LIMITATIONS OF CURRENT NONLINEAR FINITE ELEMENT METHODS IN DYNAMIC ANALYSIS OF SOLAR ARRAYS

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ABSTRACT

DEPLOYABLE SOLAR ARRAYS CONSIST OF A "BLANKET" OF SOLAR COLLECTORS, AND A MAST. THE BLANKET IS STRETCHED INTO POSITION WHEN THE ARRAY IS DEPLOYED. THE STIFFNESS OF THE ARRAY IS A FUNCTION OF THE RIGIDITY OF THE MAST AS WELL AS THE TENSION IN THE BLANKET.

CURRENT FINITE ELEMENT FREQUENCY ANALYSIS CONSISTS OF USING MSC NASTRAN SOLUTION 64 (NON-LINEAR ANALYSIS) TO OBTAIN THE TANGENTIAL STIFFNESS MATRIX OF THE ARRAY. THIS MATARIX IS THEN INPUT, USING DMAP ALTERS, INTO MSC NASTRAN SOLUTION 63 (DYNAMIC ANALYSIS) TO OBTAIN THE NATURAL FREQUENCIES OF THE ARRAY.

THE AUTHOR HAS FOUND THAT PSEUDO-FORCES ARE DEVELOPED, HOWEVER, AT THE ELEMENT LEVEL DUE TO LIMITATIONS INHERENT IN THE GEOMETRIC STIFFNESS MATRICES CURRENTLY IN ACCEPTED USE. IN PARTICULAR, THE GEOMETRIC STIFFNESS MATRICES LACK THE CAPABILITY FOR RIGID BODY ROTATIONS, ESPECIALLY WHEN THE ROTATIONS ARE LARGE.

THE AUTHOR DEMONSTRATES THE LIMITATIONS OF THE ANALYSIS, SHOWS WHERE THE ERRORS ARE INTRODUCED IN THE DERIVATION OF THE GEOMETRIC STIFFNESS MATRIX, AND EXAMINES VARIOUS TECHNIQUES EITHER TO ELIMINATE THE PSUEDO-FORCE GENERATION AND/OR IMPROVE UPON THE CONVERGENCE OF THE CURRENT ALGORITHMS.

THIS PAPER IS THE PRODUCT OF AN NASA/ASEE* SUMMER FACULTY FELLOWSHIP AND AN ON-GOING JOINT RESEARCH EFFORT BETWEEN CLEVELAND STATE UNIVERSITY AND THE NASA LEWIS RESEARCH CENTER.

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SPACE STATION SOLAR ARRAY

NASA'S SPACE STATION IS POWERED UTILIZING PHOTOVOLTAIC ARRAYS, CONSISTING OF A DEPLOYABLE TRUSS "MAST" AND BLANKET SUBSTRATES. (FIG. 1) THE STIFFNESS OF THE SPLIT-BLANKET ARRAY IS A FUNCTION OF THE RIGIDITY OF THE MAST AS WELL AS THE TENSION MAINTAINED IN THE BLANKETS. THE "BLANKETS" THEMSELF POSSESS NEGLIGIBLE STIFFNESS.

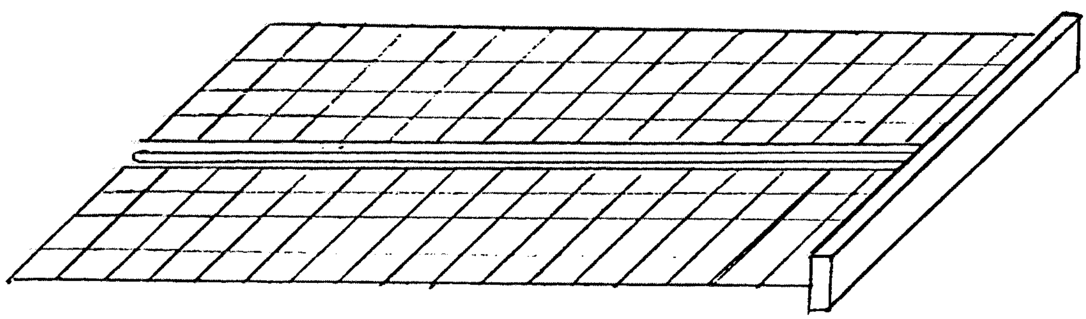

FIG. 1 SPACE STATION SPLIT-BLANKET SOLAR ARRAY

THE FREE VIBRATION CHARACTERISTICS OF THE SPLIT-BLANKET SOLAR ARRAYS WAS STUDIED USING TWO METHODS. MODE SHAPES AND FREQUENCIES WERE CALCULATED USING EQUATIONS OF CONTINUUM MECHANICS, AS WELL AS A FINITE ELEMENT SOLUTION USING MSC NASTRAN (REF. 1 AND 2).

