A Finite Element Technique for Tape-Head Interaction Problems in High Speed Recording: The Steady-State Case

by

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ABSTRACT

This paper presents a finite element technique to model the steady-state response of a flexible tape during high-speed recording. The tape is modeled as a flexible plate, including geometric non-linearities due to large deflections. The air-film pressure field is described using a modified version of Reynolds equation for compressible fluids. A new nonlinear finite element that couples both air-film and tape mechanics is described in detail. This element, which has been incorporated into a standard finite element code (MSC/NASTRAN®), allows one to determine the pressure field and the air-film thickness due to tape-head interaction. An example demonstrates the usefulness of this technique.

INTRODUCTION

An important problem in the development of recording devices is the study of tape-head mechanics. Abrupt changes in geometry in the vicinity of the recording head produce significant variations in the pressure field. This, in turn, deflects the recording tape and modifies the thickness of the air-film between the tape and the recording head. An accurate determination of both the pressure field and the air-film thickness in the vicinity of the recording head is of crucial importance for manufacturers and developers.

This is a complex problem, not only due to the nonlinear features of the governing equation, but also due to the highly irregular geometry encountered in most practical situations. Thus, a solution using numerical techniques must be attempted.

Several authors have investigated this problem in the past. Stahl et al. [1] and Greenberg [2] proposed algorithms based on finite difference techniques. Hori et al. [3] presented a finite element approach assuming an infinitely wide tape and an incompressible fluid. Ono et al. [4,5] suggested a combined finite difference/Newton-Raphson method modeling the tape using linear plate theory. Wolf et al. [6] presented another finite difference algorithm similar to Greenberg's.

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However, in spite of this progress, there is still room for improvement. Finite difference algorithms, although effective, require significant CPU time and many iterations to converge. In addition, second-order effects due to large deformations are important to consider when describing the deformed shape of the tape. In some cases the deflections of the tape are comparable to its thickness, thus violating the assumptions of linear plate theory.

This paper is an attempt to model more accurately the mechanics of the tape-head interaction. It is based on a 2-D nonlinear finite element that couples both the tape and the air-film pressure field. The tape is modeled as a plate, including membrane effects and differential stiffness. The air-film pressure field is modeled using Reynolds equation for compressible fluids, as modified by Burgdorfer [7]. This modified equation accounts for the fact that the molecular mean free path of the fluid might be significant compared to a typical flow passage.

STATEMENT OF PROBLEM

A more formal statement of the problem is required at this stage.

Figure 1A shows a typical tape-head interaction problem. The tape is assumed to be moving in the y-direction with constant velocity, V. It is also assumed that the tape is under a constant tension T (force per unit of length). The head geometry is described as a protuberance defined by a known function $\Delta(y,z)$. The "spacing" or "air-film thickness", that is, the distance between the tape and the bottom surface, is denoted as h. The tape deflection normal to its plane is denoted as u_x. It is further assumed that far from the protuberance, h takes the reference value h_o. Figure 1B shows a top view of the tape. There are no constraints on the geometry of the protuberance, as long as it can be described as a function of two variables (y and z). Data of the problem include:

E,	Young's Modulus of tape material;
h_t,	tape thickness;
ρ,	tape material density (mass per unit volume);
ν,	Poisson's ratio for tape;
P_a,	ambient pressure;
μ,	viscosity coefficient of air;
λ_a,	molecular mean free path of air;
T,	tape tension (force/length);
V,	tape velocity;
h_o,	reference thickness; and
$\Delta(y,z)$,	function that defines the protuberance. This function may be defined in a discrete fashion.

The problem is solved in a finite domain, representing a segment of the tape. It is assumed that $h=h_o$ at both ends, and that the pressure takes the value P_a along the boundary (see Figure 2). The problem consists of determining: a) the pressure field inside the domain, and b) the deformed geometry of the tape.

