AN APPLICATION OF NEW TECHNIQUES FOR INTEGRATING ANALYTICAL AND EXPERIMENTAL STRUCTURAL DYNAMIC MODELS

Dr. John C. O'Callahan, Peter Avitabile, Robert Riemer

Modal Analysis and Controls Laboratory

University of Lowell

One University Avenue

Lowell, Massachusetts 01854

ABSTRACT

Several techniques are presented which are used in a variety of ways to integrate analytical and experimental modal data bases. These techniques span the range of correlation of analytical and experimental modal data, estimation of rotational degrees of freedom for experimental modal data to be used in system modelling and modification studies, analytical model improvement based on the measured modal vectors, and nonlinear response techniques using reduced models. General theoretical background is presented for the techniques identified. A simple structure is used for demonstration of the integration of the analytical and experimental modal data bases.

INTRODUCTION

For many years, finite element modelling (FEM) has been used for the characterization of dynamic systems. The FEM process has been very successful in the early design stages of the development of prototypes and allows for optimization of specific design parameters. Thus, before hardware is manufactured, the majority of the desired design characteristics can be included and tuned to some degree. Once a design has been finalized, a prototype can be manufactured and tested to verify that the desired characteristics have been obtained. Experimental modal analysis (EMA) techniques have become very popular in recent years due to the affordability of modal test systems and ease of use of their associated software packages. However, FEM and EMA have rarely been used to complement each other in an effort to maximize the benefits inherent in each technique. Over the past several years, a tremendous amount of effort has been devoted to the development, verification, and improvement of component and system modal data bases.

Generally, an analytical model of a complicated component or system is available for early design studies. This model is generally developed from FEM techniques such as those of MSC/NASTRAN. The FEM will almost always contain many more degrees of freedom (dof) than are necessary for a description of the dynamic behavior of the component due to the discretization necessary for the FEM process to produce accurate results. With so many dofs available, any correlation with an EMA with a limited set of dofs is extremely difficult. The accurate reduction of a FEM to the set of test dof is of paramount importance.

For most model reductions, Guyan condensation has been widely used and accepted. However, this reduction technique was generated mainly as an eigensolution economizer and not as a general mapping transformation between the full set of analytical dofs and the reduced set of test dofs. Since Guyan reduction is a static condensation technique, the reduced mass matrix may be in serious error and may be of little use for cross orthogonality checks between the analytical and experimental modal vectors. A newer technique referred to as the System Equivalent Reduction Expansion Process (SEREP) (1) allows for an exact mapping between the full analytical model set of dof and the reduced set of experimental dof for an arbitrary set of modes and an arbitrary set of dof. This salient feature of SEREP allows it to be used as a mapping between the analytical and experimental modal models.

The experimental modal models are extremely valuable in that they contain estimates of the real world structure under consideration. However, their use is limited in that generally the EMA models do not contain rotational dofs

(Rdof). These Rdof are critical to the success of structural dynamic modification (2) and component system modelling studies. Without the Rdof, the use of the experimental models is limited. The SEREP process can be used not only as a reduction of the analytical models to a reduced set of dof but also as an expansion process to estimate the Rdof as well as the unmeasured translation dof (Tdof) (3,4).

Once the analytical model has been reduced to the set of test dof or the experimental model expanded to the full set of analytical dof, correlation and orthogonality studies (5) of the modal vectors can be accomplished using the analytical mass matrix. Since the mass matrix is reduced using the SEREP process which preserves the selected modes at the selected dofs, the mass matrix and analytical modal vectors are consistently related. This is not true of other model reduction techniques such as Guyan condensation where the inertia properties of the system are not preserved in the reduction process.

Generally, there is never perfect correlation between the analytical and experimental modal data bases. Providing that a good experimental modal survey has been performed, then the experimental modal vectors are expected to accurately depict the true system modal character and can be used to update the analytical mass and stiffness matrices. The analytical model improvement (AMI) process (6) will produce updated component mass and stiffness matrices which reflect the same frequencies and mode shapes observed during test. It is important to note that the improvement of these system matrices can be performed at either the reduced set of test dof or at the full set of analytical dof.

Now system models can be developed using either the expanded experimental component modal models or the improved component physical mass and stiffness matrices (7). Both models can be used to study system model characteristics due to the combination of component models. Once a system model is developed, linear response studies can be performed to determine the response due to applied loads.

However, the physical improved system models offer an advantage over the modal models in that forced response studies need not be limited to linear response. Since physical equations are available, discrete nonlinear springs and dashpots can be easily incorporated into the system model and used to predict nonlinear response (8,9) and direct integration of the equations of motion can be performed.

The processing of data discussed above is displayed in the schematic shown in Figure 1. The theory of each of these techniques is briefly discussed in the next section followed by application of these techniques to a simple structure comprised of two components.

