# DERIVATION OF AN EQUIVALENT BEAM MODEL FROM A STRUCTURAL FINITE ELEMENT MODEL

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ABSTRACT

An equivalent beam model is derived from an MSC/NASTRAN finite element model of a horizontal stabilizer structure. An automated interactive graphics program has also been developed to obtain the elastic axis and beam stiffness of the surface.

## INTRODUCTION

In order to build a wind tunnel model for flutter testing, one is often required to derive the equivalent structural beam stiffness, EI and GJ, and the elastic axis from an existing finite element model of a wing-like structure. In addition, to reduce the cost of flutter analysis it is necessary to reduce a large finite element model to an equivalent beam element model if it matches the critical mode shapes and frequencies.

Historically there has been much confusion in the definition of an elastic axis of a wing- or fuselage-like structure. In this paper, the elastic axis is obtained by applying a unit torque at the tip of a finite element model of a stabilizer surface, and obtaining the locus of zero deflection on the surface.

It should be noted that the elastic axis location is dependent on the support condition, as well as the applied load. Structures with significant shear lag effects, redundant supports, tapered cross-section, or composite material may not yield an elastic axis that can be used as equivalent beam representation for vibration analysis. It must be emphasized that this method is inherently very approximate and must, therefore, be used with caution and judgment. No universal application of an elastic axis is possible, while a particular application may be successfully made. In general, the locus of the shear centers of a wing-like structure is not coincidental with the elastic axis of the surface.

In this paper an MSC/NASTRAN finite element model of a stabilizer is used to derive the elastic axis and the beam stiffness of an equivalent beam.

The interpolation element RBE3 is used to obtain the slope and twist at various spanwise stations on the axis. Then, a check is made on the viability of the elastic axis by applying a concentrated load, which should produce no twisting or negligible twist of the beam. A normal mode analysis of the beam model is performed for comparison with the frequencies and mode shapes of the finite element model and ground vibration test data.

The program to evaluate the elastic axis and beam stiffness is written in an automated interactive graphics environment which is built around the MSC/NASTRAN program.

## EVALUATION OF THE ELASTIC AXIS

The elastic axis of an airplane wing is defined as the spanwise line along which transverse loads must be applied in order to allow bending only with negligible torsion of the wing at any station along the span.

An MSC/NASTRAN model of a composite stabilizer surface (as shown in Figure 1) is subjected to an equivalent beam reduction procedure as described in Figure 2. The surface is cantilevered from the centerline of the airplane by means of a torque tube which attaches to the root rib of the stabilizer. A unit torque is applied at the free end and the coordinates of the locus of the zero transverse deflection points are obtained. The elastic axis is assumed to lie along the locus of the points.