GOVERNING EQUATIONS

Pressure Field

The pressure field is described using Reynolds equation for compressible fluids, modified by Burgdorfer [7]. That is,

$$\nabla \cdot (\phi \nabla P) = 6\mu V \frac{\partial (Ph)}{\partial y} \tag{1}$$

with

$$\phi = \left(1 + \frac{6\lambda_a P_a}{Ph}\right) Ph^3 \tag{2}$$

The second term in the equation that defines ϕ (Equation 2), accounts for the fact that the molecular mean free path of the fluid might be significant compared to a characteristic flow passage. If $\lambda_a = 0$ in Equation 2, the standard Reynolds equation for compressible fluids is recovered. For convenience, rewrite P as

$$P = P_a + p \tag{3}$$

where p represents the deviation with respect to the ambient pressure P_a. The air-film thickness, h, can be expressed as (see Figure 1A),

$$h = h_o - \Delta + u_x \tag{4}$$

Thus, using Equations 3 and 4, Equation 1 can be reformulated as

$$\nabla \cdot (\phi \nabla p) = 6\mu V \left\{ (P_a + p)\left(\frac{\partial u_x}{\partial y} - \frac{\partial \Delta}{\partial y}\right) + (h_o - \Delta + u_x) \frac{\partial p}{\partial y} \right\} \tag{5}$$

Tape Equation

The moving tape can be modeled as a 2-dimensional thin plate including membrane effects. Due to the tape motion, the substantial time derivative operator is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V \frac{\partial}{\partial y} \tag{6}$$

This operator expresses, in terms of a fixed reference frame, the time rate of change of a function associated with a particle fixed to the tape. The lateral acceleration experienced by a fixed particle is

$$\frac{D}{Dt}\left(\frac{Du_x}{Dt}\right) = \frac{\partial^2 u_x}{\partial t^2} + 2V\frac{\partial^2 u_x}{\partial y \partial t} + V^2\frac{\partial^2 u_x}{\partial y^2} \tag{7}$$

Thus, the equation of motion for the moving tape is

$$D\nabla^4 u_x - T\frac{\partial^2 u_x}{\partial y^2} + \rho t\left(\frac{\partial^2 u_x}{\partial t^2} + 2V\frac{\partial^2 u_x}{\partial y \partial t} + V^2\frac{\partial^2 u_x}{\partial y^2}\right) = P - P_a = p \tag{8}$$

which, for the steady-state case, reduces to

$$D^4\nabla^4 u_x + (-T + \rho h_t V^2)\frac{\partial^2 u_x}{\partial y^2} = p \tag{9}$$

D is the tape stiffness, defined as

$$D = \frac{Eh_t^3}{12(1 - v^2)} \tag{10}$$

Hence, Equation 9 is simply the static equation of a plate loaded with a pressure p, and subjected to an effective tension equal to $T - \rho h_t V^2$.

Variational Statement for Pressure Field

Equation 5 does not have a classical variational statement. A Galerkin technique, however, can be employed to find a variational statement needed to formulate the problem in finite element terms.

Let δp represent the variation of p. By multiplying Equation 5 by δp and integrating over the domain of interest, Ω, the following relationship is obtained

$$\int_\Omega \nabla \cdot (\phi\, \nabla p)\, \delta p\, d\Omega = 6\mu v \int_\Omega \left[(P_a + p)\left(\frac{\partial u_x}{\partial y} - \frac{\partial \Delta}{\partial y}\right) + (h_o - \Delta + u_x)\frac{\partial p}{\partial y}\right] \delta p\, d\Omega \tag{11}$$

Recall that

$$(\nabla \cdot a)\, b = -\nabla b \cdot a + \nabla \cdot (ba) \tag{12}$$

then

$$\int_\Omega (\nabla \cdot a)\, b\, d\Omega = -\int_\Omega a \cdot \nabla b\, d\Omega + \int_\Gamma ba\, d\Gamma \tag{13}$$

where Γ represents the boundary of Ω. Invoking Equation 13 with $a = \phi \nabla p$ and $b = \delta p$, the left-hand side of Equation 11 can be expressed as

$$\int_\Omega \nabla \cdot (\phi \nabla p) \delta p \, d\Omega = -\int_\Omega \phi \nabla p \cdot \delta \nabla p \, d\Omega \tag{14}$$

since $\delta p = 0$ along Γ.

Finally, using Equation 14, Equation 11 can be expressed as

$$\begin{aligned} &\int_\Omega \phi \nabla p \cdot \delta \nabla p \, d\Omega + \\ &6\mu V \int_\Omega \left[P_a \frac{\partial u_x}{\partial y} + p \left(\frac{\partial u_x}{\partial y} - \frac{\partial \Delta}{\partial y} \right) + (h_o - \Delta + u_x) \frac{\partial p}{\partial y} \right] \delta p \, d\Omega = \\ &6\mu V P_a \int_\Omega \frac{\partial \Delta}{\partial y} \delta p \, d\Omega \end{aligned} \tag{15}$$

This result provides the fundamental variational relationship required to formulate the "fluid" part of the finite element to be presented.

