SPC] = [CST]
8) Perform two multiplications: [CSTT][K], then ([CSTT][K])[CST] = [KR]
9) Multiply [CSTT][P] = [PR].
10) Solve for [uR] from [KR][uR] = [PR].
11) Multiply [CSTT][uR] = [u].
12) Read the DOF/row table and [u] to create a STAT2 displacement input file.
13) Run STAT2 and read the STAT2 input file for post processing.

The next section illustrates the above procedure.

An Example Problem of MPC's Used for Freebody Analysis

The loads shown on the torque box in Figure 1 are similar to a deployed flap forcing the lift center of pressure far aft. Since the forward spar is stiffer, the lift load must move forward to the front spar. This is accomplished by an interaction of the skins and the ribs where the ribs transmit the lift forward, causing a couple on each rib. Each couple is then balanced by in-plane shear (shear flow) from the skins. The loads on the rib in Figure 2 may be interpreted as two couples, one of which is caused by vertical shear, and the other by skin shear flow.

Figure 3 shows freebody loads from Figure 2 applied to the detailed freebody using MPC's. Since in-plane quadrilaterals generated the freebody loads, Figure 5 recommends using the truss member MPC equation.

The left edge, bottom edge, and right-most edge MPC's are straight-forward. The slanted edge and right edge MPC's need some discussion. The in-plane quadrilateral MPC maintains a straight (unwarped) edge. In a sense, this is a local coordinate system where the MPC accepts and distributes loads parallel and perpendicular to the edge. When the MPC does not end at an independent DOF, the MPC must be connected to an independent DOF through a system of MPC's in order to accept and distribute load while maintaining this local coordinate system. This is similar to loading the edge through a system of linkages. For example, point A is the intersection of the slanted edge MPC and the right-most edge MPC. The slanted edge MPC accepts load on the right from point A via the right-most edge MPC, and this MPC is connected to independent points.

The MPC's are written using the detail geometry in Figure 6. The detail tabulations are shown in Tables 2. Figure 6 also shows that statically determinate weak springs are added to prevent rigid body motion. These spring stiffnesses should be about 1.E-5 times the average member stiffness to prevent matrix ill-conditioning while only balancing any small imbalance in the freebody loads. In PAL2, springs must have two points of attachment so that the SPC's, in this example, only constrain the springs to ground. Now, the methods of the previous sections are used, and the results are shown in Figure 7.

In Figure 7, the edges with MPC's are straight (not warped) as required. However, the lines around the hole are warped since there were no constraints there.

From a stress analysis/design standpoint, the rib could now be further modified. For instance, a frame structure equal in shear stiffness to the missing quadrilateral could be added around the hole to increase the rib's shear efficiency. Once satisfactorily modified, the detail model could now be converted to a NASTRAN model via ADCAP2, sent to a main frame computer, converted to a NASTRAN super element, and inserted into the overall torque box model. Once the next overall run is completed, the rib's boundary displacements are returned to the stress analyst, and applied to the detailed rib model to produce a new set of freebody loads. These new loads can then be used as in this example for further rib improvements.

Conclusions

1. Using MPC's with PAL2, freebody loads from an overall coarse model can be applied to a detail freebody finite element model resulting in a balanced detail freebody.

2. MPC's have a load distribution property which distributes the freebody loads to the detail freebody points.

3. This author prefers the MPC coefficients which are consistent with the elements that originally created the freebody loads; namely, using the element shape function itself.

4. MPC's are not implemented in PAL2. However, using the PAL2 facility ADCAP2, and the appropriate user written programs running outside of PAL2, MPC's can be implemented.

5. Once a detail freebody with MPC's, SPC's and freebody loads has been arranged, the stress analyst may improve the design. Once complete, the model can be converted to a NASTRAN model, passed to a main frame computer, converted to super element, and added to the overall model. Once the next overall run is completed, the detail freebody displacements can be passed to the stress analyst to obtain a new set of freebody loads and further detail freebody improvement.

References:

1. R. H. MacNeal, "MSC NASTRAN Theoretical Manual", 1972, pp 3.5-1 thru 3.5-3.

2. Robert D. Cook, "Concepts and Applications of Finite Element Analysis", second edition, John Wiley and Sons Inc., 1981, pp 78.

