APPLICATIONS OF A SELF-ADAPTIVE ALGORITHM TO NONLINEAR FINITE ELEMENT ANALYSES

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Abstract

The accuracy and effectiveness of a nonlinear finite element solution rely upon several critical procedures: spatial discretization, material modeling, incremental time (or load) stepping, and equilibrium iteration. Performance and user friendliness can be improved if these procedures have intelligence to cope with any numerical difficulties without requiring user discretion. Adaptive mesh generation and refinement has become a major research topic to automate finite element modeling or spatial discretization. MSC/NASTRAN facilitates the adaptation of the constitutive relations by using a subincremental scheme in the material processing. This paper focuses on the adaptive incremental and iterative solution technique.

A self-adaptive algorithm for the time-stepping and equilibrium iteration was recently implemented for the nonlinear transient analysis in MSC/NASTRAN. This algorithm adopts Newmark's one-step integrator, which is suitable for the adaptive implicit integration. Before each step of direct integration, a proper time increment is estimated based on the dominant frequency of vibration. Equilibrium is attained by the Newton's iteration process at each time step. The iteration method employs expedient procedures such as the quasi-Newton update and line seach technique. Divergence problems are overcome by the systematic stiffness matrix updates and the bisection process. This adaptive procedure is also applicable to static analysis by simply ignoring the inertia and damping effects.

Several numerical examples are illustrated to demonstrate the applicability of the present method to a wide variety of nonlinear analyses. Geometric nonlinear effects are included in a shallow dome with an apex load, an elliptic cylinder subjected to an internal pressure, and a column excited with an axial follower force. A contact problem is introduced in an elastic rod subjected to an impact in the longitudinal direction. The material nonlinearity is combined with the geometric nonlinear effects in an elasto-plastic beam subjected to an impulse and a z-shaped frame loaded with a static force. The last example is a static problem which is analyzed without inertia and damping effects, and the automatic time step adjustment is bypassed.

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HIGHLIGHTS IN SOL 99

- Two-point Implicit Direct Time Integration
- Automatic time stepping
- Bisection algorithm
- Iteration uses BFGS update and Line search
- Follower forces
- Static analysis capability added to SOL 99
- Restart capabilities enhanced and expanded

IMPLICIT DIRECT TIME INTEGRATION

- **Newmark's Time Integrator** $(\gamma = 0.5, \beta = 0.25)$

- **Iteration for Equilibrium at Each Time Step**

$$\left[\frac{4}{\Delta t^2} M + \frac{2}{\Delta t} C + \tilde{K}\right] \Delta U^{i+1} = R^i_{n+1}$$

- **Two-point Recurrent Formula:**
  Easy Starting, Restarting, Ending

- **Residual error carried over effectively**

- **Initial Equilibrium satisfied**

$$M \ddot{U}_0 + C \dot{U}_0 = P_0 - F_0.$$

- **Acceleration Need Not be computed during time stepping**

AUTOMATIC TIME STEP ADUSTMENT

- **For Accuracy, Efficiency and User Friendliness**
- **Predict Δt based on the Dominant frequency in the incremental deformation pattern**

$$\omega_n^2 = \frac{\Delta U_n^T K \Delta U_n}{\Delta U_n^T M \Delta U_n} = \frac{\Delta U_n^T \{F_n - F_{n-1}\}}{\Delta U_n^T M \Delta U_n}$$

- **Number of steps (MSTEP) for a period is adaptive**
  based on the stiffness ratio:

$$r = \frac{\Delta t_{n+1}}{\Delta t_n} = \frac{1}{m} \frac{2\pi}{\omega_n} \frac{1}{\Delta t_n}$$

- **Thrashing prevented by stepping function**

$$\Delta t_{n+1} = f(r) \Delta t_n$$

- **Bounds for Δt adjustment:**

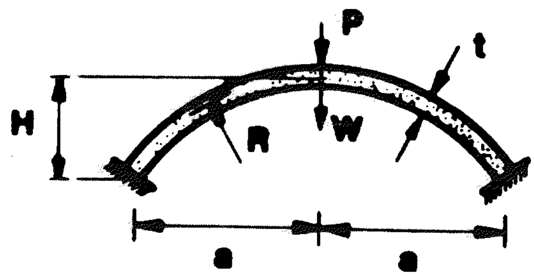
$$DT/MAXR < \Delta t < DT * MAXR$$

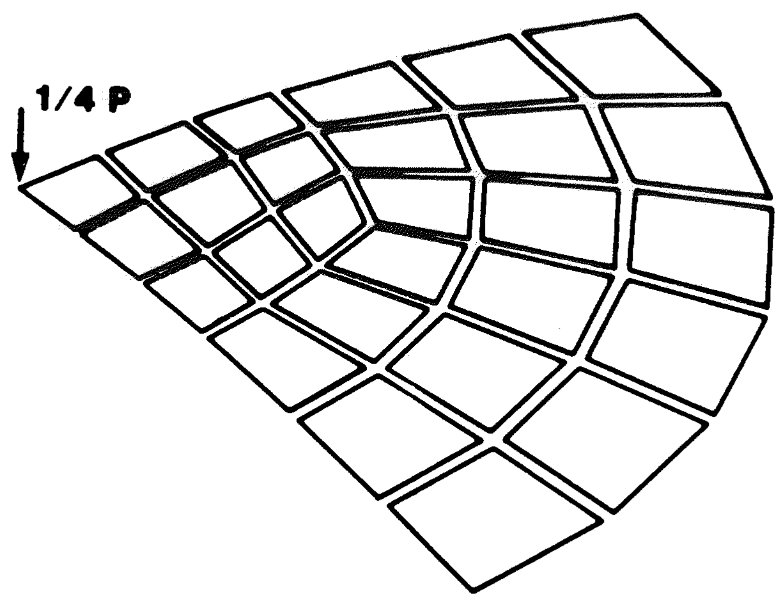
- **Undesirable effects due to GAP, plasticity, large mass, massless points, etc. are filtered out**

BISECTION ALGORITHM

- To overcome divergent problems due to nonlinearity
- Activated when diverence occurs
- Activated when MAXITER is reached
- Activated when excessive $\Delta\sigma$ is detected
- Decomposition at every bisection
- Update [K] at every KSTEP-th converged bisection
- Bisection continues until solution converges or MAXBIS is reached
- If MAXBIS is reached:
  Reiteration procedure is activated
  to select the best attainable solution

SHALLOW SPHERICAL CAP UNDER APEX LOAD





FINITE ELEMENT MESH

$R = 4.75$ in.

$a = 0.90$ in.

$H = 0.08589$ in

$t = 0.01576$ in.

