Application of Design Sensitivity Analysis to Improve Correlations Between Analytical and Test Modes

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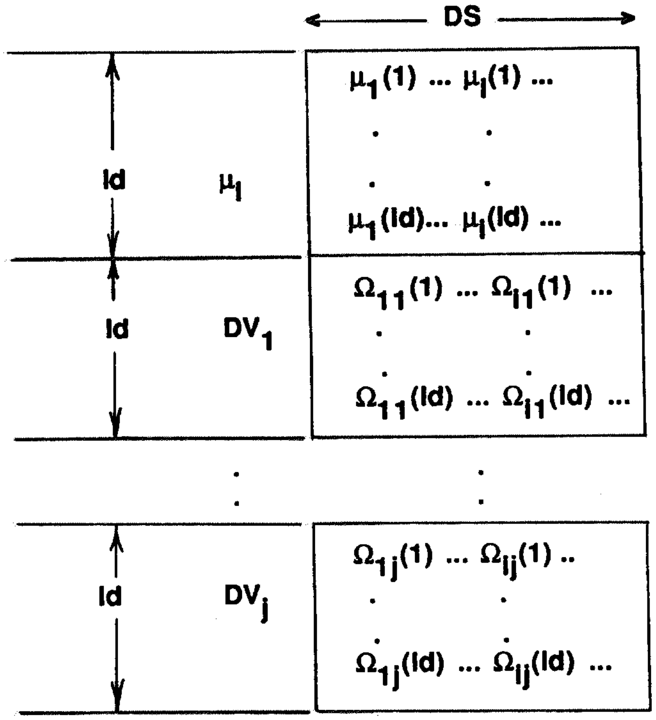
A procedure is presented for obtaining better correlation between analytical and test modes. MSC/NASTRAN Modal Design Sensitivity Analysis was employed for this application. The derivatives of the constraints, in this case the eigenvalues, with respect to the design variables (such as cross sectional properties) indicate the significances each design variable has on all the modes of interest. This procedure eliminates the necessities for performing costly multiple trial and error mode runs. The Bell AH1G model was used for this particular study. This work was performed during the fourth quarter of 1987 under a Bell Helicopter Textron Inc. contract study as part of the NASA DAMVIBS program. The results were presented at NASA Langley Research Center in May of 1988 under contract NAS1-17496. This task represented a six man-week effort. In addition, design sensitivity can be an excellent tool for providing quick inputs for design modifications during flight test stages of typical aircraft programs.

As part of the NASA DAMVIBS program, a scheme was set up to provide solutions for improved correlations between test and analytical data. MSC NASTRAN Design Sensitivity Analysis Capabilities were employed for this task. Three modes of vibrations of the Bell Helicopters AH1G were chosen for this purpose; specifically, the fuselage first vertical bending mode, the landing gear mode, and the fuselage second vertical bending mode. The task was initiated in 1987 and the results were presented at NASA Langley DAMVIBS meeting in 1988. This task represented a six weeks effort.

Current MSC NASTRAN supports three types of solutions sequences; namely, static, modal and buckling. In the case of static analysis, solution 51 is run after solution 61. The constraints can be displacements, element forces, or element stresses. In the case of modal analysis, solution 53 is run after solution 63. The constraints can be eigenvalues or frequencies. In the case of buckling analysis, solution 55 is run after solution 65. The constraint is the buckling load factor.

The NASTRAN output will also provide matrices of derivatives of the design constraints with respect to the design variables for each loading condition. The design sensitivity coefficient matrix generates outputs such as μ_i and Ω_{ij}. The i^{th} constraint, μ_i, can be displacement, element force, stress, eignenvalue, frequency, or buckling load factor. The elements presented in figure 1 are called the sensitivity coefficients of the constraint, μ_i, with respect to their corresponding design variables, DV_j. The design variable, DV_j, can be either a cross sectional property, such as area of a rod, or material property. The load case, LD, can be a static load case or normal mode analysis.

<u>Design Sensitivity Coefficient Matrix</u>



where

μ_i - i^{th} constraint

DV_j - j^{th} design variable

ld - load case

Ω_{ij} - sensitivity coefficient of constraint ith with respect to the jth design variable

<u>figure 1.</u>

The first step in performing a Design Sensitivity Analysis for the Bell AH1G is to carry out a normal superelement solution 63 run. The next step is to establish a list of design variables to be used for the sensitivity study. At this point, a solution 53 run is performed to generate the sensitivity coefficient matrix, $[\Omega_{ij}]$. The magnitude of the various sensitivity coefficients, Ω_{ij}, determines the effect each design variable with respect to each of the three modes of interest. The constraint values, μ_i, is also be calculated. The sign of the constraint will indicate whether the constraint has been violated. The magnitude of the constraint will indicate how much it has been violated by.

After reviewing the Design Sensitivity Coefficient Matrix, $[\Omega_{ij}]$, one can select a set of design variables that has the most significant effect on the design constraint matrix, $[\mu_i]$. The design variables are then altered until the constraints approach zero. In other words, vary ΔB_j such that

$$\{\Delta u_i\}_{3x1} + \{u_i\}_{3x1} \longrightarrow \{0\}_{3x1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

$$\{\Delta\mu_i\} = [\Omega_{ij}]_{3xN} \{\Delta B_j\}_{Nx1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

$[\Omega_{ij}]$ = Design Sensitivity Coefficient Matrix

$\{\mu_i\}$ = Design Constraint Matrix

$[\Delta B_j]$ = Finite Incremental Change in the jth normalized design variable

N = # of design variables

These simple iterations can be accomplished using spreadsheet programs such as Lotus 1-2-3 or external Fortran Program. Once this is accomplished, one additional modal analysis run can then be carried out based on the new properties generated from Design Sensitivity Analysis.