OVERVIEW OF STRUCTURAL DYNAMIC MODELLING PROCESS

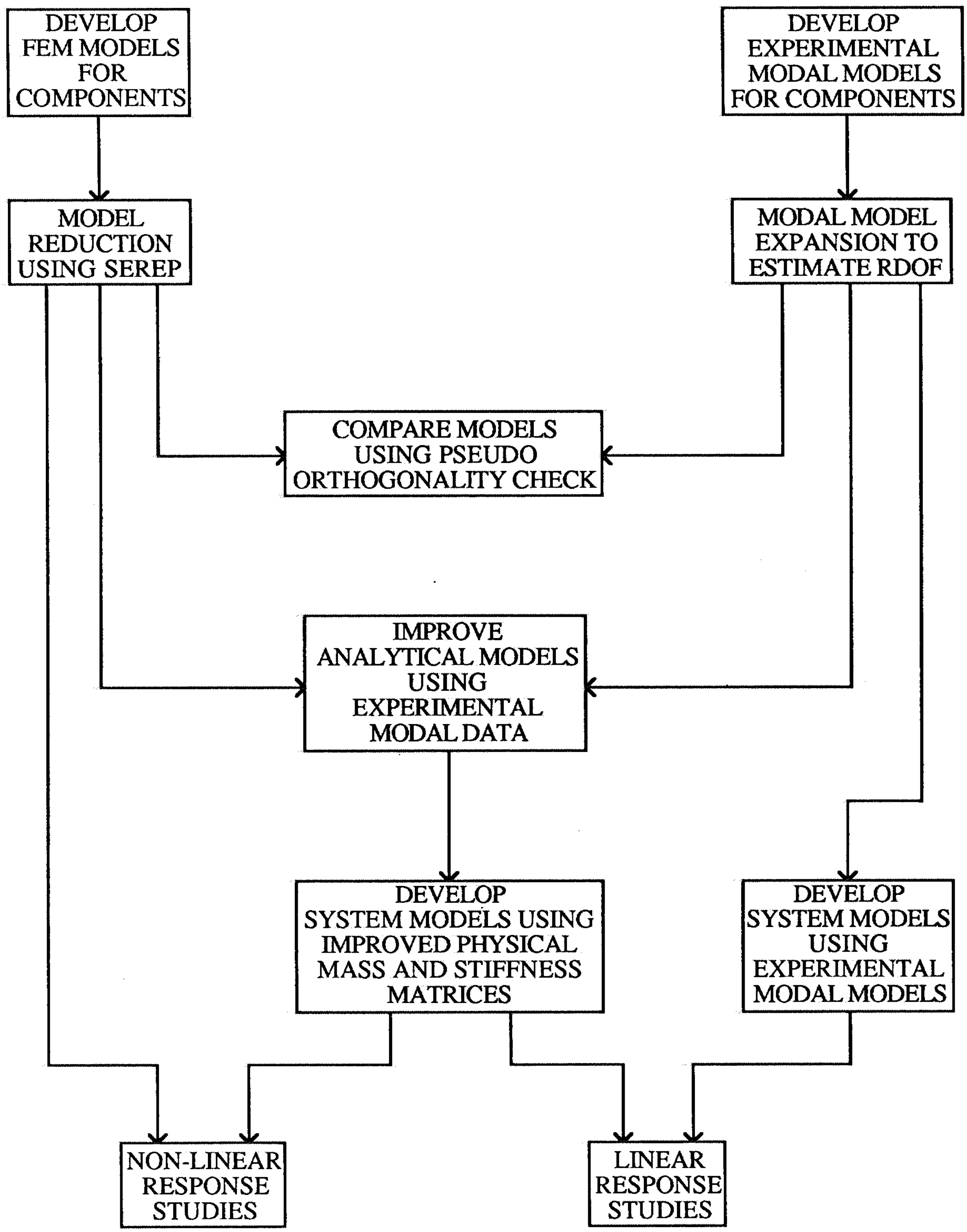


FIGURE 1

THEORY

There are three relatively new items that are used for the development of all the component and system models generated. They are the model reduction using SEREP, the model expansion to estimate Rdofs and the improvement of the analytical mass and stiffness matrices using experimental modal vectors. Other items such as development of analytical and experimental modal models, cross orthogonality of analytical and experimental modal vectors, development of system models from modal models, linear response analysis using physical or modal equations, and nonlinear response through direct integration of the equations of motion are well documented in the literature and are not discussed herein.

System Equivalent Reduction Expansion Process

The SEREP process relies on a finite element or analytical model, from which an eigensolution is performed to develop a mapping from the full set of analytical dofs to the reduced set of active dofs. Detailed information on the SEREP process is presented in reference 1. In essence, the modal vectors of the full system model can be partitioned into a set of active (master) dof (Adof) and a set of embedded (deleted) dof. The mapping of the Adof relative to the modal coordinate can be formed using a generalized inverse and can be written as

$$\{P\} = [U_a]^g \{X_a\} \tag{1}$$

where $[U_a]^g$ is the generalized inverse of $[U_a]$.