FINITE ELEMENT FORMULATION

A four-node quadrilateral element including "structural" and "fluid" degrees of freedom has been implemented. The "structural" degrees of freedom at a node i, denoted as u_i, refer to the usual rotations and displacements needed to define the deformed geometry of the tape. Thus, in its most general form

$$u_i = \begin{Bmatrix} u_{x_i} \\ u_{y_i} \\ u_{z_i} \\ \theta_{x_i} \\ \theta_{y_i} \\ \theta_{z_i} \end{Bmatrix} \tag{16}$$

The "fluid" degree of freedom at a node i is p_i, the deviation with respect to ambient pressure, P_a.

Using conventional finite element discretization for a generic element, Equation 9 reduces to

$$[K_{uu}]\{u\} = [K_\ell]\{u\} + [K_d]\{u\} = [A]\{p\} \tag{17}$$

where $[K_\ell]$, represents the linear stiffness of a plate;

$[K_d]$, represents the differential stiffness of a plate (which depends on both u and the effective membrane tension),

$[A]$, is a pressure distribution matrix that depends on the area of the finite element under consideration, and

$[K_{uu}]$, is simply $[K_\ell] + [K_d]$.

To formulate the pressure equation (Equation 5) in terms of finite elements, using the corresponding variational principle (Equation 15), both p and u_x need to be expressed in terms of their nodal values. Hence,

$$p = \sum_i N_i p_i \tag{18}$$

and

$$u_x = \sum_i N_i \, u_{x_i} \tag{19}$$

where the N_i's are the corresponding shape functions.

Then, using Equations 18 and 19, Equation 15 results in the following equilibrium equation for an individual finite element

$$\{F_p\} = \{R_p\} \tag{20}$$

where

$$\{F_p\} = \begin{Bmatrix} F_{p_1} \\ F_{p_2} \\ F_{p_3} \\ F_{p_4} \end{Bmatrix} \tag{21}$$

and

$$\{R_p\} = \begin{Bmatrix} R_{p_1} \\ R_{p_2} \\ R_{p_3} \\ R_{p_4} \end{Bmatrix} \tag{22}$$

The components of $\{F_p\}$ and $\{R_p\}$ are

$$F_{p_i} = \int_{\Omega_e} \nabla N_i \phi \nabla p \, d\Omega_e + \tag{23}$$

$$6\mu V \int_{\Omega_e} N_i \left[(h_o - \Delta + u_x) \frac{\partial p}{\partial y} + \frac{\partial u_x}{\partial y} (p + P_a) - \frac{\partial \Delta}{\partial y} p \right] d\Omega_e$$

and

$$R_{p_i} = 6\mu V \int_{\Omega_e} N_i \frac{\partial \Delta}{\partial y} P_a \, d\Omega_e \tag{24}$$

where Ω_e is the area of the quadrilateral element. In summary, the equilibrium equations for the fluid/plate quadrilateral element can be written as,

$$\begin{Bmatrix} \{F_p\} \\ \{F_u\} \end{Bmatrix} = \begin{Bmatrix} \{R_p\} \\ \{0\} \end{Bmatrix} \tag{25}$$

where $\{F_p\}$ and $\{R_p\}$ are defined according to Equations 23 and 24, and $\{F_u\}$ can be expressed (from Equation 17) as

$$\{F_u\} = [K_{uu}]\{u\} - [A]\{p\} \tag{26}$$

Equation 25 represents a nonlinear equation for p and u, which results in a nonlinear system of equations when the contributions of all the finite elements are considered. Therefore, it must be solved using an iterative technique. In this case, the well known Newton-Raphson method is employed. To this end, an approximate "tangent matrix" can be defined as follows,

$$[K_t] = \begin{bmatrix} [K_{pp}] & [K_{pu_x}] \\ -[A] & [K_{uu}] \end{bmatrix} \tag{27}$$

where $[K_{pp}]$ and $[K_{pu_x}]$ are determined from Equation 23, by computing $\partial F_p / \partial p$ and $\partial F_p / \partial u_x$. Therefore,

$$[K_{pp}]_{ij} = \int_{\Omega_e} \nabla N_i \left(\frac{\partial \phi}{\partial p} N_j \nabla p + \phi \nabla N_j \right) d\Omega_e \tag{28}$$

$$+ 6\mu V \int_{\Omega_e} N_i \left[(h_o - \Delta + u_x) \frac{\partial N_j}{\partial y} + \frac{\partial u_x}{\partial y} N_j - \frac{\partial \Delta}{\partial y} N_j \right] d\Omega_e$$

