ON SHELL ELEMENTAL FORCE/MOMENT IN NONLINEAR ANALYSIS

C.H.S. CHEN

The BFGoodrich Company

ABSTRACT

An error is detected in the elemental force and moment output of the shell element (QUAD4,TRIA3) in nonlinear static analysis (SOL 66) with antisymmetric or nonsymmetric laminate. The error appears to be due to the neglecting of the membrane-bending coupling terms. A quick simple method of remedy is presented. The linear analysis (SOL 24) is free of this error.

INTRODUCTION

In both linear and nonlinear analyses using MSC/NASTRAN, the shell element forces and moments can be requested for output. The forces and moments are very usefull engineering information. Together with the deformed shape, they provide an insight to the structural response and can be used to check the validity of the model and the severity of the service condition. They are also often used in the post-processing analysis such as the local analysis.

For the shell structure, the force and moment are expressed in the following stiffness equations

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \begin{Bmatrix} \epsilon_m \\ \chi \end{Bmatrix} \qquad (1)$$

where $\{N\} = [N_x \;\; N_y \;\; N_{xy}]$ = in-plane normal and shear forces

$\{M\} = [M_x \;\; M_y \;\; M_{xy}]$ = bending and twisting moments

$\{\epsilon_m\} = [\epsilon_{x,m} \;\; \epsilon_{y,m} \;\; \epsilon_{xy,m}]$ = mid-plane strains

$\{\chi\} = [\chi_x \;\; \chi_y \;\; \chi_{xy}]$ = curvatures

[A] = membrane rigidity
[D] = bending rigidity
[B] = membrane-bending coupling rigidity

and $\{\;\} = [\;]^T$ = matrix transposed

In NASTRAN's notation, $A=TG_1$, $B=T^2G_4$, $D=IG_2$ where T is the total thickness of the laminate and I is the moment of inertia of the laminate cross section. If the laminate is single-layered or symmetric, then [B]=0. [B] is non-zero if the laminate is antisymmetric or nonsymmetric.

Equations (1) can be inverted as

$$\begin{bmatrix} \epsilon_m \\ \chi \end{bmatrix} = \begin{bmatrix} A^* & B^* \\ B^{*T} & D^* \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} \qquad (2)$$

Equation (2) is a convenient starting point for obtaining ply-to-ply information. The analysis output of the element forces and moments as well as the strains and curvatures should satisfy both the equations (1) and (2). Specifically, the substitution of the NASTRAN strains and curvatures output in the right hand side of equation (1) and the substitution of the NASTRAN forces and moments output in the right hand side of equation (2) should yield, repectively, the forces and moments, and the strains and curvatures consistent with the NASTRAN output values. In the following, the NASTRAN outputs will be subjected to this test.

TEST PROBLEM 1

The first problem chosen for the test is the internally pressurized thin spherical shell of a 6-ply laminate with 90°/0°/90°/0°/90°/0° fiber orientation. The characteristics of this problem is the uniform expansion of the spherical shell and zero curvature associated with this deformation. The lamina has the following fictitious orthotropic stress(σ)-strain(ϵ) relations

$$[\sigma] = \begin{bmatrix} 1E{+}6 & 1000 & 0 \\ 1000 & 2000 & 0 \\ 0 & 0 & 500 \end{bmatrix} [\epsilon] \qquad (3)$$

The lamina is 0.1 in. thick, thus T=0.6 in. and I=0.018 in^4. for the laminate.

The matrices of the laminate elastic constants are:

$$[G_1]=[G_2]= \begin{bmatrix} 5.01E{+}5 & 1000 & 0 \\ 1000 & 5.01E{+}5 & 0 \\ 0 & 0 & 500 \end{bmatrix} \qquad [G_4]= \begin{bmatrix} -41583 & 0 & 0 \\ 0 & 41583 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (4)$$

The laminate stiffness matrices are then:

$$[A]= \begin{bmatrix} 300600 & 600 & 0 \\ 600 & 300600 & 0 \\ 0 & 0 & 300 \end{bmatrix} \qquad [B]= \begin{bmatrix} -14969.88 & 0 & 0 \\ 0 & 14969.88 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[D]= \begin{bmatrix} 9018 & 18 & 0 \\ 18 & 9018 & 0 \\ 0 & 0 & 9 \end{bmatrix} \qquad (5)$$

