Analysis of Water Distribution Networks using MSC/NASTRAN®

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ABSTRACT

A problem that arises very frequently in applied hydraulics is the study of steady incompressible flows in a network of pipes. This paper introduces a structural analogy that allows the problem to be treated with MSC/NASTRAN using the nonlinear statics capability.

INTRODUCTION

The analysis of steady incompressible flow problems in a network of pipes has been traditionally solved using the well-known Hardy Cross technique [1] and most recently the Newton-Raphson method [2]. Both approaches have proven to be successful. Another possibility, however, is to cast the problem as a nonlinear static problem, by making use of a structural analogy. This formulation fits very well with the current nonlinear capabilities offered by MSC/NASTRAN.

PROBLEM FORMULATION

Consider a network of pipes as shown in Figure 1. It is assumed that the geometry of the system and its material properties are known. Furthermore, the outflows at certain given nodes are also known. The problem under consideration consists of determining: a) the flow, and b) the head loss, in each one of the pipes of the system.

For the case of a steady incompressible flow, the relationship between the flow (Q) in a pipe and the head loss (h), is given by the Darcy-Weisbach [3] equation,

$$h/L = \frac{fQ^2}{2gD^5 \, (\pi/4)^2} \qquad (1)$$

where, f is the friction coefficient; L is the length of the pipe; g is the gravity acceleration and D is the diameter of the pipe. Then, the problem reduces to find the values of Q and h in each pipe, such that the following three conditions are met:

1) The Darcy-Weisbach relationship between Q and h must be satisfied in each pipe.

2) The continuity equation must be satisfied at each node (the flow into each junction must equal the flow out of the junction).

3) The algebraic sum of the pressure drops (head losses) around each circuit must be zero.

STRUCTURAL ANALOGY

The Darcy-Weisbach relationship given by Eq. (3) can be rewritten as

$$Q = \frac{\pi}{\sqrt{8}} \sqrt{\frac{g\, D^5}{L\, f}}\; h^{1/2} \qquad (2)$$

or more simply as

$$Q = R\, h^{1/2} \qquad (3)$$

where

$$R = \frac{\pi}{\sqrt{8}} \sqrt{\frac{g\, D^5}{L\, f}} \qquad (4)$$

Note that for a given pipe the constant R is completely determined once the geometry and the material properties are known.

Consider a generic pipe located between two nodes, say A and B. For convenience, the relationship between the flow and the head loss can be rewritten as

$$Q = R_{AB}\; (h_A - h_B)^{1/2} \qquad (5)$$

where h_A and h_B represent the value of the hydraulic head at each node.

Consider now a nonlinear elastic rod with a cross section equal to one, aligned in the x-direction, as shown in Figure 2. Let u_A and u_B be the displacements in the axial direction, at A and B; and let F be the axial force taken by the rod. It is desired to construct an analogy, using this nonlinear rod, that could simulate the constitutive relationship of the pipe (Eq. (5)). Furthermore, it is desired to represent the flow Q by F, the axial force taken by the rod; and the hydraulic head at each end of the pipe by the displacement at the end of the rod. This can be accomplished by defining a certain stress-strain (σ vs. ε) relationship for the rod.

Assume then that the relationship between σ and ε is given by

$$\sigma = R_{AB}\, \ell_{AB}^{1/2}\, \varepsilon^{1/2} = R_{AB}\, \ell_{AB}^{1/2} \left[\frac{u_A - u_B}{\ell_{AB}} \right]^{1/2} \quad (6)$$

where ℓ_{AB} is the length of the rod, and u_A and u_B are the nodal displacements in the x-direction at each node. Since the cross section of the rod is equal to one, it follows then that the force (F) taken by the rod is equal to

$$F = \sigma \cdot 1 = R_{AB}\, \ell_{AB}^{1/2} \left[\frac{u_A - u_B}{\ell_{AB}} \right]^{1/2}$$

$$= R_{AB} \left[u_A - u_B \right]^{1/2} \quad (7)$$

Finally, examination of Equation (5) and Equation (7) indicates that there is a complete analogy between the structural problem and the hydraulic problem. The flow in the pipe is represented by the force taken by the rod, and the hydraulic heat at each node is represented by the displacement of the rod in the x-direction. This analogy allows one to solve the problem under study, simply by modeling the network of pipes as an ensemble of nonlinear elastic rods.