The global mapping transformation matrix, which relates the reduced set of master dof (Adof) to the full set of analytical dof (Ndof), can be written as

$$[T_u] = [U_n] [U_a]^g \tag{2}$$

and is used to form the reduced mass and stiffness matrices

$$[M_a] = [T_u]^T [M_n] [T_u] \tag{3}$$

$$[K_a] = [T_u]^T [K_n] [T_u] \tag{4}$$

where $[M_a]$ and $[K_a]$ are the equivalently reduced mass and stiffness matrices, respectively; the subscript 'a' denotes that these matrices are associated with the Adof.

These reduced matrices are similar in form and function to those obtained from Guyan reduction. However, the difference lies in the fact that the generalized inverse formulation carries information pertaining to the selected modes of the analytical system and therefore, the reduction produces an equivalent set of reduced matrices whose eigensolution yields the same selected frequencies. The mode shapes, expanded back to the full set of analytical dofs, are identical to the selected modes used to form the $[T_u]$ matrix. This is not true of the Guyan reduction technique.

Estimation of Rotational Degrees of Freedom

The estimation of Rdof, as well as the unmeasured Tdof, relies heavily on the SEREP process and is discussed in Reference 3 and 4. The $[T_u]$ is used to expand the reduced variable back to the full set of Ndof from

$$\{X_n\} = [T_u]\ \{X_a\} \qquad (5)$$

Using this global mapping matrix, $[T_u]$, from the SEREP process, the experimentally measured modal vectors can be expanded to the full set of 'n' analytical dof using

$$[E_n] = [T_u]\ [E_a] \qquad (6)$$

where [E] denotes the experimental modal vectors.

The main advantage of this estimation technique is that all the system dof for all selected modes can be expanded simultaneously. Once the expansion matrix, $[T_u]$, is applied to the experimental modal component, the component offers a full complement of dof.

Analytical Model Improvement

The analytical system matrices can be improved using the measured modal vectors with a generalized inverse technique. Details of this theoretical development are presented in reference 6. Various optimization techniques can developed depending on which system parameters are chosen as reference for starting the process. Herein, it is assumed that mass is the reference and the system mass matrix can be optimized either at the full or reduced set of dofs.

The equations for improvement of the mass and stiffness matrices using the measured modal vectors are summarized as

$$[M]^I = [M] + [V]^{gT} [[I] - [E]^T[M][E]] [V]^g \quad (7)$$

$$[K]^I = [K] + [V]^{gT} [[W]^2 + [E]^T[K][E]] [V]^g - [K][E][V]^g - [[K][E][V]^g]^T \quad (8)$$

where

$[M]^I$ - improved mass matrix

$[M]$ - original analytical mass matrix

$[K]^I$ - improved stiffness matrix

$[K]$ - reduced analytical stiffness matrix

$[W]^2$ - omega squared (diagonal)

$[E]$ - set of experimental modal vectors

$[V]^g$ - generalized inverse

$[V]^g = ([E]^T [M] [E])^{-1} [E]^T [M]$

An eigensolution of the improved mass and stiffness matrices will yield the same measured frequencies and modal vectors used for the improvement process along with the other analytical modes that were not adjusted during the improvement process.

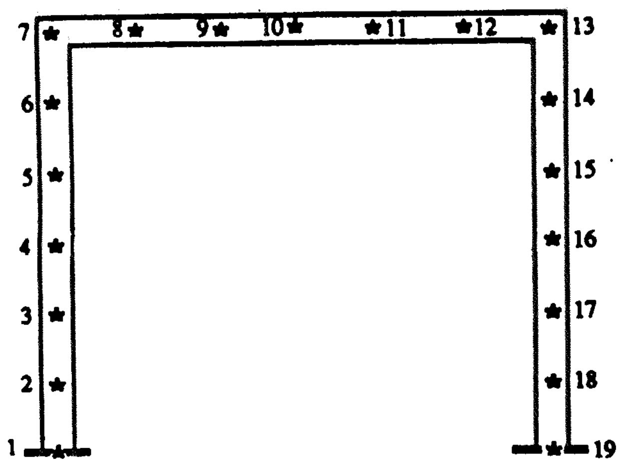
APPLICATION

The general design process shown in Figure 1 was applied to a planar frame structure for the components used to develop the system model of the frame assembly and determine forced response. The frame assembly is shown in Figure 2 along with its components, the shell and beam. All the details of this frame structure and its components are discussed in detail in Reference 10 and used as an example for seminar lecture notes in Reference 9. Unless noted otherwise, all matrix manipulations were performed using MAT_SAP/MATRIX (11) with data obtained from the finite element model and experimental modal test data bases. However, all matrix manipulations could easily have been implemented using the DMAP facilities of MSC/NASTRAN.

