Example Problems Illustrating The Effect of Multiple Cross Corrleated Excitations on the Response of Linear Systems to Gaussian Random Excitations

by

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Reference: (1) DYNAMICS OF STRUCTURES, R. W. Clough and J. Penzien, McGraw-Hill, 1975.

SUMMARY:

Several simple example problems have been provided demonstrating the application of the multiple source capability of MSC/NASTRAN rigid format solution 30. In particular, the effect of various amounts of cross correlation of the input sources is investigated. It is shown through the examples that the peak value of output dis-placements, stresses, etc. may occur for values of cross correlation other than that resulting from fully correlated sources.

DISCUSSION:

The MSC/NASTRAN computer program provide a means of obtaining both direct as well as modal solutions for complex structures subjected to arbitrary excitations. In particular, the rigid format frequency response solution 30 provides for the post-processing of frequency response data to obtain output responses of various specified variables for response to Gaussian random vibration excitation. Also, the inputs to various points can be correlated arbitrarily as the user desires. It is the purpose of this paper to present examples wherein the effect of varying the correlation of the inputs to the various support points of a structure is examined and the effect on output variables is determined.

Usually, structures subject to random vibration testing are analyzed as supported on a fixture, which is attached to a vibration exciter. If the rigidity of the fixture is high enough as to decouple its modes from the response of the structure, the vibration inputs to the structure in question are fully correlated. Thus, the analyst ignores all questions concerning the possible correlation, or lack of correlation, of the inputs. He assumes that the structure is attached to a very large seismic mass, and excites the seismic mass by a force proportional to the mass and to the desired input power spectral density.

However, if the structure is large enough so that its supports are fairly discrete from each other, than it is clear that the flexibility of the support structure must be taken into account in analyzing the response of the structure itself (i.e., the modes of the fixture and structure will be coupled). It is also apparent that the actual inputs to the support points of the structure affixed to its next assembly may be correlated to a lesser degree than the simple analysis would indicate. It is possible to examine the potential difference in output variable response as a function of the degree of correlation of the inputs, although the actual correlation of the inputs may be unknown. Thus, the bounds on output responses can be established.

In order to conduct this study, it is necessary only to depart slightly from the normal MSC/NASTRAN random analysis solution procedure. Instead of supporting the structure on one seismic mass of ten to the fourth or fifth times the mass of the structure, equal seismic masses are attached to each support point, of a combined magnitude equal to that of the usual analysis. Then, the correlation of the sources are identified on the bulk data deck input cards "RANDPS". Providing that the support offered by the separate seismic masses does not lessen the rigidity of the support offered by the usual single seismic mass ("RBARS" sometimes constrain the in-plane motion of the structure), then it can easily be demonstrated (see Appendix A) that the results of the two analyses will be identical for a positive (+1) correlation of all of the inputs. The uncorrelated case (all cross coupling terms null) is a mean solution, and the results for negatively or positively correlated inputs will be greater or lesser than the uncorrelated values.

Several examples are included in Appendix A.

APPENDIX A

ANALYSIS
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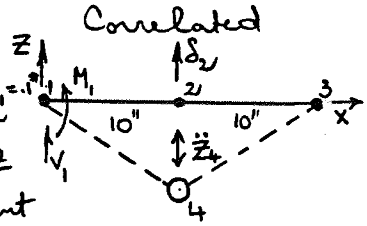
Example 1: 2 Span Beam

"Usual"

(One Mass)
Correlated

"White" Gaussian Random Z Axis Input

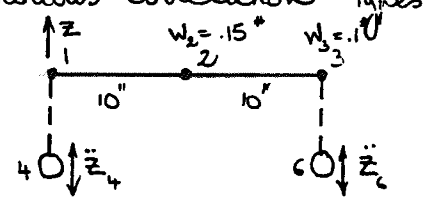
$S_{oz} = 1.0\ g^2/Hz$



1 Seismic Mass
$(W_4 = 1.+3)$

"Variable"

(Two Mass)
Various Correlation Types



½ Seismic Mass   ½ Seismic Mass

$\zeta = 2\frac{1}{2}\%$ of critical
$(Q = 20)$

$(f_n = 144.49\ Hz)$ Beam: $\begin{cases} A = .04\ in^2 \\ I = 1.333{-}4 \\ E = 10^7\ psi \end{cases}$

Results:

RMS Response

Variable	"Usual" Way	"Variable" Case ($S_{uc} = +1, 0, -1$) **		
		Correlated (+1)	Uncorrelated (0)	Correlated (-1)
M_1 (in.lbs)	25.24 (144.2)*	25.24 (144.2)	154.2 (18.66)	216.6 (8.41)
V_1 (lbs)	5.05 (144.2)	5.05 (144.2)	15.7 (33.8)	21.7 (8.41)
δ_2 (in.)	.544 (11.8)	.544 (11.8)	.385 (11.8)	No (26.7)

* RMS Response (No. positive crossing)