THE FINITE ELEMENT MODELING CONSISTED OF GENERATING A TANGENTIAL STIFFNESS MATRIX BY APPLYING THE PRE-TENSIONING LOAD IN MSC NASTRAN GEOMETRIC NON-LINEAR SOLUTION (SOLUTION 64). THE STIFFNESS MATRIX GENERATED WAS THEN INPUT INTO MSC/NASTRAN DYNAMIC ANALYSIS (SOLUTION 63) TO OBTAIN THE NATURAL FREQUENCIES AND MODE SHAPES. (REF. 3)

THE FINITE ELEMENT ANALYSIS INDICATED THAT LARGE INTERNAL "PSEUDO-FORCES" DEVELOPED WHEN RIGID BODY MOTION WAS APPLIED. AN INVESTIGATION WAS SUBSEQUENTLY MADE TO DETERMINE WHETHER THE LARGE PSEUDO-FORCES WHICH DEVELOPED WERE DUE TO USER MODELING ERRORS OR LIMITATIONS OF THE FINITE ELEMENT PROCESS.

THE GEOMETRIC STIFFNESS MATRIX UTILIZED IN MSC/NASTRAN SOLUTION 64 WAS FOUND TO BE IDENTICAL TO THE STIFFNESS MATRIX FORMULATED BY MARTIN (REF. 4). FOR SIMPLICITY, A 2-DIMENSIONAL BEAM-COLUMN ELEMENT WAS INVESTIGATED.

LIMITATIONS OF CURRENT [Kg]

TYPICALLY, FINITE ELEMENT STATIC ANALYSIS IS USED TO SOLVE LINEAR ELASTI PROBLEMS OF THE FORM

$$[Ke]\{U\} = \{R\}$$

WHERE [Ke] IS THE ELASTIC STIFFNESS MATRIX
{U} IS THE MODAL DISPLACEMENT VECTOR
{R} IS THE FORCE VECTOR

THE [Ke] MATRIX MUST POSSESS THE CAPACITY FOR RIGID BODY DISPLACEMENT. IN OTHER WORDS, THE ELEMENT MUST BE ABLE TO TRANSLATE OR ROTATE WITHOUT DEVELOPING STRESSES. (SEE FIGURE 2)

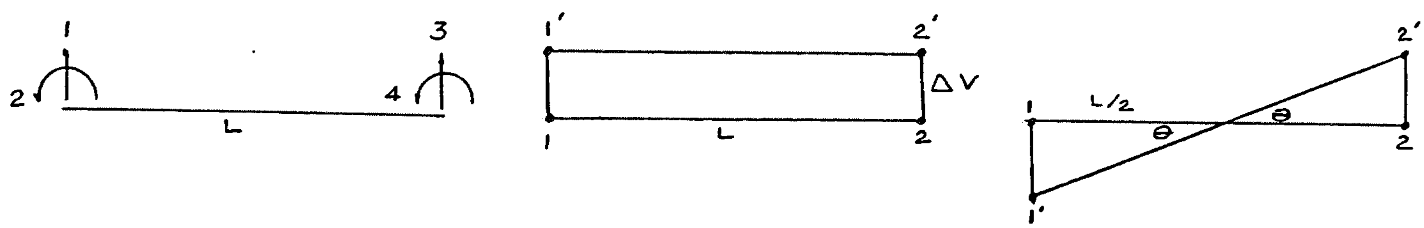

FIG.2(a) 2-NODE BERNOULLI BEAM ELEMENT WITH DEGREES OF FREEDOM SHOWN

FIG.2(b) RIGID BODY TRANSLATION IN Y-DIRECTION

FIG.2(c) RIGID BODY ROTATION ABOUT Z-AXIS

THE RIGID BODY TRANSLATION VECTOR IS $\{\Delta V, 0, \Delta V, 0\}$. SIMILARY, THE RIGID BODY ROTATION VECTOR IS APPROXIMATED BY $\{-L\theta/2, \theta, L\theta/2, \theta\}$ WHICH CAN BE WRITTEN AS ∞$\{-L/2, 1, L/2, 1\}$, WHERE ∞ IS THE ANGLE OF ROTATION. COMBINING YIELDS

$$\left\{\begin{matrix}\text{RIGID}\\\text{BODY}\\\text{MODES}\end{matrix}\right\} = \begin{bmatrix}\Delta V & -L\theta/2\\ 0 & \theta\\ \Delta V & L\theta/2\\ 0 & \theta\end{bmatrix}$$

THE ELASTIC STIFFNESS MATRIX FOR A 2-NODE BERNOULLI BEAM IS

$$[Ke] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \qquad \text{-----------------(2)}$$

WHERE

E = MODULUS OF ELASTICITY
I = MOMENT OF MERTIA OF THE CROSS SECTION
L = LENGTH OF THE ELEMENT

BY DEFINITION OF RIGID BODY MOTION,

[Ke] {RIGID BODY MODES} MUST EQUAL {0} ------------------------------(3)

SUBSTITUTING (1) AND (2) INTO (3) YIELDS

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} 1 & -L/2 \\ 0 & 1 \\ 1 & L/2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

THEREFORE, [Ke] HAS THE CAPACITY FOR RIGID BODY MOTION.

