# Using Superelements to Identify the Dynamic Properties of a Structure

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ABSTRACT

The following paper contains an MSC/NASTRAN® DMAP for Solution 63, which, with the proper partitioning into Superelements, provides an excellent method of identifying the normal modes of a model. In addition, by using parameters, the DMAP will perform a series of static checks on each superelement.

INTRODUCTION

Identifying the modes of a structure is normally done by looking at deformed plots of the mode shapes and the printed displacement output. Unfortunately, in the case of complex structures, this can be tedious if not impossible. By carefully partitioning a model into superelements and using the DMAP Alters provided with this paper, modal identification can be made much simpler.

The method presented in this paper uses approximations of the system's true dynamic properties to identify the characteristics of the system modes. It consists of determining kinetic energy and effective weight at both the system and the subsystem (Superelement) levels. These values can then be used to identify which areas of the structure are active in each mode, allowing for easy identification of the modes.

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Kinetic Energy

The expression normally used for kinetic energy is:

$$K.E. = \frac{1}{2} mv^2$$

for a point mass, m, moving with velocity, v. The kinetic energy for a system is the sum of all its components. In a finite element model, the mass is represented in matrix form utilizing the degrees of freedom at the grid points in the mass matrix [M]. This matrix is normally formed using the lumped mass assumption, that is, there are no off-diagonal terms in the matrix. In this situation, the method given in this paper is exact for the model. If the mass matrix is not diagonal, the method is an approximate, but, based on past experience, appears to work quite well.

The Alter uses the following method to calculate kinetic energy. The displacement $x_{i,n}(t)$ of any degree of freedom i in a specified mode n can be represented as

$$x_{i,n}(t) = a_{i,n} \sin\omega_n t \qquad (1)$$

where $a_{i,n}$ is the modal displacement of d.o.f. i in mode n and $\omega_n$ is the frequency of mode n.

The velocity is simply the first derivative of displacement with respect to time, or

$$v_{i,n}(t) = a_{i,n}\,\omega_n \cos\omega_n t \tag{2}$$

thus, with mass, $m_i$, associated with d.o.f. i, the kinetic energy at d.o.f. i is,

$$k.e._{i,n} = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i\, a_{i,n}^2\, \omega_n^2 \cos^2 \omega_n t \tag{3}$$

On a system level, the kinetic energy of the system for mode n is

$$K.E._n = \sum_{i=1}^{ndof} \frac{1}{2} m_i\, a_{i,n}^2\, \omega_n^2 \cos^2 \omega_n t \tag{4}$$

$$= \frac{1}{2} \omega_n^2 \cos^2 \omega_n t \sum_{i=1}^{ndof} m_i a_{i,n}^2 \tag{5}$$

this can be shown in matrix form as

$$K.E._n = \frac{1}{2} \omega_n^2 \cos^2 \omega_n t \{\phi_n\}^T [\ulcorner m_i \lrcorner] \{\phi_n\} \tag{6}$$

where

$$\{\phi_n\} = \begin{Bmatrix} a_{1,n} \\ a_{2,n} \\ a_{3,n} \\ \vdots \\ a_{ndof,n} \end{Bmatrix}$$

is the vector (mode shape) for mode n and

$$\lceil m_i \rfloor$$

is a diagonal matrix containing the masses associated with the d.o.f.

At this point, it can be noted that the mode shapes are non-dimensional quantities and can be scaled in any number of ways. Due to this, it is best to scale the kinetic energy of the individual d.o.f.'s by the total to get a number which represents the portion of the total system K.E. at each d.o.f. This is accomplished by dividing equation (3) by equation (6), which gives

$$\text{K.E. Fraction}_{i,n} = \frac{\text{k.e.}_{i,n}}{\text{K.E.}_n} = \frac{\frac{1}{2} m_i a_{i,n}^2 \omega_n^2 \cos^2 \omega_n t}{\frac{1}{2} \omega_n^2 \cos^2 \omega_n t \{\phi_n\}^T \lceil m_i \rfloor \{\phi_n\}} \tag{7}$$

or,

$$\text{KEFraction}_{i,n} = \frac{m_i a_{i,n}^2}{\{\phi_n\}^T \lceil m_i \rfloor \{\phi_n\}} \tag{8}$$

In the case where we normalize the modes to unit mass,

$$\{\phi_n\}^T \lceil m_i \rfloor \{\phi_n\} = 1.0 \tag{9}$$

Combining equations (8) and (9) gives,

$$\text{KEFraction}_{i,n} = m_i a_{i,n}^2 \tag{10}$$

for any d.o.f. i, if the mass matrix is diagonal and the modes are scaled to unit mass.

The kinetic energy fractions for all of the d.o.f. in the model can be obtained in matrix form by

$$\{KEFraction\}_n = \{\phi_n\}^T [\,\diagdown m_i \diagdown\,] \cdot \{\phi_n\} \tag{11}$$

where the • means perform a term-by-term multiply as opposed to a matrix multiply.

The kinetic energy fraction can be obtained for all of the modes by using

$$\{\phi\} = \{\phi_1 : \phi_2 : \phi_3 : \phi_4 : \cdots \phi_{n\ modes}\} \tag{12}$$

in equation 11, or

$$\{KEFraction\} = \{\phi\}^T [\,\diagdown m_i \diagdown\,] \cdot \{\phi\}$$

where each column of the KEFraction matrix represents one mode.

The explanation so far has concentrated on models with a diagonal mass matrix. As it turns out, the results of having a coupled mass matrix (such as one resulting from performing a Guyan reduction) appear to be equally as good.

## Effective Weight

This is an approximate measure of how much of the structures' weight is active in any given mode. The best way to explain this concept is to use the participation factors from a Response Spectrum analysis. In this type of analysis, the structural response of each mode is found to some type of excitation at a given point. This is done by calculating what are called participation factors and using those to assist in finding the structural response.