EXAMPLE OF APPLICATION

Consider the network of pipes depicted in Figure 3. This example has been taken from Streeter's Fluid Mechanics [4] book. The outflow at node 2, 3 and 4 is given. The geometry of this configuration (length and diameter of each pipe) as well as the friction properties of the material are known. All this information is summarized in the coefficient R (see Eq. (4)) given for each pipe. This example was solved using MSC/NASTRAN and Solution 66 (Nonlinear static analysis).

The first step to solve this problem using the structural analogy is to define an appropriate graph to specify the connectivity of the network. To this end, four nodes (1, 2, 3 and 4) are defined along the x-axis. These nodes represent the nodes of the pipe network. Note that the coordinate of each node is completely arbitrary. The only requirement is that all nodes must lie along the same axis (see Figure 3).

Then, nonlinear elastic rods are defined to model the pipes. Each rod is governed by a stress-strain relationship of the form

$$\sigma_{ij} = R_{ij} \, \ell_{ij}^{1/2} \, \varepsilon_{ij}^{1/2} \qquad (8)$$

where

$$\ell_{ij} = \left| x_i - x_j \right| \qquad (9)$$

x_i and x_j are the coordinates of the two nodes associated with that particular rod in the graph. It is clear that the location of the nodes in the graph is irrelevant, since an appropriate scaling factor ($\ell_{ij}^{1/2}$) has been introduced in the definition of the stress-strain relationship of the rod.

Thus, the following stress-strain relationships are adopted for the rods of the structural model:

$$\begin{aligned}
\sigma_{12} &= 0.500000 \, (100)^{1/2} \, \varepsilon_{12}^{1/2} \\
\sigma_{14} &= 0.707106 \, (50)^{1/2} \, \varepsilon_{14}^{1/2} \\
\sigma_{23} &= 1.000000 \, (100)^{1/2} \, \varepsilon_{23}^{1/2} \qquad (10) \\
\sigma_{24} &= 1.000000 \, (50)^{1/2} \, \varepsilon_{24}^{1/2} \\
\sigma_{34} &= 0.447213 \, (150)^{1/2} \, \varepsilon_{34}^{1/2}
\end{aligned}$$

The outflows are represented as forces (discrete loads) at the corresponding nodes. Node 1 is assumed to be constrained in the x-direction; the reaction force at this node represents the total input flow to be supplied to the network.

Table 1 compares the values for the flow in the pipes (forces) obtained with MSC/NASTRAN and the values reported by Streeter. (Streeter's values have been computed with an error of approximately 1% [4]). It can be seen that both solutions agree well. The head loss in each pipe is determined by subtracting the displacements at the end nodes. The solution to this problem is summarized in Figure 4.

CONCLUDING REMARKS

A structural analogy has been presented to solve incompressible steady flow problems in closed conduits using MSC/NASTRAN. In this approach, each pipe is modeled with a nonlinear elastic rod that has a stress-strain relationship analogous to the Darcy-Weisbach relationship of the pipe to be modeled. In this context, flows in pipes are represented by forces in rods and the structural displacement corresponds to the hydraulic head. It is clear that this approach guarantees satisfaction of the three conditions stated in the introduction. The continuity equation is now replaced by the static equilibrium equation; the Darcy-Weisbach relationship in each pipe is satisfied by virtue of the stress-strain relationship used to describe the nonlinear rod; and condition (3) is automatically enforced since the independent variable used in this formulation is the value of the hydraulic head (displacement) at each node.

In addition, MSC/NASTRAN offers a number of structural features that have a useful counterpart in hydraulics. For instance, nonlinear springs can be used to model losses due to valves, fittings or abrupt changes in diameter; rigid elements can model loss-free pipes; and GAP elements can be used to represent pipes in which the flow can go only in one direction.

REFERENCES

[1] Cross, H., "Analysis of Flow in Networks of Conduits or Conductors," University of Illinois, Bull. 286, Nov. 1946.

[2] Ralston, A. and Rabinowitz, P., "A First Course in Numerical Analysis," McGraw-Hill, 1978.

[3] Marks' Standard Handbook for Mechanical Engineers, McGraw-Hill, 7th Edition, 1967.

