The Optimal Design of Structural Systems by the Superelement Method

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ABSTRACT

Recently, the analytical methods of the optimal structual design is to utilize the sensitivity technique. This approach is then a programming problem. In order to arrive at an optimal size for a structural element by solving the programming problem, the sensitivity analysis must be repeated by computation of the structural matrices. The process may consume large amount of computer time. For practical applications, how to arrive it the most efficient mathematical model for this purpose is of majo preferest. This paper present, the superelement method and the sensitivity analysis technique for optimal structural design for this regard, personal computers using prove to be an economic means to apply there methods.

1. Introduction

In general, the analysis of large structural systems uses the matrix procedure by the finite element method, with computers using large central processing unit memory.

In the step of structural sizing, by the fail-safe concept, local structural arrangement along with element sizes will change from step to step, while the procedure of structural analysis is repeated. The process using consume very lengthy computer time. When the superelement method is used, only the superelement which is affected needs to be updated, thus computer time saving is achieved. Recently, the method of optimization of structural designs is to solve the programming problems by repeating the computation of sensitivity analysis. How to reduce the go-back procedure is an important point of computer time reduction. In this paper, a method to maximize the modular characteristics of the superelement method, and thus the computer time saving is proposal. There is a reciprocal relationship between superelement size and a amount of computer time.

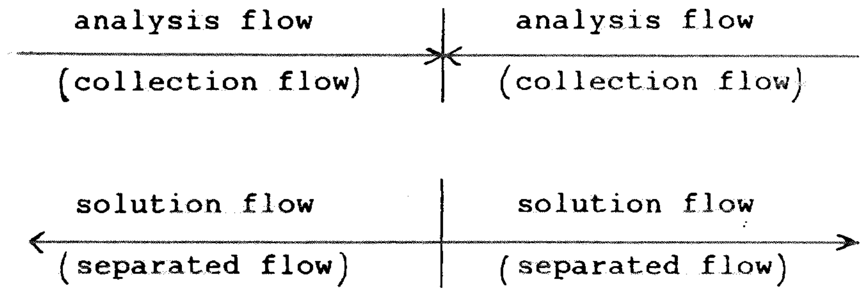
2. The Superelement method

The superelement method is the same as substructure method. The concept is to divide the structural system to several substructures, then establish the relationship at the interfaces of the substructures. Each analysis flow has only one direction. Each substructure is a module. The procedures of processing substructures are shown in section 2.1.

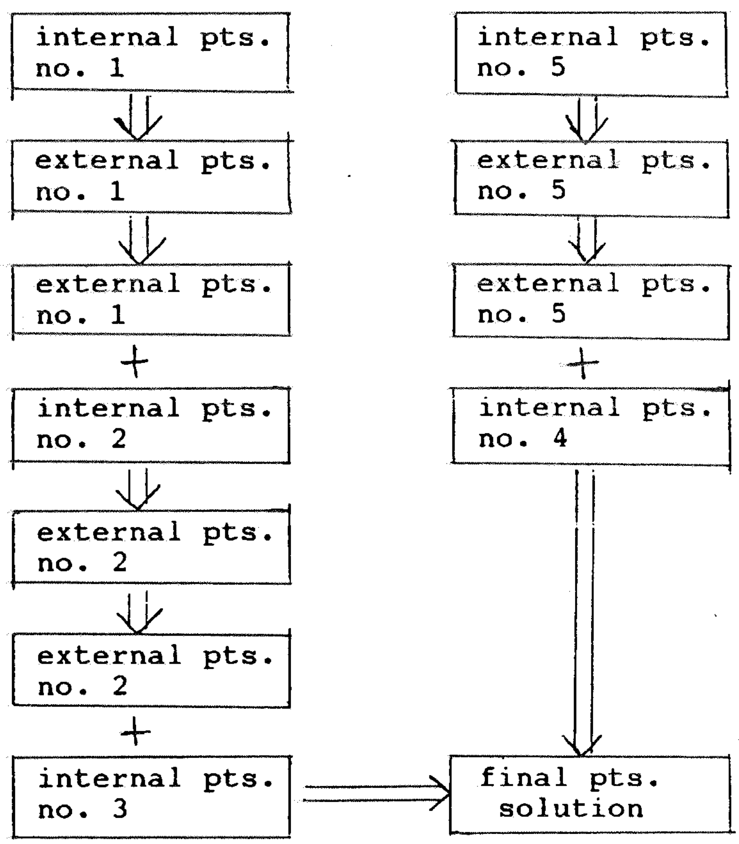
2.1 The Procedure of Substructure method Process

substr. no. 1	substr. no. 2	substr. no. 3	substr. no. 4	substr. no. 5
internal pts. no. 1	internal pts. no. 2	internal pts. no. 3	internal pts. no. 4	internal pts. no. 5

external pts. no. 1   external pts. no. 2   final pts. (residual structure)   external pts. no. 5