The participation factors are defined as follows

$$\Gamma_{in} = \{\phi_n\}^T [M] \{\phi_{iR}\} \tag{13}$$

where $\Gamma_{in}$ is the participation factor for mode n in direction i, $\{\phi_n\}$ is the mode shape for mode n, $[M]$ is the system mass matrix, and $\{\phi_{iR}\}$ is a vector representing the structural deformation resulting from moving the excitation point one unit in the direction of the excitation, i.

In the special case of a cantilever support, the participation factors can be used to give the reaction at the constraint resulting from each mode. That is,

$$R_{in} = \Gamma_{in}^2 \, g \tag{14}$$

where $R_{in}$ is the base reaction at the support in d.o.f. i for mode n, and g is gravity. Once again, this only works out if the modes have been scaled to unit mass.

As it turns out, if all of the modes of the system are obtained, the summation of $R_{in}$ over the modes is the weight of the model in direction i

$$R_i = \sum_{n=1}^{nmodes} R_{in} \tag{15}$$

The reactions $R_{in}$ for all 6 d.o.f. at the point of cantilever can be obtained for each mode, giving the portion of the system weight active in each direction for each mode.

The effective weight is calculated using this idea. Since not all structures are cantilevered from a point, the grid point specified using PARAM,GRDPNT is used as a reference point and vectors representing geometric rigid body motion about that point are used for $\{\phi_{iR}\}$. This results in "psuedo reactions" about that point, which are useful in identifying system (large effective weight) or local (small effective weight) modes.

## Use of Superelements

So far the discussion has been for the entire system model. Values of kinetic energy and effective weight at the system level allow for modal identification, but this can be improved by finding the values at a subsystem or assembly level. This is where Superelements are valuable in modal identification.

If Superelements are used to represent subsystems of the structure, then the resulting kinetic energy and effective weight can be automatically obtained on a subsystem (Superelement) level. That is, calculations can be performed on each superelement individually and also including upstream Superelements (in the case of a multi-level tree).

The advantage of this is that the kinetic energy and effective weight can be obtained for each subsystem and it is immediately known which subsystems are active in which modes, allowing for easy identification of modes. An example of this is shown in Appendix B.

An application of this is an aircraft, where the wings and tail could be individual Superelements or assemblies, allowing easy identification of "wing" or "tail" modes.

## Implementation of the Alter

The Alter in Appendix A is PARAMETER driven, that is, each feature is controlled by PARAMETERs in the Bulk Data, or can be turned on or off for each Superelement by placing the PARAMETERs in the Case Control if desired. The PARAMETERs are used as follows. All have a default of -1, and setting them to any integer > - 1 turns that feature on.

| PARAMETER | FEATURE |
|---|---|
| CHKSTIF | Check G-, N-, and A-set stiffness matrices for rigid-body motion. This prints CHKKGG, CHKKNN, and CHKKAA for the appropriate sets. The diagonals of these matrices should be 0.0 or very near. If the diagonals are not 0.0, then there is something constraining rigid-body motion of the Superelement, and the appropriate reactions (REACG, REACN, or REACA) should be |

checked to locate the constraints. These reactions represent the reactions to geometric rigid-body motion about the GRID in PARAM,GRDPNT.

| | |
|---|---|
| CHKMASS | Print G- and A-SET rigid-body mass matrix using geometry about PARAM,GRDPRNT |
| KEPRT | Calculate and Print kinetic energy for Superelements. The results include the total with (TOTENWUP) and without (TOTENOUP) upstream effects, and also the kinetic energy fractions for each d.o.f. in the Superelement with (ENERGWUP) and without (ENERNOUP) upstream contributions. |
| EFWGT | Calculate and print effective weight with (EFWWUP) and without (EFWNOUP) upstream effects. |

An example of the results of using the ALTER are in Appendix B.