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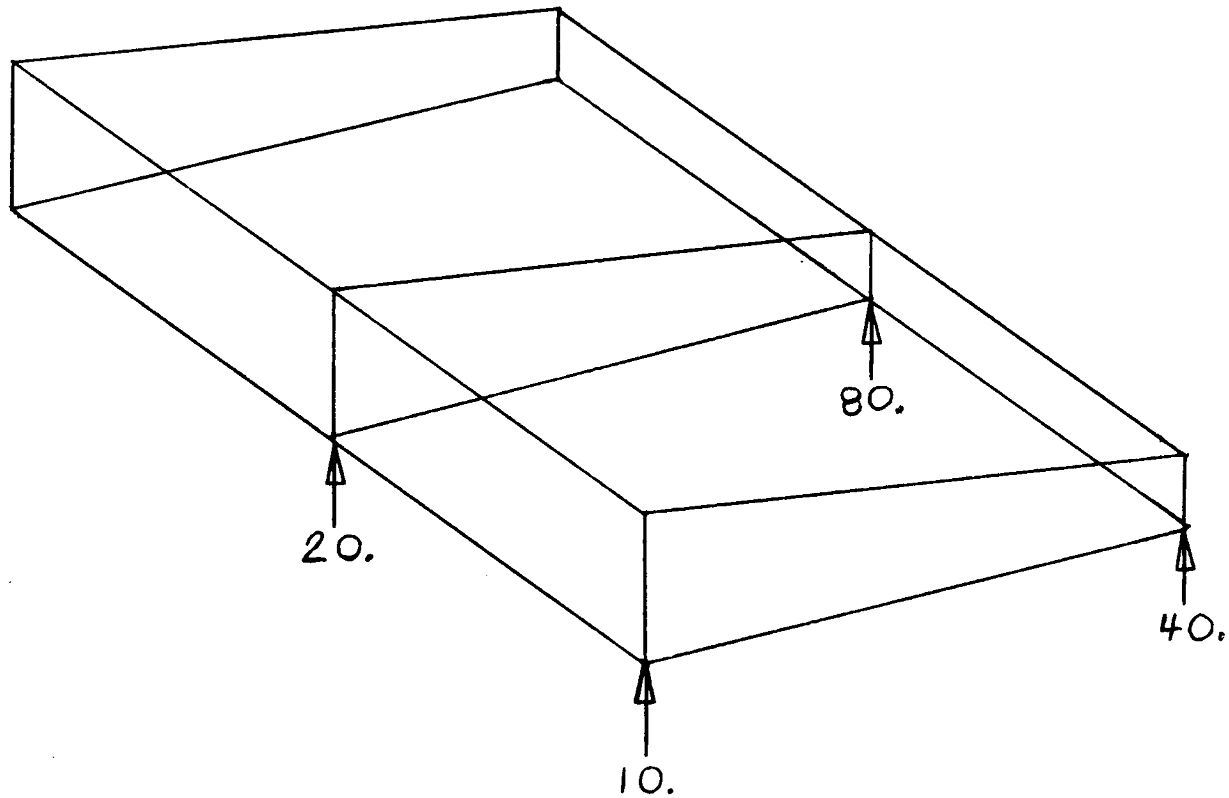


Wing Torque Box Member	Points
Front Spar	1,4,5,8,9,12
Rear Spar	2,3,6,7,10,11
Upper Skin	3,4,7,8,11,12
Lower Skin	2,1,6,5,10,9
Outboard Rib	2,1,4,3
Middle Rib	6,5,8,7
Inboard Rib (fixed)	10,9,12,11