The laminate compliance matrices are then:

$$[A^*] = \begin{bmatrix} 3.6265E\text{-}6 & -7.2385E\text{-}9 & 0 \\ -7.2385E\text{-}9 & 3.6265E\text{-}6 & 0 \\ 0 & 0 & 3.3333E\text{-}3 \end{bmatrix}$$

$$[B^*] = \begin{bmatrix} 6.01997E\text{-}6 & 0 & 0 \\ 0 & -6.01997E\text{-}6 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (6)$$

$$[D^*] = \begin{bmatrix} 1.20883E\text{-}4 & -2.41283E\text{-}7 & 0 \\ -2.41283E\text{-}7 & 1.20883E\text{-}4 & 0 \\ 0 & 0 & 0.11111 \end{bmatrix}$$

The spherical shell is modeled by one QUAD4 element or one TRIA3 element as shown in Fig.1. A spherical coordinate system is used to define the nodes at which all the degrees of freedom are constrained except the radial displacement. The spherical shell is 100 in. in diameter and the internal pressure is 2500 psi.. Both QUAD4 and TRIA3 element give practically the identical solution for both linear and nonlinear cases.

For the linear solution, NASTRAN yields

$$\{N\} = [62381.2 \quad 62381.2 \quad 0 \quad] \qquad (7)$$
$$\{M\} = [-3100.42 \quad 3100.42 \quad 0 \quad]$$

$$\{\epsilon_m\} = [0.2071089 \quad 0.2071089 \quad 0 \quad] \qquad (8)$$
$$\{\chi\} = [\quad 0 \quad 0 \quad 0 \quad]$$

The substitutions of (8) into (1) and (7) into (2) give

$$\{N\} = [62381.2 \quad 62381.2 \quad 0 \quad]$$
$$\{M\} = [-3100.4 \quad 3100.4 \quad 0 \quad]$$
$$\{\epsilon_m\} = [0.2071088 \quad 0.2071087 \quad 0 \quad]$$
$$\{\chi\} = [\quad 0 \quad 0 \quad 0 \quad]$$

which are identical to (7) and (8).

For the nonlinear solution, NASTRAN gives

$$\{N\} = [124764.2 \quad 124764.2 \quad 0 \quad] \qquad (9)$$
$$\{M\} = [\quad 0 \quad 0 \quad 0 \quad]$$

$$\{\epsilon_m\} = [0.4142238 \quad 0.4142238 \quad 0 \quad]$$
$$\{\chi\} = [\quad 0 \quad 0 \quad 0 \quad] \tag{10}$$

Substitution of (10) into (1) yields

$$\{N\} = [124764.2 \quad 124764.2 \quad 0 \quad]$$
$$\{M\} = [-6200.881 \quad 6200.881 \quad 0 \quad] \tag{11}$$

and there are disagreements in M_x and M_y when compared to (9).

Substitution of (9) into (2) yields

$$\{\epsilon_m\} = [0.4515529 \quad 0.4515529 \quad 0 \quad]$$
$$\{\chi\} = [0.751077 \quad -0.751077 \quad 0 \quad] \tag{12}$$

and there are disagreements in χ_x and χ_y and slight differences in $\epsilon_{x,m}$ and $\epsilon_{y,m}$ when compared to (10).

However, if (11) is used instead of (9), then there will be a complete agreement in all the quantities. Therefore, in this example, the NASTRAN output of nonlinear elemental moments are incorrect.

SOURCE OF ERROR

The discrepancies in the above example occur in the moment and curvature which are related to each other in (1) as

$$[M] = \{B\}[\epsilon_m] + [D][\chi] \tag{13}$$

Because of the uniform expansion of the spherical shell, we can say that the correct values for all the curvaturs are zero. Then $[D][\chi]$ is zero which is true for both linear and nonlinear cases. In addition, if $\{B\}=0$ then $[M]=0$ which agrees with the nonlinear solution but not with the linear solution. However, in this example, $\{B\}$ is non-zero as shown in (5), thus $[M]=\{B\}[\epsilon_m]$. For the linear case, this gives

$$M_x = -M_y = -14970\text{x}0.2071089 = -3100.42$$

which is exactly the same as the NASTRAN output.
For the nonlinear case, this gives

$$M_x = -M_y = -14970\text{x}0.4142238 = -6200.93$$

which should be the correct value for the nonlinear elemental moment.