## APPENDIX A - Listing of Alter

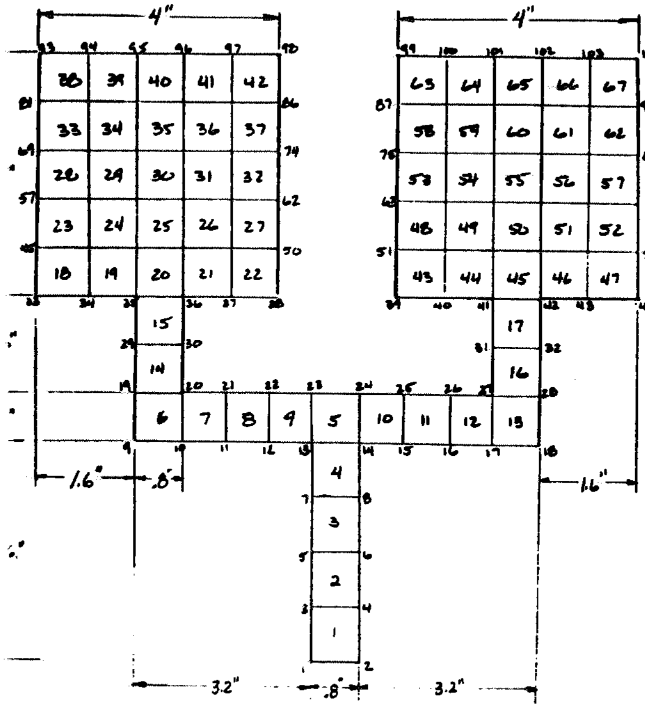
```
$
$ ALTERS FOR SOLUTION 63 TO PERFORM STIFFNESS CHECKS
$      AND CALCULATE KINETIC ENERGY AND MODAL WEIGHT BY S.E.
$      BOTH WITH AND WITHOUT UPSTREAM COONTRIBUTIONS
$
ALTER 5 $ SET DEFAULTS FOR PARAMETERS
 PARAMR  //DIV/V,N,MW/1.0/V,Y,WTMASS=1.0 $
 PARAMR  //COMPLEX//V,N,MW/0.0/V,N,MASSWT $
 PARAM   //NOP/V,Y,CHKSTIF=-1 $ SET CHKSTIF >-1 TO PRINT STIFFNESS
 CHECKS PARAM   //NOP/V,Y,CHKMASS=-1 $ SET CHKMASS >-1 TO PRINT MASS
 CHECKS PARAM   //NOP/V,Y,KEPRT=-1 $ SET KEPRT >-1 FOR KINETIC ENERGY
 PRINT PARAM   //NOP/V,Y,EFWGT=-1 $ SET EFWGT > -1 FOR MODAL WEIGHT
 CALCS
ALTER 334 $ JUST BEFORE SEMA
$
$ UPSTREAM Q-SET PARTITION VECTOR
$
  SEMA EQEXINS,SLIST,EMAP,/KPP/KLAA/MAPS/SOLID/LUSETS/SEID/V,Y,DBSET4 $
  DIAGONAL  KPP/VGUQ/COLUMN/1. $
$
ALTER 342 $ AFTER GP4
$
$ CREATE Q-SET PARTITION VECTOR & COMBINE W/UPSTREAM PARTITION
$
  VEC USETB/VGLQ/G/COMP/Q/ $
  ADD VGLQ,VGUQ/VGPQ/ $
  DBSTORE VGPQ,VGLQ//MODEL/SEID/DBSET2 $
  PARAML VGPQ//TRAILER/5/V,N,NZWDS $
  PARAM //SUB/V,N,NOPAR/NZWDS/1 $
  VECPLOT ,,BGPDTS,EQEXINS,CSTMS,,/RBTG/GRDPNT//4 $
  COND NOPART,NOPAR $
  PARTN RBTG,VGPQ,/RBX,,,/1 $
  MERGE RBX,,,,VGPQ,/RBTG/1 $
  LABEL NOPART $
  TRNSP RBTG/RBG/ $
  DBSTORE RBG//SOLID/SEID/DBSET2 $
$
 COND NOKGGCHK,CHKSTIF $
$
$ CHECK KGG FOR CONSTRAINTS IF CHKSTIF > -1
$
  MPYAD KGG,RBG,/REACG/ $
  MPYAD RBTG,REACG/CHKKGG/ $
  MATGPR GPLS,USETB,SILS,REACG//H/G//1.-2 $
  MATPRN CHKKGG // $
 LABEL NOKGGCHK $
$
ALTER 358 $ CHECK KNN IF CHKSTIF > -1
 COND NOKNNCHK,CHKSTIF $
$ CHECK KNN FOR CONSTRAINTS
```