Analysis flow



⟹ : establish the relations between internal points and external points

The size of the matrix depends on each substructure size or each interface
The number of matrix depends on the number of substructures.

Solution flow

We solve the structural problem at final points, then we obtain the displacements and stresses of substructures from the direction of counter analysis flow.

2.2 The mathematical model of substructure method

Assume a substructure no. n

$$K^n \Delta^n = P^n$$

$$\begin{bmatrix} K_{bb}^n & K_{bc}^n \\ K_{cb}^n & K_{cc}^n \end{bmatrix} \begin{Bmatrix} \Delta_b^n \\ \Delta_c^n \end{Bmatrix} = \begin{Bmatrix} P_b^n \\ P_c^n \end{Bmatrix} \qquad \text{--(1)}$$

b: internal points of substructure no. n
c: external points of substructure no. n

$$P_b^n = K_{bb}^n \Delta_b^n + K_{bc}^n \Delta_c^n$$

$$\Delta_b^n = -(K_{bb}^n)^{-1} K_{bc}^n \Delta_c^n + (K_{bb}^n)^{-1} P_b^n \qquad \text{--(2)}$$

$P_c^n = K_{cb}^n \Delta_b^n + K_{cc}^n \Delta_c^n$ substitude Δ_b^n into this equation.

$$P_c^n = K_{cb}^n [-(K_{bb}^n)^{-1} K_{bc}^n \Delta_c^n + (K_{bb}^n)^{-1} P_b^n] + K_{cc}^n \Delta_c^n$$

$$\Delta_c^n = [K_{cc}^n - K_{cb}^n (K_{bb}^n)^{-1} K_{bc}^n]^{-1} [P_c^n - K_{cb}^n (K_{bb}^n)^{-1} P_b^n]$$

$$[K_{cc}^n - K_{cb}^n (K_{bb}^n)^{-1} K_{bc}^n] \Delta_c^n = P_c^n - K_{cb}^n (K_{bb}^n)^{-1} P_b^n$$

express by

$$\hat{K}_{cc}^n \Delta_c^n = P_c^n - R_c^n \qquad \text{--(3)}$$

so $\hat{K}c c^n$ express the condensation of stiffness matrix
Rc^n express the condensation of load matrix

The equation (3) is the external points equilibrium equation.
One by one flow to the final points by top-down method. The final equilibrium equation is

$$\hat{K}_{cc} \Delta_c = Pc - Rc$$

After solving this equation, deformations are obtained at the final points of structural system, then recover to each substructure by the lower-up method, finally we get the deformations and stresses of each substructure.
For example, substructure n can be expressed as

$$\Delta_b^n = (K_{bb}^n)^{-1} P_b^n - (K_{bb}^n)^{-1} K_{bc}^n \Delta_c^n \qquad \text{--(4)}$$

