On the Shape Optimization of Large Structures

Dr. Dirschmid, AUDI AG

A method is presented which achieves an optimized shape of a structure with very many degrees of freedom, by a global consideration and by neglecting local effects.

The shape optimization of structures with very many degrees of freedom needs to consider many design variables, so this work becomes very expensive. An economic solution demands to operate with a reduced but realistic representation of the structure. The complete structure is subdivided into subregions, called macros (fig. 1), surrounded - preferebly, but not necessarely - by three or four lines which have three or four points of intersection. Each macro "n" has a stiffness matrix $\mathbf{K}_n$; the force - displacement relation can be stated as

$$\mathbf{K}_n \begin{matrix} u_a \\ u_m \end{matrix} = \begin{matrix} f_a \\ f_m \end{matrix} \qquad (1)$$

u_a and f_a represent the displacements and the forces of the corner points and accordingly, u_m and f_m relate to the remaining points of the macro. The following relation makes the displacements u_m depend on u_a

$$u_m = \mathbf{c}u_a \qquad (2)$$

The macro has the degrees of freedom of the corner points only, the related stiffness matrix is achieved by few steps of matrix manipulation

$$\mathbf{k}_n = \mathbf{C}^T \mathbf{K}_n \mathbf{C} \qquad (3)$$

with

$$C = \left[I \quad c \right]^T \tag{4}$$

I represents the identity matrix having the size of the a-set.

It is very convenient to define the constraints **c** using a shape function N(x,y) - x,y being the local coordinates - as it is usual for determining the element stiffness. Now the macro behaves like a finite element. For example, in case of a macro as a plane triangular the points inside of the macro have a linearly varying displacement state, and consequently there is a constant stress distribution in the macro. The accuracy of non-planar macros can be improved by additional points lying on the sides of the macros as it is usual in the finite element theory. Fig. 1 shows an original structure subdivided into subregions, and fig. 2 the reduced structure due to the introduction of constraints.

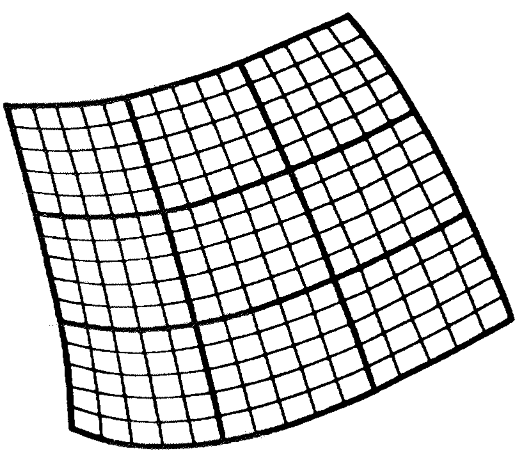

Fig. 1
Structure to be optimized, subdivided into macros

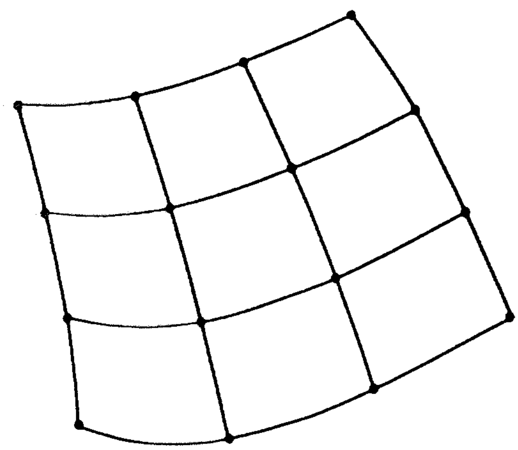

Fig. 2
Reduced structure due to the constraints

The stiffness matrices k_n are assembled in the usual way leading to a stiffness matrix k of the complete reduced system (fig. 2) which has the dimension according to the degrees of freedom of the corner points of the macros only. The actual formulation in MSC/NASTRAN is very easy; an a-set and a m-set must be defined, and the constraints c are considered by MPC-conditions.