NASA LERC ROUTINELY USES A DMAP ALTER WHICH CALCULATES

[K] {RIGID BODY MODES} = {RFORCES (PSEUDO-FORCES)}

FOR AN ELASTIC PROBLEM, THE PRESENCE OF LARGE RFORCES WOULD INDICATE THAT STRESSES ARE BEING PRODUCED DURING RIGID BODY MOVEMENT. THESE PSUEDO-FORCES ARE AN INDICATION THAT "GROUNDING" HAS OCCURRED, AND THAT THE MODEL IS NOT RELIABLE.

MANY PROBLEMS, SUCH AS THE SOLAR ARRAY, ARE NON-LINEAR PROBLEMS. FINITE ELEMENT SOLVES NON-LINEAR PROBLEMS OF THE FORM

WHERE [Ke] IS THE ELASTIC STIFFNESS MATRIX
[Kg] IS THE GEOMETRIC STIFFNESS MATRIX
{R} IS THE OUTPUT FORCE VECTOR AT THE END OF A STEP
{F} IS THE INPUT FORCE VECTOR AT THE BEGINNING OF A STEP
{ U} IS THE CHANGE IN THE DISPLACEMENT VECTOR DURING A STEP.

GRAPHICALLY, THIS IS SHOWN BY FIG. 3.

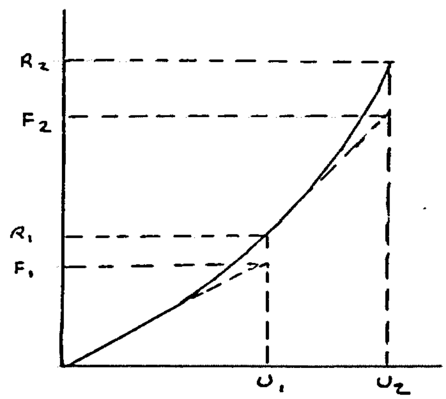

FIG. 3 NON-LINEAR STIFFENING CURVE

THE TRADITIONAL [Kg] MATRIX, DEVELOPED BY MARTIN[4] IS

$$[Kg] = Po \begin{bmatrix} 6/5L & 1/10 & -6/5L & 1/10 \\ 1/10 & 2L/15 & -1/10 & -L/30 \\ -6/5L & -1/10 & 6/5L & -1/10 \\ 1/10 & -L/30 & -1/10 & 2L/15 \end{bmatrix}$$

IF I APPLY RIGID BODY DISPLACEMENT DURING AN INCREMENTAL LOAD STEP, I OBTAIN

$$Po \begin{bmatrix} 6/5L & 1/10 & -6/5L & 1/10 \\ 1/10 & 2L/15 & -1/10 & -L/30 \\ -6/5L & -1/10 & 6/5L & -1/10 \\ 1/10 & -L/30 & -1/10 & 2L/15 \end{bmatrix} \begin{bmatrix} 1 & -L\theta/2 \\ 0 & \theta \\ 1 & L\theta/2 \\ 0 & \theta \end{bmatrix} = \begin{bmatrix} 0 & -Po\theta \\ 0 & 0 \\ 0 & Po\theta \\ 0 & 0 \end{bmatrix}$$

THUS, IT CAN BE SEEN THAT [Kg] POSSESSES THE CAPACITY FOR RIGID BODY TRANSLATION, BUT NOT RIGID BODY ROTATION. THUS, MARTIN'S [Kg] IS NOT EXACT.

IT CAN ALSO BE SHOWN THAT MSC/NASTRAN NON-LINEAR ANALYSIS (BASED ON MARTIN'S DEVELOPMENT) SIMILARILY DOES NOT HAVE AN EXACT GEOMETRIC STIFFNESS MATRIX AND WILL PRODUCE PSEUDO-FORCES.

THEREFORE, THE RFORCE CHECK DMAP ALTER FOR MSC/NASTRAN SOLUTION 64 (NON-LINEAR ANALYSIS) IS NOT SUFFICIENT CRITERIA FOR DETERMINING THE VALIDITY OF A MODEL.