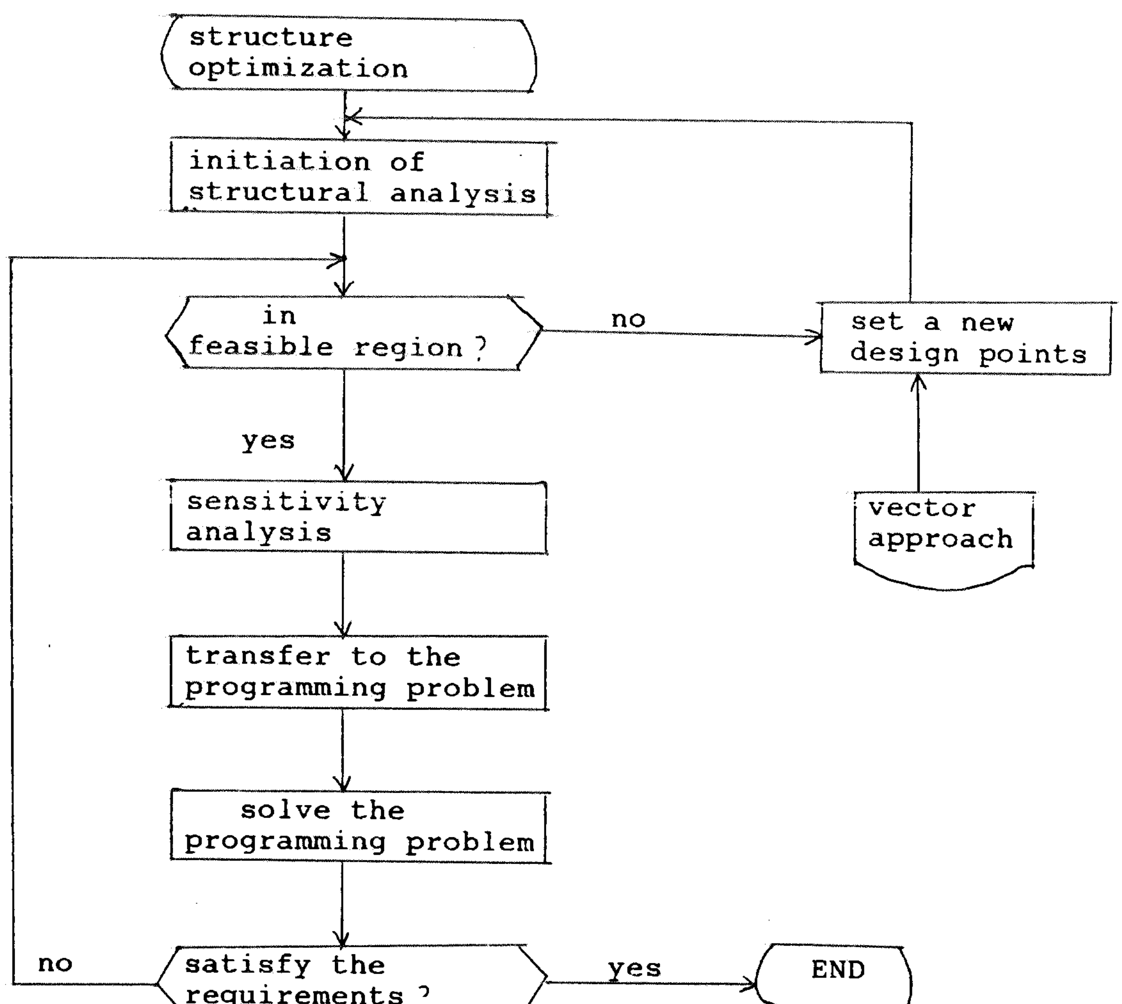
3. The procedure of the Optimization of Structural Design

Recently, the development of the nonlinear programming analysis has yielded better efficiency and reliaability. So, the sensitivity analysis technique that using finite element method and the programming solution. Then we get the optimal sizes of structural systems.

3.1 The Procedure of Structural Optimal Design

The procedures of the structural optimal design by the flow chart are expressed here:

(one level optimization)



3.2 Sensitivity Analysis transfer to Programming Problem

minimize $f(x_i)$

subject to

$$g(x)_{min} \leq g(x_{.}) + \sum \frac{\partial g}{\partial x_i}(u_i - 1) \leq g(x)_{max}$$

$$x_i > 0$$

x_i : design variables

$f(x_i)$: object function, in general, an explicit function

$g(x_i)$: constraints, in general, an implicit function on the procedure of the finite element method

The approach of linear programming will converge very quickly. This method needs one cycle of structural analysis and sensitivity analysis only, so it renders the superelement method more efficient.

3.3 Applying the Sensitivity Analysis Technique to Superelement Method

Internal points equilibrium equation

$$P_b^n = (K_{bb}^n + \Delta K_{bb}^n)(\Delta_b^n + d\Delta_b^n) + (K_{bc}^n + \Delta K_{bc}^n)(\Delta_c^n + d\Delta_c^n)$$

$$\begin{aligned} P_b^n = {} & K_{bb}^n \Delta_b^n + K_{bb}^n\, d\Delta_b^n + \Delta K_{bb}^n \Delta_b^n + \Delta K_{bb}^n\, d\Delta_b^n \\ & + K_{bc}^n \Delta_c^n + K_{bc}^n\, d\Delta_c^n + \Delta K_{bc}^n \Delta_c^n + \Delta K_{bc}^n\, d\Delta_c^n \end{aligned}$$