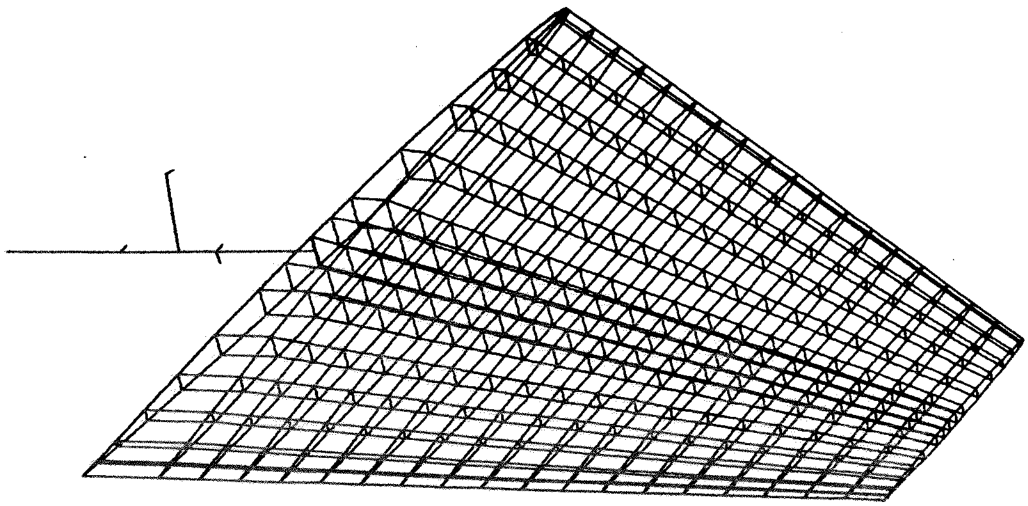


FIGURE 1. FINITE ELEMENT MODEL OF COMPOSITE STABILIZER

A check is made to determine the accuracy of the elastic axis by picking eight discrete grids on the axis and applying transverse loads to them one at a time. The torsion of the stabilizer for each applied load is found to be negligible. Eight RBE3 elements are used along the span of the surface to apply the loads on the selected elastic axis grids.

The RBE3 elements ensure proper distribution of the applied loads among the surrounding finite element grids and recover transverse deflection, bending slopes and torsion at the selected elastic axis points. The torsion of the surface is found to be negligible due to the applied loads at the axis. Such a check is necessary to maintain confidence in the elastic axis of the surface. There are engineers who believe the elastic axis to be the locus of the shear centers of the stabilizer cross-section (see Figure 3 for comparison). In this case, when transverse loads are applied at the shear