Properties in several regions of the helicopter are chosen as design variables for the sensitivity analysis. They are in the main keel beams (100 to 600 series design variables), tailboom (700 series design variables), vertical fin (800 and 900 series design variables) and landing gear (1000 series design variables).

The elements Ω_{ij}, presented in tables 1,2, and 3 are called the sensitivity coefficients of the constraint, μ_i, with respect to their corresponding design variables. The constraints μ_1, μ_2, μ_3 are the frequency constraint of the fuselage first vertical bending mode, the frequency constraint of the landing gear mode, and the frequency constraint of the fuselage second vertical bending mode, respectively. The order of magnitude of the various sensitivity coefficients, Ω_{ij}, determines the degree of significance each design variable has on μ_i. In addition, if Ω_{ij} is positive, an increase of B_j will increase μ_i. On the other hand, if Ω_{ij} is negative, an increase in B_j will decrease μ_i. In this case, B_j is the j^{th} normalized design variable. The design variables enclosed in solid rectangular boxes are the ones considered to have significant effects on μ_i. The dotted rectangular boxes are simply included due to their geometric relationships with the design variables enclosed in solid rectangular boxes, e.g., if I1 is changed for a tubular section, then I2 is automatically changed.

TABLE 1

Design Sensitivity Coefficient Matrix for Bell AH1G First Vertical Bending Mode

```
                 C O N S T R A I N T       10  FSTVERT                    FREQUENCY        10

MODE      DESIGN VARIABLE      VALUE     DESIGN VARIABLE      VALUE     DESIGN VARIABLE      VALUE     DESIGN VARIABLE      VALUE
   10        101 MAT1A      2.9436E-03      102 MAT1B      1.7300E-03      201 PSHEAR2A   3.1745E-03      202 PMEM2B     7.2856E-06
             301 MAT3A      2.6710E-03      302 MAT3B      5.2929E-03      303 MAT3C      1.2726E-04      401 PSHEAR4A   1.2878E-03
             402 PSHEAR4B   2.6419E-04      403 PSHEAR4C   3.0284E-06      404 MAT4D      1.0739E-03      501 MAT5A      5.8759E-03
             502 MAT5B      7.0416E-03      503 MAT5C      1.1703E-02      504 MAT5D      8.7152E-03      601 PSHEAR6A   8.1039E-05
             602 PSHEAR6B   2.3457E-05      603 PSHEAR6C   1.2112E-05      701 BAR7IYA    3.6835E-02      702 BAR7IYB    3.5103E-02
             703 BAR7IYC    3.8103E-02      704 BAR7IYD    4.1188E-02      705 BAR7IYE    4.3911E-02      706 BAR7IYF    3.9829E-02
             707 BAR7IYG    3.5343E-02      708 BAR7IYH    1.2531E-02      709 BAR7IYI    1.1970E-02      801 BAR8I1A    3.0083E-03
             802 BAR8I1B    2.2053E-03      803 BAR8I1C    1.4357E-03      804 BAR8I1D    6.7133E-04      805 BAR8I1E    2.9105E-04