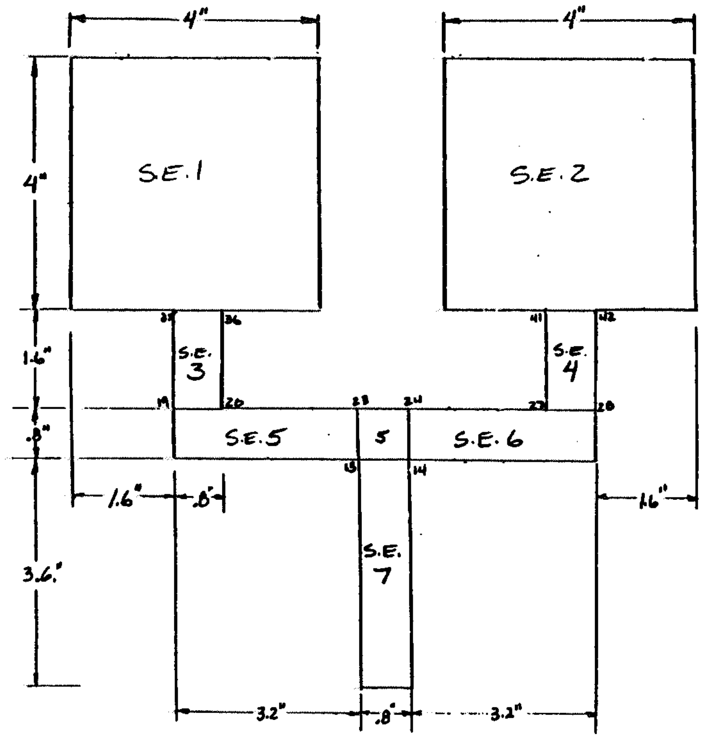
```
  UPARTN USETB,RBG/RBN,,,/G/N/M/1 $
  TRNSP RBN/RBTN $
  MPYAD KNN,RBN,/REACN/ $
  MPYAD RBTN,REACN/CHKKNN/ $
  MATGPR GPLS,USET,SILS,REACN//H/N//1.-2 $
  MATPRN CHKKNN // $
 LABEL NOKNNCHK $
$
ALTER 437 $ AFTER DBSTORE KAA
 UPARTN USET,RBG/RBA,,,/G/A/O/1 $
 COND NOKAACHK,CHKSTIF $
$ CHECK KAA FOR CONSTRAINTS IF CHKSTIF > -1
  TRNSP RBA/RBTA $
  MPYAD KAA,RBA,/REACA/ $
  MPYAD RBTA,REACA/CHKKAA/ $
  MATGPR GPLS,USET,SILS,REACA//H/A//1.-2 $
  MATPRN CHKKAA // $
 LABEL NOKAACHK $
$
ALTER 473 $ AFTER MGG STORE
 PARAMR //DIV/V,N,MW/1.0/V,Y,WTMASS=1.0 $
 PARAMR //COMPLEX//V,N,MW/0.0/V,N,MASSWT $
 COND NOMGGCK,CHKMASS $
$
$  CHECK MGG IF CHKMASS > -1
$
  MPYAD MGG RBG,/MGRB/ $
  MPYAD RBTG,MGRB,/MASS/ $
  ADD MASS,/WGHT/MASSWT $
  MATPRN WGHT// $
  PRTPARM //0/C,N,MASSWT $
  PRTPARM //0/C,N,GRDPNT $
 LABEL NOMGGCK $
$
ALTER 535 $ AFTER MAA STORE
 COND NOMAACK,CHKMASS $
$
$  CHECK MAA IF CHKMASS > -1
$
  MPYAD MAA,RBA,/MARB $
  TRNSP RBA/RBTA/ $
  MPYAD RBTA,MARB,/MASSA/ $
  ADD MASSA,/WGHTA/MASSWT $
  MATPRN WGHTA// $
  PRTPARM //0/C,N,MASSWT $
  PRTPARM //0/C,N,GRDPNT $
 LABEL NOMAACK $
$
ALTER 960 $ AFTER SDR1 - SYSTEM K.E.
 PARAM //AND/V,N,KEEFM/KEPRT/EFWGT $
```

```
 COND NOKEEFW,KEEFW $ CHECK IF K.E. AND EFW ARE WANTED
$
$ CALC EFFECTIVE WEIGHT AND KINETIC ENERGY IF KEEFW > -1
$
   DBFETCH /LUSETD,EQEXINS,GPLS,SILS,USET/SOLID/SEID/DBSET3 $
   DBFETCH /MGG,RBG,MJJ,VGPQ,/SOLID/SEID/DBSET3 $
$ PARTITION OUT Q-SET DOF FROM UGVS
$PARTN UGVS,,VGPQ/UGVX,,,/1 $
$$MATPRN UGVX,U1// $
$MERGE UGVX,,,,,VGPQ/UGXS/1 $
   MPYAD UGVS,MGG,/PHITM/1//// $
  COND NOWGT1,EFWGT $
    MPYAD PHITM,RBG,/MER///// $
    TRNSP MER/MERT $
    TRNSP MER/MERTA $
    ADD MERT,MERTA/EFWWUP/MASSWT//1 $
    MATPRN EFWWUP// $
  LABEL NOWGT1 $
$
  COND NOKE1,KEPRT $
   TRNSP PHITM/PHITMT $
   ADD PHITMT,UGVS/ENERGWUP///1 $
   MPYAD PHITM,UGVS,/TOTEN1/ $
   DIAGONAL TOTEN1/TOTENWUP $
   MATPRN TOTENWUP// $
   MATGPR GPLS,USET,SILS,ENERGWUP//H/G//1.-2 $ 1% FILTER ON ENERGY
  LABEL NOKE1 $
$
  MPYAD UGVS,MJJ,/PHITMJ/1//// $
  COND NOWGT2,EFWGT $
   MPYAD PHITMJ,RBG,/MERJ///// $
   TRNSP MERJ/MERJT $
   TRNSP MERJ/MERJTA $
   ADD MERJT,MERJTA/EFWNOUP/MASSWT//1 $
   MATPRN EFWNOUP// $
  LABEL NOWGT2 $
  COND NOKE2,KEPRT $
   TRNSP PHITMJ/PHITMJT $
   ADD PHITMJT,UGVS/ENERNOUP///1 $
   MPYAD PHITMJ,UGVS,/TOTEN2/ $
   DIAGONAL TOTEN2/TOTENOUP $
   MATPRN TOTENOUP// $
   MATGPR GPLS,USET,SILS,ENERNOUP//H/G//1.-2 $ 1% FILTER ON ENERGY
  LABEL NOKE2 $
 LABEL NOKEEFW $
$
$ END OF ALTER FOR KINETIC ENERGY BASED ON SUPERELEMENTS
$
```

APPENDIX B - Sample Problem

The following model was analyzed using SOL 63 with the alters. A brief summary of the results follows.




## Mass Check - Superelement 1

The following is the output from the Grid Point Weight Generator, followed by the weight check output for the A-set, showing that the reduction retained the mass properties of the original.

```
S.E. SAMPLE PROBLEM 1 - USING KE AND EFM ALTERS                          DECEMBER  24, 1987  MSC/NASTRAN 11/27/85
 S.E. DYNAMICS - USE DIFFERENT REDUCTION TECHNIQUES                                                SUPERELEMENT 1

                  O U T P U T   F R O M   G R I D   P O I N T   W E I G H T   G E N E R A T O R
                                      REFERENCE POINT =        0
                                               M O
        *  2.264000E-01  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00 -1.811200E+00 *
        *  0.000000E+00  2.264000E-01  0.000000E+00  0.000000E+00  0.000000E+00 -7.244799E-01 *
        *  0.000000E+00  0.000000E+00  2.264000E-01  1.811200E+00  7.244799E-01  0.000000E+00 *
        *  0.000000E+00  0.000000E+00  1.811200E+00  1.481561E+01  5.795839E+00  0.000000E+00 *
        *  0.000000E+00  0.000000E+00  7.244799E-01  5.795839E+00  2.644352E+00  0.000000E+00 *
        * -1.811200E+00 -7.244799E-01  0.000000E+00  0.000000E+00  0.000000E+00  1.745997E+01 *
                                               S
                         *  1.000000E+00  0.000000E+00  0.000000E+00 *
                         *  0.000000E+00  1.000000E+00  0.000000E+00 *
                         *  0.000000E+00  0.000000E+00  1.000000E+00 *
           DIRECTION
      MASS AXIS SYSTEM (S)     MASS             X-C.G.        Y-C.G.        Z-C.G.
               X            2.264000E-01     0.000000E+00  8.000000E+00  0.000000E+00
               Y            2.264000E-01    -3.200000E+00  0.000000E+00  0.000000E+00
               Z            2.264000E-01    -3.200000E+00  8.000000E+00  0.000000E+00
                                              I(S)
                         *  3.260155E-01  0.000000E+00  0.000000E+00 *
                         *  0.000000E+00  3.260162E-01  0.000000E+00 *
                         *  0.000000E+00  0.000000E+00  6.520319E-01 *
                                              I(Q)
                         *  3.260155E-01                             *
                         *                3.260162E-01               *
                         *                              6.520319E-01 *
                                               Q
                         *  1.000000E+00  0.000000E+00  0.000000E+00 *
                         *  0.000000E+00  1.000000E+00  0.000000E+00 *
                         *  0.000000E+00  0.000000E+00  1.000000E+00 *
```

```
   S.E. SAMPLE PROBLEM 1 - USING KE AND EFM ALTERS                          DECEMBER  24, 1987  MSC/NASTRAN 11/27/85
    S.E. DYNAMICS - USE DIFFERENT REDUCTION TECHNIQUES                                                 SUPERELEMENT 1

      MATRIX MGHTA     (GINO NAME 101 ) IS A DB  PREC        6 COLUMN X      6 ROW SQUARE   MATRIX.
COLUMN        1        ROWS       1 THRU      6      ------------------------------------------------------
ROW
     1)    2.2640D-01  1.2035D-15  0.0000D+00  0.0000D+00  0.0000D+00 -1.8112D+00
COLUMN        2        ROWS       1 THRU      6      ------------------------------------------------------
ROW
     1)    1.1930D-15  2.2640D-01  0.0000D+00  0.0000D+00  0.0000D+00 -7.2448D-01
COLUMN        3        ROWS       3 THRU      5      ------------------------------------------------------
ROW
     3)    2.2640D-01  1.8112D+00  7.2448D-01
COLUMN        4        ROWS       3 THRU      5      ------------------------------------------------------
ROW
     3)    1.8112D+00  1.4816D+01  5.7958D+00
COLUMN        5        ROWS       3 THRU      5      ------------------------------------------------------
ROW
     3)    7.2448D-01  5.7958D+00  2.6444D+00
COLUMN        6        ROWS       1 THRU      6      ------------------------------------------------------
ROW
     1)   -1.8112D+00 -7.2448D-01  0.0000D+00  0.0000D+00  0.0000D+00  1.7460D+01
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =        3
THE DENSITY OF THIS MATRIX IS  50.00 PERCENT.
```