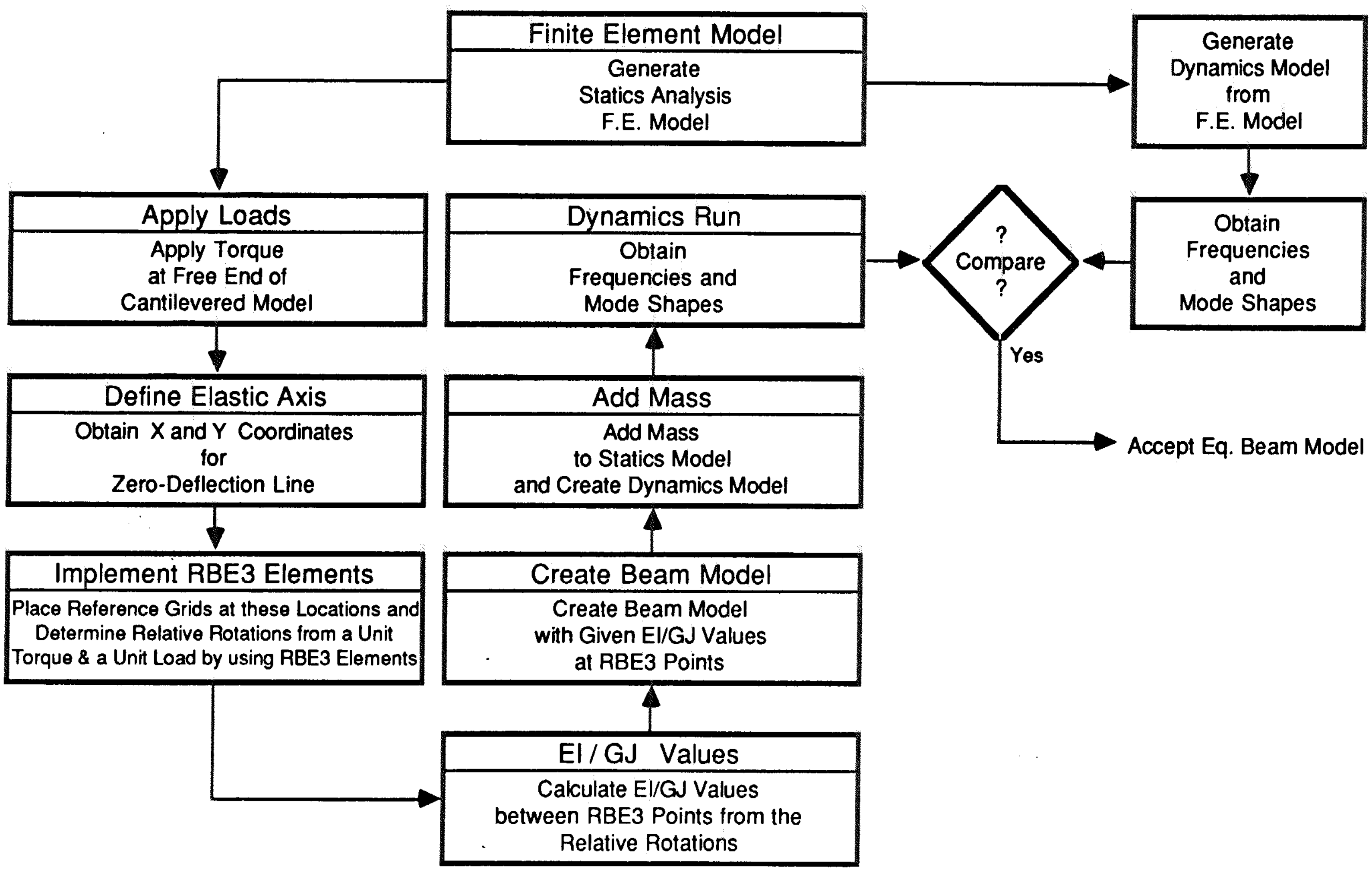



FIGURE 2. EQUIVALENT BEAM REDUCTION PROCEDURES

center points, the surface twists are irregular and higher at the tip than the derived elastic axis (see Figure 4 for comparison). Consequently, the locus of the shear centers should not be assumed to be the elastic axis of the structure.