The momnet values just calculated above can be checked using the ply stress output using PCOMP option. If the PCOMP option is used in both the linear and nonlinear analyses, the ply stresses can be obtained as shown in Fig.2. Multiplying the ply streses by the ply cross sectional area (0.1 inxin in our example) will give the ply forces. By taking the moment of the ply forces about y and x axes, we get the element moment as for linear case: $M_x = -M_y = -(207316-621.327)x0.15x0.1 = -3100.42$

for nonlinear case: $M_x = -M_y = -(414638-1242.67)x0.15x0.1 = -6200.93$

and they both agree with those computed by $[M]=\{B\}[\epsilon_m]$.

Hence, it appears that the NASTRAN output for the nonlinear elemental moments fails to include the membrane-bending couplng term $\{B\}[\epsilon_m]$.

TEST PROBLEM 2

In the previous problem of spherical shell expansion, the membrane effect is isolated by letting the bending/twisting curvatures diminish. The second problem being considered is that of a cylindrical bending of a flat panel without going stretching, Fig.3. Thus the membrane effect is suppressed and the bending effect is isolated. Again, the panel is modeled by one QUAD4 alement of 10 in. by 10 in.. The basic rectangular coordinate system is used to define the four nodes at which all the degrees of freedom are constrained except the rotation about the y axis(O). A moment of M =100 in.-lb. is applied at the four nodes.

For the linear case, NASTRAN gives

$$\{N\} = [-33.19761 \quad 0 \quad 0 \quad]$$
$$\{M\} = [\quad 20 \quad 0.03992016 \quad 0 \quad] \qquad (14)$$

$$\{\epsilon_m\} = [\quad 0 \quad 0 \quad 0 \quad]$$
$$\{\chi\} = [0.002217786 \quad 0 \quad 0 \quad] \qquad (15)$$

The substitutions of (15) into (1) and (14) into (2) yield

$$\{N\} = [-33.20005 \quad 0 \quad 0 \quad]$$
$$\{M\} = [\quad 20 \quad 0.03992022 \quad 0 \quad]$$
$$\{\epsilon_m\} = [\quad 0 \quad 0 \quad 0 \quad]$$
$$\{\chi\} = [0.0022178 \quad 0 \quad 0 \quad]$$

which are practically the same as (14) and (15).

For the nonlinear case, NASTRAN gives

$$\{N\} = [\quad 0 \quad 0 \quad 0 \quad]$$
$$\{M\} = [\quad 19.2 \quad 0.03832335 \quad 0 \quad] \qquad (16)$$

$$\{\epsilon_m\} = [\quad 0 \quad 0 \quad 0 \quad]$$
$$\{\chi\} = [0.002129 \quad 0 \quad 0 \quad] \qquad (17)$$

where χ_x is obtained by $\chi_x = M_x/D_{11} = 19.2/9018$. The substitutions of (17) into (1) and (16) into (2) yield

$$\{N\} = [-31.87087 \quad 0 \quad 0 \quad]$$
$$\{M\} = [19.19932 \quad 0.038322 \quad 0 \quad] \qquad (18)$$

$$\{\epsilon_m\} = [0.0001156 \quad 0 \quad 0 \quad]$$
$$\{\chi\} = [0.0023209 \quad 0 \quad 0 \quad] \qquad (19)$$

and there are discrepancies in N_x and $\epsilon_{x,m}$. If $N_x = -31.87087$ is used istead of $N_x=0$ in (16), then there will be a complete agreement after the substitutions. Therefore, the NASTRAN output of nonlinear element forces are incorrect in this example. Calculating form PCOMP results, we have $N_x = -33.2003$, $N_y = 0$ for both linear and nonlinear cases.