## Stiffness Check - Superelement 1

The next table is the results of the "free-free" stiffness check. Note that the terms in CHKKGG are essentially 0.0, indicating that Superelement 1 is not constrained internally against rigid-body motion.

```
   S.E. SAMPLE PROBLEM 1 - USING KE AND EFM ALTERS                          DECEMBER  24, 1987  MSC/NASTRAN 11/27/85
    S.E. DYNAMICS - USE DIFFERENT REDUCTION TECHNIQUES                                                 SUPERELEMENT 1

      MATRIX CHKKGG    (GINO NAME 101 ) IS A DB  PREC        6 COLUMN X      6 ROW SQUARE   MATRIX.
COLUMN        1        ROWS       1 THRU      6      ------------------------------------------------------
ROW
     1)    1.4734D-10  9.5497D-12  0.0000D+00  0.0000D+00  0.0000D+00 -1.1043D-09
COLUMN        2        ROWS       1 THRU      6      ------------------------------------------------------
ROW
     1)    2.2283D-11  4.7294D-11  0.0000D+00  0.0000D+00  0.0000D+00  3.0286D-10
COLUMN        3        ROWS       3 THRU      5      ------------------------------------------------------
ROW
     3)   -2.6375D-11 -2.1598D-10 -7.0054D-11
COLUMN        4        ROWS       3 THRU      5      ------------------------------------------------------
ROW
     3)   -1.6223D-10 -1.3626D-09 -4.2452D-10
COLUMN        5        ROWS       3 THRU      5      ------------------------------------------------------
ROW
     3)   -1.6712D-11 -1.3521D-10 -1.8872D-11
COLUMN        6        ROWS       1 THRU      6      ------------------------------------------------------
ROW
     1)    3.9463D-10  1.5243D-09  0.0000D+00  0.0000D+00  0.0000D+00 -1.0091D-08
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =        3
THE DENSITY OF THIS MATRIX IS  50.00 PERCENT.
```

In this case, the reactions, REACG were all small and none were printed. If they had been non-trivial, a list of DOF and their reactions (> .01) would have been printed also.

## Results - Residual Structure

The system level analysis solved for 9 modes. The eigenvalue table is shown below. Note that the Generalized Mass is 1.0 for the modes, indicating the modes are normalized to unit mass.

S.E. SAMPLE PROBLEM 1 - USING KE AND EFM ALTERS      DECEMBER 24, 1987 MSC/NASTRAN 11/27/85
S.E. DYNAMICS - USE DIFFERENT REDUCTION TECHNIQUE      SUPERELEMENT 0

R E A L   E I G E N V A L U E S

| MODE NO. | EXTRACTION ORDER | EIGENVALUE | RADIANS | CYCLES | GENERALIZED MASS | GENERALIZED STIFFNESS |
|---|---|---|---|---|---|---|
| 1 | 1 | 1.186271E+03 | 3.444228E+01 | 5.481659E+00 | 1.000000E+00 | 1.186271E+03 |
| 2 | 2 | 3.557532E+03 | 5.964505E+01 | 9.492805E+00 | 1.000000E+00 | 3.557532E+03 |
| 3 | 3 | 4.184140E+04 | 2.045517E+02 | 3.255542E+01 | 1.000000E+00 | 4.184140E+04 |
| 4 | 4 | 6.280658E+04 | 2.506124E+02 | 3.988620E+01 | 1.000000E+00 | 6.280658E+04 |
| 5 | 6 | 1.216821E+05 | 3.488297E+02 | 5.551797E+01 | 1.000000E+00 | 1.216821E+05 |
| 6 | 7 | 2.412955E+05 | 4.912184E+02 | 7.817984E+01 | 1.000000E+00 | 2.412955E+05 |
| 7 | 5 | 2.444724E+05 | 4.944415E+02 | 7.869281E+01 | 1.000000E+00 | 2.444724E+05 |
| 8 | 9 | 1.353530E+06 | 1.163413E+03 | 1.851629E+02 | 1.000000E+00 | 1.353530E+06 |
| 9 | 8 | 1.471050E+06 | 1.212868E+03 | 1.930340E+02 | 1.000000E+00 | 1.471050E+06 |

The effective weight results for the entire structure are contained in the Table EFFWWUP for Superelement 0. In this table, each column represents a mode, and the rows represent X, Y, Z, Xrot, Yrot, and Zrot respectively. Normally, the terms for the translations are used for quick identification (example: mode 1 is active in the Z direction), but rotational terms help also (example: mode 2 has no translational weight and a large Y rotation term, implying an internally balanced mode). Significant terms for the 9 modes are underlined below (note, this is a small, light structure, so terms which might be negligible for a larger model are significant here. They should be compared to the grid point weight generator output.)