Figure 1: Wing Torque Box

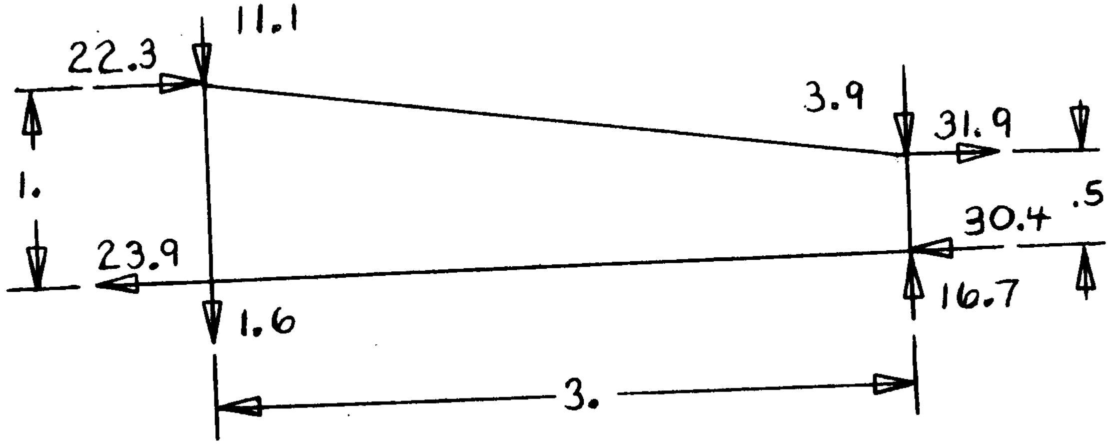


Figure 2a: Freebody Loads and Freebody from Figure 1

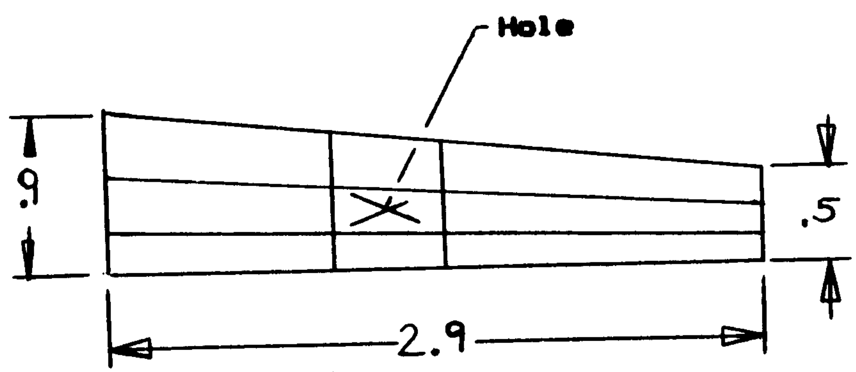
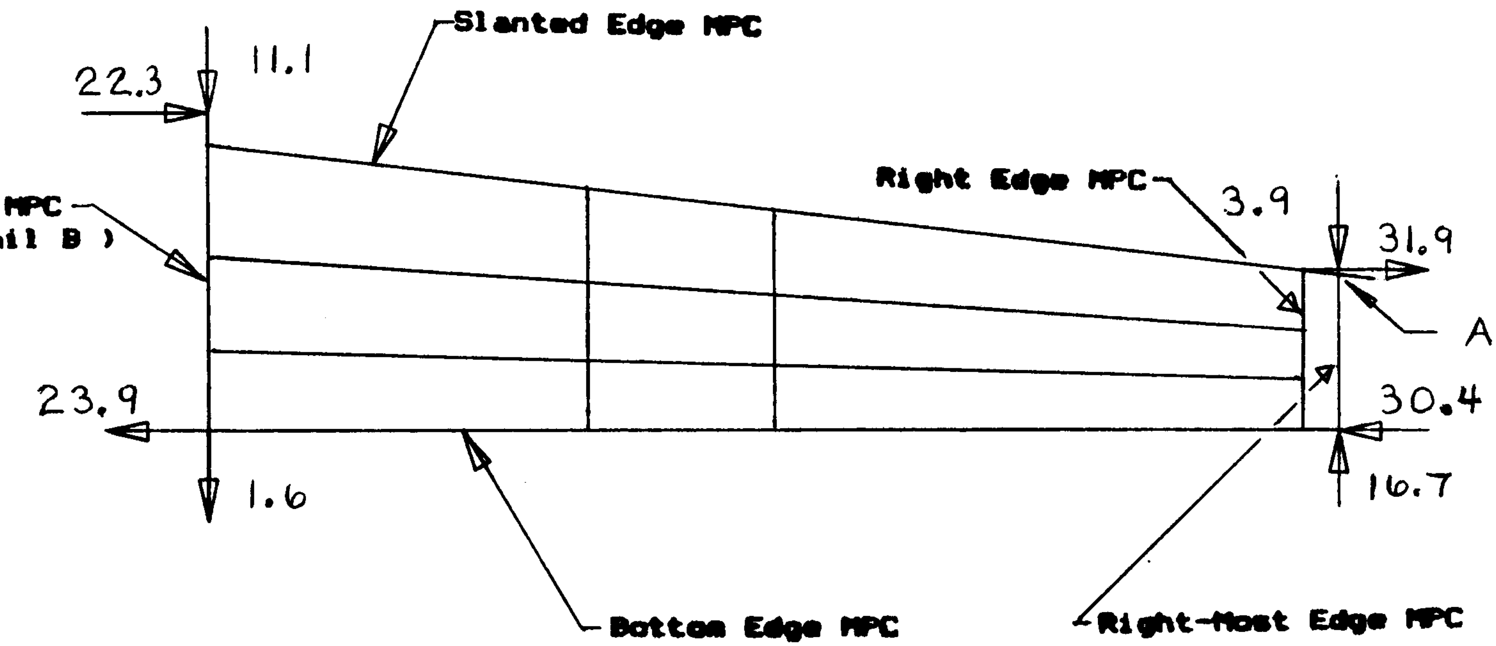
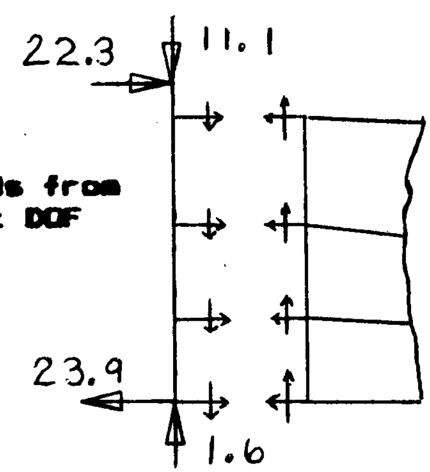