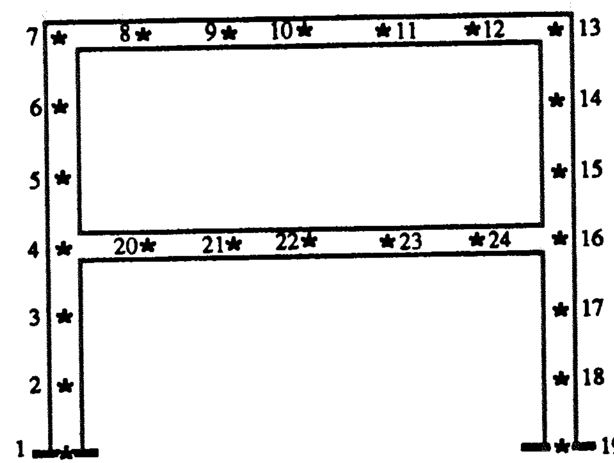
Analytical Models for Individual Components

All of the analytical component models were developed using standard finite element beam elements such as those available in MSC/NASTRAN in order to obtain frequencies and mode shapes for the components. The individual models are shown in Figure 2 and are described below.

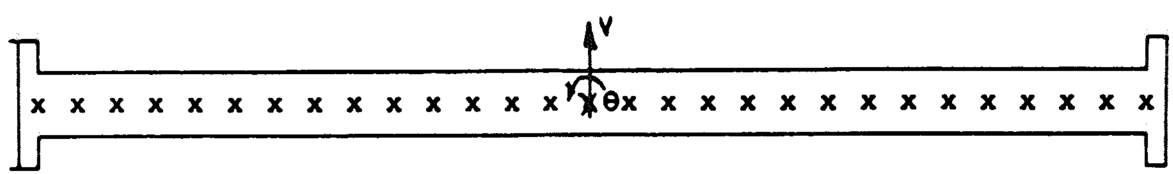
The frame was modelled in both the free-free and built-in states. The mass and stiffness matrices were assembled using



SHELL FEM MODEL



FRAME FEM MODEL



BEAM FEM MODEL

FIGURE 2

using 24 elements and 24 nodes with 3 dof per node, as shown. For the free-free case, the full 72 dof mass and stiffness matrices were used to extract eigenvalues and eigenvectors. For the built in case, stiff boundary springs were applied at nodes 1 and 19.

The shell was modelled in both the free-free and built-in states. The mass and stiffness matrices were assembled using 18 elements and 19 nodes with 3 dof per node, as shown. For the free-free case, the full 57 dof mass and stiffness matrices were used to extract eigenvalues and eigenvectors. For the built-in case, stiff boundary springs were applied at nodes 1 and 19.

The beam was modeled in the free-free state only. The mass and stiffness matrices were assembled using 28 elements and 29 nodes with 3 dof per node, as shown. The full 87 dof mass and stiffness matrices were used to extract the eigenvalues and eigenvectors.

SEREP Process

In order to illustrate the SEREP process, the frame analytical model (with the ends built in) was reduced to preserve arbitrarily selected sets of modes at arbitrarily selected sets of dofs. The results are shown in Table 1.

Comparison of Frequencies for Arbitrary Selection of DOF

and Arbitrary Selection of Modes

Mode	Ref.	Model 2	Model 3	Model 4	Model 5	Model 6
1	32.8	32.8			32.8	
2	109.5	109.5	109.5			109.5
3	116.4	116.4		116.4	116.4	
4	129.7	129.7	129.7	129.7		129.7
5	310.0	310.0			310.0	310.0
6	355.6		355.6			355.6
7	458.7			458.7		
8	580.0		580.0			
9	610.9			610.9		
10	701.0		701.0			
11	765.8					
12	802.7			802.7		