EFFECTIVE WEIGHT for S.E. 0 with upstream contributions:

```
 S.E. SAMPLE PROBLEM 1 - USING KE AND EFM ALTERS                        DECEMBER  24, 1987  MSC/NASTRAN 11/27/85
  S.E. DYNAMICS - USE DIFFERENT REDUCTION TECHNIQUES                                                SUPERELEMENT 0

     MATRIX EFMMUP   (GINO NAME 101 ) IS A DB  PREC      9 COLUMN X      6 ROW RECTANG  MATRIX.
COLUMN      1       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    1.3709D-30  2.4018D-34  5.1122D-01  3.1764D+01  2.0600D-25  1.?122D-28
COLUMN      2       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    2.4328D-35  1.9775D-34  5.2685D-26  2.2904D-24  5.7288D+00  3.4981D-33
COLUMN      3       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    2.7854D-34  3.0650D-31  1.0593D-24  5.8341D-26  2.7408D-01  8.4459D-32
COLUMN      4       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    4.7019D-27  6.2497D-33  7.2185D-03  1.3580D-02  4.0994D-23  4.2408D-25
COLUMN      5       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    1.1833D-25  1.3846D-30  6.2242D-02  2.9838D-01  1.8723D-23  1.0843D-23
COLUMN      6       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    6.0495D-27  6.6496D-28  3.3995D-23  1.6179D-22  2.4243D-02  5.7572D-26
COLUMN      7       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    3.9436D-01  9.7914D-28  3.5601D-26  1.5527D-27  3.7638D-28  3.7462D+01
COLUMN      8       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    1.2944D-25  1.2518D-27  4.2116D-03  1.3403D-02  1.1319D-25  1.8071D-23
COLUMN      9       ROWS       1 THRU      6       -----------------------------------------------
ROW
      1)    8.5921D-27  4.5766D-02  2.3216D-29  7.3583D-29  1.8603D-25  8.5342D-25
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =        6
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
```

This table is presented more clearly in the following spreadsheet.

EFFECTIVE WEIGHT FOR S.E. 0 - WHOLE MODEL

| MODE | X | Y | Z | Rx | Ry | Rz |
|---|---|---|---|---|---|---|
| 1 | 0.000 | 0.000 | 0.511 | 31.764 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 5.729 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.274 | 0.000 |
| 4 | 0.000 | 0.000 | 0.007 | 0.014 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.062 | 0.298 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.024 | 0.000 |
| 7 | 0.394 | 0.000 | 0.000 | 0.000 | 0.000 | 37.462 |
| 8 | 0.000 | 0.000 | 0.004 | 0.013 | 0.000 | 0.000 |
| 9 | 0.000 | 0.046 | 0.000 | 0.000 | 0.000 | 0.000 |
| TOTAL | 0.394 | 0.046 | 0.585 | 32.089 | 6.027 | 37.462 |
| SYSTEM WGT | 0.586 | 0.586 | 0.586 | 32.091 | 6.029 | 38.123 |
| FRACTION | 0.673 | 0.078 | 0.998 | 1.000 | 1.000 | 0.983 |

Where the values can be compared to the total system weight. A popular way to look at this data is to divide the values by the corresponding totals and thus have numbers which represent the portion of the system active in a mode.

The matrix EFWNOUP (effective weight without upstream contributions) for Superelement 0 isn't shown here, but has nearly all 0's in it, indicating that the results for the upstream superelements should be used for identification of which subsystems are active in which modes.

Matrix EFWNOUP is shown for Superelement 1 below with significant terms underlined.

```
 S.E. SAMPLE PROBLEM 1 - USING KE AND EFM ALTERS                    DECEMBER  24, 1987  MSC/NASTRAN 11/27/85
  S.E. DYNAMICS - USE DIFFERENT REDUCTION TECHNIQUES                                          SUPERELEMENT 1

    MATRIX EFMNOUP  (GINO NAME 101 ) IS A DB  PREC     9 COLUMN X     6 ROW RECTANG  MATRIX.
COLUMN      1        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    3.3976D-31  1.3447D-36  1.0421D-01  7.0856D+00  1.0804D+00  3.0148D-29
COLUMN      2        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    3.7373D-30  1.9637D-36  9.9275D-02  6.6447D+00  1.2082D+00  3.3150D-28
COLUMN      3        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    4.7597D-28  3.6222D-32  4.1567D-03  5.2937D-01  6.3830D-03  4.2296D-26
COLUMN      4        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    1.1215D-27  1.3900D-29  6.8165D-08  1.6805D-02  1.3066D-01  1.0251D-25
COLUMN      5        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    2.6856D-26  1.3317D-27  1.1053D-05  6.9732D-02  2.8320D-02  2.5293D-24
COLUMN      6        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    3.1565D-26  3.4266D-29  1.6389D-03  1.1966D-02  1.8847D-02  2.7664D-24
COLUMN      7        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    8.1178D-02  1.8535D-02  8.6266D-27  3.3845D-25  7.5514D-26  8.1813D+00
COLUMN      8        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    7.9994D-27  2.7766D-25  4.0067D-03  7.9916D-02  1.8064D-02  2.5013D-24
COLUMN      9        ROWS        1 THRU      6    ----------------------------------------------
ROW
     1)    8.4853D-02  8.5799D-03  5.3905D-25  1.3191D-2[illegible]  6.4826D-24  8.1438D+00
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =        6
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
```

These values can be compared to the weight of Superelement 1 in a manner similar to above. This results in the following table.