IN SPITE OF ITS DEFICIENCIES, MARTIN'S [Kg] PROVIDES ACCEPTABLE RESULTS FOR SOLVING STATICS PROBLEMS DUE TO THE ITERATON PROCESS. (ALTHOUGH [Kg] IS NOT EXACT, THE PROCESS CONVERGES TO THE EXACT SOLUTION.)

IN DYNAMIC ANALYSIS, HOWEVER, FINITE ELEMENT IS USED TO SOLVE EQUATIONS OF THE FORM

$$[M]\{\ddot{U}\} + [K]\{U\} = \{R\} \quad \text{------------------------------} (4)$$

WHERE

[M] IS THE MASS MATRIX

AND

{U} IS THE DISPLACEMENT VECTOR

$\{\ddot{U}\}$ IS THE SECOND DERIVATIVE WITH RESPECT TO TIME OF THE DISPLACEMENT VECTOR.

[K] IS THE STIFFNESS MATRIX (Ke OR Ke+Kg, WHEN APPLICABLE)

THERE IS NO APPARENT GUARANTEE THAT THE NATURAL FREQUENCIES OF VIBRATION FROM EQ. (4) ARE ACCURATE, WHEN [Kg] IS KNOWN TO BE IN-EXACT.

LARGE ROTATION EFFECTS

THE RIGID BODY ROTATION VECTOR PREVIOUSLY USED IS $C=\{-L\alpha/2,\alpha,L\alpha/2,\alpha\}$. FIG. 4 ILLUSTRATES RIGID BODY ROTATION OF A BEAM WITH AN AXIAL LOAD. (REF. 5)

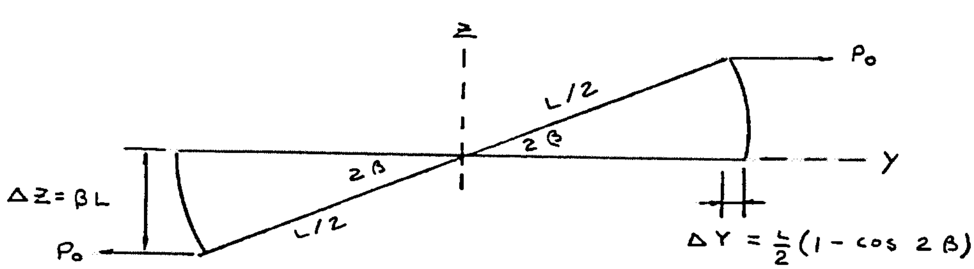

FIG. 4 RIGID BODY ROTATION OF A BEAM SUBJECTED TO AXIAL TENSION Po

BASED ON THE ABOVE $Q = \beta\{-L,2,L,2\}^T$.

CONSIDER THE WORK/ENERGY RELATIONSHIP OF FIG. 4

WORK DONE BY Po = PoL(1-COS 2β)

BUT, $(1-\cos 2\beta)/2 = \sin^2\beta \approx \beta^2 + \varnothing(\beta^4) + - - -$

THEREFORE, THE WORK DONE $= 2P_oL\beta^2 + \varnothing(\beta^4) = -V = Q^TKQ/2$,
WHERE $-V$ IS THE LOSS OF POTENTIAL ENERGY.

SIMILARLY

$$Q^TKQ/2 = P_o\beta^2/2 \, [-2 \;\; 0 \;\; 2 \;\; 0] \begin{Bmatrix} -\frac{L}{2} \\ \frac{L}{2} \end{Bmatrix} \; \frac{P_o\beta^2 \, [4L]}{2} = 2P_oL\beta^2$$

THEREFORE, MARTIN'S [Kg] PROVIDES A CORRECT ENERGY RELATIONSHIP FOR THE REPRESENTATION SHOWN.

IT SHOULD BE NOTED, HOWEVER, THAT THE DISPLACEMENT IN THE Y DIRECTION (AXIAL DIRECTION IN THE ORIGINAL GEOMETRY) HAS BEEN NEGLECTED WHEN WE LET

$$Q = \begin{Bmatrix} -\frac{L}{2} \\ \frac{L}{2} \end{Bmatrix} \quad \text{, WHICH IS REALLY} \quad \begin{Bmatrix} 0 \\ -\frac{L}{2} \\ 0 \\ \frac{L}{2} \end{Bmatrix} \quad \text{--------------(5)}$$

THE ZERO TERMS IN (5) NEGATE ANY CONTRIBUTION TO THE EQUATION FROM AXIAL TERMS IN THE STIFFNESS MATRIX. (IF THERE WERE ANY AXIAL TERMS). SINCE SIGNIFICANT AXIAL LOADING OCCURS IN THE SOLAR ARRAY (AS WELL AS OTHER BEAM/ COLUMN PROBLEMS), AND THESE AXIAL LOADS SIGNIFICANTLY AFFECT THE STIFFNESS, IT INTUITIVELY SEAMS UNREASONABLE TO ARBITRARILY NEGLECT THE CONTRIBUTION OF AXIAL STIFFNESS TERMS AND AXIAL DISPLACEMENTS.