$[K_{pu_x}]$ has nonzero terms only on those columns that correspond to u_x, since F_p does not depend on u_y, u_z, θ_x, θ_y, or θ_z. Hence

$$[K_{pu_x}]_{ij} = \int_{\Omega_e} \nabla N_i \left(\frac{\partial \phi}{\partial u_x} N_j \nabla p \right) d\Omega_e + \\ 6\mu V \int_{\Omega_e} \left[N_i \frac{\partial p}{\partial y} + (p + P_a) \frac{\partial N_j}{\partial y} \right] d\Omega_e \qquad (29)$$

An isoparametric quadrilateral element was developed following the derivation presented here and incorporated into a standard finite element code (MSC/NASTRAN). The "plate" features of the element are those of the MSC/NASTRAN QUAD4 element [8]. This element uses five degrees of freedom at each node, namely, u_x, u_y, u_z, θ_y, and θ_z; it features plate and membrane effects; and supports geometric nonlinearity through a differential stiffness matrix that is updated at every iteration. The "fluid" terms in the equilibrium equation (F_p and R_p), as well as those of the "tangent matrix" (K_{pp} and K_{pu_x}), are computed using the appropriate formulae and standard Gauss quadrature.

The performance of the MSC/NASTRAN QUAD4 element has been validated by several tests [9]. The performance of the "fluid" part of this fluid/plate element has been validated with some examples reported elsewhere [10].

EXAMPLE OF APPLICATION

Consider the situation depicted in Figure 3. Figure 3A shows a longitudinal view of a typical tape-head interaction problem. Figure 3B shows a front view of the same problem. The tape is moving in the positive y-direction with a constant velocity $V = 2.54$ m/s. The data for this problem are given below.

Tape Properties

ρ = 3284 kg/m^3
h_t = 2.286×10^{-5} m
E = 4.1×10^{9} N/m^2
ν = .30
tape width = 0.0127 m

Fluid Properties

μ = 1.50×10^{-5} N-s/m^2
λ_a = 3.81×10^{-8} m

In addition, $P_a = 100482$ N/m^2, $T = 139$ N/m, and $h_o = 3.81 \times 10^{-4}$ m.

A 0.0762 m tape segment was considered in this analysis. A regular mesh of 10 x 60 = 600 square elements of side equal to 1.27×10^{-3} m was used. The function $\Delta(y,z)$, which defines the geometry of the protuberance, was defined in a discrete fashion by giving its value at the nodal points. Linear interpolation was used to determine the value of Δ at the intermediate points.

The value of p was constrained to 0 along the boundary of the tape. At y = 0, the tape (plate) displacements were set to 0; only rotations were permitted. At y = 0.0762 m, u_x and u_z were constrained but u_y was left free to permit the application of an effective tension equal to $T - \rho h_t V^2 = 138.4$ N/m (rotations were permitted). This is accomplished simply by applying at those nodes located at y = 0.0762 m a force F consistent with the tape tension. Thus, $F = 1.27 \times 10^{-3} \times 138.4 = 0.176$ N, except for the nodes located at the corners that will be applied half that force.

The problem was solved using MSC/NASTRAN SOL 66, which corresponds to a Newton-Raphson iteration technique for nonlinear transient problems. Figure 4 shows the pressure field, p (deviation with respect to ambient pressure). Figure 5 depicts the deformed shape of the tape. As expected, the pressure increases and then decreases sharply in the vicinity of the protuberance.

The maximum deformation of the tape (u_x) is equal to 1.19×10^{-5} m. This is significant since the original gap (distance between the tape and the head when the tape is at rest) is 1.27×10^{-5} m. In other words, the deformation of the tape has almost doubled the size of the gap in the vicinity of the recording head. These changes are relevant in the design of recording devices. It should also be noted that the maximum deflection of the tape is about one half its thickness. This shows how important it is to consider nonlinear geometric effects when modeling the structural features of the tape; linear theory does not hold for such large deformations.