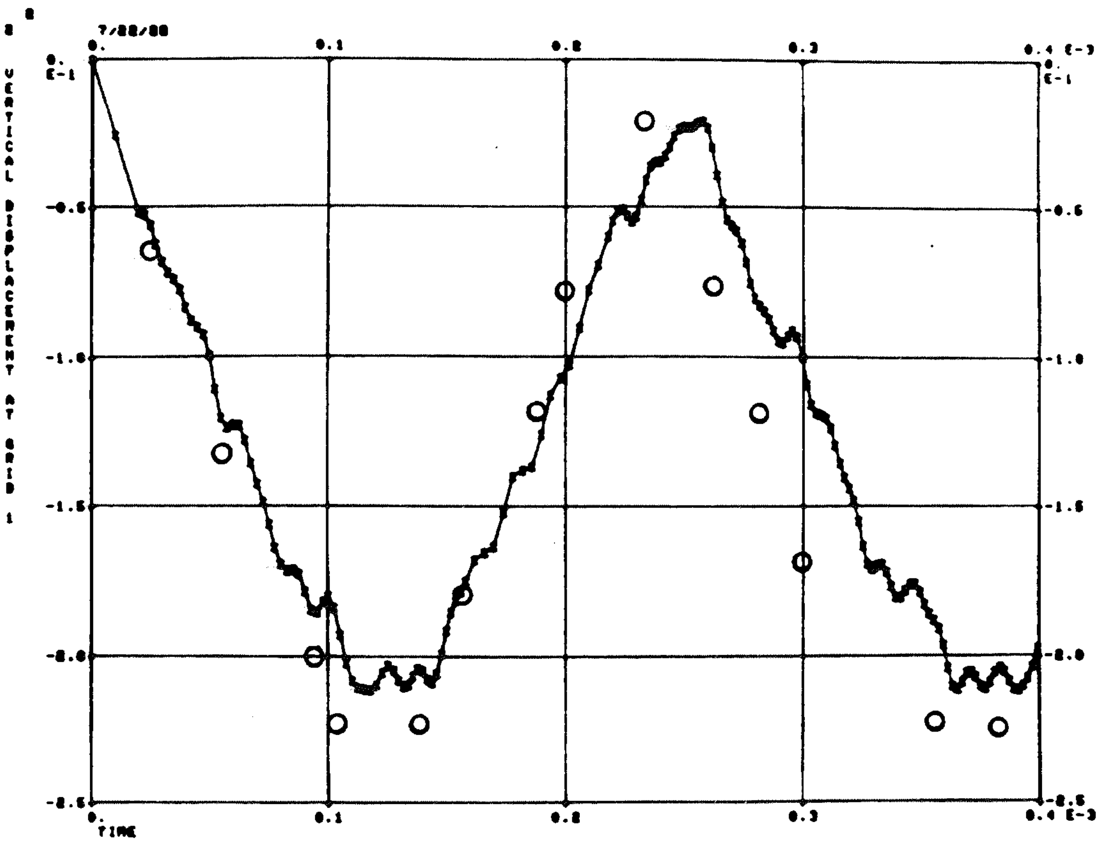
$E = 10000$ ksi. Young's modulus

$\nu = 0.3$, Poisson's ratio

$\rho = 2.45 \times 10^{-4}$ lb-sec^2/in^4

$P = 100.$ lbs., constant applied force

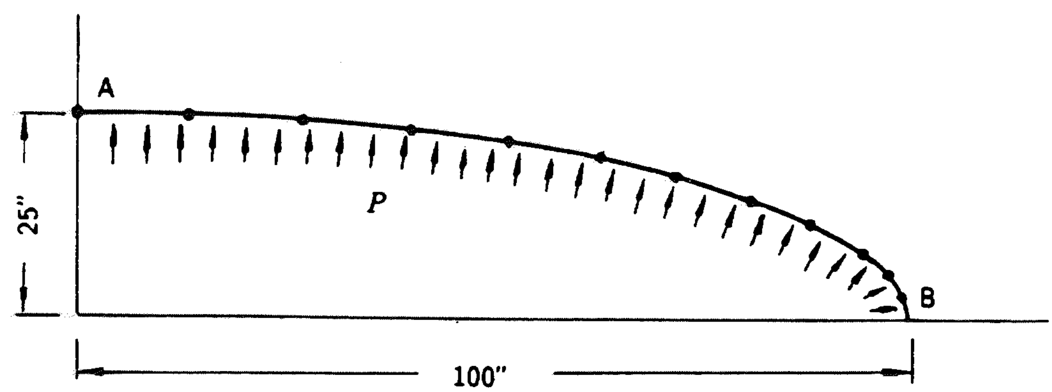
SPHERICAL CAP: APEX DISPLACEMENT RESPONSE

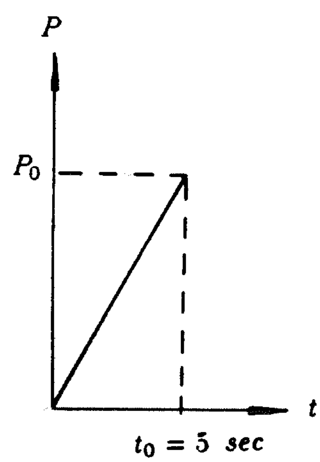


—✕— MSC/NASTRAN
○ Mondkar and Powell[9]