THE EXACT RIGID BODY ROTATION VECTOR IS

$$Q_{exact} = \begin{Bmatrix} L/2 \, (1-\cos 2\beta) \\ -L/2 \, \sin \beta \\ 2\beta \\ -L/2 \, (1-\cos 2\beta) \\ L/2 \, \sin 2\beta \\ 2\beta \end{Bmatrix} \quad \text{--------------------------------(6}$$

SERIES EXPANSION, AND TRUNCATION OF HIGHER ORDER TERMS YIELDS

$$Q = \begin{Bmatrix} L\beta^2 \\ -L\beta \\ 2\beta \\ -L\beta^2 \\ L\beta \\ 2\beta \end{Bmatrix} \quad \text{--(7}$$

APPLICATION OF EQ.(7) TO [Kg] AND [Ke] YIELDS

$$Po\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6/5L & 1/10 & 0 & -6/5L & 1/10 \\ 0 & 1/10 & 2L/15 & 0 & -1/10 & -L/30 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6/5L & -1/10 & 0 & 6/5L & -1/10 \\ 0 & 1/10 & -L/30 & 0 & -1/10 & 2L/15 \end{bmatrix}\begin{bmatrix} L\beta^2 \\ -\beta L \\ 2\beta \\ -L\beta^2 \\ \beta L \\ 2\beta \end{bmatrix} = \begin{bmatrix} 0 \\ -Po(2\beta) \\ 0 \\ 0 \\ 0 \\ Po(2\beta) \end{bmatrix} \quad \text{-------(8)}$$

AND

$$\frac{EI}{L^3}\begin{bmatrix} AL^2/I & 0 & 0 & -AL^2/I & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -AL^2/I & 0 & 0 & AL^2/I & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} L\beta^2 \\ -L\beta \\ 2\beta \\ -L\beta^2 \\ L\beta \\ 2\beta \end{bmatrix} = \begin{bmatrix} 2AE\beta^2 \\ 0 \\ 0 \\ -2AE\beta^2 \\ 0 \\ 0 \end{bmatrix} \quad \text{-----(9)}$$

FROM (8) AND (9) WHEN THE MORE EXACT RIGID BODY ROTATION VECTOR IS USED, NEITHER [Ke] NOR [Kg] POSSESSES RIGID BODY ROTATION CAPABLITIES, ALTHOUGH FROM (8), $2AE\beta^2$ APPROACHES 0 AS HALF THE ANGLE OF ROTATION GETS VERY SMALL. A SIMILAR PROCEDURE SHOWS THAT BOTH POSSESS RIGID BODY TRANSLATION CAPABILITY IN TWO DIRECTIONS.

BUT, $(1-\cos 2\beta)/2 = \sin^2\beta \approx \beta^2 + \phi(\beta^4) + - - -$

THEREFORE, THE WORK DONE $= 2P_oL\beta^2 + \phi(\beta^4) = -V = Q^TKQ/2$,
WHERE $-V$ IS THE LOSS OF POTENTIAL ENERGY.

SIMILARLY

$$Q^TKQ/2 = P_o\beta^2/2 \; [-2 \;\; 0 \;\; 2 \;\; 0] \begin{Bmatrix} -\frac{L}{2} \\ \frac{L}{2} \end{Bmatrix} \cdot \frac{P_o\beta^2}{2}[4L] = 2P_oL\beta^2$$

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IT SHOULD BE NOTED, HOWEVER, THAT THE DISPLACEMENT IN THE Y DIRECTION (AXIAL DIRECTION IN THE ORIGINAL GEOMETRY) HAS BEEN NEGLECTED WHEN WE LET

$$Q = \begin{Bmatrix} -\frac{L}{2} \\ \frac{L}{2} \end{Bmatrix} \text{, WHICH IS REALLY } \begin{Bmatrix} 0 \\ -\frac{L}{2} \\ 0 \\ \frac{L}{2} \end{Bmatrix} \text{ -------------- (5)}$$

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$$Q_{exact} = \begin{Bmatrix} L/2\,(1-\cos 2\beta) \\ -L/2\,\sin \beta \\ 2\beta \\ -L/2\,(1-\cos 2\beta) \\ L/2\,\sin 2\beta \\ 2\beta \end{Bmatrix} \text{ -------------------------------- (6}$$