             806 BAR8I1F    4.2657E-05      811 BAR8I2A    7.2256E-07      812 BAR8I2B    2.6412E-06      813 BAR8I2C    5.7700E-06
             814 BAR8I2D    1.0131E-05      815 BAR8I2E    1.8519E-05      816 BAR8I2F    1.2228E-04      901 BAR9JA     1.3002E-03
             902 BAR9JB     9.8083E-04      903 BAR9JC     1.1725E-03      904 BAR9JD     1.4228E-03      905 BAR9JE     1.6668E-03
             906 BAR9JF     1.2749E-03     1001 BT10I1A    1.5014E-04     1002 BT10I1B    4.1519E-06     1003 BT10I1C    6.1834E-05
            1004 BT10I1D    1.0798E-04     1005 BT10I1E    1.6292E-05     1006 BT10I1F    3.1896E-05     1007 BT10I1G    5.4034E-05
            1011 BR10I1A    4.7331E-04     1012 BR10I1B    1.3089E-05     1013 BR10I1C    1.9493E-04     1014 BR10I1D    3.4041E-04
            1015 BR10I1E    5.1360E-05     1016 BR10I1F    1.0055E-04     1017 BR10I1G    1.7034E-04     1021 BT10I2A    3.6029E-05
            1022 BT10I2B    8.3015E-05     1023 BT10I2C    7.0226E-06     1024 BT10I2D    1.9314E-05     1025 BT10I2E    2.8596E-05
            1026 BT10I2F    2.1639E-06     1027 BT10I2G    1.8543E-07     1031 BR10I2A    1.1358E-04     1032 BR10I2B    2.6171E-04
            1033 BR10I2C    2.2139E-05     1034 BR10I2D    6.0888E-05     1035 BR10I2E    9.0147E-05     1036 BR10I2F    6.8217E-06
            1037 BR10I2G    5.8457E-07     1041 BT10JA     7.1352E-05     1042 BT10JB     1.0418E-06     1043 BT10JC     1.7363E-06
            1044 BT10JD     2.4151E-07     1045 BT10JE     5.3150E-06     1046 BT10JF     2.6653E-07     1051 BR10JA     2.2494E-04
            1052 BR10JB     3.2844E-06     1053 BR10JC     5.4737E-06     1054 BR10JD     7.6136E-07     1055 BR10JE     1.6755E-05
            1056 BR10JF     8.4024E-07
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TABLE 2

Design Sensitivity Coefficient Matrix for Bell AH1G Landing Gear Mode

C O N S T R A I N T 11 LNDGEAR FREQUENCY 11

MODE	DESIGN	VARIABLE	VALUE	DESIGN	VARIABLE	VALUE	DESIGN	VARIABLE	VALUE	DESIGN	VARIABLE	VALUE
11	101	MAT1A	-3.1108E-04	102	MAT1B	-1.5259E-04	201	PSHEAR2A	-2.9964E-04	202	PMEM2B	-6.2712E-07
	301	MAT3A	-5.4096E-05	302	MAT3B	-5.0215E-05	303	MAT3C	-9.5379E-06	401	PSHEAR4A	-3.6971E-05
	402	PSHEAR4B	-1.0241E-06	403	PSHEAR4C	-4.7347E-06	404	MAT4D	-7.2042E-05	501	MAT5A	-6.0487E-07
	502	MAT5B	-8.6016E-07	503	MAT5C	-3.7174E-07	504	MAT5D	-1.4263E-06	601	PSHEAR6A	-1.6305E-06
	602	PSHEAR6B	-2.5744E-07	603	PSHEAR6C	-2.8519E-07	701	BAR7IYA	-9.5675E-07	702	BAR7IYB	-1.3328E-06
	703	BAR7IYC	-2.0034E-06	704	BAR7IYD	-2.7433E-06	705	BAR7IYE	-3.4617E-06	706	BAR7IYF	-3.4681E-06
	707	BAR7IYG	-3.1513E-06	708	BAR7IYH	-1.1133E-06	709	BAR7IYI	-1.0409E-06	801	BAR8I1A	-2.6949E-07
	802	BAR8I1B	-2.1725E-07	803	BAR8I1C	-1.4610E-07	811	BAR8I2A	-1.4270E-06	812	BAR8I2B	-5.6113E-07