[4] Streeter, V., "Fluid Mechanics," McGraw-Hill, 1962.

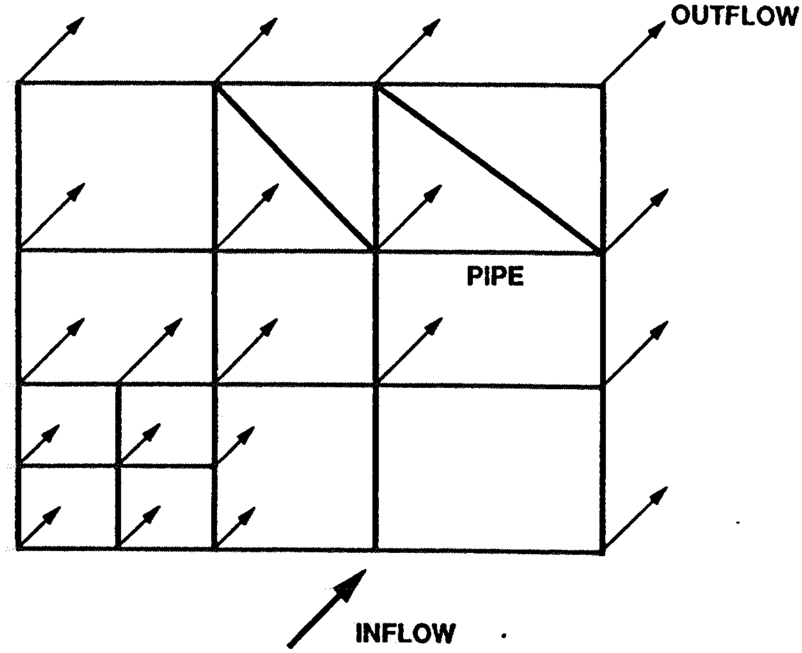


Figure 1. Typical Hydraulic Network.