SERIES EXPANSION, AND TRUNCATION OF HIGHER ORDER TERMS YIELDS

$$Q = \begin{Bmatrix} L\beta^2 \\ -L\beta \\ 2\beta \\ -L\beta^2 \\ L\beta \\ 2\beta \end{Bmatrix} \text{ -- (7}$$

DEVELOPMENT OF MODIFIED [Kg]

USING RIGID BODY TRANSLATION RELATIONSHIPS, AND THE EXPANDED RIGID BOD ROTATION VECTOR, I CAN SOLVE FOR ADDITIONAL TERMS IN THE [Kg] MATRIX WHICH ENFORCES THE RIGID BODY CAPABILITIES.

USING RIGID BODY TRANSLATION CONSTRAINTS, [Kg] HAS THE FORM

$$Kg = Po \begin{bmatrix} A & B & C & -A & -B & C \\ B & 6/5L & 1/10 & -B & -6/5L & 1/10 \\ C & 1/10 & 2L/15 & -C & -1/10 & -L/30 \\ -A & -B & -C & A & B & -C \\ -B & -6/5L & -1/10 & B & 6/5L & -1/10 \\ C & 1/10 & -L/30 & -C & -1/10 & 2L/15 \end{bmatrix} \quad \text{---------------(10)}$$

MULTIPLYING EQ.(10) BY EQ.(7) AND SETTING THE PRODUCT EQUAL TO THE ZER VECTOR YIELDS

$$Po \begin{bmatrix} A & B & C & -A & -B & C \\ B & 6/5L & 1/10 & -B & -6/5L & 1/10 \\ C & 1/10 & 2L/15 & -C & -1/10 & -L/30 \\ -A & -B & -C & A & B & -C \\ -B & -6/5L & -1/10 & B & 6/5L & -1/10 \\ C & 1/10 & -L/30 & -C & -1/10 & 2L/15 \end{bmatrix} \begin{bmatrix} L\beta^2 \\ -\beta L \\ 2\beta \\ -L\beta^2 \\ \beta L \\ 2\beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{---(11}$$

EXPANDING ROW 3 YIELDS

$$Po\ [2CL\beta^2 - \beta L/5 + 4\beta L/15 - 2\beta L/30] = 0$$

$$Po2CL\beta^2 = 0$$

$$C=0 \quad \text{---(12)}$$

EXPANDING ROW 2 YIELDS

$$Po\ [2BL\beta^2 - 12\beta L/5 + 4\beta/10] = 0$$

$$2BL\beta - 2 = 0$$

$$B = 1/L\beta \quad \text{---(13)}$$

EXPANDING ROW 1 YIELDS

$$Po\ [2AL\beta^2 - 2] = 0$$

$$AL\beta^2 - 1 = 0$$

$$A = 1/L\beta^2 \quad \text{---(14)}$$

THUS, SUBSTITUTING EQ.(12), EQ.(13), AND EQ.(14) INTO EQ.(10) YIELDS

$$\mathbf{Kg} = \begin{bmatrix} 1/L\beta^2 & 1/L\beta & 0 & -1/L\beta^2 & -1/L\beta & 0 \\ 1/L\beta & 6/5L & 1/10 & -1/L\beta & -6/5L & 1/10 \\ 0 & 1/10 & 2L/15 & 0 & -1/10 & -L/30 \\ -1/L\beta^2 & -1/L\beta & 0 & 1/L\beta^2 & 1/L\beta & 0 \\ -1/L\beta & -6/5L & -1/10 & 1/L\beta & 6/5L & -1/10 \\ 0 & 1/10 & -L/30 & 0 & -1/10 & 2L/15 \end{bmatrix} \quad \text{----------(15)}$$

THE STRAIN ENERGY (U) SHOULD EQUAL ZERO. IT CAN BE CALCULATED FROM THE EQUATIO

$$\mathbf{U} = Po\beta^2/2\ [L\beta, -L, 2, -L\beta, L, 2]\ [\mathbf{Kg}] \begin{Bmatrix} L\beta \\ -L \\ 2 \\ -L\beta \\ L \\ 2 \end{Bmatrix} \quad \text{------------------------(16)}$$

PERFORMING THE MATRIX MULTIPLICATIONS IN EQ.(16) YIELDS $[0,0,0,0,0,0]^T$

THEREFORE, NO STRAIN ENERGY OCCURRS DURING RIGID BODY ROTATION USING [**Kg**].

INABILITY TO APPLY MODIFACTION PROCEDURES TO [Ke]

USING RIGID BODY TRANSLATION RELATIONSHIPS, AND THE EXPANDED RIGID BODY ROTATION VECTOR, AN ATTEMPT WAS MADE TO MODIFY [Ke].