	813	BAR8I2C	-4.1576E-07	814	BAR8I2D	-2.7035E-07	815	BAR8I2E	-1.6991E-07	816	BAR8I2F	-1.1210E-07
	901	BAR9JA	-2.5428E-07	902	BAR9JB	-1.9104E-07	903	BAR9JC	-2.2879E-07	904	BAR9JD	-2.7766E-07
	905	BAR9JE	-3.2472E-07	906	BAR9JF	-1.4280E-07	1001	BT10I1A	-5.6496E-02	1002	BT10I1B	-1.0195E-02
	1003	BT10I1C	-2.4624E-03	1004	BT10I1D	-5.3421E-04	1005	BT10I1E	-1.8696E-02	1006	BT10I1F	-1.0385E-01
	1007	BT10I1G	-1.3568E-01	1011	BR10I1A	-1.7810E-01	1012	BR10I1B	-3.2140E-02	1013	BR10I1C	-7.7628E-03
	1014	BR10I1D	-1.6841E-03	1015	BR10I1E	-5.8938E-02	1016	BR10I1F	-3.2739E-01	1017	BR10I1G	-4.2773E-01
	1021	BT10I2A	-3.3712E-02	1022	BT10I2B	-1.1815E-02	1023	BT10I2C	-2.8504E-04	1024	BT10I2D	-6.8170E-04
	1025	BT10I2E	-1.0160E-02	1026	BT10I2F	-7.0915E-03	1027	BT10I2G	-4.8528E-03	1031	BR10I2A	-1.0628E-01
	1032	BR10I2B	-3.7246E-02	1033	BR10I2C	-8.9859E-04	1034	BR10I2D	-2.1491E-03	1035	BR10I2E	-3.2031E-02
	1036	BR10I2F	-2.2356E-02	1037	BR10I2G	-1.5299E-02	1041	BT10JA	-5.2595E-02	1042	BT10JB	-8.1077E-05
	1043	BT10JC	-1.4879E-04	1045	BT10JE	-6.0474E-03	1046	BT10JF	-4.7462E-05	1051	BR10JA	-1.6581E-01
	1052	BR10JB	-2.5559E-04	1053	BR10JC	-4.6907E-04	1055	BR10JE	-1.9065E-02	1056	BR10JF	-1.4962E-04

TABLE 3

Design Sensitivity Coefficient Matrix for Bell AH1G Second Vertical Bending Mode

CONSTRAINT 14 SECVERT FREQUENCY 14

MODE	DESIGN VARIABLE	VALUE	DESIGN VARIABLE	VALUE	DESIGN VARIABLE	VALUE	DESIGN VARIABLE	VALUE
14	101 MAT1A	1.3673E-02	102 MAT1B	7.3238E-03	201 PSHEAR2A	1.1517E-02	202 PMEM2B	3.7534E-05
	301 MAT3A	5.8121E-03	302 MAT3B	1.1351E-02	303 MAT3C	3.1058E-04	401 PSHEAR4A	1.7466E-03
	402 PSHEAR4B	2.9395E-04	403 PSHEAR4C	2.1463E-04	404 MAT4D	3.8118E-04	501 MAT5A	9.1445E-04
	502 MAT5B	1.7945E-03	503 MAT5C	4.3829E-03	504 MAT5D	1.5521E-03	601 PSHEAR6A	1.0976E-03
	602 PSHEAR6B	1.0295E-03	603 PSHEAR6C	4.9713E-04	701 BAR7IYA	5.4605E-05	702 BAR7IYB	7.1016E-04
	703 BAR7IYC	3.5269E-03	704 BAR7IYD	9.1206E-03	705 BAR7IYE	1.8091E-02	706 BAR7IYF	2.6068E-02
	707 BAR7IYG	3.3717E-02	708 BAR7IYH	1.6123E-02	709 BAR7IYI	1.9182E-02	801 BAR8I1A	5.8248E-03
	802 BAR8I1B	4.7265E-03	803 BAR8I1C	3.2634E-03	804 BAR8I1D	1.6422E-03	805 BAR8I1E	7.9327E-04
	806 BAR8I1F	1.3586E-04	811 BAR8I2A	1.4716E-03	812 BAR8I2B	6.4305E-04	813 BAR8I2C	5.3584E-04
	814 BAR8I2D	4.1795E-04	815 BAR8I2E	3.5896E-04	816 BAR8I2F	2.7472E-05	901 BAR9JA	3.7724E-03
	902 BAR9JB	2.8492E-03	903 BAR9JC	3.4040E-03	904 BAR9JD	4.1303E-03	905 BAR9JE	4.8432E-03
	906 BAR9JF	5.2718E-03	1001 BT10I1A	1.5137E-03	1002 BT10I1B	3.3366E-04	1003 BT10I1C	3.9404E-04
	1004 BT10I1D	4.9071E-04	1005 BT10I1E	7.8434E-04	1006 BT10I1F	1.0281E-03	1007 BT10I1G	6.5748E-04
	1011 BR10I1A	4.7720E-03	1012 BR10I1B	1.0519E-03	1013 BR10I1C	1.2422E-03	1014 BR10I1D	1.5470E-03
	1015 BR10I1E	2.4726E-03	1016 BR10I1F	3.2412E-03	1017 BR10I1G	2.0727E-03	1021 BT10I2A	2.7042E-03