Superelement 1 EFWNOUP/Superelement 1 weight

| MODE | X | Y | Z | Rx | Ry | Rz |
|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.46 | 0.48 | 0.41 | 0.00 |
| 2 | 0.00 | 0.00 | 0.44 | 0.45 | 0.46 | 0.00 |
| 3 | 0.00 | 0.00 | 0.02 | 0.04 | 0.00 | 0.00 |
| 4 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.00 |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 |
| 6 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 |
| 7 | 0.36 | 0.08 | 0.00 | 0.00 | 0.00 | 0.47 |
| 8 | 0.00 | 0.00 | 0.02 | 0.01 | 0.01 | 0.00 |
| 9 | 0.37 | 0.04 | 0.00 | 0.00 | 0.00 | 0.47 |

This table indicates how active Superelement 1 is in any of the system modes. As a rule, the response is similar to the system response, that is, in the same direction. However, mode 9 demonstrates why it is valuable to have results at a subsystem (Superelement) level. At the system level, this mode seems unimportant, that is, the effective weight is nearly 0.0 for all d.o.f., which normally indicates a local mode. The results for Superelement 1 reveal that it is active in the X translation and Z-rotation directions, indicating a "tuning fork" mode where the mode is self equilibrating, but has noticeable motion.

## Kinetic Energy

Perhaps the easiest way to identify a mode is to look at the kinetic energy and use it to locate which parts of the structure are moving in a mode. The alter gives the kinetic energy both by subsystem (Superelement) and by d.o.f. The kinetic energy by d.o.f. is good for coarse models using lumped masses in that the kinetic energy will be concentrated at the masses, allowing for quick modal identification. The terms are filtered, so that only d.o.f. with > 1% of the system kinetic energy are printed, so for fine mesh models, the energy may be distributed such that few, if any, individual d.o.f. have more than 1% of the system total. In this case, the total by Superelement will allow for quick identification of which subsystems (Superelements) are active in which modes. The following table, made up of the kinetic energy by Superelement, allows rapid identification of which subsystems are active in each mode.

TOTAL KINETIC ENERGY BY SUPERELEMENT - TOTENOUP

| S.E. | MODE 1 | MODE 2 | MODE 3 | MODE 4 | MODE 5 | MODE 6 | MODE 7 | MODE 8 | MODE 9 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.002 | 0.000 | 0.000 | 0.000 | 0.039 | 0.001 | 0.001 | 0.095 | 0.000 |
| 1 | 0.480 | 0.474 | 0.407 | 0.452 | 0.250 | 0.369 | 0.473 | 0.199 | 0.492 |
| 2 | 0.480 | 0.474 | 0.407 | 0.452 | 0.250 | 0.369 | 0.473 | 0.199 | 0.492 |
| 3 | 0.011 | 0.016 | 0.038 | 0.023 | 0.054 | 0.051 | 0.014 | 0.011 | 0.006 |
| 4 | 0.011 | 0.016 | 0.038 | 0.023 | 0.054 | 0.051 | 0.014 | 0.011 | 0.006 |
| 5 | 0.008 | 0.010 | 0.055 | 0.025 | 0.154 | 0.080 | 0.012 | 0.161 | 0.002 |
| 6 | 0.008 | 0.010 | 0.055 | 0.025 | 0.154 | 0.080 | 0.012 | 0.161 | 0.002 |
| 7 | 0.001 | 0.000 | 0.000 | 0.000 | 0.046 | 0.001 | 0.001 | 0.164 | 0.000 |
| | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

This table indicates that, with the exception of modes 5 & 8, Superelements 1 & 2 are the dominant features with generally over 80% of the system kinetic energy between them.

Using this information and then looking at the kinetic energy for Superelement 1 below, it is easy to characterize the system modes.