This problem was run in a VAX 750 machine. It took 45 minutes of CPU time to converge (6 iterations).

CONCLUDING REMARKS

A new nonlinear quadrilateral element to model the behavior of a flexible tape passing above a protuberance has been developed and implemented. This element can be thought as an assembly of two basic elements: a thin plate/membrane element which supports geometric nonlinearity and represents the structural properties of the tape; and a pressure element based on a modified form of Reynolds equation for compressible fluids. This finite element has been incorporated into a special release of a general purpose finite element code (MSC/NASTRAN), and successfully used in combination with state-of-the-art nonlinear iteration routines. This new technique constitutes a powerful analysis tool in the design of recording devices.

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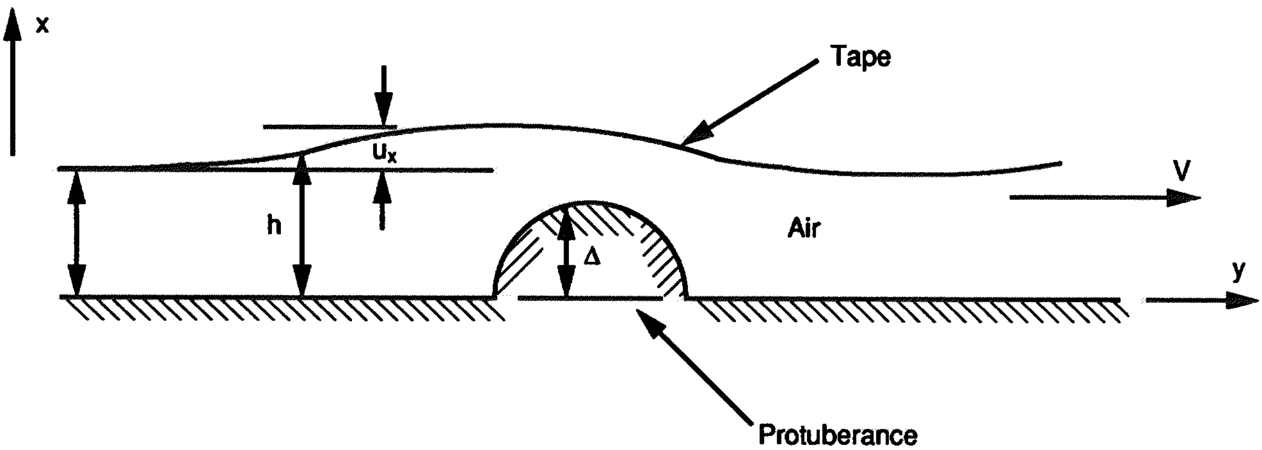

Figure 1A. Lateral View.

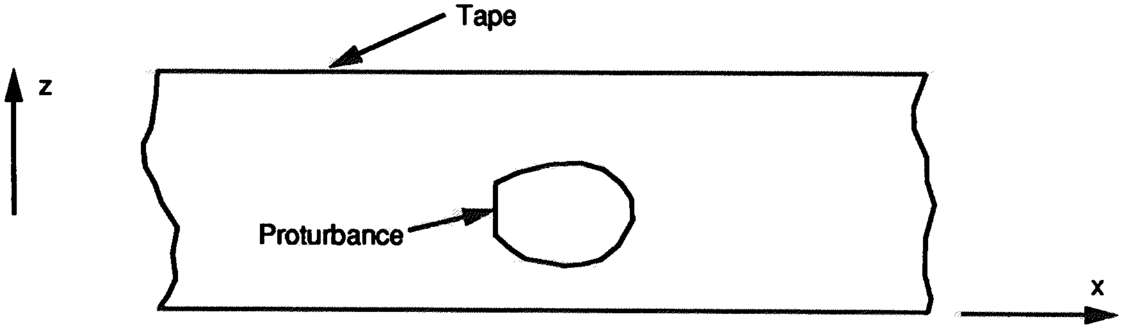

Figure 1B. Top View.

Figure 1. Typical Tape-head Interaction Problem.