Spherical Cap: Apex Displacement Response

ELLIPTIC CYLINDER SUBJECTED TO AN INTERNAL PRESSURE



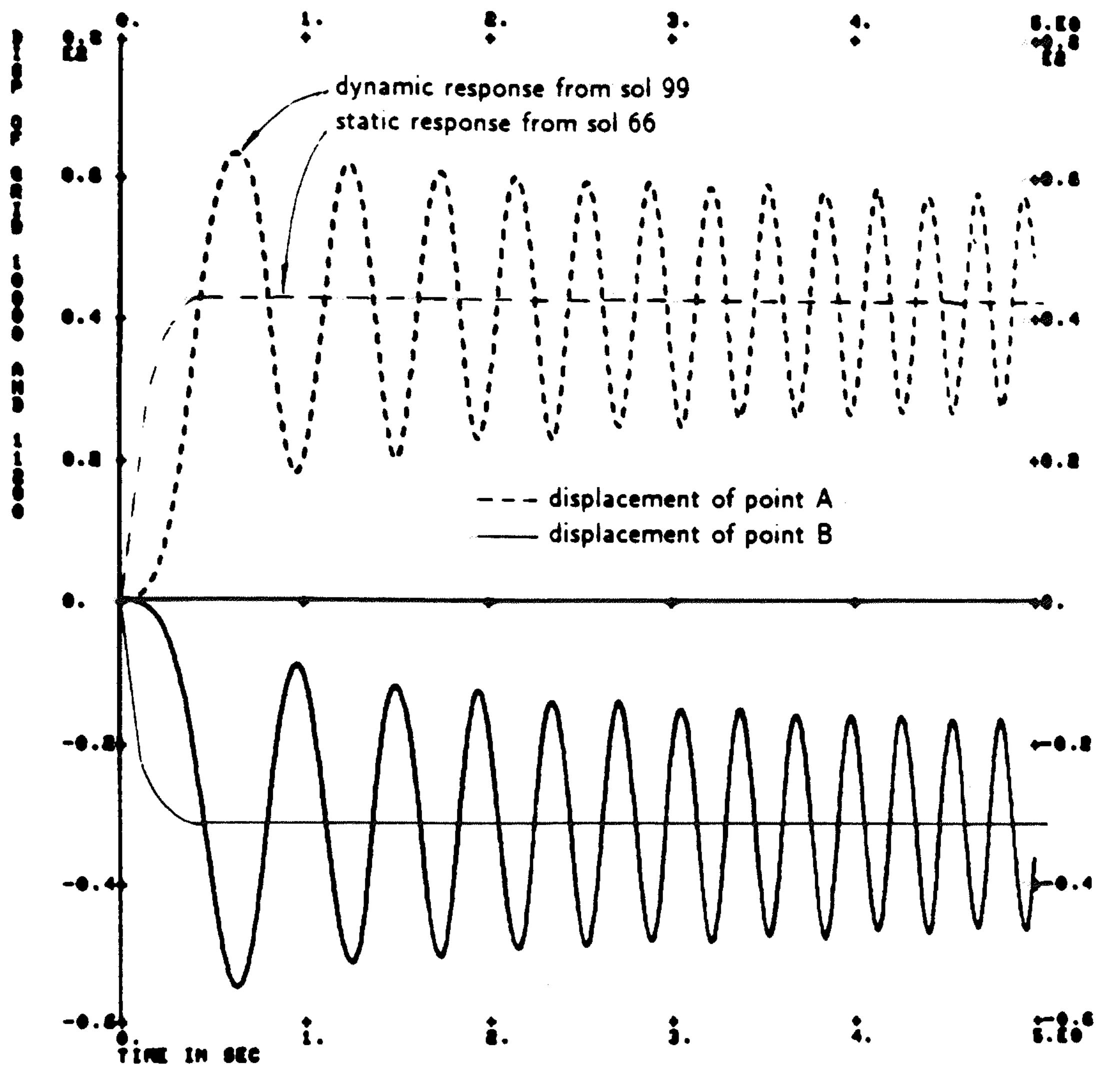


$E = 30 \times 10^6$ psi, Young's modulus

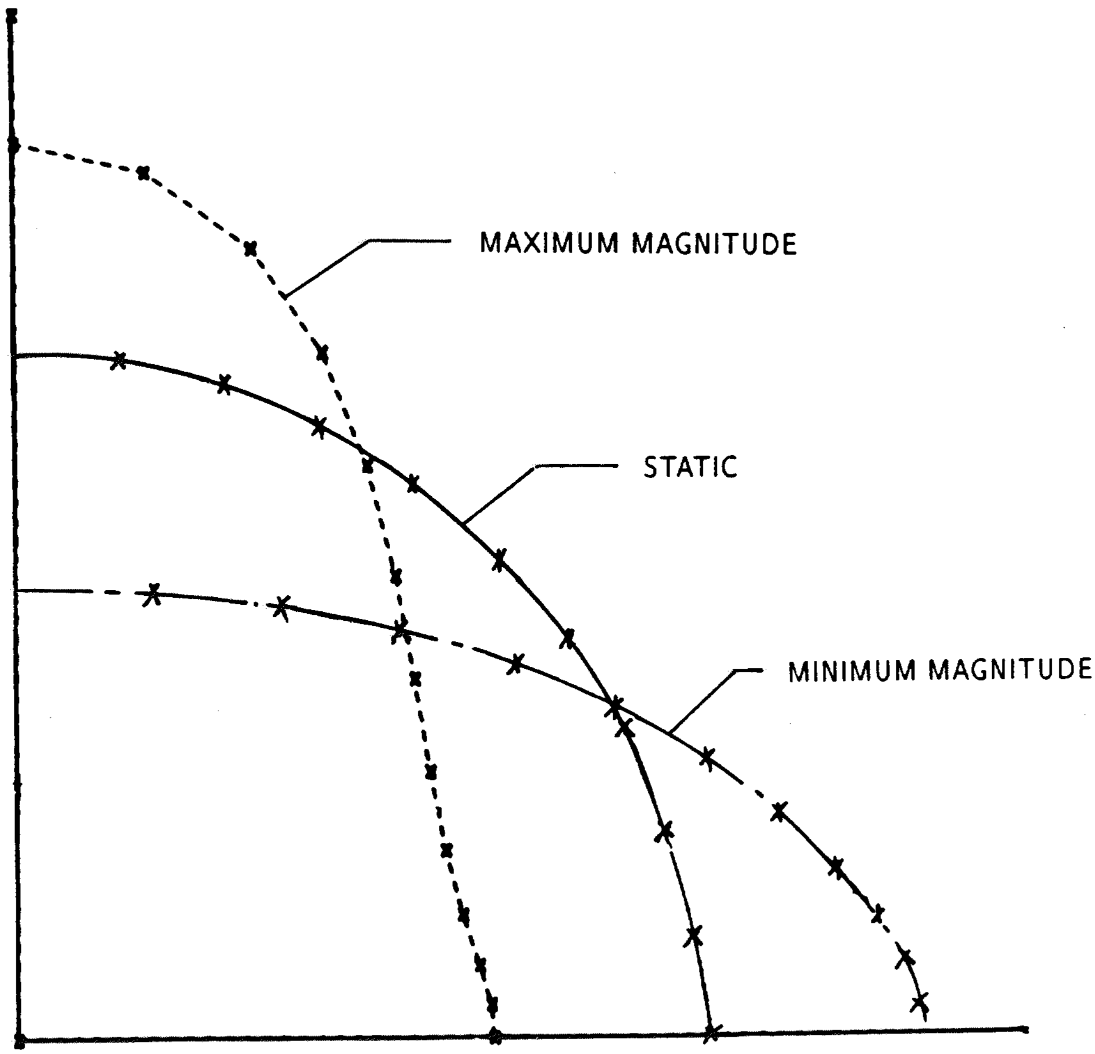
$F_y = 5 \times 10^5$ psi, yield stress in tension

$H = 3 \times 10^5$ psi, slope of stress vs. plastic strain

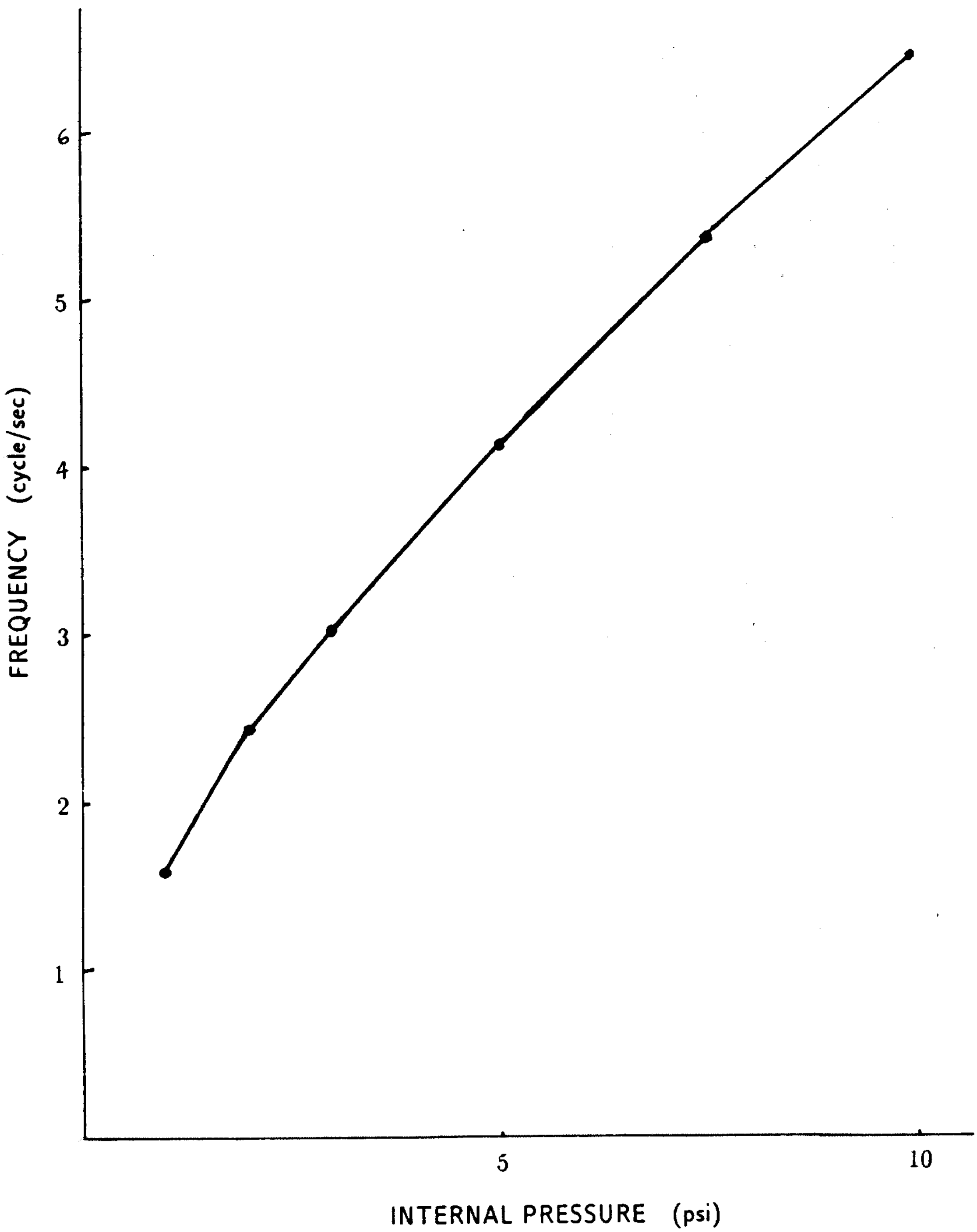
ELLIPTIC CYLINDER: DISPLACEMENT FOR $P_0 = 5$ psi ($\Delta t = 0.01 - - - 0.0025$)