	1022 BT10I2B	1.0084E-03	1023 BT10I2C	1.4135E-04	1024 BT10I2D	3.8788E-04	1025 BT10I2E	5.0748E-04
	1026 BT10I2F	4.5348E-04	1027 BT10I2G	7.1678E-04	1031 BR10I2A	8.5251E-03	1032 BR10I2B	3.1789E-03
	1033 BR10I2C	4.4560E-04	1034 BR10I2D	1.2228E-03	1035 BR10I2E	1.5998E-03	1036 BR10I2F	1.4296E-03
	1037 BR10I2G	2.2597E-03	1041 BT10JA	2.2396E-04	1042 BT10JB	1.0155E-07	1043 BT10JC	5.1515E-05
	1044 BT10JD	2.0041E-07	1045 BT10JE	2.3594E-04	1046 BT10JF	7.3118E-05	1047 BT10JG	1.1134E-07
	1051 BR10JA	7.0603E-04	1052 BR10JB	3.2014E-07	1053 BR10JC	1.6240E-04	1054 BR10JD	6.3178E-07
	1055 BR10JE	7.4381E-04	1056 BR10JF	2.3050E-04	1057 BR10JG	3.5100E-07		

This is due to the fact that both I1 and I2 are related to the design variables t (thickness of the tube) and r(mean radius of the tube). Based on the magnitudes of the DSA coefficients, the landing gear mode is reasonably decoupled from the fuselage first vertical bending mode. However, there are some slight couplings between the landing gear mode and the fuselage second vertical bending mode.

Table 4 summarizes the locations of areas where changes in the design variables are most effective. The regions in this table include the tailboom, vertical fin and the main beams in the forward, center and aft fuselage. Similarly, table 5 summarizes the locations of areas where changes in the design variables are most effective for the landing gear. Also included in the tables are the actual percentage change applied to each area for the revised modal run.

The frequency for mode number 10, first vertical bending mode, of the existing AH1G analytical model is 8.21 hz. A frequency of 7.93 hz is obtained through testing. After implementing the changes to the model, as a result of the Design Sensitivity Analysis, the revised frequency is 7.85 hz. Their respective mode shapes are displayed in figure 2. Similarly, the frequency for mode number 11, landing gear mode, of the existing AH1G analytical model is 13.44 hz. A frequency of 14.61 hz is obtained through testing. After implementing the changes to the model, as a result of the design sensitivity analysis, the revised frequency is 14.63 hz. Their respective mode shapes are displayed in figure 3. Finally, the frequency for mode number 14, second vertical bending mode, of the existing AH1G model is 17.75 hz. A frequency of 16.77 hz is obtained by testing. After implementing the changes to the model, as a result of the Design Sensitivity Analysis, the revised frequency of the second vertical bending mode becomes 16.54 hz. However, the mode has shifted from the 14th to the 13th mode. Their respective mode shapes are displayed in figure 4. A summary of comparisons for the first vertical bending, landing gear, and second vertical bending modes between the revised NASTRAN run, original NASTRAN run, and the test modes are