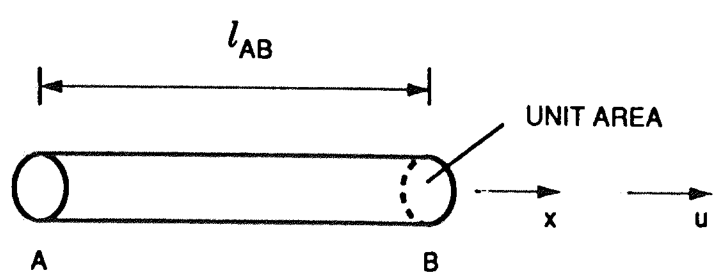
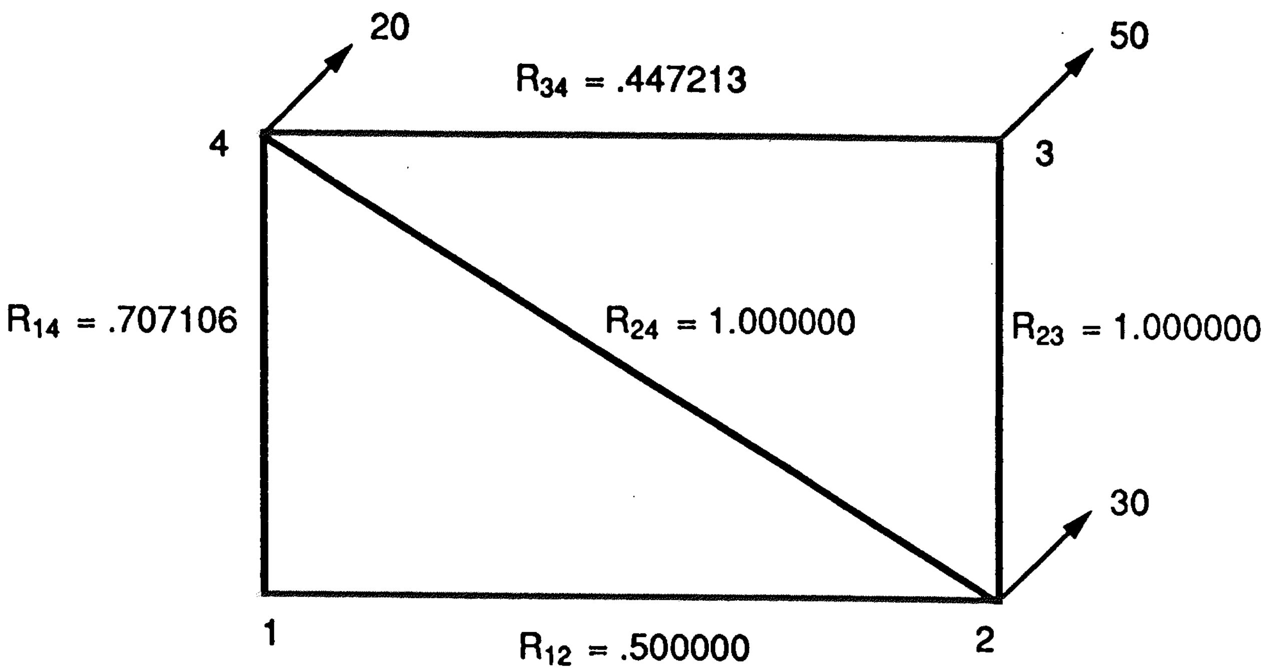
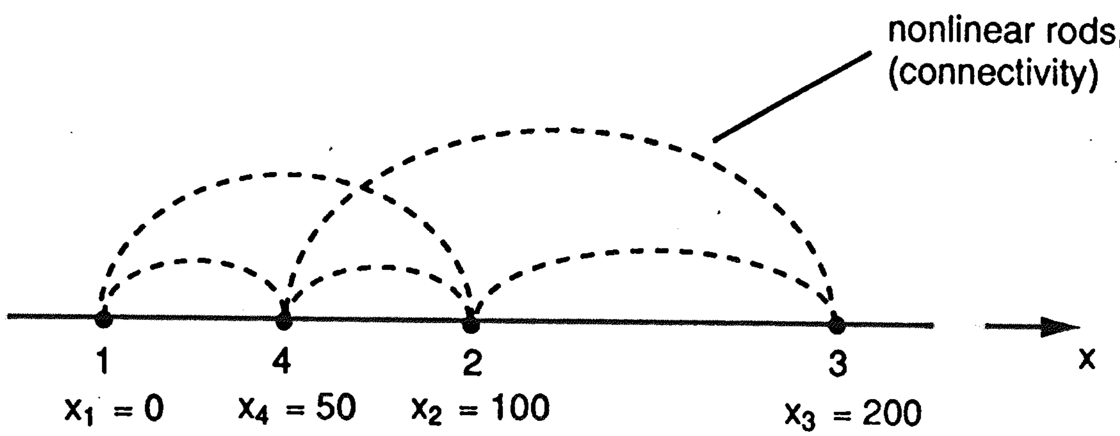


Figure 2. Nonlinear Elastic Rod.



Hydraulic Network



Equivalent Structural Graph

Figure 3. Example of Application.

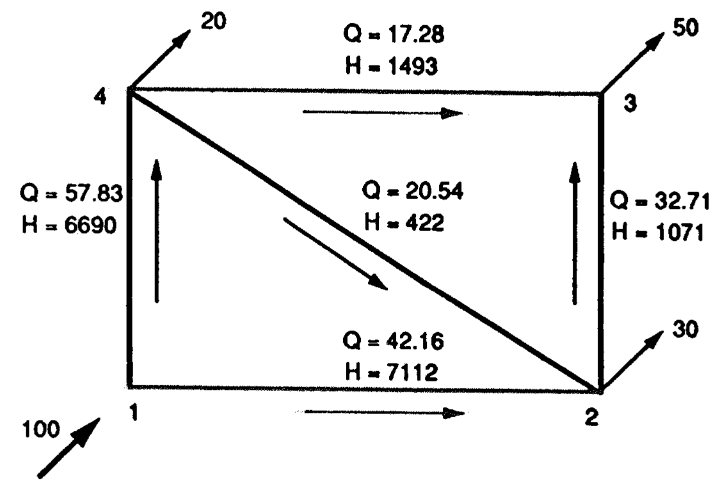


Figure 4. MSC/NASTRAN Solution.

TABLE 1. - Flows in Pipes

Pipe	MSC/NASTRAN	Streeter
1-2	42.16	42.
1-4	57.83	58.
2-3	32.71	33.
4-2	20.54	21.
4-3	17.28	17.