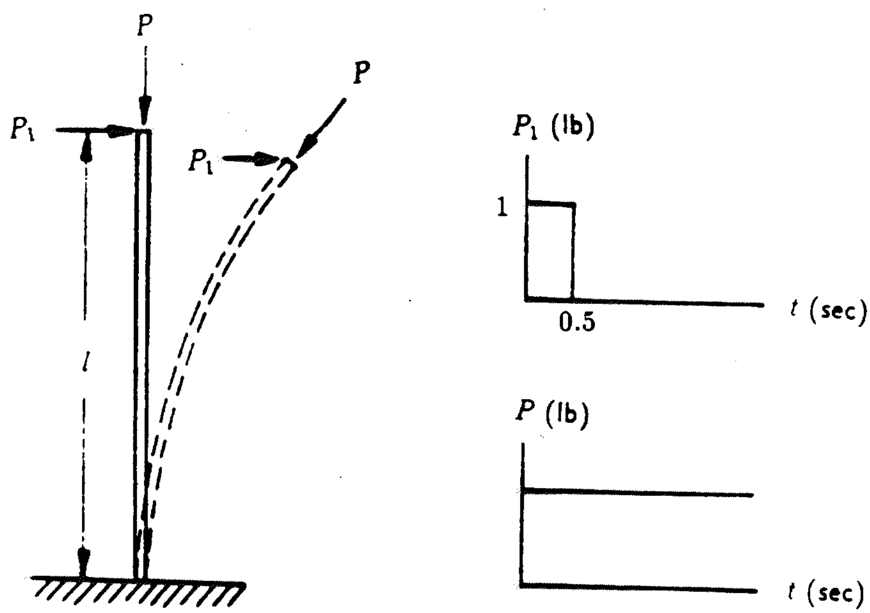
ELLIPTIC CYLINDER: DEFORMED SHAPE



ELLIPTIC CYLINDER: FREQ. VS. PRESSURE



LATERAL VIBRATION OF A COLUMN UNDER AXIAL FOLLOWER FORCES



l = 1000 in, column length

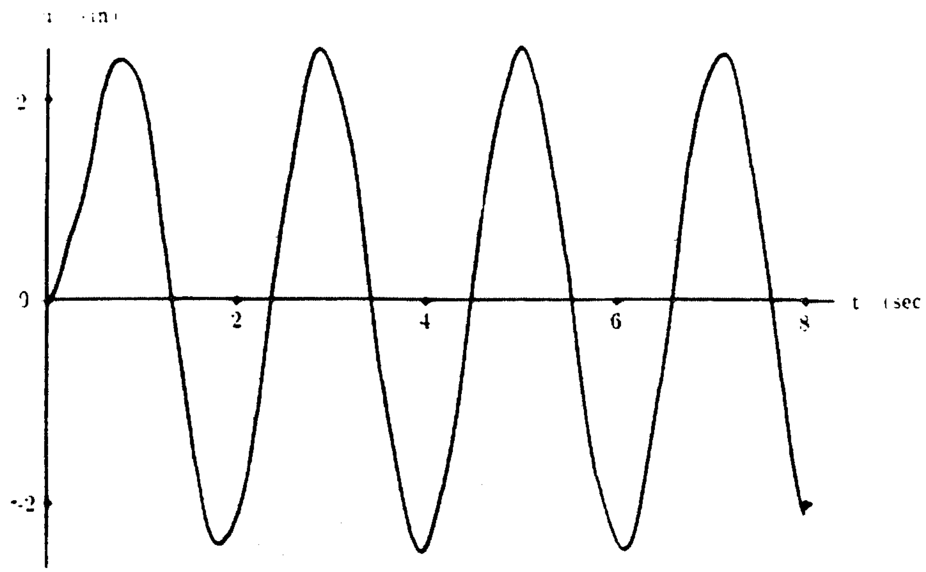
A = 1000 in^2, cross-sectional area of the column

I = 1.0 in^4, inertia moment

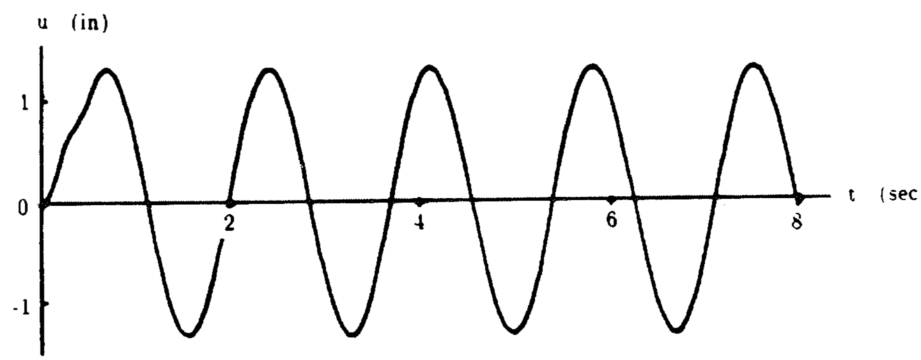
E = 1×10^7 psi, Young's modulus

ρ = 2×10^8 lb-sec^2/in, mass density

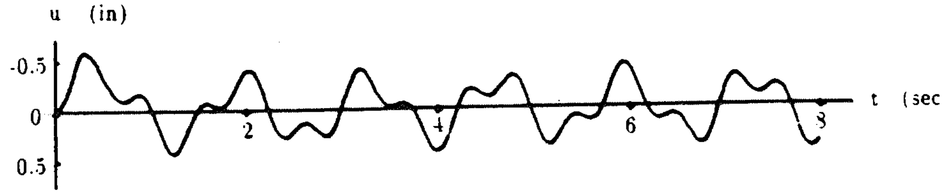
HORIZONTAL DISPLACEMENT VS. TIME AT $P = 50$, $P = 100$, AND $P = 150$ LBS.