presented in table 6. As one can see,the frequencies of the revised NASTRAN run correlate with testing much better than the original run. It should be emphasized that the improved results are obtained based on only one additional modal run using informations generated through design sensitivity analysis. This is probably no longer true if the number of modes of interest is increased. This problem will be addressed later on in the paper. A listing of all the pertinent design sensitivity cards is included in appendix A.

Since all the masses of the AH1G are represented by conm2, the effects due to the variation of structural mass are not accounted for in this study. The effects due to mass variations can, however, be easily accounted for if the conm2s are substituted with the appropriate densities on the material cards.

In this particular study, only three modes are considered. Therefore, only the effects that the design variables have on these particular three modes are considered. The effects they have on the rest of the modes are not considered. If the number of modes of interest are increased, then one would probably have to increase the number of design variables in order to avoid violating the constraints. In addition, since NASTRAN Design Sensitivity is based on linear perturbation about the current design point, unacceptables amount of approximations can be introduced if the variations of the design variables are large. This, however, can be avoided if we update the sensitivity coefficients based on the updated design variables. The procedures, however, are identical. The eigenvector derivative may also have to be employed in addition to mode plots for tracking the appropriate modes.

TABLE 4

Design Variable Changes in Fuselage and Tail Section

Location of Design Variables	Design Variable ID	Percent Change (ΔB_j)
Fwd Fus, Lower Cap, Main Beam	101 MAT1A	-0.4
Fwd Fus, Upper Cap, Main Beam	102 MAT1B	-0.4
Fwd Fus, Inner Skin of Main Beam	201 PSHEAR2A	-0.4
Center Fus, Lower Cap, Main Beam	301 MAT3A	-0.4
Center Fus, Upper Cap, Main Beam	302 MAT3B	-0.4
Aft Fus, Upper Cap, Right Main Beam	503MAT5C	-0.02
Tailboom EI_y	701 BAR7IYA	-0.02
	702 BAR7IYB	-0.02
	703 BAR7IYC	-0.02
	704 BAR7IYD	-0.02
	705 BAR7IYE	-0.02
	706 BAR7IYF	-0.02
	707 BAR7IYG	-0.3
	708 BAR7IYH	-0.3
	709 BAR7IYI	-0.3
Vertical Fin EI_y	801 BAR8I1A	-0.4
	802 BAR8I1B	-0.4
	803 BAR8I1C	-0.4
Vertical Fin GJ	901 BAR9JA	-0.4
	902 BAR9JB	-0.4
	903 BAR9JC	-0.4
	904 BAR9JD	-0.4
	905 BAR9JE	-0.4
	906 BAR9JF	-0.4

TABLE 5

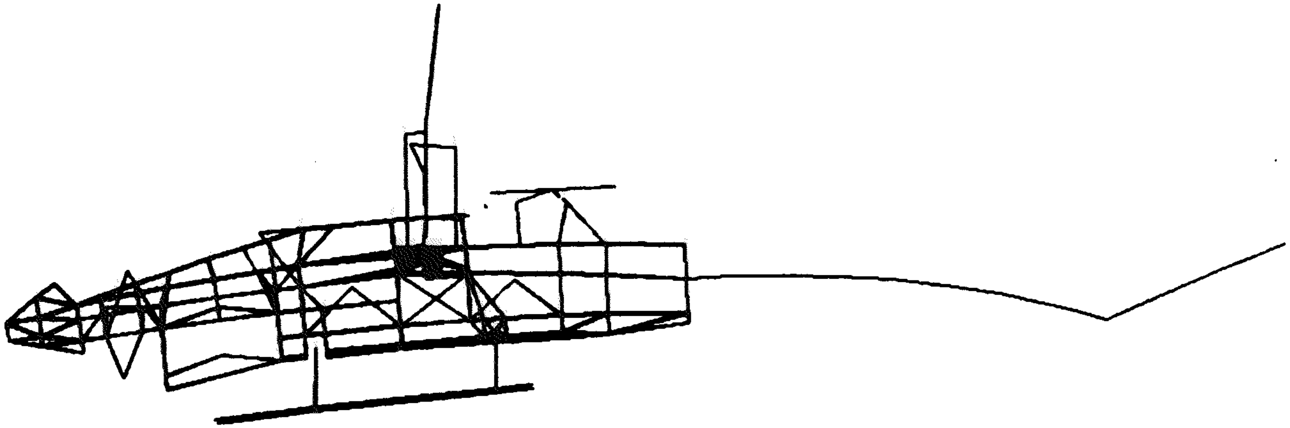
Design Variable Changes in Landing Gear Section

Location of Design Variables	Design Variable ID	Percent Change (ΔB_j)
Landing Gear	1001 BT10I1A	0.046
	1006 BT10I1F	0.046
	1007 BT10I1G	0.046
	1011 BR10I1A	0.046
	1015 BR10I1E	0.046
	1016 BR10I1F	0.046