Eliminate the items same as $\Delta K\, d\Delta$

$$P_b^n = K_{bb}^n \Delta_b^n + K_{bb}^n\, d\Delta_b^n + \Delta K_{bb}^n \Delta_b^n + K_{bc}^n \Delta_c^n + K_{bc}^n\, d\Delta_c^n + \Delta K_{bc}^n \Delta_c^n$$

$$K_{bb}^n\, d\Delta_b^n + K_{bc}^n\, d\Delta_c^n = -\Delta K_{bb}^n \Delta_b^n - \Delta K_{bc}^n \Delta_c^n$$

$$d\Delta_b^n = (K_{bb}^n)^{-1}(-K_{bc}^n)\, d\Delta_c^n - (K_{bb}^n)^{-1}(\Delta K_{bb}^n)\Delta_b^n - (K_{bb}^n)^{-1}(\Delta K_{bc}^n)\Delta_c^n$$

The equilibrium equation of external points

$$P_c^n = K_{cb}^n \Delta_b^n + K_{cb}^n\, d\Delta_b^n + \Delta K_{cb}^n \Delta_b^n + K_{cc}^n \Delta_c^n + K_{cc}^n\, d\Delta_c^n + \Delta K_{cc}^n \Delta_c^n$$

$$\begin{aligned} P_c^n = {} & K_{cb}^n \Delta_b^n + K_{cb}^n\, d\Delta_b^n + \Delta K_{cb}^n \Delta_b^n + \Delta K_{cb}^n\, d\Delta_b^n \\ & + K_{cc}^n \Delta_c^n + K_{cc}^n\, d\Delta_c^n + \Delta K_{cc}^n \Delta_c^n + \Delta K_{cc}^n\, d\Delta_c^n \end{aligned}$$

Eleminate the items same as $\Delta K d\Delta$

$$P_c^n = K_{cb}^n \Delta_b^n + K_{cb}^n\, d\Delta_b^n + \Delta K_{cb}^n \Delta_b^n + K_{cc}^n \Delta_c^n + K_{cc}^n\, d\Delta_c^n + \Delta K_{cc}\Delta_c^n$$

$$K_{cb}^n\, d\Delta_b^n + \Delta K_{cb}^n \Delta_b^n + K_{cc}^n\, d\Delta_c^n + \Delta K_{cc}^n \Delta_c^n = 0 \qquad \text{--(6)}$$

Substitude $d\Delta_b^n$ into equation (6)

$$\begin{aligned} & K_{cb}^n (K_{bb}^n)^{-1}(-K_{bc}^n)\, d\Delta_c^n - K_{cb}^n (K_{bb}^n)^{-1}(\Delta K_{bb}^n)\Delta_b^n \\ & - K_{cb}^n (K_{bb}^n)^{-1}(\Delta K_{bc}^n)\Delta_c^n + \Delta K_{cb}^n \Delta_b^n + K_{cc}^n\, d\Delta_c^n + \Delta K_{cc}^n \Delta_c^n = 0 \end{aligned}$$

Assume

$$K_1^n = K_{cb}^n (K_{bb}^n)^{-1} K_{bc}^n - K_{cc}^n$$
$$K_2^n = \Delta K_{cb}^n - K_{cb}^n (K_{bb}^n)^{-1} (\Delta K_{bb}^n)$$
$$K_3^n = \Delta K_{cc}^n - K_{cb}^n (K_{bb}^n)^{-1} (\Delta K_{bc}^n)$$

K_1^n, K_2^n, K_3^n are the matrice of condensation by sensitivity analysis.

So $K_1^n \, d\Delta_c^n = K_2^n \Delta_b^n + K_3^n \Delta_c^n$
assume $\Delta P_c^n = K_2^n \Delta_b^n + K_3^n \Delta_c^n$
final, we obtain

$$K_1^n \, d\,\Delta_c^n = \Delta P_c^n \qquad \text{--(7)}$$

The equation (7) use to the equilibrium equation of external points of the variable stiffness (ΔK) in substructure n. Then condense to substructure (n+1).

$$K_1^n = \begin{bmatrix} \Delta K_{bb}^{n+1} & \Delta K_{bc}^{n+1} \\ \Delta K_{cb}^{n+1} & \Delta K_{cc}^{n+1} \end{bmatrix}$$

The matrix K_1^n is the stiffness matrix of the external points which transfer from the internal points by sensitivity analysis.