Figure 2b: Detail Freebody

Figure 2: Middle Rib Freebodies





Detail B: Distribution of Freebody Loads from Independent DOF to Dependent DOF

Figure 3: MPC's Used to Distribute the Freebody Loads to the Detail Freebody

Note:

1. E = Young's Modulus
2. A = Truss Area

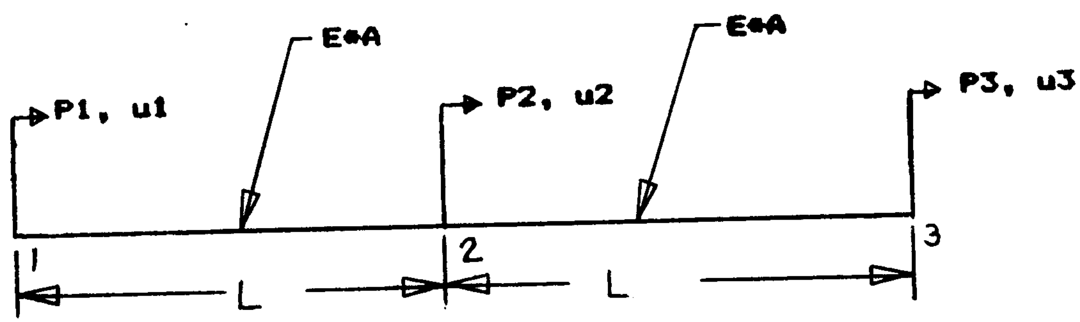
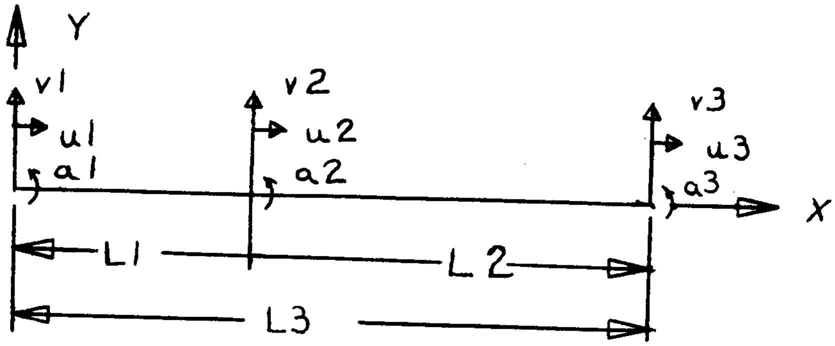