	1017 BR10I1G	0.046
	1021 BT10I2A	0.046
	1026 BT10I2F	0.046
	1027 BT10I2G	0.046
	1031 BR10I2A	0.046
	1035 BR10I2E	0.046
	1036 BR10I2F	0.046
	1037 BR10I2G	0.046
	1041 BT10JA	0.046
	1046 BT10JF	0.046
	1051 BR10JA	0.046
	1055 BR10JE	0.046
	1056 BR10JF	0.046

FIGURE 2

AH1G First Vertical Bending Mode

Original Model
Mode No. 10
8.21 Hz



Revised Model
Mode No. 10
7.85 Hz

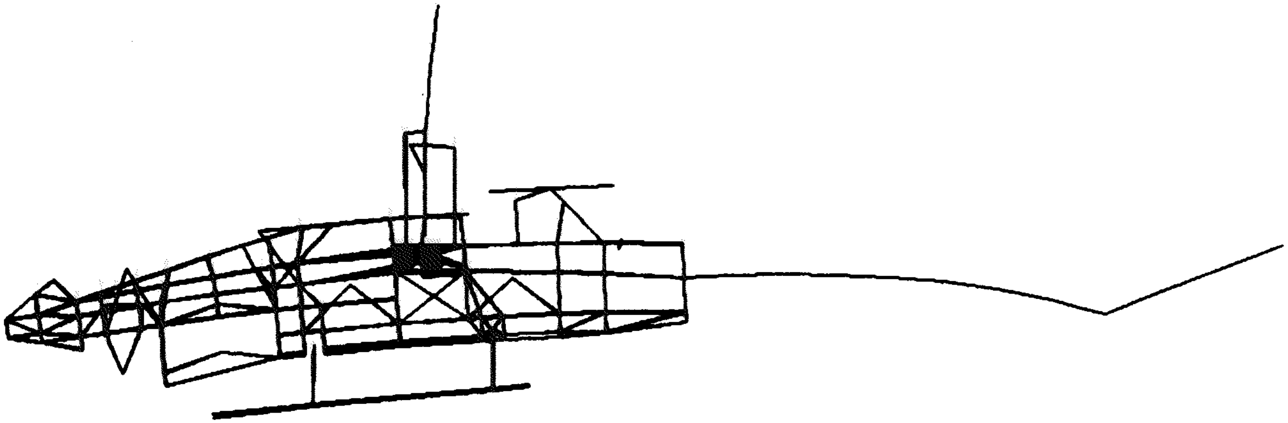
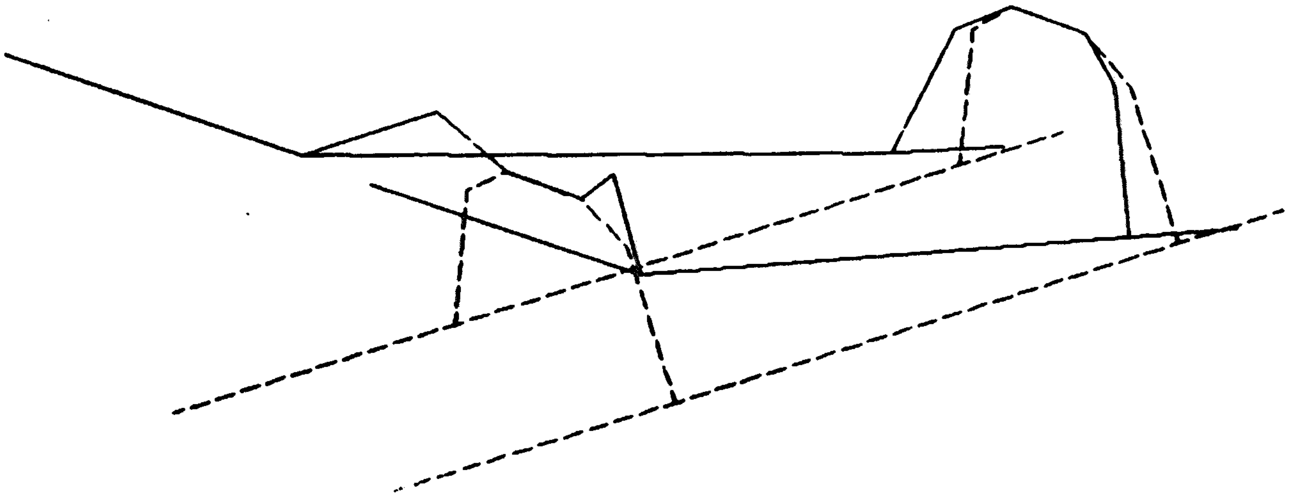


FIGURE 3

AH1G Landing Gear Mode

Original Model
Mode No. 11
13.44 Hz



Revised Model
Mode No. 11
14.63 Hz

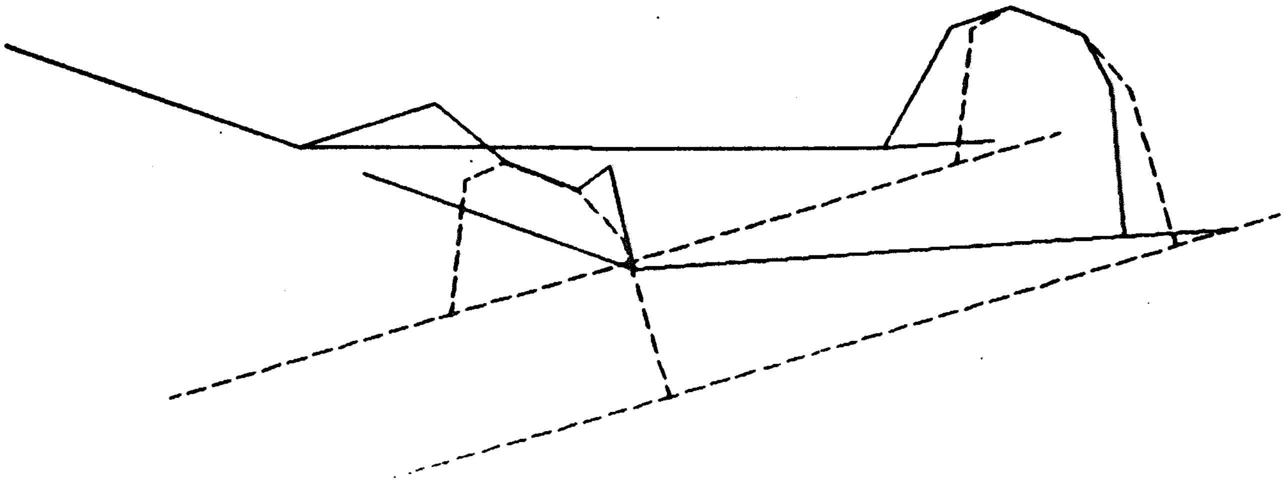
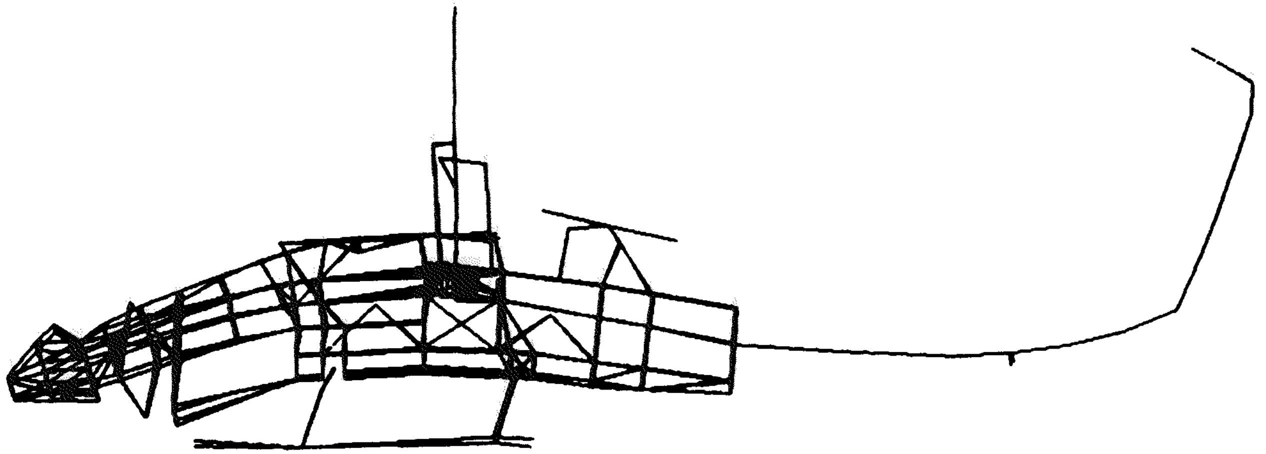


FIGURE 4

AH1G Second Vertical Bending Mode

Original Model
Mode No. 14
17.75 Hz



Revised Model
Mode No. 13
16.54 Hz

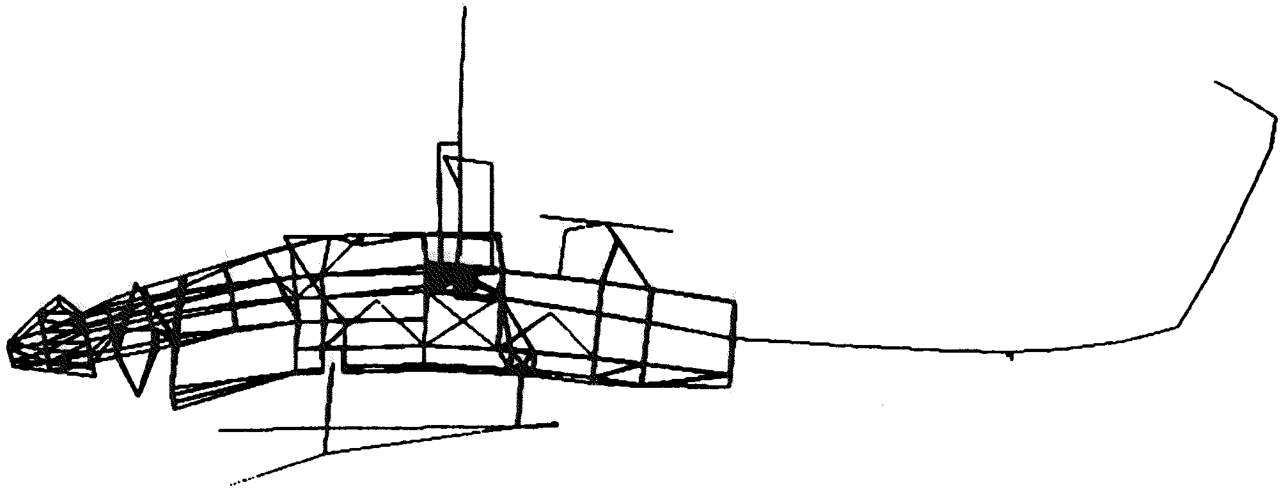
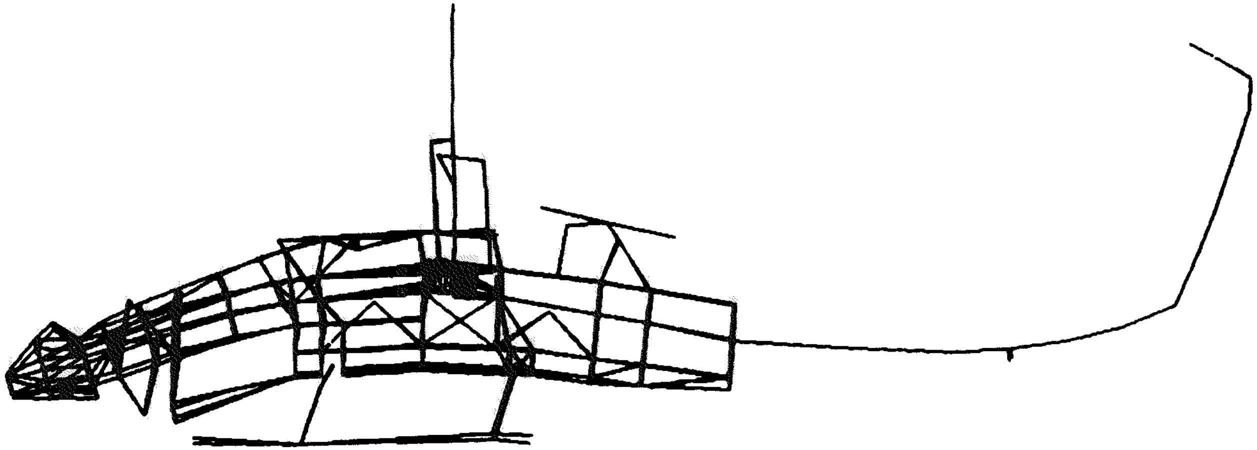


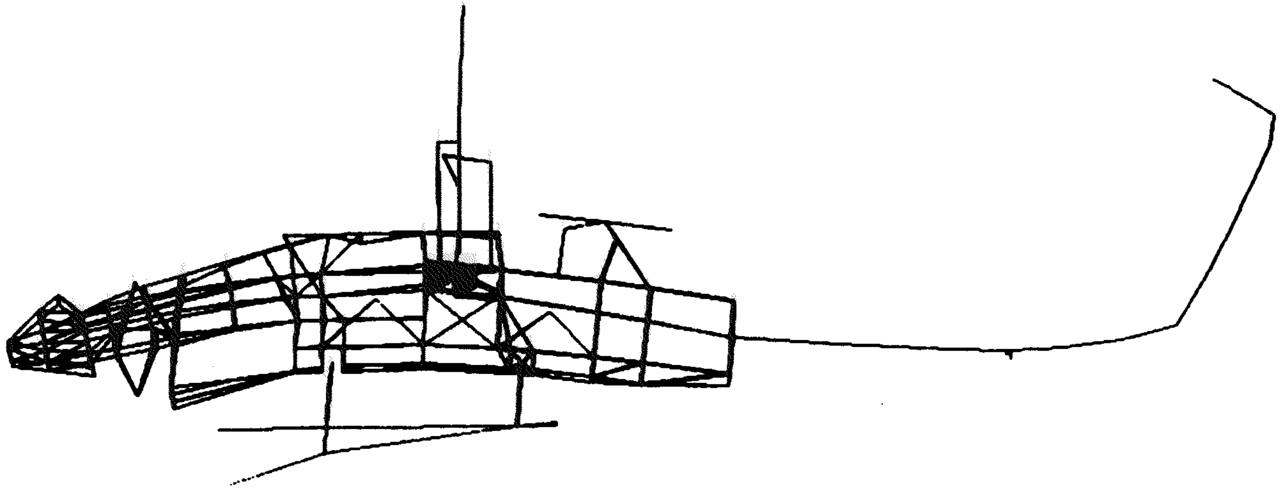
FIGURE 4

AH1G Second Vertical Bending Mode

Original Model
Mode No. 14
17.75 Hz



Revised Model
Mode No. 13
16.54 Hz



As an extension, design sensitivity could be an excellent tool for providing design modifications during flight test stages of typical aircraft program. Typically, most new aircrafts undergo several years of flight test, and it is very common that they encounter various vibration problems during this period. However, with the availability of the sensitivity coefficient matrix, one can quickly identify the structural elements to modify in order to alleviate this vibration problem. More importantly, the effects these changes have on the rest of the modes are known prior to performing any additional modal runs. The only additional computations required are simple matrix multiplications of equations 1 and 2. This eliminates the necessity for performing multiple trial and error modal runs. In addition, the sensitivity coefficient matrix can be updated periodically as part of the revised modal run without requiring any additional preparation.

CONCLUSION

Applications and procedures for implementing MSC NASTRAN Modal Design Sensitivity Analysis as a tool for correlating between analytical and test modes is presented in this paper. The sensitivity coefficient matrix indicates the significances each design variable has on all the modes of interest. This alleviates the necessities for costly multiple trial and error mode runs. In addition, design sensitivities could be an excellent tool for providing quick inputs for design modifications during flight test stages of typical aircraft program. Applications of design sensitivity and optimization to "real life" structures are becoming practical with the availibility of the design sensitivity coefficients and ever increasing speed of computers.

ACKNOWLEDGEMENTS

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APPENDIX A