| | POINT | ENEPGNUP VALUE | POINT | VALUE | POINT | VALUE | POINT | VALUE | POINT | VALUE |
|---|---|---|---|---|---|---|---|---|---|---|
| COLUMN 1 | 46 T3 | 1.21298E-02 | 47 T3 | 1.18004E-02 | 48 T3 | 1.14765E-02 | 49 T3 | 1.11583E-02 | 58 T3 | 1.65476E-02 |
| | 59 T3 | 1.61635E-02 | 60 T3 | 1.57839E-02 | 61 T3 | 1.54090E-02 | 69 T3 | 1.10527E-02 | 70 T3 | 2.16630E-02 |
| | 71 T3 | 2.12236E-02 | 72 T3 | 2.07882E-02 | 73 T3 | 2.03570E-02 | 81 T3 | 1.39855E-02 | 82 T3 | 2.74728E-02 |
| | 83 T3 | 2.69779E-02 | 84 T3 | 2.64867E-02 | 85 T3 | 2.59992E-02 | 86 T3 | 1.27676E-02 | 94 T3 | 1.69873E-02 |
| | 95 T3 | 1.67120E-02 | 96 T3 | 1.64385E-02 | 97 T3 | 1.61667E-02 | | | | |
| COLUMN 2 | 45 T3 | 1.36792E-02 | 46 T3 | 2.07983E-02 | 47 T3 | 1.51282E-02 | 48 T3 | 1.03632E-02 | 57 T3 | 1.59705E-02 |
| | 58 T3 | 2.48069E-02 | 59 T3 | 1.85706E-02 | 60 T3 | 1.32366E-02 | 69 T3 | 1.84561E-02 | 70 T3 | 2.92067E-02 |
| | 71 T3 | 2.23986E-02 | 72 T3 | 1.64917E-02 | 73 T3 | 1.14867E-02 | 81 T3 | 2.11316E-02 | 82 T3 | 3.39827E-02 |
| | 83 T3 | 2.66013E-02 | 84 T3 | 2.01204E-02 | 85 T3 | 1.45414E-02 | 93 T3 | 1.19953E-02 | 94 T3 | 1.95632E-02 |
| | 95 T3 | 1.55854E-02 | 96 T3 | 1.20576E-02 | | | | | | |
| COLUMN 3 | 33 T3 | 1.68240E-02 | 34 T3 | 1.83968E-02 | 45 T3 | 2.26961E-02 | 46 T3 | 2.13140E-02 | 57 T3 | 1.37484E-02 |
| | 61 T3 | 1.03127E-02 | 62 T3 | 1.41279E-02 | 73 T3 | 2.31554E-02 | 74 T3 | 2.40270E-02 | 84 T3 | 1.83490E-02 |
| | 85 T3 | 4.12217E-02 | 86 T3 | 3.65759E-02 | 96 T3 | 1.73128E-02 | 97 T3 | 3.22473E-02 | 98 T3 | 2.58638E-02 |
| COLUMN 4 | 33 T3 | 1.96192E-02 | 34 T3 | 1.91634E-02 | 45 T3 | 3.22232E-02 | 46 T3 | 2.88353E-02 | 50 T3 | 1.28308E-02 |
| | 57 T3 | 2.56997E-02 | 58 T3 | 2.01798E-02 | 61 T3 | 1.20679E-02 | 62 T3 | 1.87916E-02 | 69 T3 | 1.96499E-02 |
| | 70 T3 | 1.27240E-02 | 73 T3 | 2.02976E-02 | 74 T3 | 2.58535E-02 | 81 T3 | 1.42160E-02 | 85 T3 | 3.06055E-02 |
| | 86 T3 | 3.39418E-02 | 97 T3 | 2.14863E-02 | 98 T3 | 2.15123E-02 | | | | |
| COLUMN 5 | 37 T3 | 1.67518E-02 | 38 T3 | 1.23540E-02 | 49 T3 | 1.82038E-02 | 50 T3 | 1.49427E-02 | 81 T3 | 1.61729E-02 |
| | 82 T3 | 2.02685E-02 | 83 T3 | 1.09249E-02 | 93 T3 | 1.42962E-02 | 94 T3 | 2.02341E-02 | 95 T3 | 1.32489E-02 |
| | 36 T3 | 1.06896E-02 | | | | | | | | |
| COLUMN 6 | 37 T3 | 2.23687E-02 | 38 T3 | 1.90866E-02 | 48 T3 | 1.41775E-02 | 49 T3 | 3.15747E-02 | 50 T3 | 2.90246E-02 |
| | 61 T3 | 1.88970E-02 | 62 T3 | 2.04236E-02 | 69 T3 | 1.38742E-02 | 70 T3 | 1.06163E-02 | 74 T3 | 1.25172E-02 |
| | 81 T3 | 2.53594E-02 | 82 T3 | 2.54134E-02 | 93 T3 | 2.00435E-02 | 94 T3 | 2.33050E-02 | 95 T3 | 1.08966E-02 |
| | 36 T3 | 1.20186E-02 | | | | | | | | |
| COLUMN 7 | 58 T1 | 1.23827E-02 | 59 T1 | 1.23827E-02 | 60 T1 | 1.23827E-02 | 61 T1 | 1.23827E-02 | 70 T1 | 1.64459E-02 |
| | 71 T1 | 1.64459E-02 | 72 T1 | 1.64459E-02 | 73 T1 | 1.64460E-02 | 81 T1 | 1.05423E-02 | 82 T1 | 2.10847E-02 |
| | 83 T1 | 2.10847E-02 | 84 T1 | 2.10847E-02 | 85 T1 | 2.10847E-02 | 86 T1 | 1.05424E-02 | 94 T1 | 1.31495E-02 |
| | 95 T1 | 1.31495E-02 | 96 T1 | 1.31495E-02 | 97 T1 | 1.31495E-02 | | | | |
| COLUMN 8 | 48 T3 | 1.00291E-02 | 49 T3 | 1.35318E-02 | 50 T3 | 1.06579E-02 | 94 T3 | 1.46644E-02 | 95 T3 | 1.07969E-02 |
| COLUMN 9 | 58 T1 | 1.19226E-02 | 59 T1 | 1.19227E-02 | 60 T1 | 1.19226E-02 | 61 T1 | 1.19228E-02 | 70 T1 | 1.84117E-02 |
| | 71 T1 | 1.84118E-02 | 72 T1 | 1.84120E-02 | 73 T1 | 1.84121E-02 | 81 T1 | 1.31527E-02 | 82 T1 | 2.63054E-02 |
| | 83 T1 | 2.63055E-02 | 84 T1 | 2.63056E-02 | 85 T1 | 2.63057E-02 | 86 T1 | 1.31529E-02 | 94 T1 | 1.78017E-02 |

In this table, each column represents a system mode and the terms printed represent the kinetic energy fractions > 1% contained in Superelement 1. For example, for mode 1 (column 1), the term "46 T3 1.21298E-02" means that 1.21% of the kinetic energy for mode 1 is at grid point 43 in the Z- translation direction. Nothing that all the terms printed for mode 1 are in the T3 direction, it's easy to determine that Superelement 1 is moving primarily in the Z-direction in mode 1 and has 48% of the system kinetic energy. Superelement 2 has similar output.

By looking at this data, one can see that 96% of the system kinetic energy for mode 1 is in Superelement 1 & 2 and is primarily in the Z-direction. Looking back at the effective weight for Superelement 1, one sees that it has a positive Z value, indicating motion in the +Z direction. The output for Superelement 2 is similar, indicating that mode 1 is +Z motion of the system, as verified by the following plot.