Figure 4: Example Truss Member Problem

1-Dimensional:



Truss

Dep DOF	Indep DOF	MPC eqn
u2	u1, u3	-1.*u2 + L2/L3*u1 + L1/L3*u3 = 0.

Beam

Dep DOF	Indep DOF	MPC eqn [2]
v2	v1,a1,v2,a2	-1.*v2 + M1*v1 +M2*a1 + M3*v2 + M4*a2 = 0.
a2	v1,a1,v2,a2	-1.*a2 + N1*v1 +N2*a1 + N3*v2 + N4*a2 = 0.

For:

```
M1 = ( 1. - 3.*L1**2./L3**2. + 2.*L1**3./L3**3. )*v1

M2 = ( L1 - 2.*L1**2./L3 + L1**3./L3**2. )*a1

M3 = ( 3.*L1**2./L3**2. - 2.*L1**3./L3**3. )*v2

M4 = ( -1.*L1**2./L3 + L1**3./L3**2. )*a2

N1 = ( -6.*L1/L3**2. + 6.*L1**2./L3**3. )*v1

N2 = ( 1. - 4.*L1/L3 + 3.*L1**2./L3**2. )*a1

N3 = ( 6.*L1/L3**2. - 6.*L1**2./L3**3. )*v1

N4 = ( -2.*L1/L3 - 3.*L1**2./L3**2. )*a1
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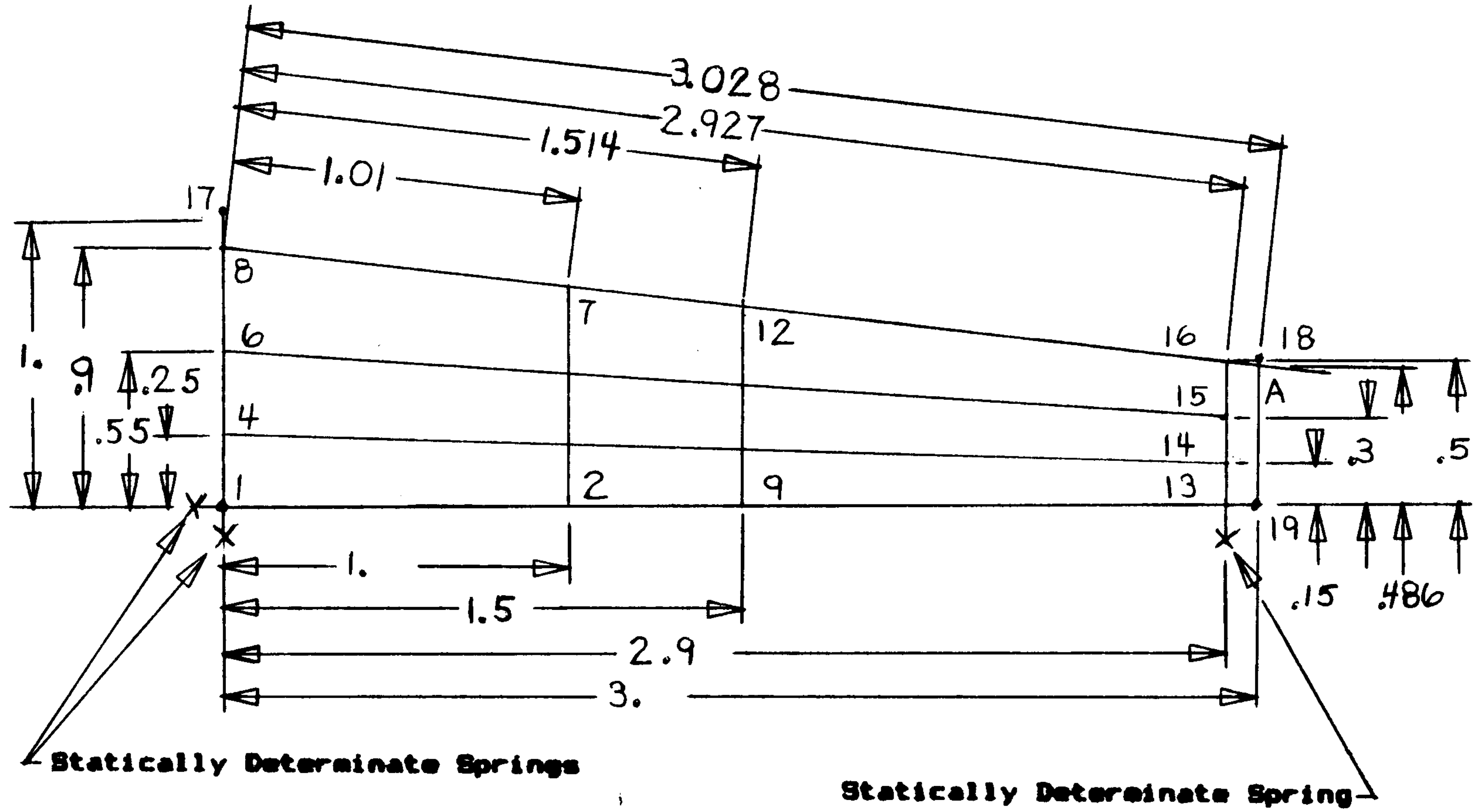
2 - Dimensional: For triangular and quadrilateral edges, use the truss relationship for in-plane behavior and the beam relationship for bending behavior.

Figure 5: Recommended MPC Equations for PAL2 Elements



Notes:

1. • Denotes freebody load application point.
2. X Denotes grounded spring attachment point.