(a) P = 50 lb

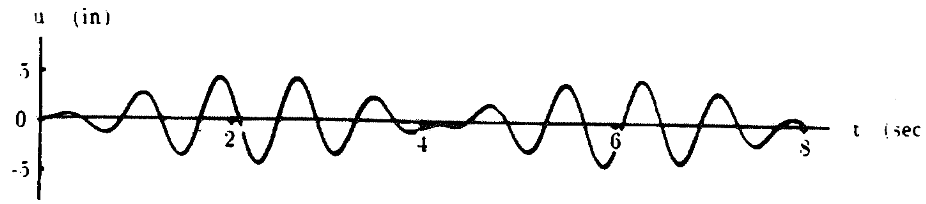


(b) P = 100 lb

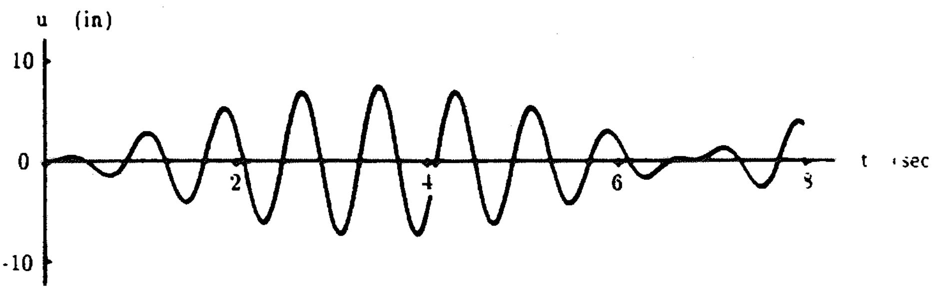


(c) P = 150 lb

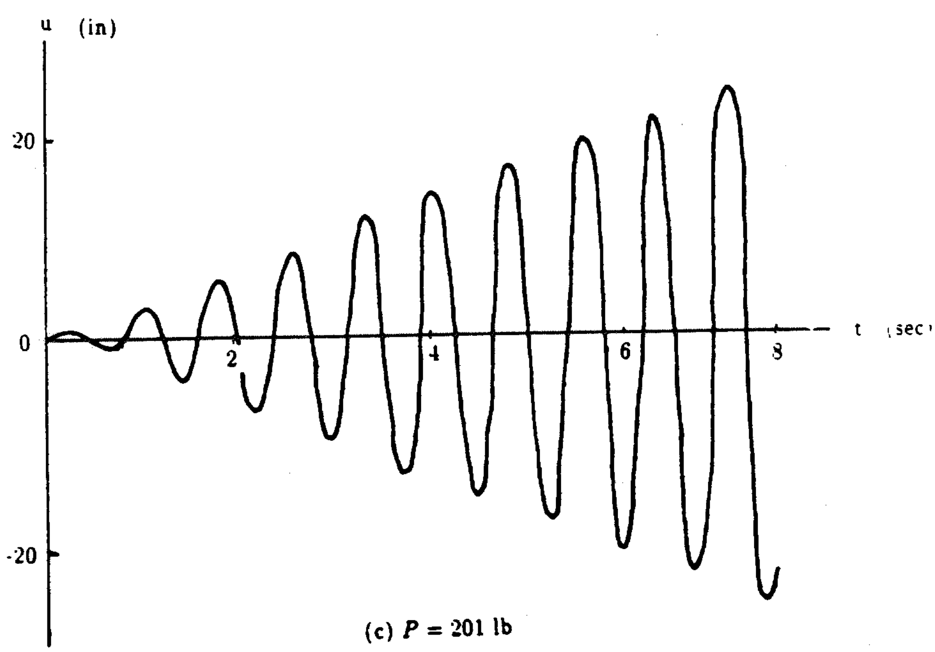
HORIZONTAL DISPLACEMENT VS. TIME IN THE VICINITY OF THE CRITICAL LOAD



(a) P = 198.2 lb

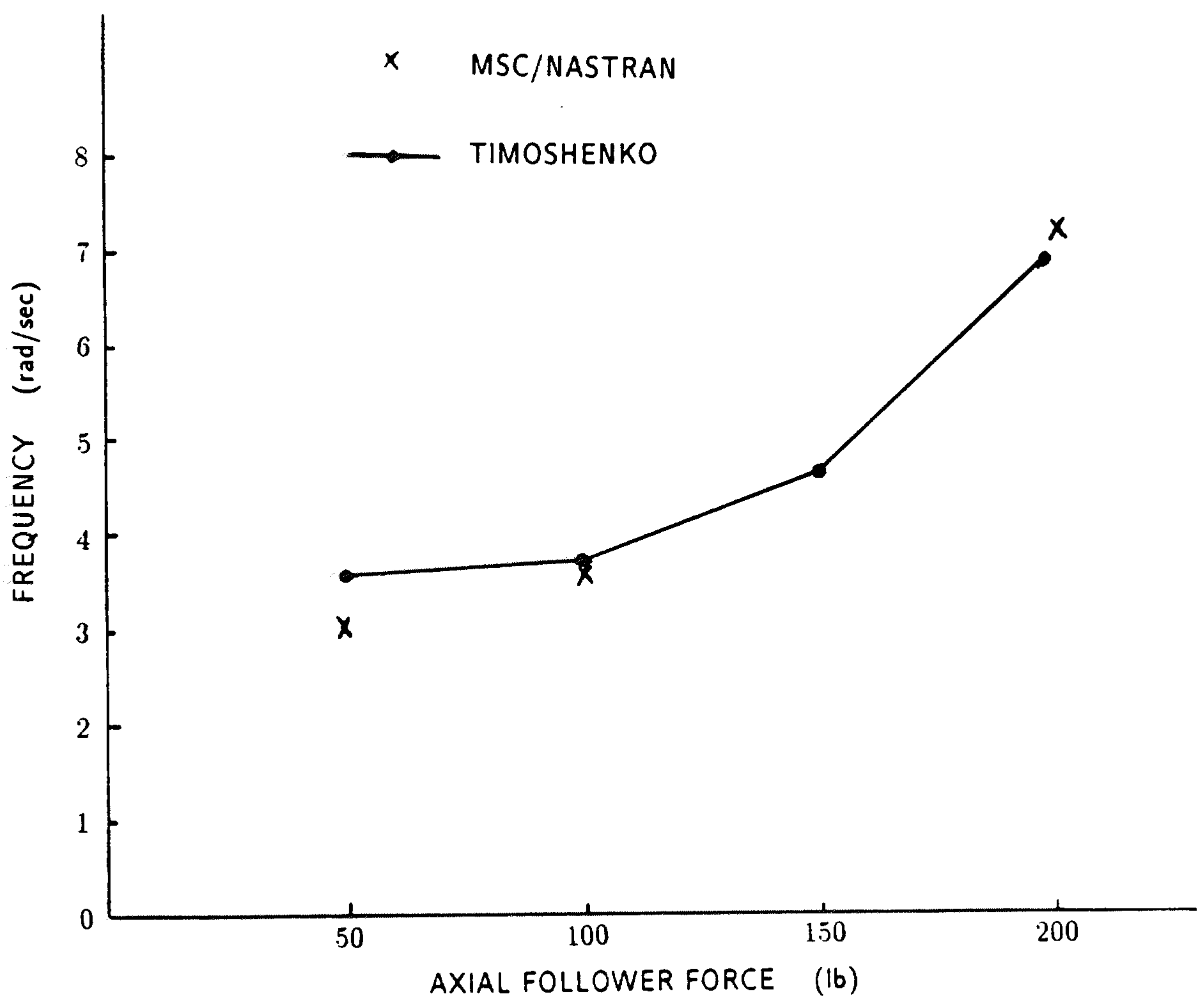


(b) P = 200 lb

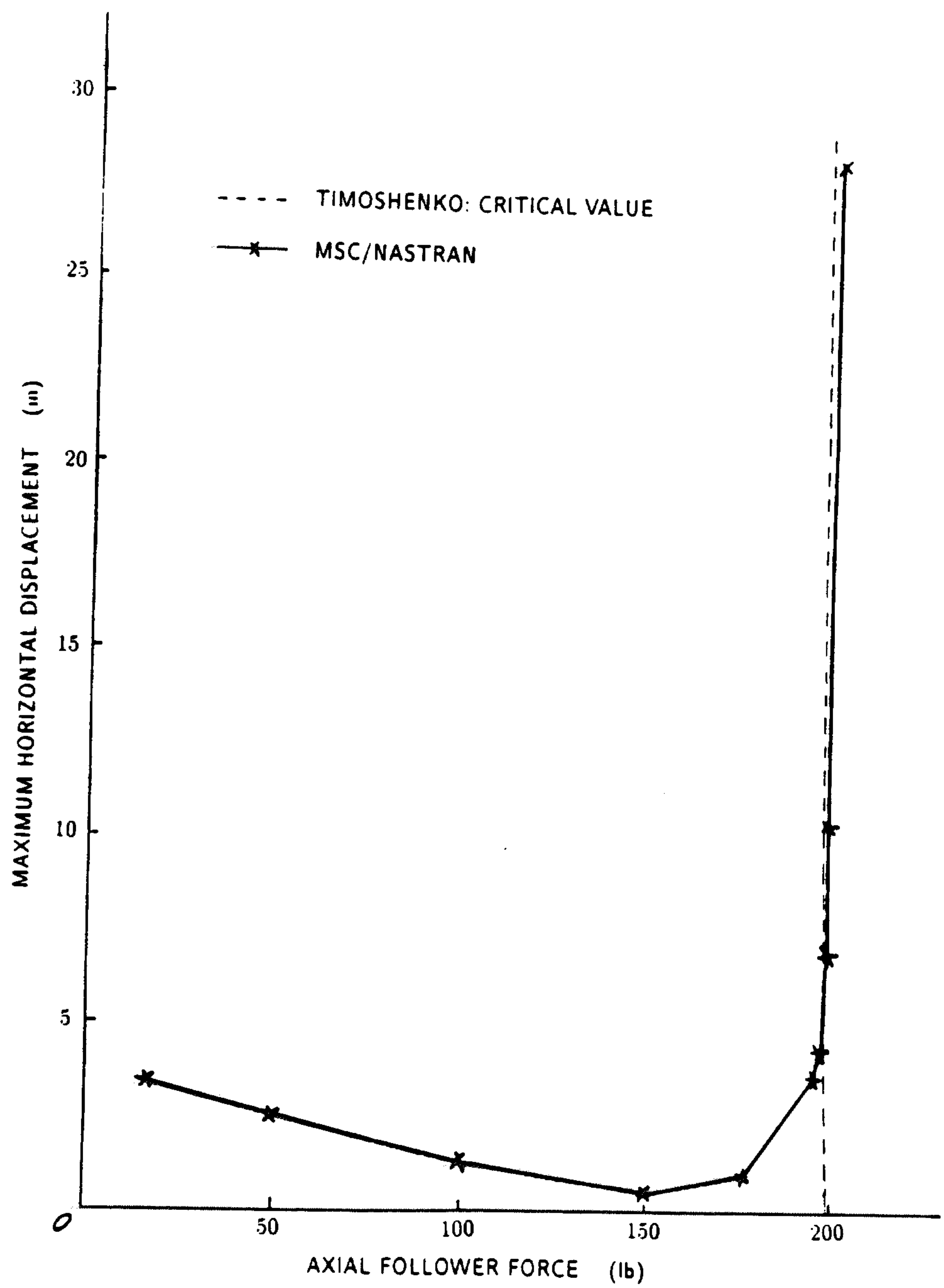


(c) P = 201 lb

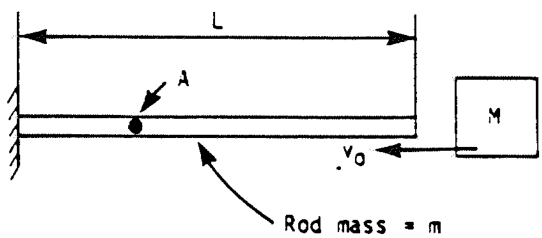
COLUMN VIBRATION: FREQ. VS. AXIAL FORCE



COLUMN VIBRATION: MAX. AMPLITUDE VS. AXIAL FORCE