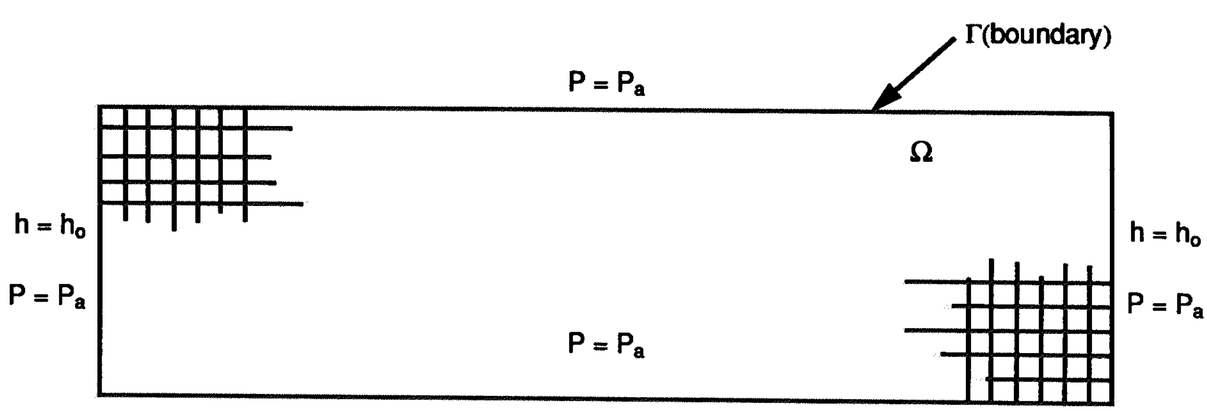

Figure 2. Tape Segment, Domain of Analysis.

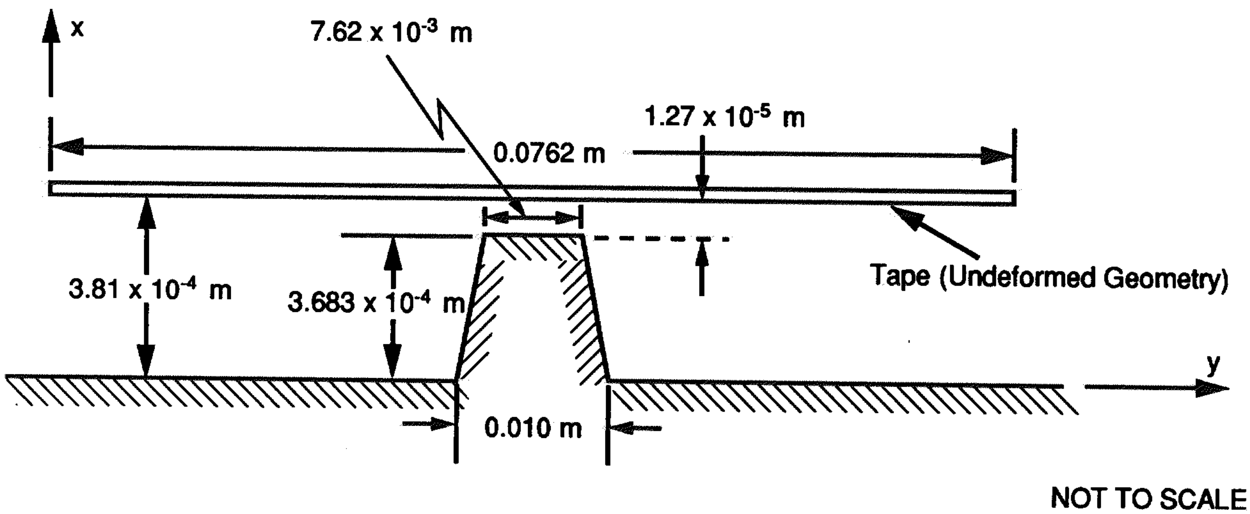

Figure 3A. Longitudinal View.