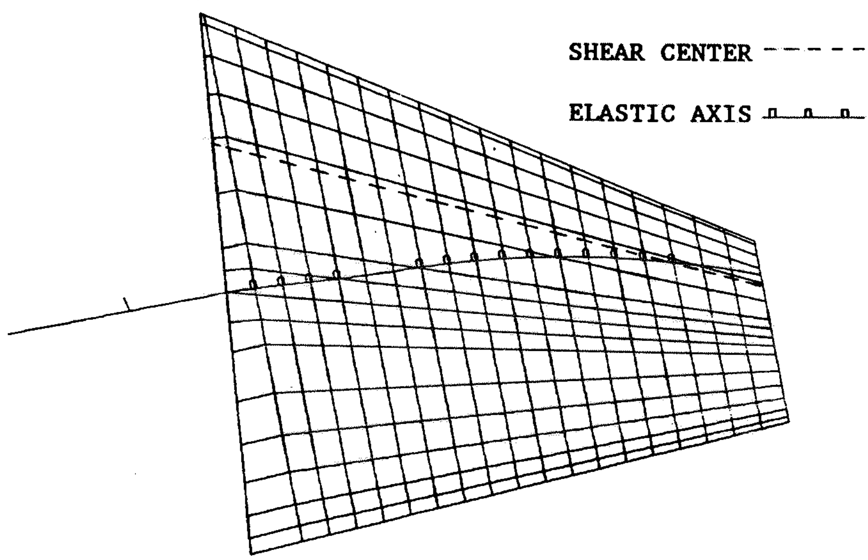



FIGURE 3. THE CALCULATED ELASTIC AXIS

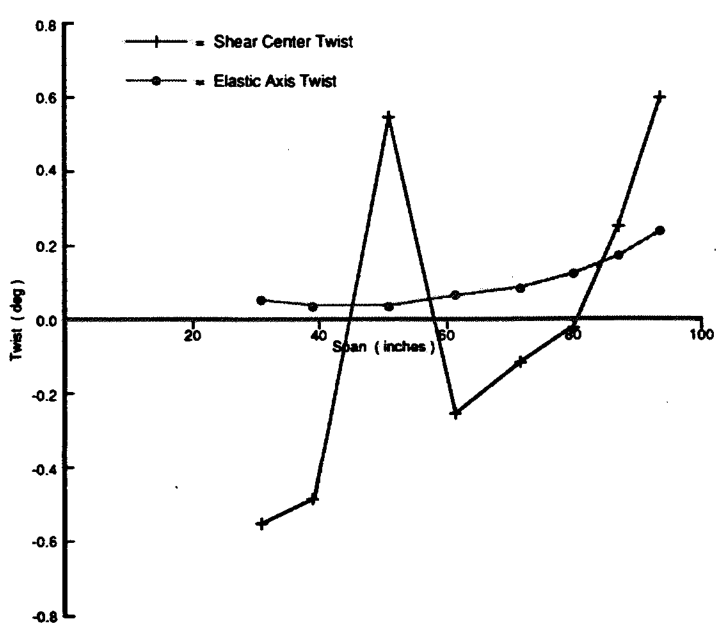
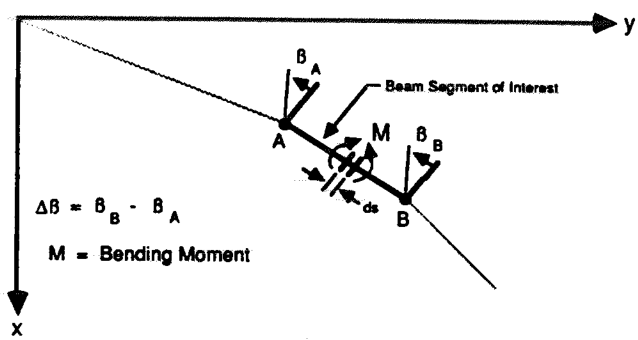



FIGURE 4. CHORD TWIST VS. SPAN

COMPUTATION OF BENDING AND TORSION STIFFNESS

Once the elastic axis has been defined and RBE3 elements have been incorporated in the finite element model of the stabilizer, the slopes and deflections are directly output from the MSC/NASTRAN analysis for a single transverse load applied at the tip of the surface. Essentially, back substitution in the simple beam formula, Equation 1, provides the bending stiffness, EI.

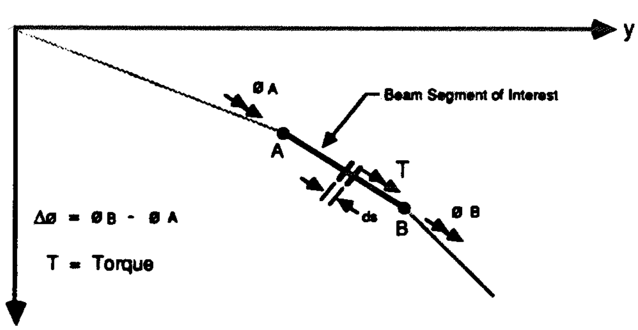



For transverse force (P) at the tip

$$\Delta\beta = \frac{1}{EI_{AB}} \int_A^B M ds \qquad \text{Eq. 1}$$

Where $EI_{AB}$ is the constant bending stiffness for the element $_{AB}$.

$$ds = \sqrt{dx^2 + dy^2}$$



Similarly, back substitution in the simple torsion formula, Equation 2, provides the torsional stiffness, GJ.

$$\Delta\phi = \frac{1}{GJ_{AB}} \int_A^B T ds \qquad \text{Eq. 2}$$

Where $GJ_{AB}$ is the constant torsional stiffness for the element $_{AB}$.

Figure 5 shows the distribution of the bending and torsion stiffness along the axis.

Variation of similar beam formulae with proper loading and boundary conditions may be used as an alternate method of estimating the beam stiffness.