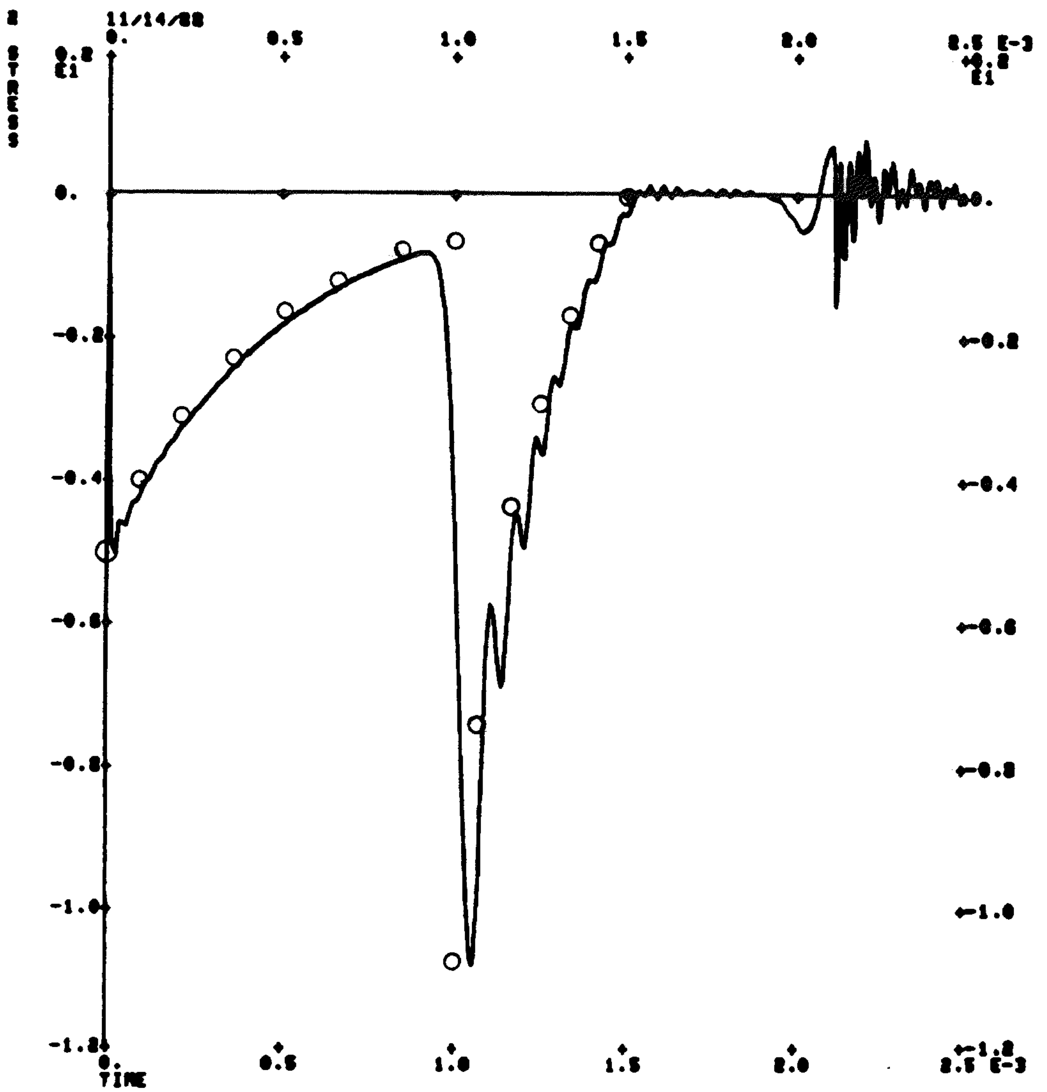


STRESS WAVE PROPAGATION IN AN ELASTIC ROD

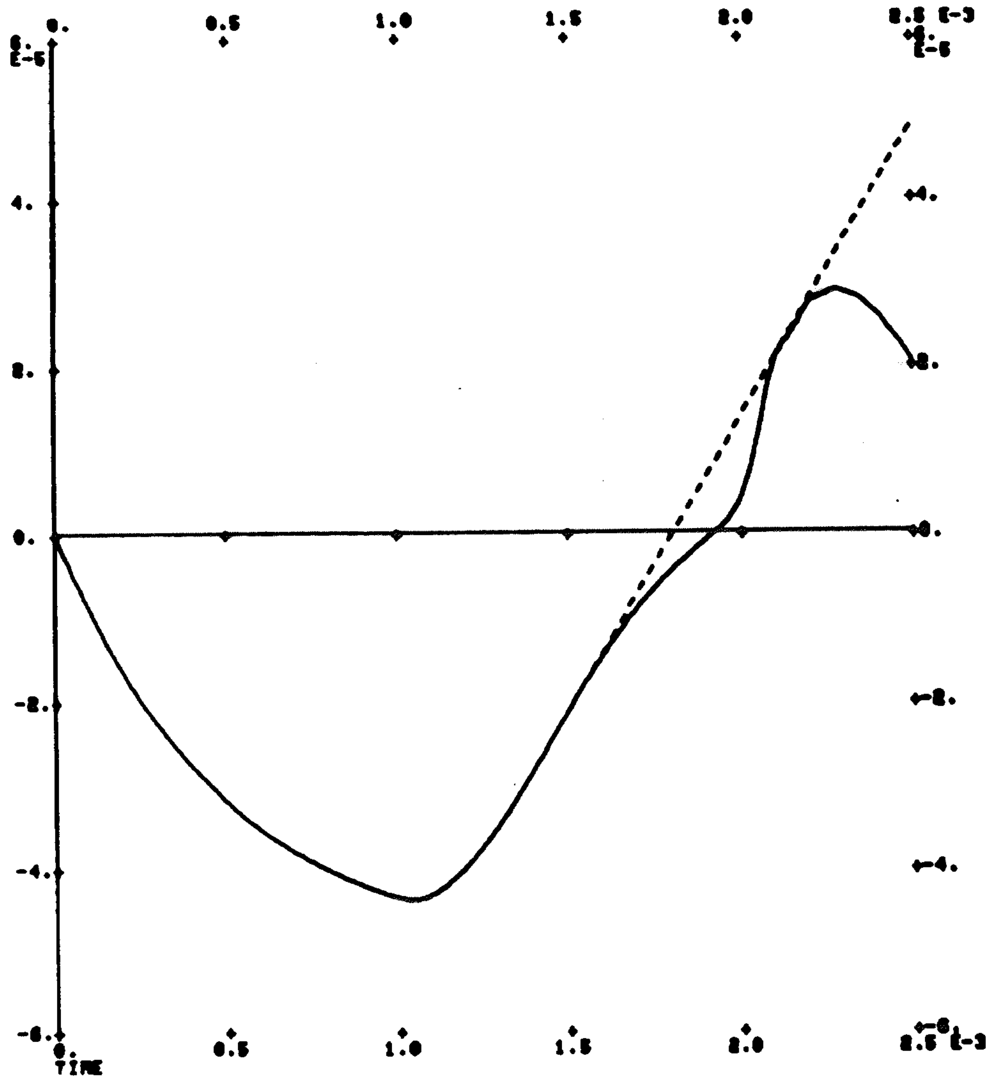


M	=	.025 lb-sec/in, mass
L	=	100.0 in, rod length
A	=	1. in^2, cross-sectional area of the rod
E	=	10×10^6 psi, Young's modulus
ν	=	0.3, Poisson's ratio
ρ	=	0.00025 lb-sec/in^4, mass density

Rod Subjected to a Longitudinal Impact

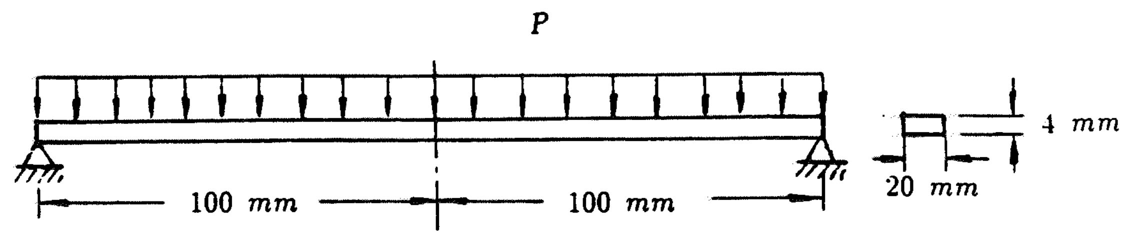


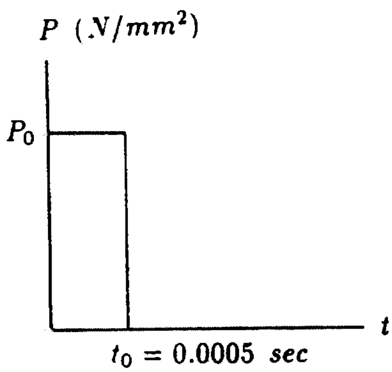
Rod Impact: Free End Stress Time History