[**Ke**] HAS THE FORM

$$\mathbf{Ke} = \frac{EI}{L^3} \begin{bmatrix} AL^2/I & K_{12} & K_{13} & -AL^2/I & -K_{12} & K_{16} \\ K_{12} & 12 & 6L & -K_{12} & -12 & 6L \\ K_{13} & 6L & 4L^2 & -K_{13} & -6L & 2L^2 \\ -AL^2/I & -K_{12} & -K_{13} & AL^2/I & K^{12} & -K_{16} \\ -K_{12} & -12 & -6L & K_{12} & 12 & -6L \\ K_{16} & 6L & 2L^2 & -K_{16} & -6L & 4L^2 \end{bmatrix} \quad \text{----------(17)}$$

MULTIPLYING EQ.(17) BY EQ.(7) AND SETTING THE PRODUCT EQUAL TO THE ZERO VECTO
YIELDS

$$\begin{bmatrix} AL^2/I & K_{12} & K_{13} & -AL^2/I & -K_{12} & K_{16} \\ K_{12} & 12 & 6L & -K_{12} & -12 & 6L \\ K_{13} & 6L & 4L^2 & -K_{13} & -6L & 2L^2 \\ -AL^2/I & -K_{12} & -K_{13} & AL^2/I & K_{12} & -K_{16} \\ -K_{12} & -12 & -6L & K_{12} & 12 & -6L \\ K_{16} & 6L & 2L^2 & -K_{16} & -6L & 4L^2 \end{bmatrix} \begin{bmatrix} L\beta^2 \\ -\beta L \\ 2\beta \\ -L\beta^2 \\ \beta L \\ 2\beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(18}$$

EXPANDING ROW 2 YIELDS

$$2K_{12}L\beta^2 - 24L\beta + 24L\beta = 0$$

$$2K_{12}L\beta^2 = 0 \qquad \text{------------------------------------(19)}$$

THEREFORE, K_{12} MUST EQUAL 0.

EXPANDING ROW 3 YIELDS

$$-2K_{13}L\beta^2 - 12L^2\beta + 8L^2\beta + 4L^2\beta = 0$$

$$-2K_{13}L\beta^2 = 0 \qquad \text{------------------------------------(20)}$$

THEREFORE, K_{13} MUST EQUAL 0.

IF K_{12} AND K_{13} EQUAL 0, NO ADDITIONAL COEFFICIENTS APPEAR IN LINE 1 OF THE [Ke] MATRIX.

SUPPOSE I ADD A CORRECTION TERM TO K_{11}.

$$(AL^2/I + C)L\beta^2 - (AL^2/I + C)(-L\beta^2) = 2(AL^2/I + C) \qquad \text{-------------(21)}$$

FROM EQ.(21) IT CAN BE SEEN THAT ADDING A CORRECTION TERM TO K_{11} INCREASES THE ERROR.

THUS, [Ke] CAN NOT BE MODIFIED TO OBTAIN RIGID BODY ROTATION CAPABILITIES FOR LARGE ROTATIONS UTILIZING THE PROCEDURE USED TO MODIFY [Kg]. THE ONLY POSSIBILITY FOR IMPROVING [Ke] MUST INCLUDE CORRECTIONS TO EXISTING TERMS.

CONCLUSIONS

CONTINUING EFFORT IS BEING MADE ON IMPROVING THE CAPABILITIES OF THE ELEMENT STIFFNESS MATRICES USED IN THE SOLAR ARRAY DYNAMIC ANALYSIS. THE MODIFIED [Kg] DEVELOPED REDUCES THE PSEUDO-FORCES PRODUCED IN THE BEAM-COLUMN STIFFENING PROBLEM, PROVIDED THAT THE ANGLE OF ROTATION IS LESS THAN TWO RADIANS. MODIFICATIONS MUST BE DEVELOPED, HOWEVER, TO ELIMINATE THE THE PSEUDO-FORCE CONTRIBUTIONS FROM [Ke] WHICH RESULT FROM UTILIZING THE EXPANDED RIGID BODY ROTATION VECTOR. THIS WOULD PERMIT THE TANGENTIAL STIFFNESS MATRIX [Kt] TO POSSESS THE THREE ZERO EIGENVALUES ASSOCIATED WITH RIGID BODY ROTATIONS OF ANY MAGNITUDE.

IT WAS DISAPPOINTING THAT THE [Kg] DEVELOPED DID NOT IMPROVE UPON THE RELATIVELY SLOW CONVERGENCE RATE OF THE STABILITY PROBLEM. FURTHER INVESTIGATION IS NEEDED TO DETERMINE WHETHER A MODIFIED [Ke] + [Kg] WOULD IMPROVE UPON THIS CONVERGENCE RATE.