Figure 6: Detail Freebody Geometry Required for MPC Equation Coefficients

Point	CMi	ui	Pi	RMi	RSi	PEXT
1	.5	0.	0.	0.	-1.	-1.
2	-1.	1.	0.	0.	-	0.
3	.5	2.	1.	0.	-	1.

QM = 0.   QS = -1.

Table 1a: Summary of Calculations for Eqn 7

Point	CMi	ui	Pi	RMi	RSi	PEXT
1	-	0.	0.	0.	-1.	-1.
2	-1.	1.	0.	1.	-	1.
3	1.	1.	1.	-1.	-	0.

QM = -1.   QS = -1.

Table 1b: Summary of Calculations for Eqn 8

Note:

1. RMi = QM*CMi
2. RSi = QS*CSi
3. PEXTi = Pi + RMi + RSi

Table 1: Summary of Calculations for Eqns 7 and 8

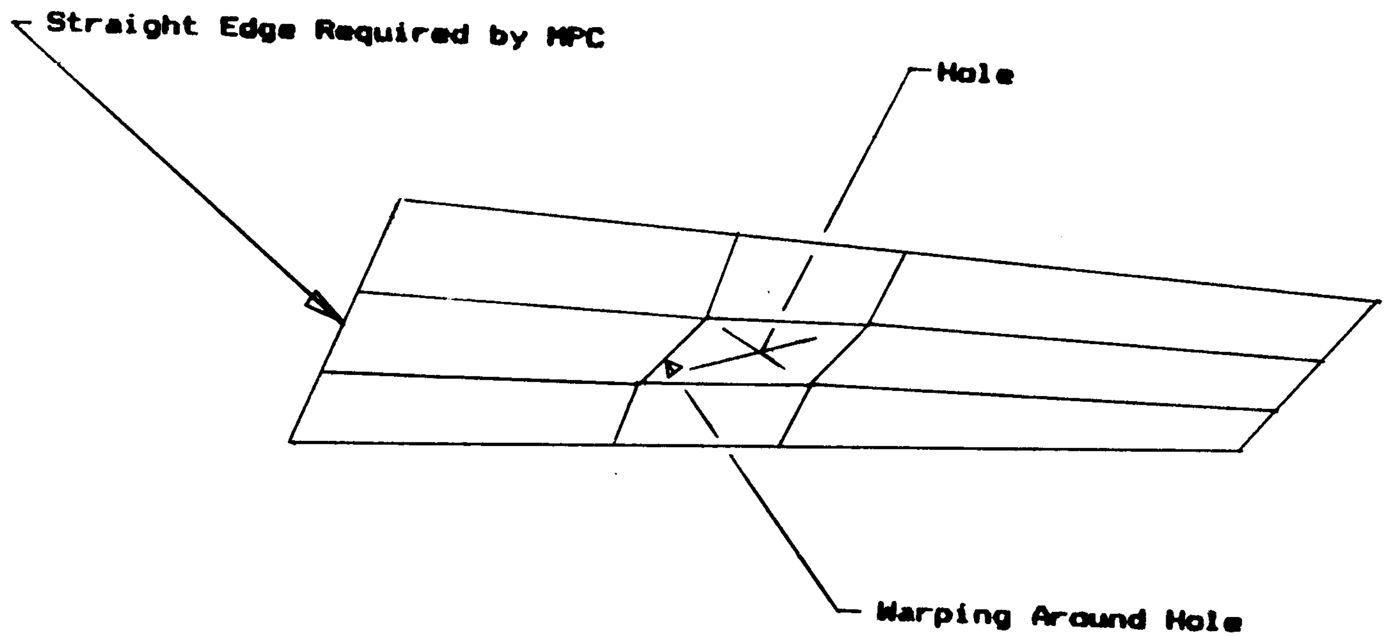


Figure 7: Deformed Detail Freebody

Left Edge MPC's

Dependent Point	% Distance to Indep. Point 1	% Distance to Indep. Point 17	MPC Equation
4	25.	75.	D4 = .25*D17 + .75*D1
6	55.	45.	D6 = .55*D17 + .45*D1
8	90.	10.	D8 = .90*D17 + .10*D1

Bottom Edge MPC's

Dependent Point	% Distance to Indep. Point 1	% Distance to Indep. Point 19	MPC Equation
2	33.3	66.7	D2 = .333*D19 + .667*D1
9	50.	50.	D9 = .500*D19 + .500*D1
13	96.7	3.33	D13 = .967*D19 + .033*D1

Right-Most Edge MPC's

Dependent Point	% Distance to Indep. Point 18	% Distance to Indep. Point 19	MPC Equation