** The cross correlation terms in MSC/NASTRAN are invariant with frequency

Example 2 : "┬" Frame

"White" Gaussian

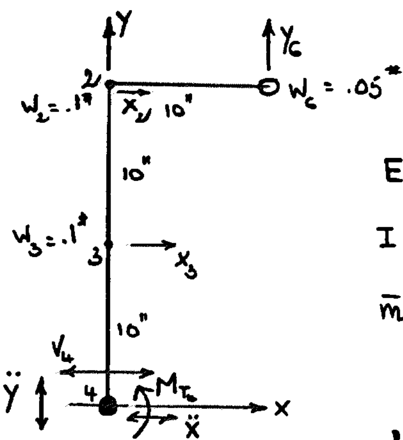
Random X and Y

Inputs :

$$\begin{cases} S_{a_x} = 1.0\ g^2/Hz \\ S_{a_y} = 0.5\ g^2/Hz \end{cases}$$

$\zeta = 2\tfrac{1}{2}\%$ of Critical

($Q = 20$)



Seismic Mass
$W = 1. \times 10^3$

$E = 10^7$ psi

$I = 1.333 \times 10^{-4}$, $A = .04$ in^2

$\bar{m} = \dfrac{.01}{386}$ (Mass/Unit length)

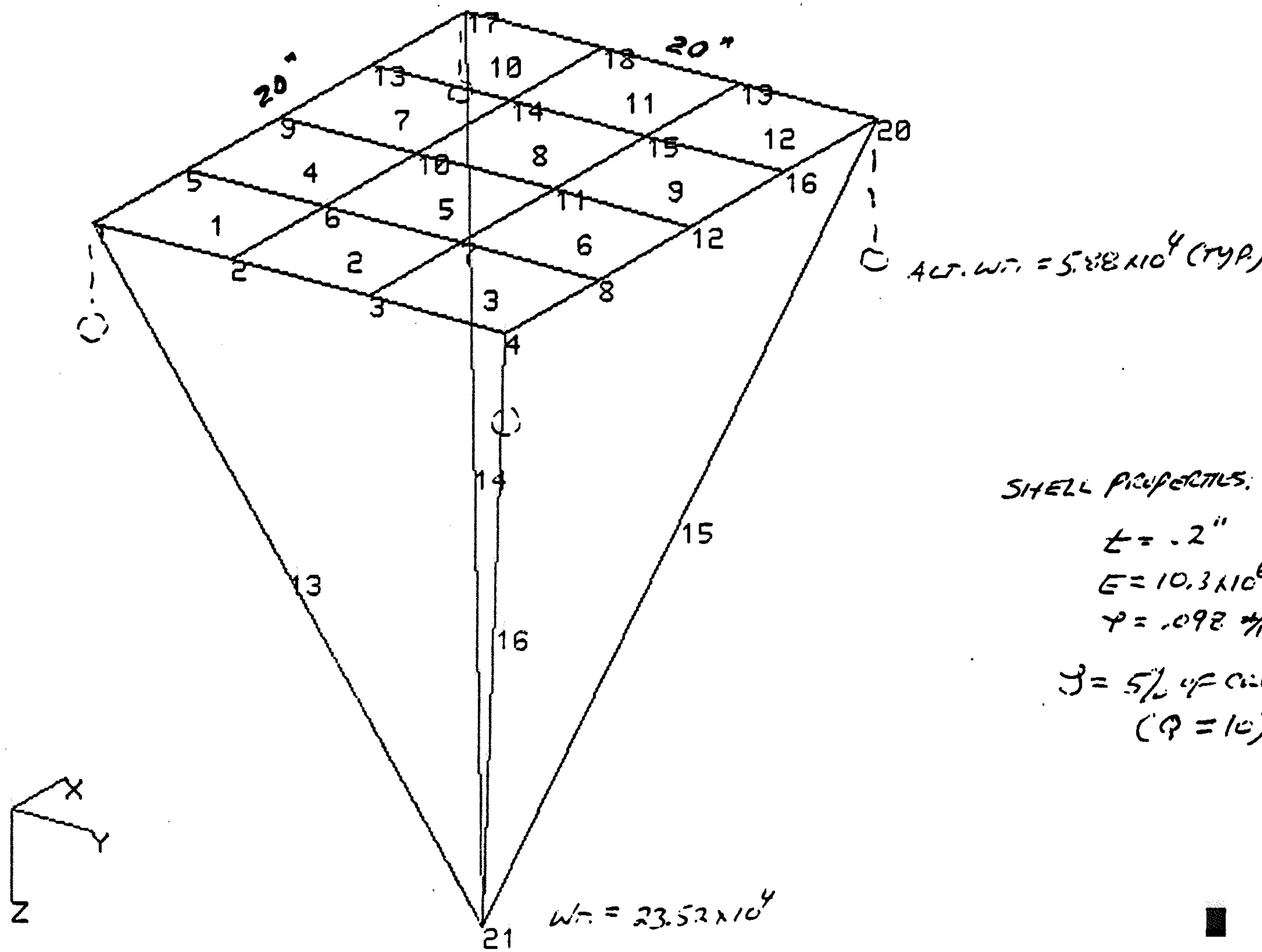
$$f_n = \begin{cases} 5.07\ Hz \\ 18.30\ Hz \\ 53.39\ Hz \end{cases}$$