Model 2 – Modes : 1,2,3,4,5
ADOF : 2X,4X,6X,8Y,10Y,12Y,14X,16X,18X,20Y,21Y,23Y

Model 3 – Modes : 2,4,6,8,10
ADOF : 2X,4X,6X,8Y,10Y,12Y,14X,16X,18X,20Y,21Y,23Y

Model 4 – Modes : 3,4,7,9,12
ADOF : 2X,4X,6X,8Y,10Y,12Y,14X,16X,18X,20Y,21Y,23Y

Model 5 – Modes : 1,3,5
ADOF : 14X,15X,16X,17X,18X,24Y

Model 6 – Modes : 2,4,5,6
ADOF : 8Y,9Y,11Y,20Y,21Y,23Y

TABLE 1

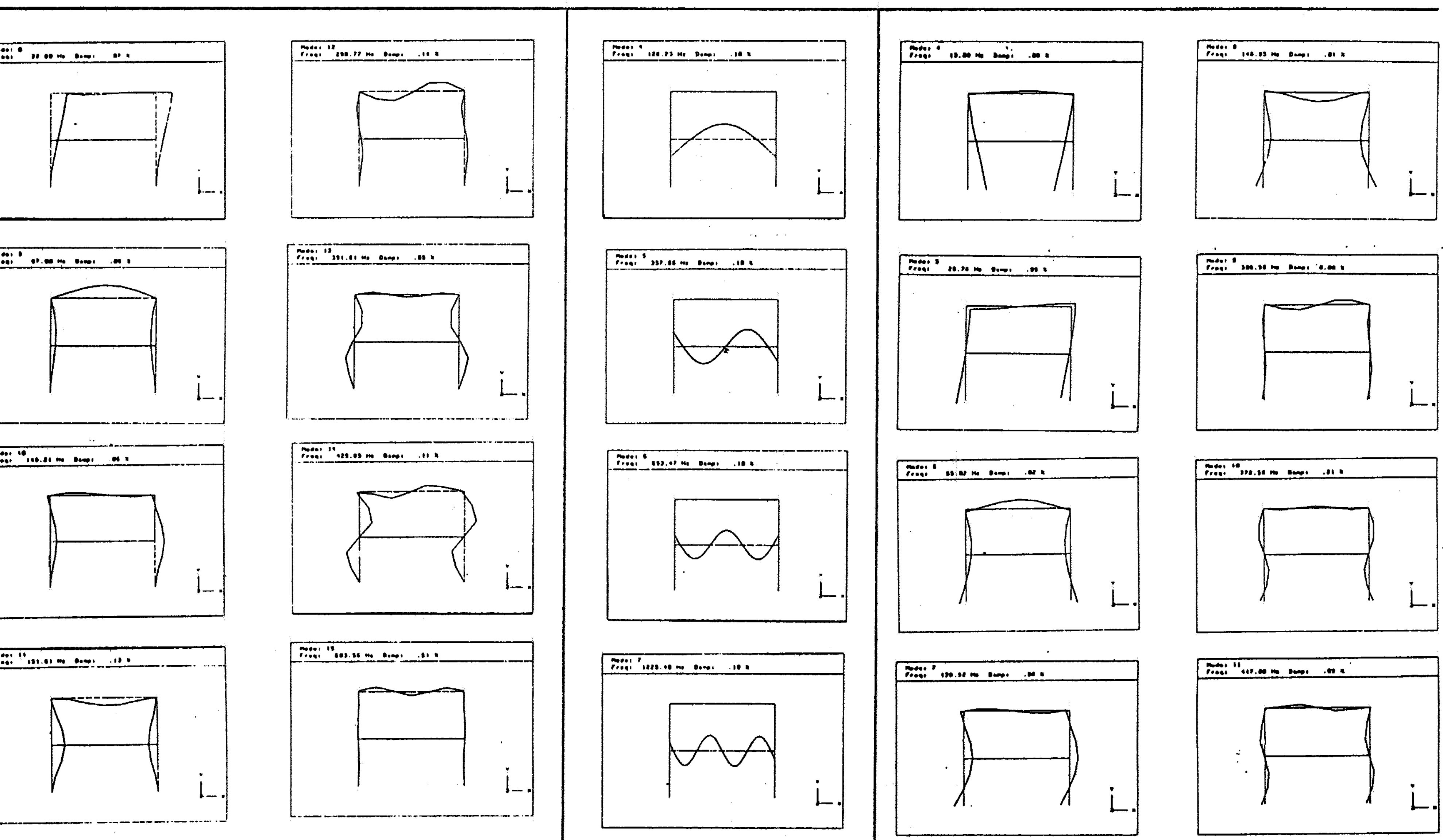
The models reduced using the SEREP process preserved all the specified modes at the selected sets of dofs. No other reduction technique can offer this unique ability.

Experimental Modal Models for Individual Components

Experimental modal models were developed for the beam and shell as well as for the assembled frame structure. Tests were performed using burst random and burst sine chirp excitations with the components and assembled frame in both the free free and built-in conditions. Modal models were developed using SMS MODAL 3.0SE modal software package (12). Typical mode shapes of the unassembled components as they appear in the frame structure are shown in Figure 3.

Pseudo Orthogonality Check Using SEREP

In order to illustrate the advantages of a reduced mass matrix developed using SEREP as opposed to Guyan condensation, the results of one test case for the frame structure with the ends built in (from Reference 5) is included herein. Using simulated experimental data which was only slightly perturbed from the full FEM solution, a pseudo orthogonality check of the analytical and experimental modal vectors with a reduced mass matrix was performed.