$\Delta P_c^n = \Delta P_b^{n+1}$
The internal points equilibrium of substructure (n+1) are expressed as

$$(\Delta P_b^n + P_b^n) = (K_{bb}^{n+1} + \Delta K_{bb}^{n+1})(\Delta_b^{n+1} + d\Delta_b^{n+1}) + (K_{bc}^{n+1} + \Delta K_{bc}^{n+1})(\Delta_c^{n+1} + d\Delta_c^{n+1})$$
$$K_{bb}^{n+1} d\Delta_b^{n+1} + K_{bc}^{n+1} d\Delta_c^{n+1} = \Delta P_b^{n+1} - \Delta K_{bb}^{n+1} \Delta_b^{n+1} - \Delta K_{bc}^{n+1} \Delta_c^{n+1}$$
$$d\Delta_b^{n+1} = (K_{bb}^{n+1})^{-1} (\Delta P_b^{n+1}) - (K_{bb}^{n+1})^{-1} (-K_{bc}^{n+1}) d\Delta_c^{n+1}$$
$$-(K_{bb}^{n+1})^{-1} (\Delta K_{bb}^{n+1}) \Delta_b^{n+1} - (K_{bb}^{n+1})^{-1} (\Delta K_{bc}^{n+1}) \Delta_c^{n+1} \qquad \text{--(8)}$$

The equilibrium equation of external points of substructure (n+1)

$$K_{cb}^{n+1} d\Delta_b^{n+1} + \Delta K_{cb}^{n+1} \Delta_b^{n+1} + K_{cc}^{n+1} d\Delta_c^{n+1} + \Delta K_{cc}^{n+1} \Delta_c^{n+1} = 0 \qquad \text{--(9)}$$

substitute equation (8) into equation (9)

$$K_1^{n+1} d\Delta_c^{n+1} = K_2^{n+1} \Delta_b^{n+1} + K_3^{n+1} + K_4^{n+1} \Delta P_b^{n+1}$$
$$K_1^{n+1} = K_{cb}^{n+1} (K_{bb}^{n+1})^{-1} K_{bc}^{n+1} - K_{cc}^{n+1} - K_{cc}^{n}$$
$$K_2^{n+1} = \Delta K_{cb}^{n+1} - K_{cb}^{n+1} (K_{bb}^{n+1})^{-1} (\Delta K_{bb}^{n+1})$$
$$K_3^{n+1} = \Delta K_{cc}^{n+1} - K_{cb}^{n+1} (K_{b}^{n+1})^{-1} (\Delta K_{bc}^{n+1})$$
$$K_4^{n+1} = K_{cb}^{n+1} (K_{bb}^{n+1})^{-1}$$
$$K_2^{n+1} \Delta_b^{n+1} + K_3^{n+1} + K_4^{n+1} \Delta P_b^{n+1} = \Delta P_c^{n+1}$$
$$K_1^{n+1} d\,\Delta_c^{n+1} = \Delta P_c^{n+1} \qquad \text{--(10)}$$

The matrix K_1^{n+1} may use the initial datum from substructural analysis. If matrice ΔK_{cc}^{n+1} ΔK_{bc}^{n+1} and ΔK_{cb}^{n+1} are null matrice, then matrix K_2^{n+1} shall be a null matrix.
With the similarity of above procedures, it arrive(top-down) the final residual structure.

$K_1 \, d\Delta_c = \Delta P_c$
$d\Delta_c = K_1^{-1} \Delta P_c$

Any varible stiffness by sensitivity analysis belongs to the substructure will be top-down to residual structure only.

After solving the residual structure, the stresses can be recovered, and then it comes out the structural elements stresses by the sensitivity analysis.

The expression of stress constraints is

$$\sigma_{min} \leq Fo + \Delta F(u_i - 1) \leq \sigma_{max}$$

The expression of deformation constraints is

$$\Delta min \leq \Delta_o + d\Delta(u_i - 1) \leq \Delta max$$

4. Conclusion

The usage of module characterization of superelement method will be more efficient and reliable. It reduces the risk of unstable computer systems. The computer time and memory size may be exchanged, that decides by the superelement dimension. With the personal computer, this is an effective method.

5. Referances

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