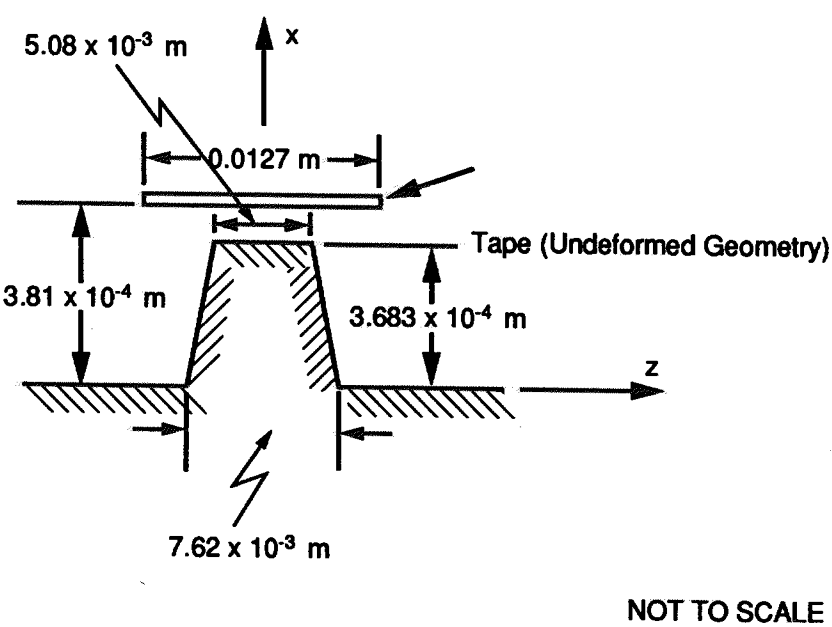

Figure 3B. Front View.

Figure 3. Demonstration Problem

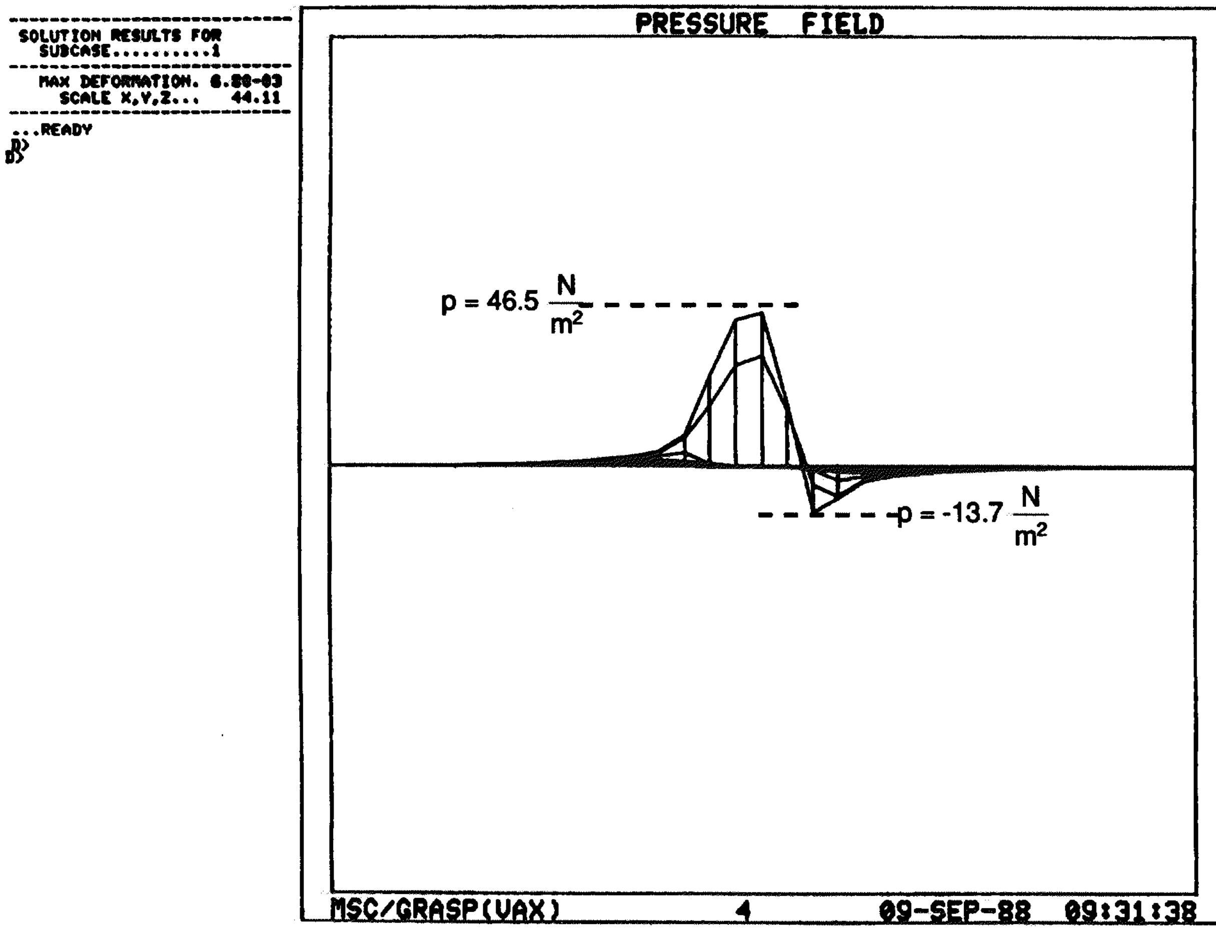

Figure 4. Pressure (p) Field.