SHELL BUILT IN SHAPES

BEAM FREE-FREE SHAPES

SHELL FREE-FREE SHAPES

EXPERIMENTAL MODAL DATA
FIGURE 3

For the modal data, it was assumed that data was only acquired at every other point on the structure on the outer shell of the frame and forms the basis for an incomplete set of measured modal data. The pseudo orthogonality check results are shown in Table 2 using a Guyan and SEREP reduced mass matrix. The results clearly show the superiority of SEREP over Guyan.

Detailed evaluation using the pseudo orthogonality check was performed for all data bases as described in Reference 14. Due to space restrictions, this data is not presented herein.

Estimation of Rotational DOF for Experimental Modal Components and Improvement of Analytical Mass and Stiffness Matrices Using Measured Modal Data

Using the analytical models, the expansion matrix $[T_u]$ was formulated and used to expand the Rdof (as well as the unmeasured Tdof) for the beam and shell component. The shell experimental modal component was expanded to a full set of 57 dof from the 38 measured dof for the 8 measured flexural modes; expansion was preformed for both the free free and built-in models. The beam experimental modal component was expanded to a full set of 87 dof from the 15 measured dof for the 3 measured free free flexural modes.

$[E_a]^T \; [M_a]^{GR} \; [U_a]$ Using Guyan Reduced Mass Matrix

1.000	0.000	-0.001	0.000	-0.020	0.053	0.000	0.018
0.000	0.863	0.000	-0.337	0.000	0.000	-0.135	0.000
-0.001	0.000	0.994	0.000	-0.002	-0.022	0.000	0.009
0.000	0.337	0.000	-0.154	0.000	0.000	0.122	0.000
-0.020	0.000	-0.002	0.000	0.698	0.307	0.000	0.202
0.053	0.000	-0.022	0.000	0.307	0.247	0.000	-0.167
0.000	0.136	0.000	0.122	0.000	0.000	-1.513	0.000
0.018	0.000	0.009	0.000	0.202	-0.168	0.000	0.721

$[E_a]^T \; [M_a] \; [U_a]$ Using Equivalent Reduced Mass Matrix

1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	-1.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	-0.001	1.002	0.000	0.001
0.000	0.025	0.000	0.060	0.000	0.000	-0.993	0.000
0.000	0.000	0.000	0.000	0.001	-0.003	0.000	1.001

Cross Orthogonality using Guyan Reduced Mass

and Pseudo Orthogonality Check

for Model without Cross Member at the Reduced Set of Test DOF

TABLE 2

Using these expanded experimental modal vectors, the original component analytical mass and stiffness matrices were updated using the analytical model improvement technique described. For reference, an eigensolution was performed for these updated models. The resulting frequencies are shown in Table 3 with the original analytical and experimental frequencies. As expected, the frequencies of the improved model reflect the same frequencies observed from test.

It is important to note that the boundary conditions of the FEM shell model (with built-in boundary) are likely to be the main point of discrepancy between the analytical and experimental data bases. Both the modal data and improved matrices reflect the actual boundary conditions observed in the experimental modal data.

System Model Development Using Expanded Experimental Modal Components

System models of the frame (built-in and free free) were assembled using expanded experimental modal components of the shell and beam. Models were developed using SMS SDM modal software package (13). In both system models, the beam and shell were tied together with translational stiffnesses of 1E5 lb/in and rotational stiffness of 1E6 lb/in.

	Analytical Model	Experimental Modal Model	Improved Physical Model
	25.2	22.8	22.8
	92.2	87.0	87.0
	164.2	149.2	149.2
SHELL	173.0	151.6	151.6
(built in)	324.0	298.8	298.8
	441.1	391.6	391.6
	497.8	429.9	429.9
	673.2	603.6	603.6
	907.9		907.9
	911.6		911.6
	13.9	13.1	13.1
	31.4	28.8	28.8
	98.0	95.0	95.0
SHELL	147.3	139.5	139.5
(free free)	165.1	149.0	149.0
	328.9	310.0	310.0
	401.3	372.6	372.6
	467.0	417.9	417.9
	696.1		696.1
	845.5		845.6
	128.4	127.8	127.8
BEAM	366.3	357.6	357.6
(free free)	733.8	692.4	692.4
	1229.4	1103.9	1103.9

COMPARISON OF ANALYTICAL, EXPERIMENTAL AND

IMPROVED ANALYTICAL COMPONENT MODELS

TABLE 3

The beam component consisted of 3 rigid body modes and 4 expanded experimental flexural modes. The shell component consisted of 8 expanded, experimental flexural modes for the built in and free free models; 3 rigid body modes were included for the free free component. The results of the built in and free free frame assembled system models are shown in Table 4 in column 2.