For highly curved macros a different, more expensive, way can be used. The points inside of the macros are eliminated by Guyan reduction leading to the structure in fig. 3; the remaining points are eliminated by spline functions (fig. 4). It is a very straightforward procedure in MSC/NASTRAN using RSPLINE elements for the points lying on the sides of the macros and specifying the points inside as the o-set.

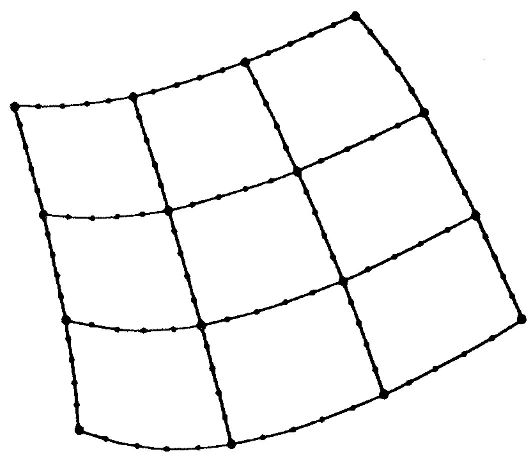

Fig. 3
Guyan reduction of the points inside of the macros

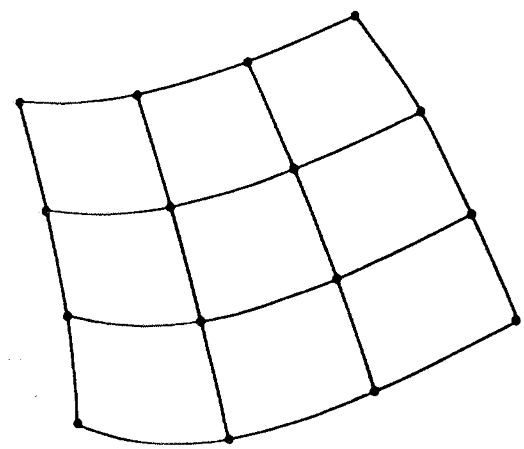

Fig. 4
Reduced structure using spline-elements for the points on the sides

Using the shape function N(x,y) a matrix **N**(x,y) can be determined which represents the constraints, so we have

$$\mathbf{c} = \mathbf{N}(x,y) \qquad (5)$$

The shape optimization process is applied to the reduced structure (fig. 2 or 3). The related matrix **k** represents the stiffness behaviour of the original structure (fig. 1) in a global manner. Because of the limited number of grid points, variations of the shape of the structure can be performed easily. Structural variations can be found by artificial loads (/1/); an eigenvalue solution allows to investigate independent variations, so the number of these can be reduced and with that the number of design variables.

$$(\mathbf{k} - \mu I)x = 0 \tag{6}$$

where I being the identity matrix. The variation of the coordinates X_a (a-set) of the structure to be optimized may be stated as

$$X_{a,var} = X_{a,old} + a_1 x_1 + a_2 x_2 + \ldots + a_j x_j \tag{7}$$

a_i represents the design variables, "x_i" the eigenvectors worked out by eq. 6, "j" the number of eigenvectors employed for the variation. Figs. 6 and 7 show the perturbations found out by an eigenvalue analysis of the structure of a cantilever beam (fig. 5) consisting of 8 macros.