```
DSCONS  10      FSTVERT FREQ    10                      7.934   MAX
DSCONS  11      LNDGEAR FREQ    11                      14.606  MIN
DSCONS  14      SECVERT FREQ    14                      16.774  MAX
DVAR    101     MAT1A   .05     101
DVAR    102     MAT1B   .05     102
DVAR    201     PSHEAR2A.05     201
DVAR    202     PMEM2B  .05     202
DVAR    301     MAT3A   .05     301
DVAR    302     MAT3B   .05     302
DVAR    303     MAT3C   .05     303
DVAR    401     PSHEAR4A.05     401
DVAR    402     PSHEAR4B.05     402
DVAR    403     PSHEAR4C.05     403
DVAR    404     MAT4D   .05     404
DVAR    501     MAT5A   .05     501
DVAR    502     MAT5B   .05     502
DVAR    503     MAT5C   .05     503
DVAR    504     MAT5D   .05     504
DVAR    601     PSHEAR6A.05     601
DVAR    602     PSHEAR6B.05     602
DVAR    603     PSHEAR6C.05     603
DVAR    701     BAR7IYA .05     701
DVAR    702     BAR7IYB .05     702
DVAR    703     BAR7IYC .05     703
DVAR    704     BAR7IYD .05     704
DVAR    705     BAR7IYE .05     705
DVAR    706     BAR7IYF .05     706
DVAR    707     BAR7IYG .05     707
DVAR    708     BAR7IYH .05     708
DVAR    709     BAR7IYI .05     709
DVAR    801     BAR8I1A .05     801
DVAR    802     BAR8I1B .05     802
DVAR    803     BAR8I1C .05     803
DVAR    804     BAR8I1D .05     804
DVAR    805     BAR8I1E .05     805
DVAR    806     BAR8I1F .05     806
DVAR    811     BAR8I2A .05     811
DVAR    812     BAR8I2B .05     812
DVAR    813     BAR8I2C .05     813
DVAR    814     BAR8I2D .05     814
DVAR    815     BAR8I2E .05     815
DVAR    816     BAR8I2F .05     816
DVAR    901     BAR9JA  .05     901
DVAR    902     BAR9JB  .05     902
DVAR    903     BAR9JC  .05     903
DVAR    904     BAR9JD  .05     904
DVAR    905     BAR9JE  .05     905
DVAR    906     BAR9JF  .05     906
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DVAR    1015    BR10I1E .05     1015
DVAR    1016    BR10I1F .05     1016
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DVAR    1023    BT10I2C .05     1023
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DVAR    1027    BT10I2G .05     1027
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DVAR    1044    BT10JD  .05     1044
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DVAR    1052    BR10JB  .05     1052
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DVSET   301     PROD    3       301             1590052 1590053 1610191
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DVSET   302     PROD    3       302             1570017 1600051 1600052
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DVSET   403     PSHEAR  4               1.0     137075
DVSET   404     PROD    3       404             1800191 1800171
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MAT1    0076    3.2+6   0.8+6   0.32
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MAT1    301     10.815+6
MAT1    302     10.815+6
MAT1    303     10.815+6
MAT1    404     11.025+6
MAT1    501     10.815+6
MAT1    502     10.815+6
MAT1    503     10.815+6
MAT1    504     10.815+6
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MAT1    2024    10.5+6  4.0+6
MAT1    4130    29.0+6  11.0+6
MAT1    4340    29.0+6  11.0+6
MAT1    4620    29.0+6  11.0+6
MAT1    7075    10.3+6  3.9+6
MAT1    9046    17.5+6  6.5+6
MAT1    12024   10.5+6
MAT1    17075   10.3+6
MAT1    27075   10.3+6
MAT1    37075   10.3+6
MAT1    47075   10.3+6
MAT1    57075   10.3+6
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