System Model Development Using Improved Component Physical Mass and Stiffness Matrices

System models of the frame (builtin and free free) were assembled (as in the finite element assembly process) using the improved component mass and stiffness matrices. The mass and stiffness matrices of the shell and beam were improved using the 8 measured flexural modes of the shell and the 3 measured flexural modes of the beam as stated previously.

An eigensolution of the two assembled improved component superelements was performed. Results of the free free and built in frame assembled system models are shown in Table 4 in column 3.

	Assembled Frame Test Data	Expanded Experimental Modal Model	Improved M & K Model
	27.5	28.1	28.4
B	97.7	100.7	97.4
U	100.5	101.5	100.9
I	119.3	121.2	125.3
L	288.7	287.9	291.3
T	323.6	324.4	339.2
	409.2	415.9	414.8
I	485.3	501.6	515.6
N	532.3	528.6	555.0
		611.3	664.9
	49.8	47.4	49.2
F	60.3	57.9	57.9
R	107.4	107.4	106.4
E	107.7	109.5	110.2
E	145.2	152.9	156.5
	290.8	290.1	296.9
F	356.0	352.9	366.5
R	403.8	394.8	393.7
E	477.4	487.1	489.9
E	514.7	545.6	529.5
	631.7	643.4	677.8
	663.9	735.8	755.9

COMPARISON OF SYSTEM MODELS DEVELOPED USING

EXPANDED, EXPERIMENTAL MODAL MODELS

AND

IMPROVED ANALYTICAL MASS AND STIFFNESS MATRICES

TABLE 4

Comparison of System Model Results

Both techniques provide very good results when compared to the actual assembled frame test results in Table 4 in column 1. Note that these models could not have been developed without SEREP for the expansion of the Rdof for the experimental modal data.

It is important to note that the model developed from improved component physical mass and stiffness matrices is generally stiffer at the component connection point than the model developed from the expanded experimental modal components due to the nature of the finite element assembly process. Relaxing this condition to include a more realistic joint stiffness would yield closer correlation with the actual tested results.

Forced Response With Nonlinear Spring/Gap

Using the physical mass and stiffness matrices of the built in frame structure, a pulse was applied at the middle of the lower cross piece of the frame in the vertical direction. The pulse consisted of a symmetric $\pm$100 lb triangular pulse acting over 0.006 seconds. A central difference scheme was used for the direct integration of the equations of motion. This integration scheme was developed as a user defined subroutine and linked to the MAT_SAP / MATRIX analysis program. The time step was 0.00001 sec and 10000 time steps were evaluated for a total time of 0.1 seconds of response.

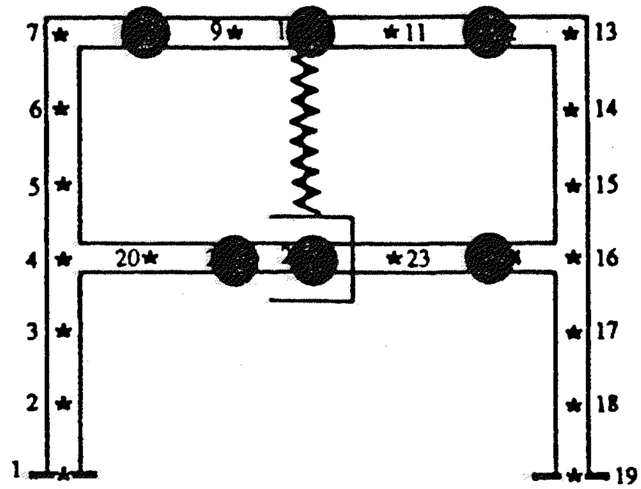
The frame model was reduced to six dofs at 8Y, 10Y, 12Y, 21Y, 22Y and 24Y and was subjected to the pulse applied at node 22Y. However, a nonlinearity was introduced into the model. For this study, a nonlinear gap/spring element was added across nodes 10Y and 22Y. When the response was such that the relative displacement of nodes 10Y and 22Y was greater than or equal to 0.01 inch, a 50000 lb/inch spring was engaged. This nonlinear model would generally require a transient dynamic analysis with many physical dofs in order to properly characterize the actual system response.