EXTENSION OF THE MODIFICATIONS TO THE STIFFNESS MATRICES OF OTHER ELEMENTS, PARTICULARY PLATE ELEMENTS, WILL ALSO BE DEVELOPED.

FINALLY, TESTING OF THE PERFORMANCE OF THE MODIFIED MATRICES IN THE ACTUAL SOLAR ARRAY MODEL, AND COMPARISON WITH THE CONTINUUM MECHANICS APPROACH, WILL BE PERFORMED.

REFERENCES

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3 JOSEPH, J.A. ED.: MSC/NASTRAN APPLICATIONS MANUAL, THE MACNEAL-SCHWENDLER CORP., 1934

4 MARTIN, H.C.: ON THE DERIVATION OF STIFFNESS MATRICES FOR THE ANALYSIS OF LARGE DEFLECTION AND STABILITY PROBLEMS. AFFDL-TR-66-80

VERIFICATION OF MODIFIED [Kg]

SEVERAL TESTS OF THE MODIFIED STIFFNESS METHODS WERE UNDERTAKEN. SUMMARY OF THE TESTS FOLLOW.

a. RFORCE CHECK

ONE AND TWO ELEMENT BEAM STIFFNESS MATRICES WERE MULTIPLIED WITH THE EXPANDED RIGID BODY MATRIX. IN ALL CASES, NO PSEUDO-FORCES WERE PRODUCED FROM RIGID BODY TRANSLATION. THE EXPANDED RIGID BODY ROTATION VECTOR YIELDED THE FOLLOWING PSEUDO-FORCES.

MATRIX		PSEUDO-FORCES
[Ke]	ONE ELEMENT	$[2A\beta^2E,0,0,-2A\beta^2E,0,0]$
	TWO ELEMENTS	$[2A\beta^2E,0,0,0,0,0,0,-2A\beta^2E,0,0]$
[Kg]	ONE ELEMENT	$[0,-2\beta Po,0,0,2\beta Po,0]$
	TWO ELEMENTS	$[0,-2\beta Po,0,0,0,0,0,2\beta Po,0]$
[Kg]	ONE ELEMENT	$[0,0,0,0,0,0]$
	TWO ELEMENTS	$[0,0,0,0,0,0,0,0,0]$
[Ke+Kg]	ONE ELEMENT	$[2A\beta^2E,-2\beta Po,0,-2A\beta^2E,2\beta Po,0]$
	TWO ELEMENTS	$[2A\beta^2E,-2\beta Po,0,0,0,0,-2A\beta^2E,2\beta Po,0]$
[Ke+**Kg**]	ONE ELEMENT	$[2A\beta^2E,0,0,-2A\beta^2E,0,0]$
	TWO ELEMENTS	$[2A\beta^2E,0,0,0,0,0,-2A\beta^2E,0,0]$

BASED ON THE ABOVE, THE MODIFIED **[Kg]** MATRIX ELIMINATES THE β ERROR TERMS GENERATED USING [Kg] STANDARD. THE β^2 TERMS GENERATED FROM [Ke] REMAIN. THUS, WHEN β (HALF THE ANGLE OF ROTATION) IS LESS THAN ONE RADIAN, THE TOTAL ERROR IS REDUCED. THE ERROR IS ASSOCIATED WITH A PSEUDO-AXIAL FORCE ONLY.

b. EIGENVALUES

USING [Ke] AND [Kg] STANDARD, IT WAS FOUND THAT ONLY TWO ZERO EIGENVALUES EXIST. **[Kg]** HAS THREE ZERO EIGENVALUES WHICH CORRESPOND TO THE THREE RIGID BODY MODES.

[Ke + **Kg**] HAS TWO ZERO EIGENGVALUES, PLUS ONE EIGENVALUES WHICH IS VERY CLOSE TO ZERO. THE EIGENVALUES ARE CLOSE TO ZERO EVEN WHEN ONLY ONE OR TWO ELEMENTS ARE USED.

c. STABILITY ANALYSIS

[Kg] WAS USED IN THE SOLUTION OF A SIMPLY SUPPORTED BEAM SUBJECTED TO AN AXIAL LOAD. THE CRITICAL BUCKLING LOAD WAS CALCULATED, AND COMPARED WITH THE TRADITIONAL SOLUTION USING MARTIN'S [Kg], AS WELL AS THE EXACT SOLUTION.

WHEN THE BOUNDARY CONDITION, β_1 EQUALS $-\beta_2$, AS OCCURS DURING BUCKLING, WAS APPLIED, THE SOLUTION USING