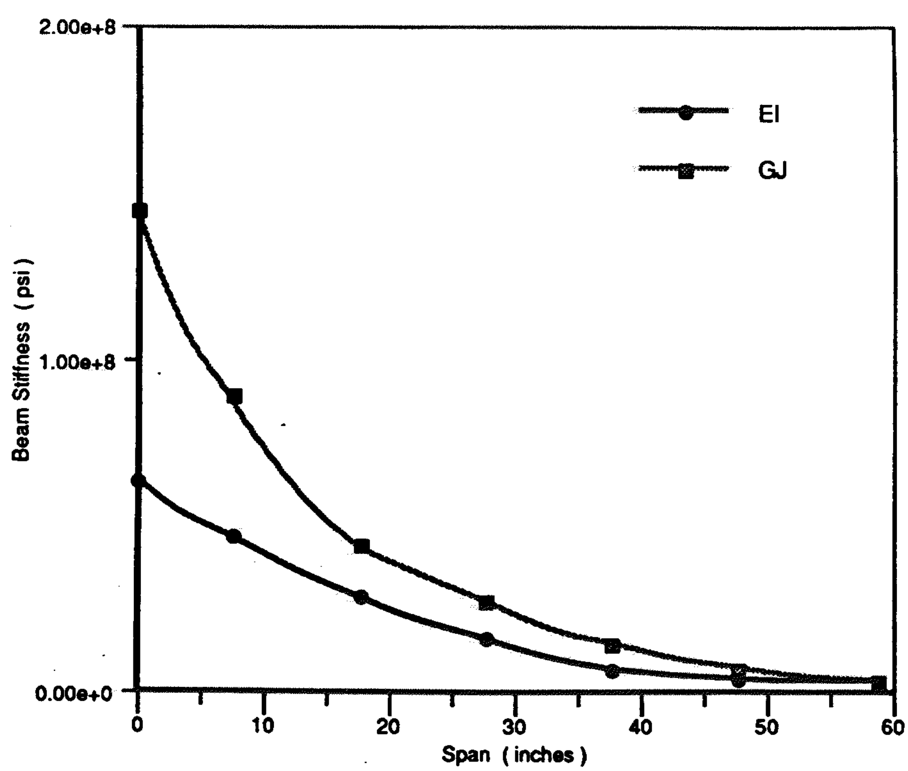



FIGURE 5. BENDING AND TORSION STIFFNESS

## COMPARISON OF NORMAL MODES ANALYSIS

Normal modes analysis of the finite element (CQUAD4) and the equivalent beam element models is performed and compared with the ground vibration test results in Table 1. The equivalent beam element model is made of nine (9) CBAR elements which are placed along the elastic axis and the stabilizer mass is lumped, by proper transformation, on the grids of the beam.

| NORMAL MODES ANALYSIS AND TEST | BENDING FREQUENCY (HZ) | TORSION FREQUENCY (HZ) |
|---|---|---|
| Finite Element Model | 18.4 | 40.90 |
| Beam Element Model | 18.34 | 41.48 |
| Ground Vibration Test | 18.80 | 40.72 |

TABLE 1. NORMAL MODES ANALYSIS AND TEST RESULTS

The agreement among the finite element model, beam model and test results for the natural frequencies is good. In this case, an equivalent beam representation of the surface is possible for reducing the degrees of freedom of the dynamics model and building a reliable wind tunnel model.

## SUMMARY

The purpose of deriving the equivalent beam for the stabilizer surface described in this paper is solely to reduce the cost of flutter and gust response analysis and to provide an elastic axis location and stiffness data for building a wind tunnel model. An automated interactive graphics procedure to derive the beam from a detailed MSC/NASTRAN finite element model has been developed.

It is important to note that in general, a unique elastic axis for all purposes does not exist, as it depends on the support system and loading condition of the structure. However, a particular application of the equivalent beam in normal modes analysis or other coarse elastic deformation applications may be possible. Engineering is an art of approximation as well as an exact science. Whenever possible, reduction of the engineering problem or simplification of manufacturing considerations should be taken into account to accomplish the engineering task.