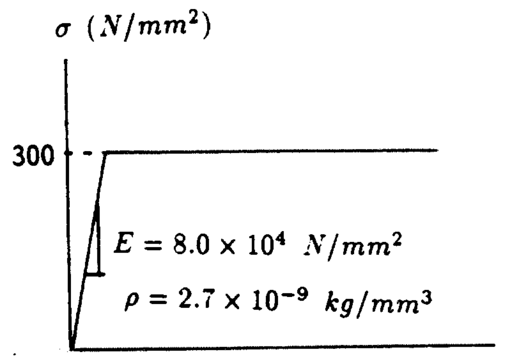


Rod Impact: Free End Displacement Time History

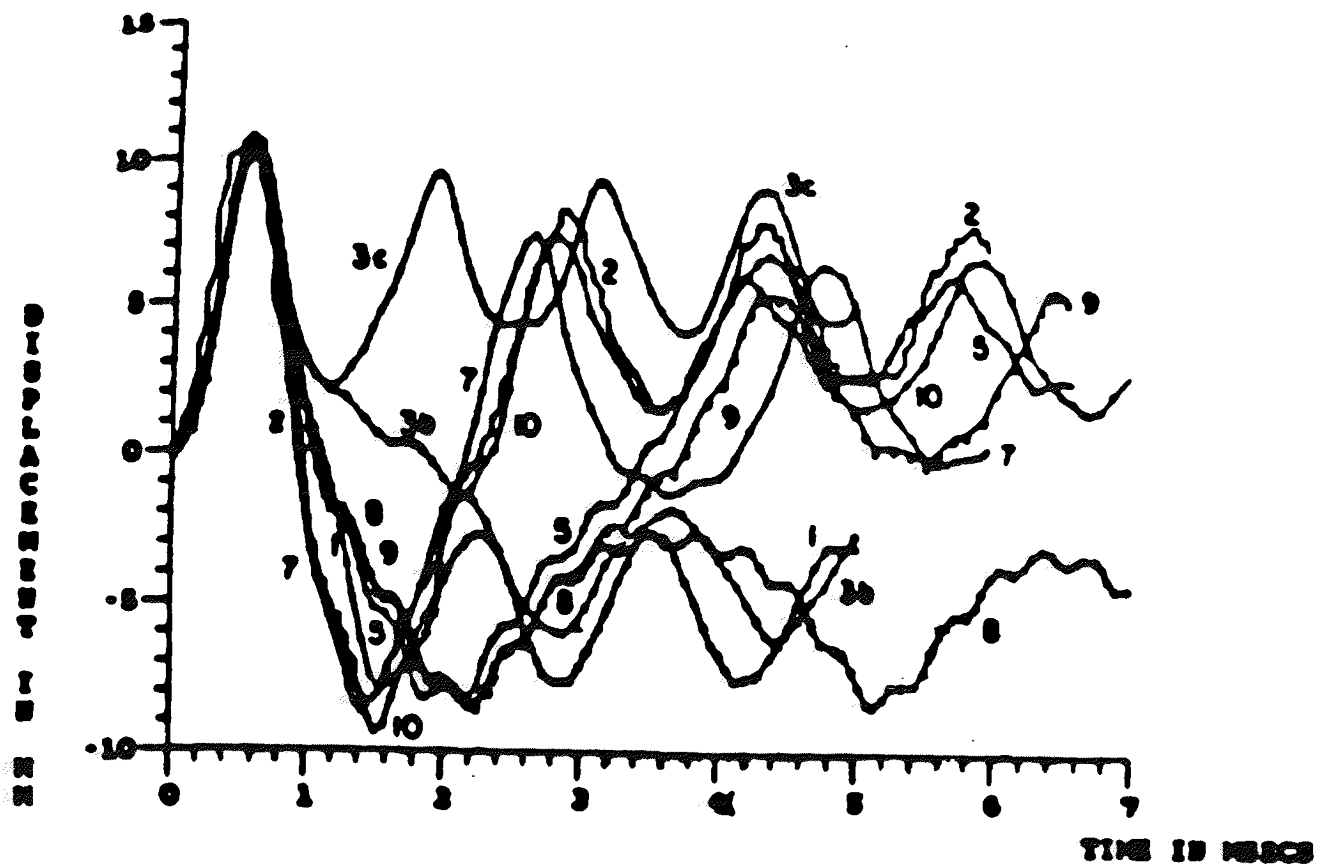
COUNTERINTUITIVE VIBRATION OF ELASTIC-PLASTIC BEAM