Results :

Variable	Correlated "Usual" Way ; Input			Variable Correlation (S_{xy} = +.707, 0, −.707)		
	X only	Y only	Both	Pos. Corr +707	No Corr 0	Neg. Corr −.707
M_{T_4} (in.lbs)	61.8	7.6	57.6	57.6	62.3	66.6
V_4 (lbs)	3.34	0.59	3.25	3.25	3.39	3.52
X_3 (in)	2.16	0.22	2.02	2.02	2.17	2.31
Y_c (in)	5.07	0.75	4.68	4.68	5.13	5.54
$X_3 + Y_c$ (in.)	7.15	0.92	6.57	6.57	7.21	7.80
$X_3 - Y_c$ (in)	3.10	0.60	2.95	2.95	3.16	3.36
$\langle X_3 \cdot Y_c \rangle^{**}$	3.22	0.35	2.94	2.94	3.24	3.52

* RMS response (No. positive crossings)

$$** \quad \langle X_3 \cdot Y_c \rangle = \frac{\left(\sigma_{X_3+Y_c}^2 - \sigma_{X_3-Y_c}^2\right)^{1/2}}{2}$$



$$f_n = \begin{cases} 18.41 \text{ Hz} \\ 33.04 \text{ Hz} \\ 45.21 \text{ Hz} \\ -45.34 \text{ Hz} \\ \vdots \end{cases}$$

SHELL PROPERTIES:

$t = .2''$

$E = 10.3 \times 10^6$ PSI

$\rho = .098$ #/in^3

$\zeta = 5\%$ of critical

($Q = 10$)

EXAMPLE 3

SQUARE PLATE

Example 3: "Square Plate"

Table I

Correlation Coefficient vs. Case

Case No.	Load Point No.	Associated Pt. No.	Correlation Coeff.	Description
1	1	1	1.	"Usual" one
2	1	1	1.	mass correlated
	1	2	1.	(+1) with four
	1	3	1.	seismic masses
	1	4	1.	
	2	2	1.	
	2	3	1.	
	2	4	1.	
	3	3	1.	
	3	4	1.	
	4	4	1.	
3	1	1	1.	Uncorrelated
	2	2	1.	input with
	3	3	1.	four seismic
	4	4	1.	masses
4	1	1	1	Negative
	1	2	-1.	correlated
	1	3	-1.	inputs
	1	4	-1.	
	2	2	1.	
	2	3	-1.	
	2	4	-1.	
	3	3	1.	
	3	4	-1.	
	4	4	1.	

Table I (Cont.)

Case No.	Load Point No.	Associated Pt. No.	Correlation Coeff.	Description
5	1	1	1.	Correlated on opposite edges
	1	2	1	
	1	3	-1.	
	1	4	-1.	
	2	2	1.	
	2	3	-1.	
	2	4	-1.	
	3	3	1	
	3	4	1	
	4	4	1.	
6	1	1	1.	Correlated on opposite corners
	1	2	-1.	
	1	3	-1.	
	1	4	1.	
	2	2	1.	
	2	3	1.	
	2	4	1.	
	3	3	-1.	
	3	4	-1.	
	4	4	1	

Results For Example 3. (Square Plate)

Table 2

1.correlated input with one mass

point	rms value*	# positive crossings**
6	1.17	13.0
7	1.17	13.0
10	1.34	14.5
11	1.34	14.5
14	1.17	13.0
15	1.14	13.0

2.correlated input with four masses

point	rms value	# positive crossings
6	1.17	13.0
7	1.17	13.0
10	1.34	14.5
11	1.34	14.5
14	1.17	13.0
15	1.17	13.0

3.uncorrelated input

point	rms value	# positive crossings
6	.672	13.1
7	.672	13.1
10	.691	14.4
11	.691	14.4
14	.672	13.1
15	.672	13.1

4.negatively correlated input

point	rms value	# positive crossings
6	1.17	13.0
7	1.14	13.0
10	1.34	14.5
11	1.34	14.5
14	1.17	13.0
15	1.17	13.0

5.correlated on opposite edges

point	rms value	# positive crossings
6	.522	14.0
7	.522	14.0
10	1.97e-4	18.7
11	1.97e-4	15.2
14	.522	14.0
15	.522	14.0

6.correlated on opposite corners

point	rms value	# positive crossings
6	.200	11.6
7	.200	11.6
10	2.20e-4	14.2
11	1.98e-4	15.2
14	.200	11.6
15	.200	11.6

* Displacement in z direction (inches)

** Measure of Frequency