SOURCE OF ERROR

Similar to the previous example, the first equation of (1) is expaneded as

$$[N] = [A][\epsilon_m] + [B][\chi] \qquad (20)$$

Since $[\epsilon_m]=0$, $[N]=[B][\chi]$ which is non-zero unless $[\chi]=0$. Thus from (17) and (20) we get

$$N_x = B_{11}\chi_x = -14969.88x0.002129 = -31.87087$$

which is exactly the force omitted in (16). Therefore, it appears that the membrane-bending coupling term $[B][\chi]$ is also missing from the NASTRAN's computations of the elemental forces in the nonlinear analysis.

TEST PROBLEM 3

In the previous two problems, either the strain or curvature is suppressed and the other is isolated. In the third example, a problem with both the strain and curvature are presented will be tested. The problem chosen is that of a fixed supported square flat plate of 100 in. by 100 in. under uniform normal pressure loading of 25 psi.. Due to symmetry, only one quarter of the plate is modeled as shown in Fig.4 with appropriate boundary conditions. Element No.2 is arbitrary chosen for examination.

For linear solution, NASTRAN gives

$$\{N\} = [-11.87838 \quad 9.835756 \quad 0.003597834]$$
$$\{M\} = [\ 4741.824 \quad 5045.577 \quad -1.564868\ ] \qquad (21)$$

$$\{\epsilon_m\} = [0.02850007 \quad -0.03033589 \quad 0.000011993]$$
$$\{\chi\} = [0.5719094 \quad 0.608713 \quad -0.1738742] \qquad (22)$$

The substitutions of the above into (1) and (2) yield

$$\{N\} = [-12.49609 \quad 10.49121 \quad 0.0035979\,] \qquad (23)$$

$$\{M\} = [\,4741.793 \quad 5045.543 \quad -1.56486\,]$$

$$\{\epsilon_m\} = [0.0285025 \quad -0.03033848 \quad 0.000011993] \qquad (24)$$

$$\{\chi\} = [0.5719168 \quad 0.608721 \quad -0.1738742]$$

which are practically identical to the NASTRAN output except a slight deviation for N_x and N_y which is probably due to different methods of force recovery. From PCOMP results we get N_x=-8.678, N_y=6.6245, M_x=4598.083 adn M_y=4892.958.

For the nonlinear solution, NASTRAN gives

$$\{N\} = [3380.325 \quad 2929.109 \quad -0.0971702] \qquad (25)$$

$$\{M\} = [47.87434 \quad 26.31254 \quad -0.00991641]$$

$$\{\epsilon_m\} = [0.01122585 \quad 0.0097218 \quad -0.0003239] \qquad (26)$$

$$\{\chi\} = [0.0055239 \quad 0.00302832 \quad -0.000956443]$$

The substitutions of the above into (1) and (2) yield

$$\{N\} = [3297.633 \quad 2974.442 \quad -0.0971703] \qquad (27)$$

$$\{M\} = [-118.1806 \quad 172.943 \quad -0.00860799]$$

$$\{\epsilon_m\} = [0.01252571 \quad 0.01043951 \quad -0.0003239] \qquad (28)$$

$$\{\chi\} = [0.02613031 \quad -0.01446397 \quad -0.01101823]$$

The most notable discrepancy is seen between the moments and between the curvatures. In accordance with the previous examples, the "correct" moments are calculated as:

$$M_x = 0.01122585\mathrm{x}(-14969.88)+47.87434 = -120.17528 \qquad (29)$$

$$M_y = 0.0097218\mathrm{x}14969.88+26.31254 = 171.84671$$

Using (29) in place of their counterparts in (25) yields, after the substitutions

$$\{N\} = [\,3297.633 \quad 2974.442 \quad -0.0971703\,] \qquad (30)$$

$$\{M\} = [-118.1806 \quad 172.943 \quad -0.00860799\,]$$

$$\{\epsilon_m\} = [0.01151406 \quad 0.0095634 \quad -0.0003239\,] \qquad (31)$$

$$\{\chi\} = [0.005780854 \quad 0.003169183 \quad -0.001101823]$$

These values are very close to the input values but deviation is still found in every quantity which is due to the incorrect membrane force values. The "correct" membrane forces should be:

$$N_x = 0.0055239x(-14969.88)+3380.325 = 3297.633$$
$$N_y = 0.00302832x14969.88+2929.109 = 2974.443 \quad (32)$$