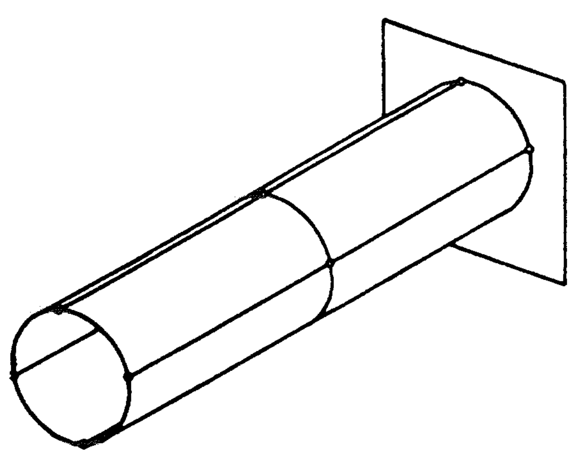

Fig. 5
Cantilever beam

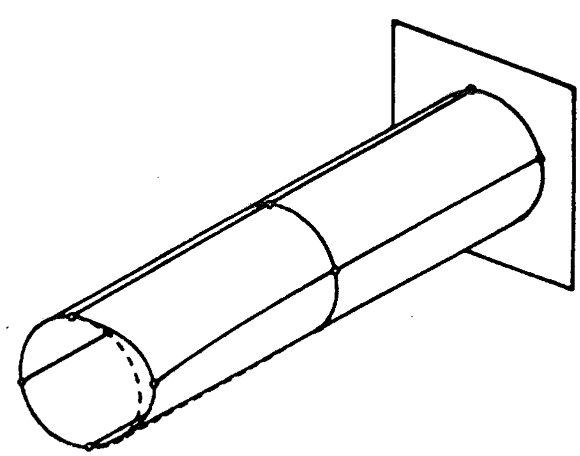

Fig. 6
Local perturbation

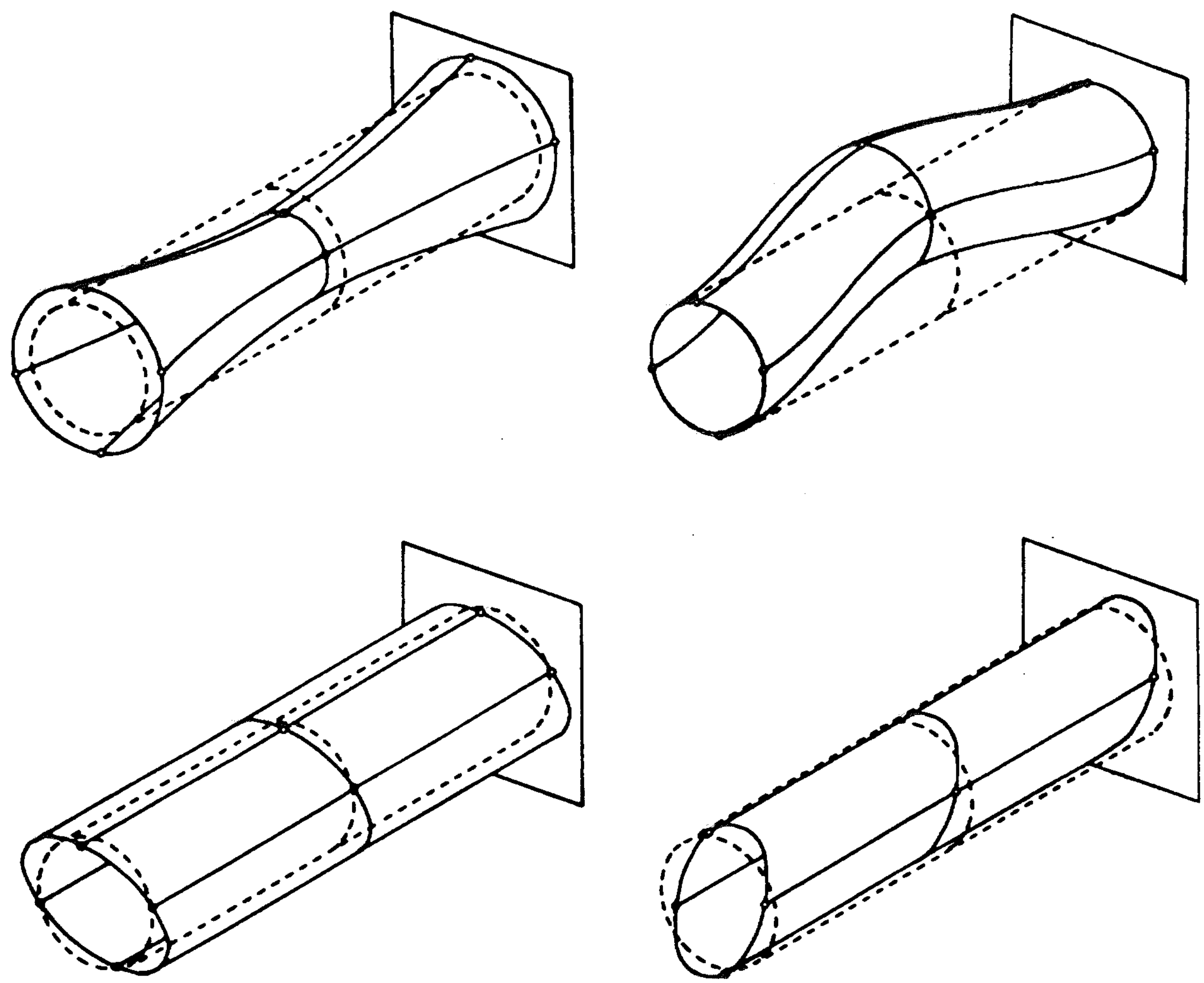

Fig.7

Possible structural variations carried out by an eigenvalue analysis

Of course, it is also possible to use additional, "conventional", design variables t_i like the thickness of the elements inside of the macros.

The eigenvalue solution (eq. 6) provides relevant perturbations (which influence the optimization process) and non-relevant ones like local modes (fig. 6) which have to be removed. The application of macros allows to select this modes without much computational effort. Let us denote for an arbitrary eigenvector x the components of the points belonging to the macro "n" by x_n, we can calculate

$$H_n = x_n^T \mathbf{k}_n x_n \tag{8}$$

$\mathbf{k}_n$ represents the stiffness matrix of the macro, x_n the variation of the coordinates. H_n indicates the distortion of the macro. So local modes can be identified and modes which relate to regions of the structure that should not be changed.

An other relationship allows one to estimate whether or not the stresses inside of the macro are high.

$$S_n = u_a^T \mathbf{k}_{n,var} u_a \tag{9}$$

u_a represents the displacements due to the external load. If S_n is below a specific limit, the derivatives of the stresses need not to be calculated.

There are also global modes which don't influence the optimization procedure. They are identified in the first step of optimization if the design variables a_i are close to zero, and they are removed in the following steps by the orthogonality check. Being $x_{1,rel}$ a relevant mode in the first step, so x_2 in the following step is only considered if E is greater than a specific limit