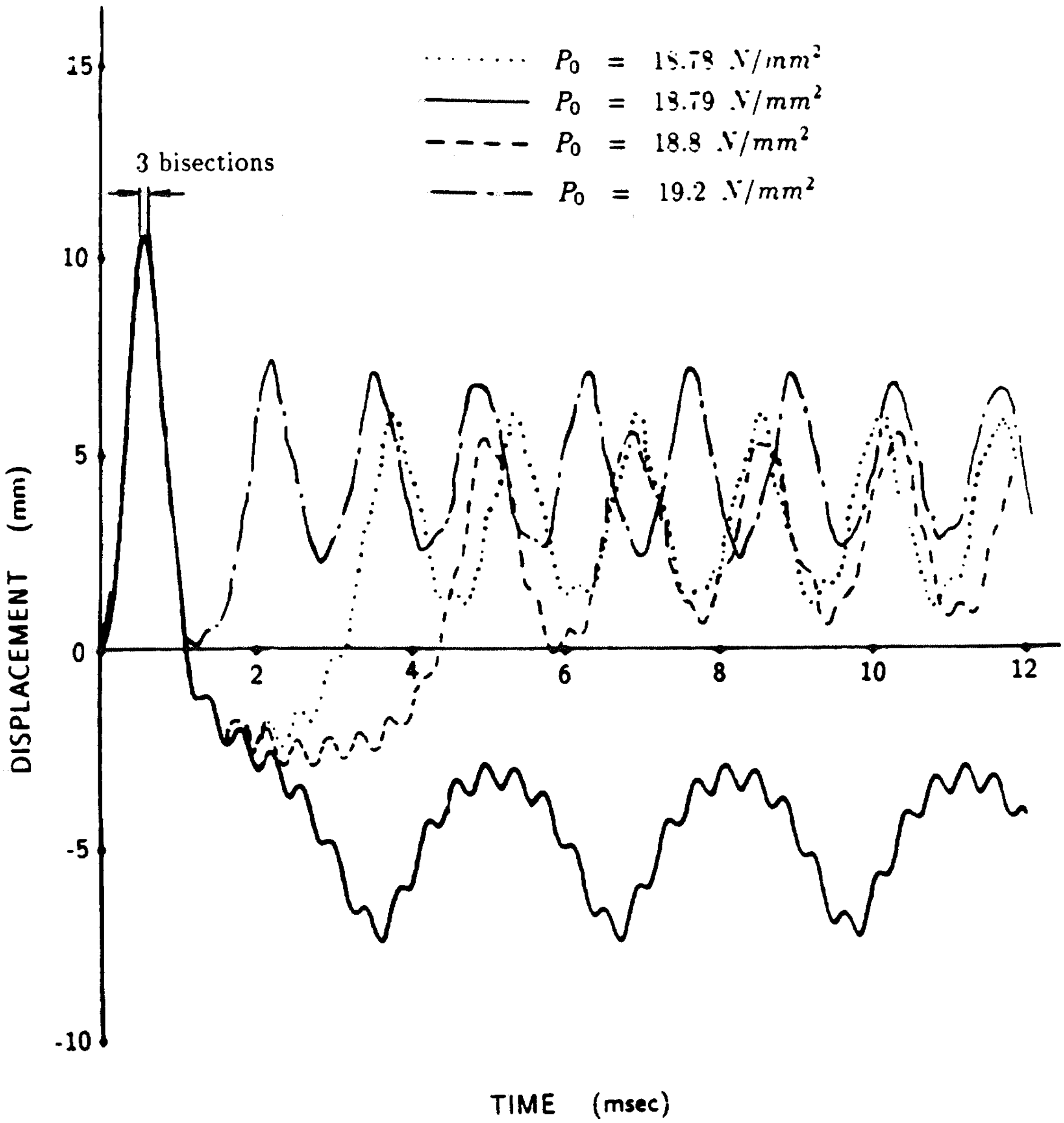




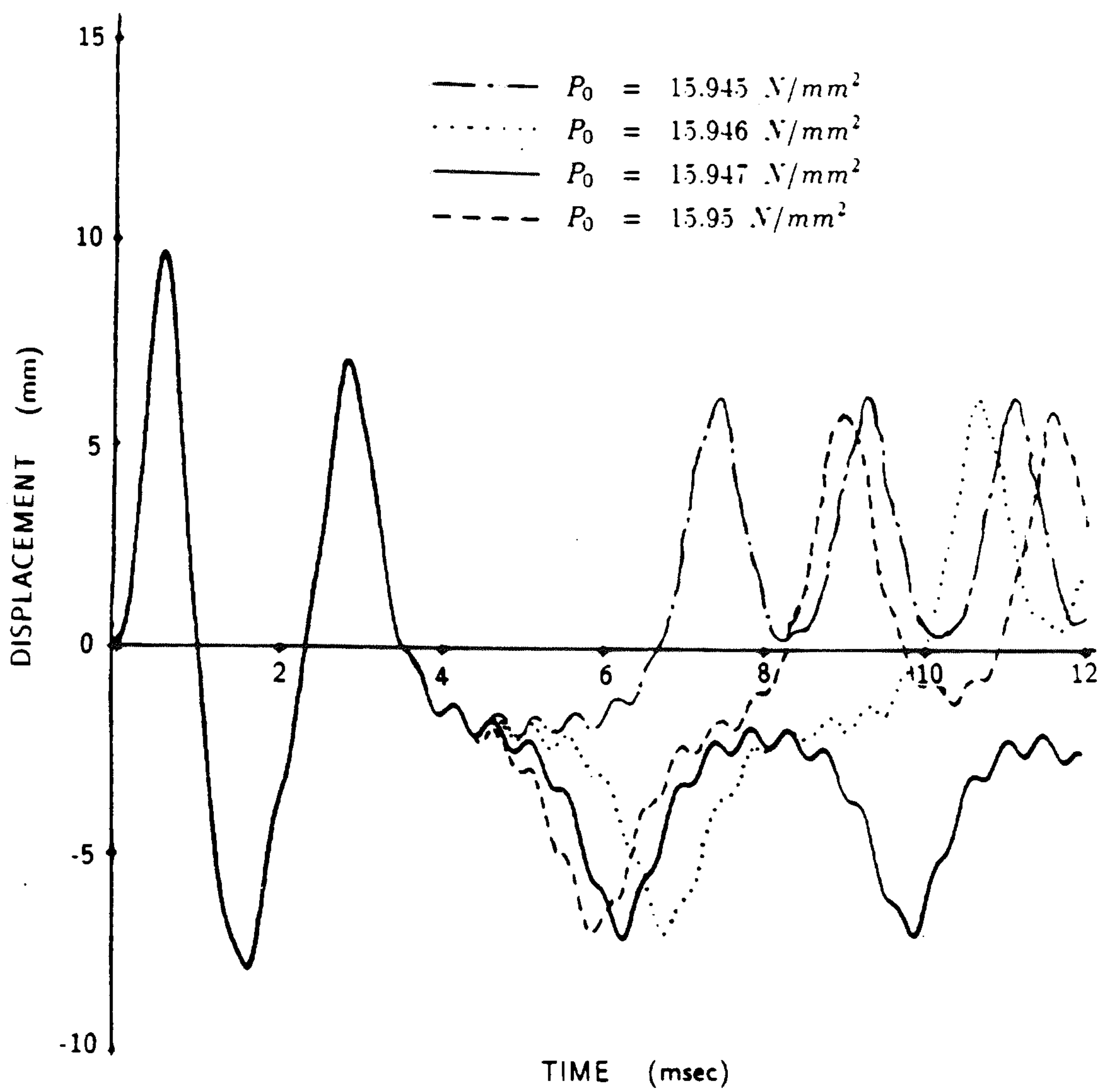
COUNTERINTUITIVE BEAM VIBRATION: MID-SPAN DISPLACEMENT (J. OF APPLIED MECHANICS)



COUNTERINTUITIVE BEAM VIBRATION: MID-SPAN VIBRATION AROUND $P = 19\ N/mm^2$

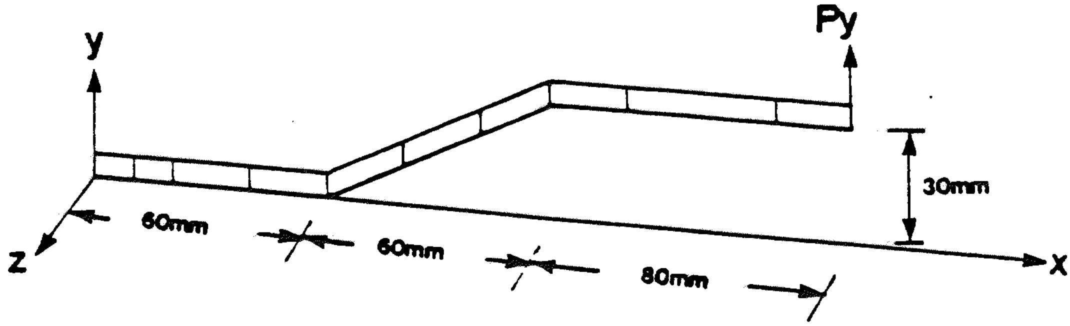


COUNTERINTUITIVE BEAM VIBRATION: MID-SPAN VIBRATION AROUND $P = 16\ N/mm^2$



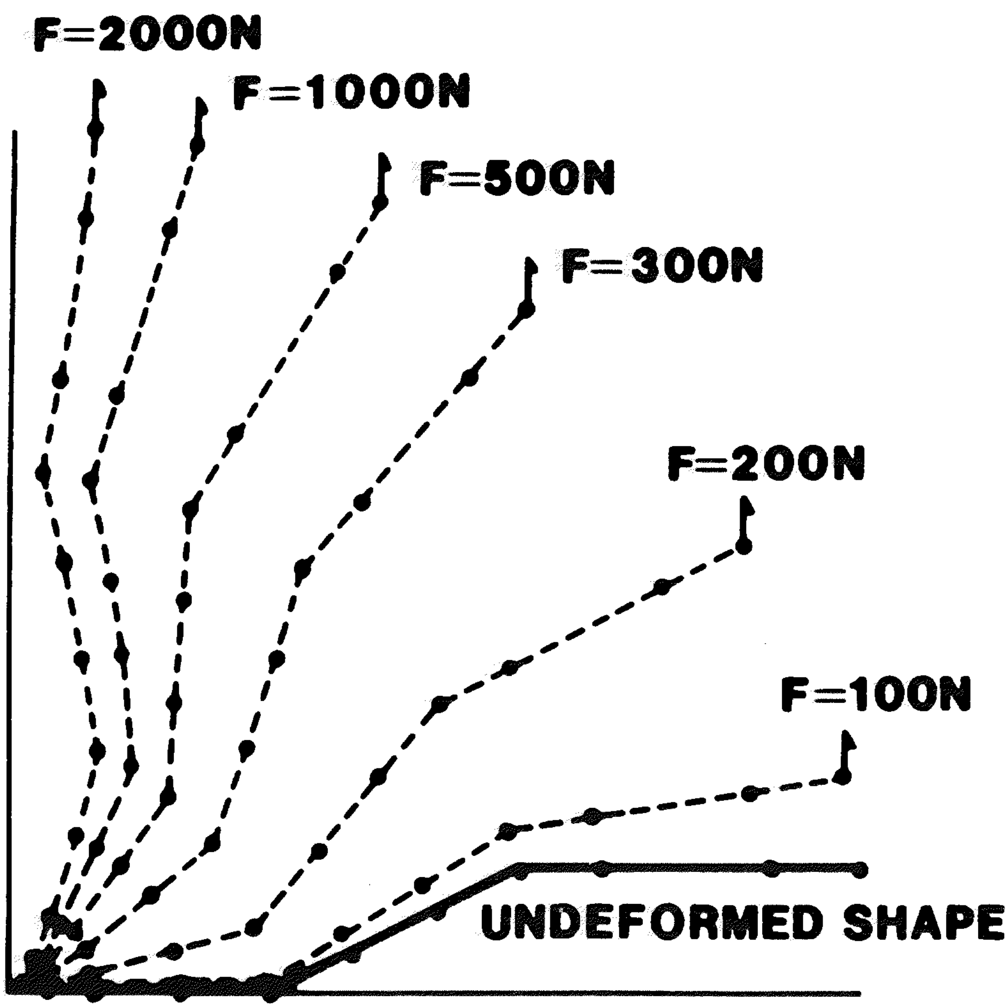
STATIC ANALYSIS IN SOL 99

THE "COLOGNE CHALLENGE"



10 BEAM elements
$A = 68 \ mm^2$
$I = 65.5 \ mm^4$
$E = 2 \times 10^5 \ N/mm^2$
$F_y = 450 \ N/mm^2$

COLOGNE CHALLENGE: DEFORMED SHAPE



COLOGNE CHALLENGE: LOAD-DEFLECTION CURVE