Replacing (25) with (29) and (32), we then have

$$\{N\} = [3297.633 \quad 2974.443 \quad -0.09717017]$$
$$\{M\} = [-120.16322 \quad 171.83621 \quad -0.00991641] \quad (33)$$

The substitutions of (26) and (33) into (1) and (2) yield

$$\{N\} = [3297.633 \quad 2974.442 \quad -0.0971703\ ]$$
$$\{M\} = [-118.1806 \quad 172.943 \quad -0.00860799]$$
$$\{\epsilon_m\} = [0.01121385 \quad 0.0097284 \quad -0.0003239\ ] \quad (34)$$
$$\{\chi\} = [0.00528305 \quad 0.00289627 \quad -0.00110182]$$

Although not exact, a close agreement is seen between (26),(33) and (34). From PCOMP results, we get N_x=3294.07, N_y=2974.442, M_x=-119.583 and M_y=172.183.

CONCLUSIONS

The three exercises presented herewith, strongly suggest that the elemental force and moment outputs for QUAD4 and TRIA3 shell elements in the nonlinear analysis (SOL 66) fail to include the membrane-bending coupling terms $[B][\chi]$ and $\{B\}[\epsilon_m]$ respectively. Before the possible error is fixed, the correct values for the forces and moments can be obtained by computing the membrane-bending coupling terms as shown in the examples and add them to the corresponding NASTRAN output values.

ACKNOWLEDGEMENT

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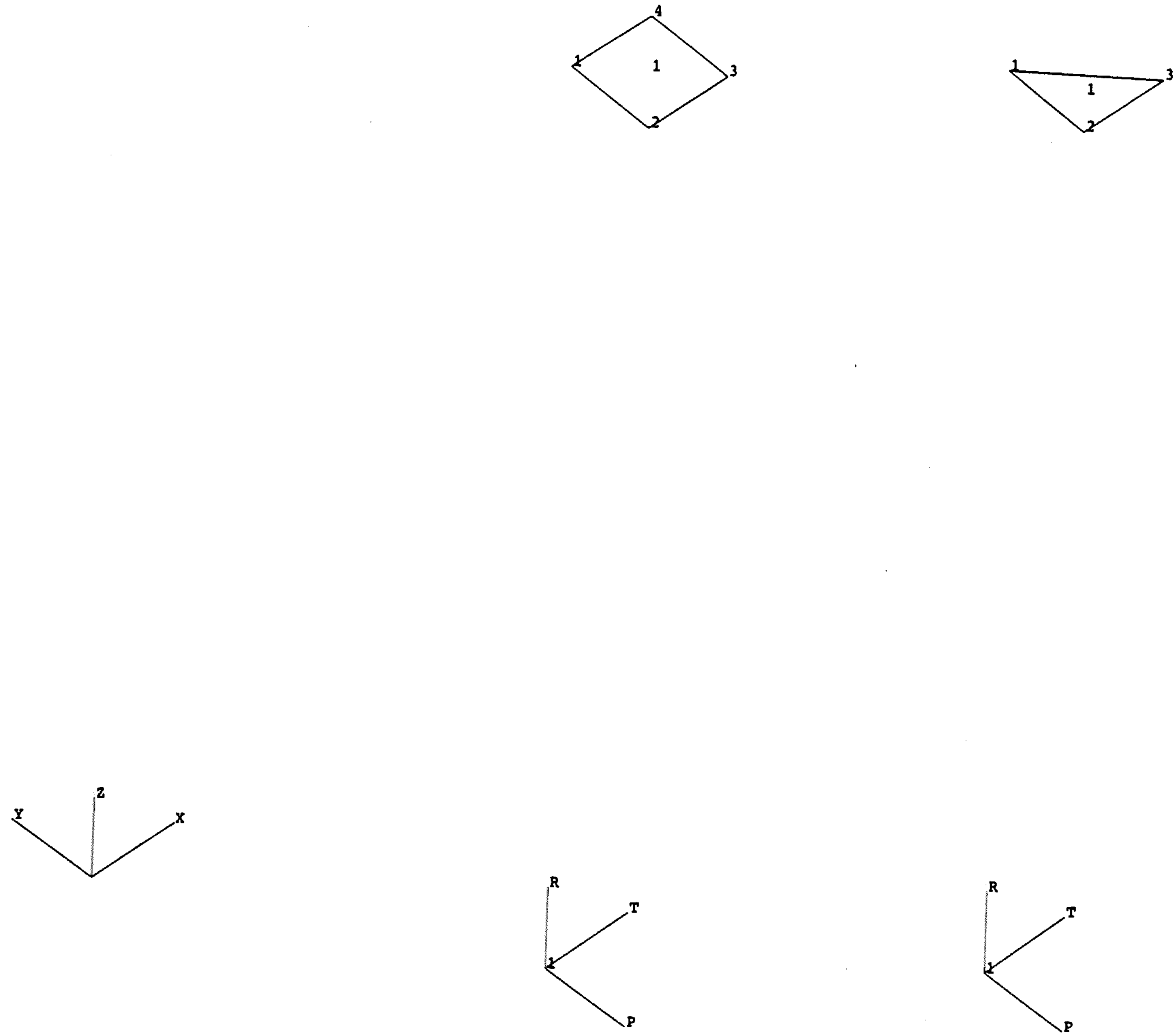