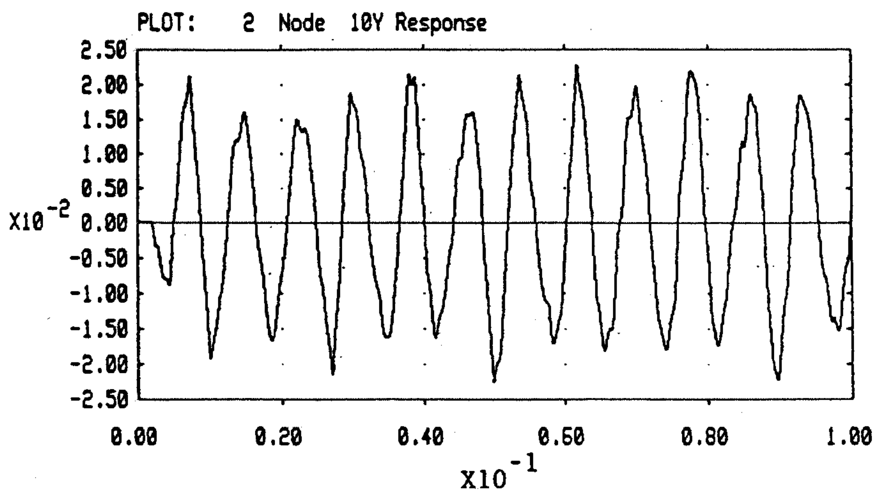
Guyan reduction of a model of this type to a smaller set of dofs would not produce accurate results. However, the SEREP reduction process preserves the modes which are expected to participate in the response at the specified set of dofs. The reduced SEREP model will produce accurate results providing that a sufficient set of proper modes are included in the reduction process. While this study was performed on the original analytical model, the model could have been the system model developed from the improved component mass and stiffness matrices. Many types of different discrete non-linearities could also have been evaluated (ie, gap, spring/dashpot, bilinear elements, etc.) using the reduced system mass and stiffness matrices.

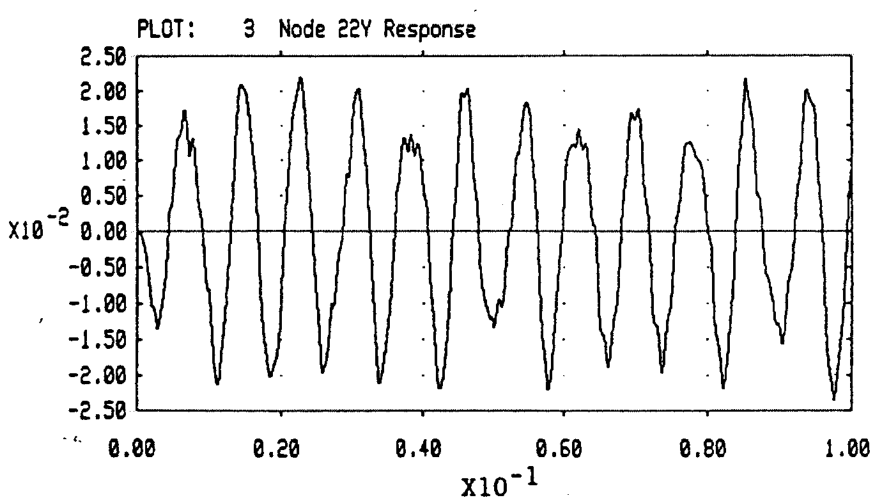
The modes that were considered critical to the response were modes 2,4,7,9,10,12. Using this collection of modes for the specified set of 6 dofs, the forced response results produced extremely good results when compared with the solution for the full model. Response is shown in Figure 4.

The solution time for the SEREP reduced system matrices was several orders of magnitude less than that of the full 72 dof physical model. The accuracy of the results clearly indicate that the SEREP reduction process preserved sufficient system dynamics to properly characterize the system.

It is extremely important to note again that the selection of dofs is arbitary and that the selction of modes to be included is arbitrary. However, the proper modes for the system response must be included but <u>all</u> of the modes up to the maximum frequency of concern need not be included in the reduced system matrices. It is this important fact that separates the SEREP process from more traditional condensation techniques.







NONLINEAR RESPONSE MODEL RESULTS

FIGURE 4

SUMMARY

Several techniques were presented which are used in a variety of ways that integrate the data bases obtained from analytical and experimental sources. These techniques cover a wide range of modal analysis and structural dynamic analysis topics. Model reduction using the SEREP process will exactly preserve the full system dynamics in the reduced model for an arbitrary set of modes at an arbitrary set of dofs. Estimation of rotational dof for an experimental modal data base can also be accomplished using the SEREP process. This enables the full utilization of experimental modal data in system modelling studies. Pseudo Orthogonality Checks between analytical and experimental modal vectors and an analytical mass matrix can be accomplished without contamination of the mass matrix at a reduced set of test dof. Techniques for improvement of the analytical mass and stiffness matrices based on experimentally measured modal vectors were presented. Development of system models using either expanded, experimental modal models or using improved physical component mass and stiffness matrices was presented. Nonlinear response using dramatically reduced analytical models or using system models developed from improved physical component mass and stiffness matrices was presented.

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12) SMS MODAL 3.0SE - Modal Analysis System software program, Structural Measurement Systems, Inc., San Jose, Calif.

13) SMS SDM - Structural Dynamic Modification software program, Structural Measurement Systems, Inc., San Jose, Calif.

14) Study of Model Correlation Techniques, University of Lowell, MACL Report 87001, December 1987