$$E = x_2^T x_{1,rel} \tag{10}$$

The remaining eigenvectors determine the variations of the structure. In order to perform the optimization process the derivatives of $\mathbf{k}_n$ with respect to the design variables a_i and t_i must be prepared. For that the stiffness matrices $\mathbf{k}_{n,var}$ of the macros must be reanalysed considering the new coordinates $X_{m,var}$ and the design variables t_i. The coordinates $X_{m,var}$ of the points which have been eliminated by the constraints $\mathbf{c}$ can be defined easily by a mesh generation program employing the macro as "patch". For small perturbations the constraint equations can be used very effectively /1/

$$X_{m,var} = X_{m,old} + c(X_{a,var} - X_{a,old}) \quad (11)$$

Now the stiffness matrices $\mathbf{K}_{n,var}$ of the macros after the variation and, using eq. 3, the reduced matrices $\mathbf{k}_{n,var}$ can be worked out. A finite difference process results the derivatives $\partial \mathbf{k}_n/\partial a_i$ of the stiffness matrices of the macros by means of $\mathbf{k}_n$ and $\mathbf{k}_{n,var}$. The derivatives of the displacements u_a are determined as usual by

$$\partial u_a/\partial a_i = -\mathbf{k}^{-1} \, \partial \mathbf{k}_n/\partial a_i \, u_a \quad (12)$$

and the derivatives of the degrees of freedom of the m-set yield

$$\partial u_m/\partial a_i = \mathbf{c} \, \partial u_a/\partial a_i \quad (13)$$

Now the data is prepared for performing a step of the optimization process in the well known manner /2/.

References

/1/ I. Raasch, A. Irrgang, Shape Optimization with MSC/NASTRAN. Proceedings XVth MSC/NASTRAN European users´ conference 1988, Rome

/2/ G. Vanderplaats, Numerical Optimization Techniques for Engineering Design. McGraw-Hill, 1984