Fig. 1

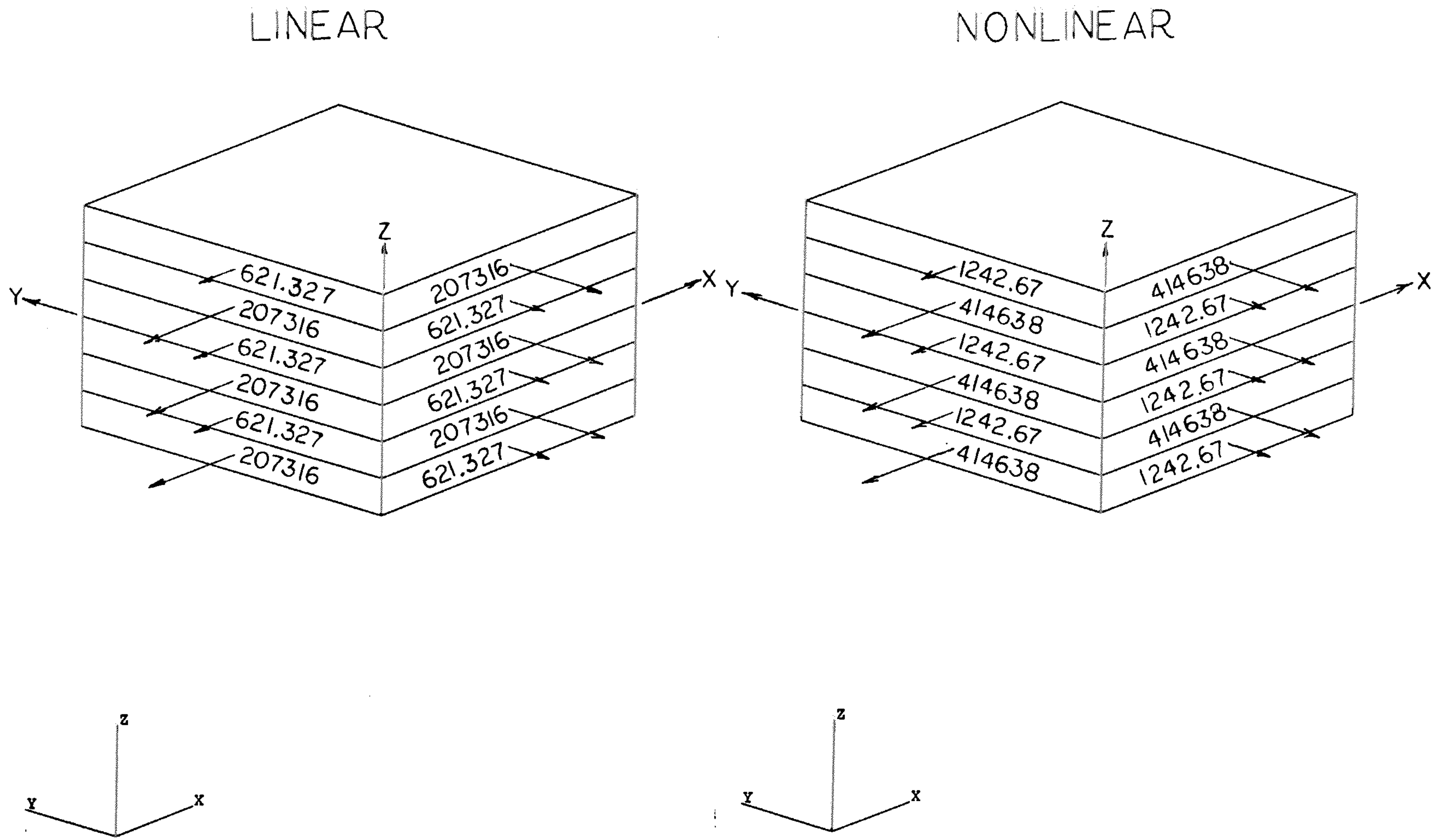
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Fig. 2