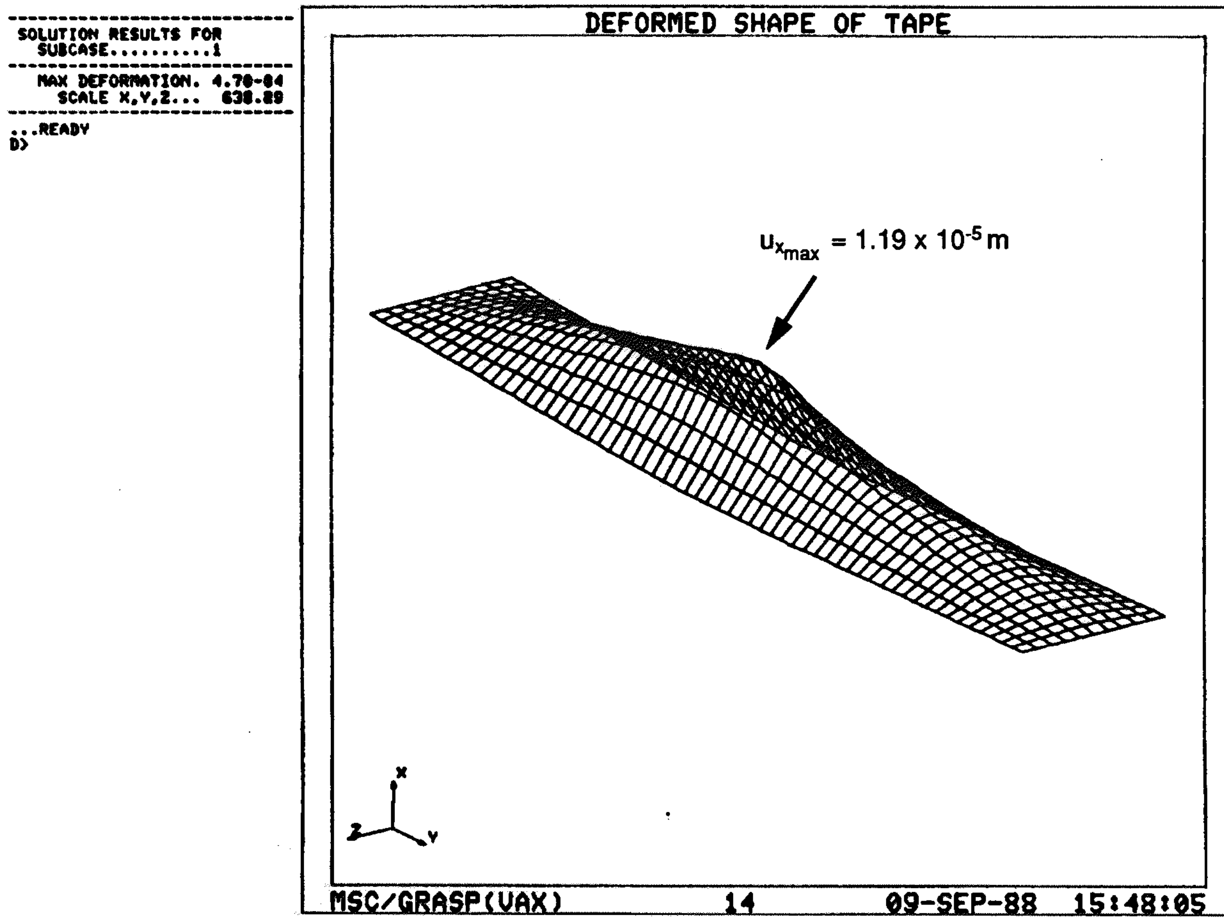

Figure 5. Deformed Shape of the Tape.