A	2.8	97.2	DA = .028*D19 + .972*D18

Table 2a: Straight-Forward MPC's

Right Edge MPC's

Dependent Point	% Distance to Indep. Point 13	% Distance to Indep. Point 16	MPC Equation
14	30.	70.	D14 = .300*D16 + .700*D13
15	60.	40.	D15 = .600*D16 + .400*D13

Inserting:

D13 = .967*D19 + .0333*D1   DA = .028*D19 + .972*D18

D16 = .966*DA + .0334*D8   D8 = .900*D17 + .100*D1

Dependent Point	MPC Equation
14	D14 = .0243*D1 + .685*D19 + .00902*D17 + .282*D18
15	D15 = .0153*D1 + .403*D19 + .01800*D17 + .563*D18

Slanted Edge MPC's

Dependent Point	% Distance to Indep. Point 8	% Distance to Indep. Point A	MPC Equation
7	33.3	66.6	D7 = .333*DA + .666*D8
12	50.	50.	D12 = .500*DA + .500*D8
16	96.6	3.3	D16 = .966*DA + .033*D8

Inserting:

D8 = .900*D17 + .1000*D1

DA = .028*D19 + .9720*D18

Dependent Point	MPC Equation
7	D7 = .0666*D1 + .599*D17 + .324*D18 + .009*D19
12	D12 = .0500*D1 + .450*D17 + .486*D18 + .014*D19
16	D16 = .0033*D1 + .030*D17 + .939*D18 + .027*D19

Table 2b: MPC's Connected to the Freebody Loads Through Other MPC's