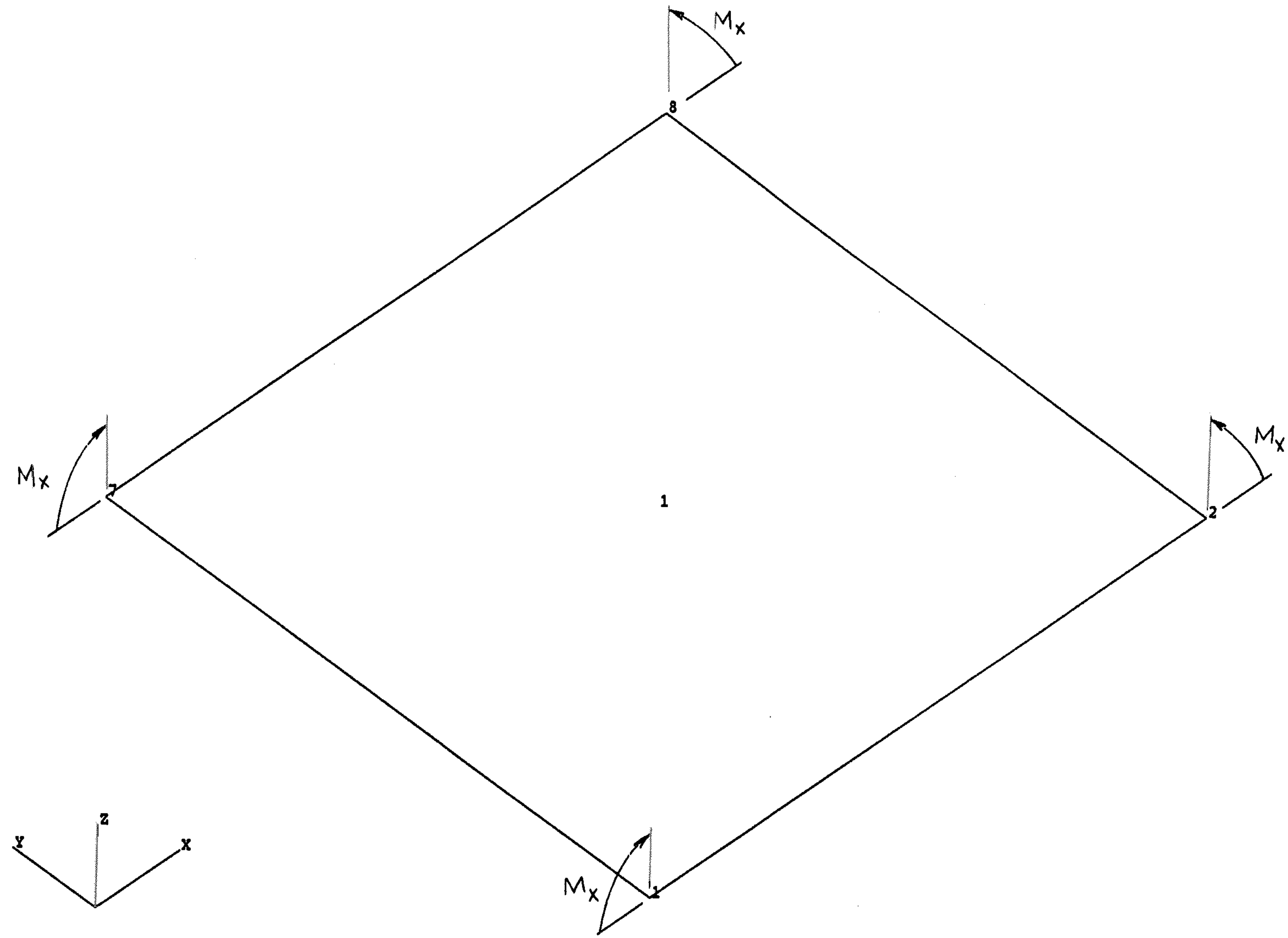

Fig. 3

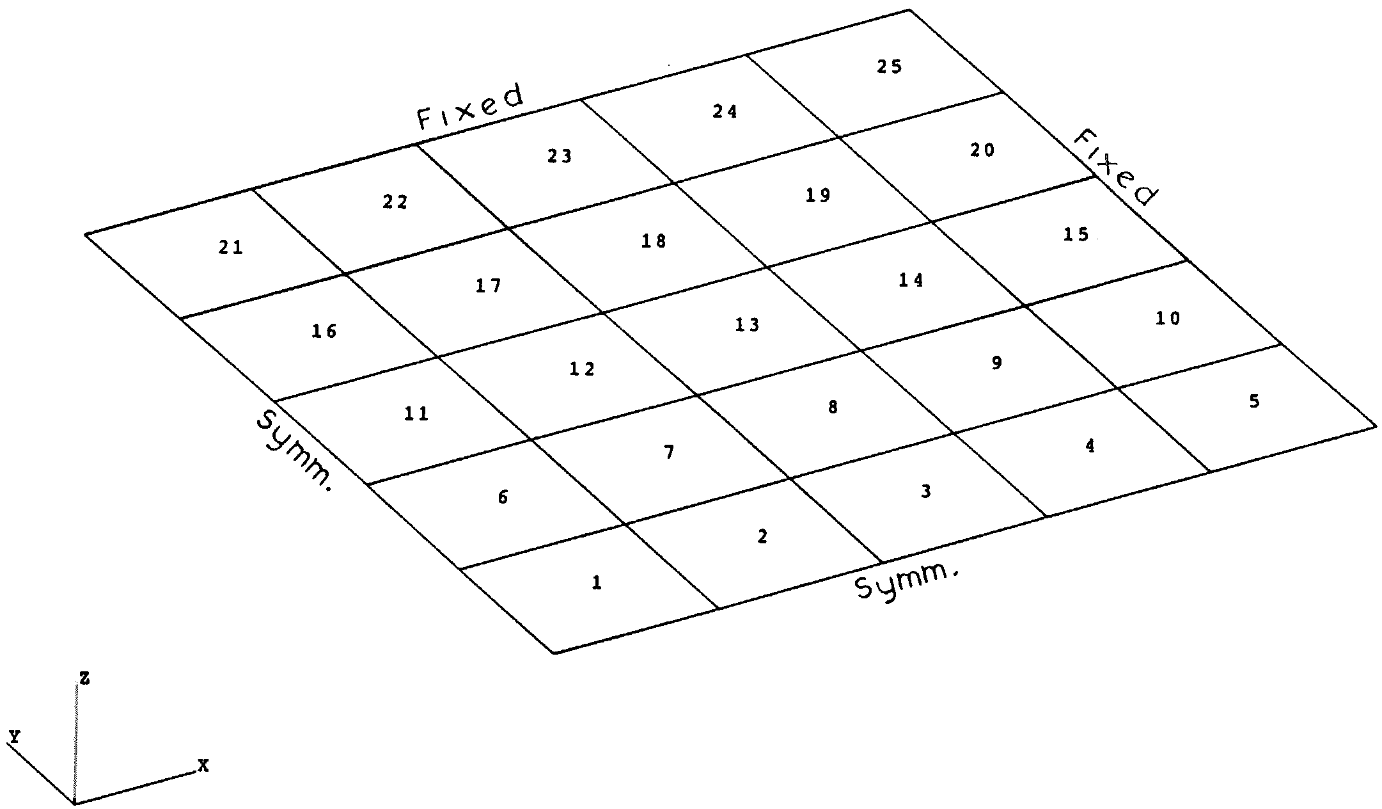
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Fig. 4