RELIABLE FINITE ELEMENT MODELING FOR ENGINEERING DESIGN

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INTRODUCTION

Over the past thirty years the finite element method has evolved to the point that it is now a commonly applied engineering analysis tool. However, its impact on the design process is typically limited to the later stages of design, or to design verification. This restricted application of finite element techniques is not due to the cost of the computation required, it is due to the cost of performing meaningful finite element analysis, particularly on a partially completed design. The reliable use of the finite element method by design engineers, who are not analysis specialists, requires capabilities past the finite element analysis program. This paper discusses the additional capabilities needed for the reliable application of finite element methods in engineering design.

The reliability of a finite element result is dependent on the ability to qualify and control the approximations involved in going from the statement of the problem to the idealization analyzed by the finite element program. The idealization steps common to this process include [22]:

1. mathematical model selection
2. domain simplifications
3. dimensional reductions
4. material property specification
5. boundary condition specification
6. mesh discretization

Current finite element analysis codes have focused on the accurate solution of the finite element discretization that results from this idealization process. Historically, it has been the responsibility of the analyst to perform the six idealization steps relying primarily on their expertise to guide them in that process. Techniques to control these idealization steps need to be an integral part of engineering design systems.

The ability to control the various analysis idealizations vary greatly ranging form analytical techniques, that can provide accurate bounds on an error contribution, to heuristic rules based on previous experience. This paper presents an approach to support the various levels of idealization control within an engineering modeling environment that is consistent with the systems and methodologies currently under development to support engineering design.

Progress on the development of idealization error control will consider two specific applications. The first application will be the control of the dimensional reductions common to the structural analysis of airframes by the application of rule-based techniques. The second application will be the control of mesh discretization errors in two-dimensional stress analysis through the application of automated adaptive finite element analysis.

IDEALIZATION IN ANALYSIS AND DESIGN

The application of engineering analysis typically employs a number of idealizations to reduce a physical behavior to a set of algebraic equations that can be solved manually or on a computer. Each step of idealization used in an engineering analysis process introduces some level of approximation. The reliability of an analysis depends on the ability to understand and control the approximation errors introduced by each step of idealization [1], [2], [22], [30].

Analysis idealizations are an important part of the design process for highly engineered artifacts. In such cases the evolution of the design often parallels the evolution of the analysis models. For example, consider a simplified view of the evolution of the airframe structure for a new aircraft. The process of defining the structural members will typically begin with a basic outer shape of the aircraft. This shape is needed to perform an initial aeroloads analysis to determine the distribution and magnitudes of the loads the airframe structure must support. The first idealization of the frame itself may be one beam running down the fuselage and one across the wings. This idealization can be analyzed to determine an estimate of the overall stiffness properties that will be required. This analysis provides the structural designer with a small number of basic parameters to assess the feasibility of the design and to guide the next step, which is the selection of the type of structural framework and its basic configuration within the inner and outer envelopes of the aircraft. The next level of structural idealization may be a set of beam and truss members to represent the structural configuration. An iterative structural analysis/design process on these members can be used to obtain good estimates of the section properties needed by the structural members. After the individual members are detailed, a more appropriate set of structural idealizations [12] can be used for static and dynamic analysis. These analyses may also be followed by more detailed local analyses for particular situations. For example, full three-dimensional analyses may be performed to determine the fatigue characteristics of critical components and to guide their detailed design.

The above example of the various structural analyses used in an airframe indicates that the level of error control needed for a particular analysis model depends on the information expected to be extracted from the analysis. The structural designer knows that the initial simple beam structure analysis only provides a rough approximation of the distribution of loads in the structure and understands that this information can only provide an overall guide in the determination of the structural configuration. These issues often become less clear later in the design process when more sophisticated analysis tools are being applied. The sophistication of the design and analysis tools may lull the inexperienced structural designer into thinking that the analysis results provide more information that can be reliably extracted from them.

CONTROLLING IDEALIZATION ERRORS

The techniques available to aid in the control of idealization errors include:

1. Analytically-based error estimation
2. Analytically-based results for ideal situations

3. Model improvement through hierarchic model comparisons
4. Sensitivity analysis
5. Statistical methods
6. Comparison to known physical limits
7. Comparison to test results
8. Comparison to reasonable limits
9. Rules based on experience and intuition

The ability of these techniques to reliably control idealization errors varies greatly [23]. The preferred method of controlling idealizations is to use the best methods available for each. These idealization control techniques can contribute to the control of the various levels of idealizations as outlined below.

Mathematical model. The derivation of a mathematical model employs physical laws, mathematical manipulations, and behavioral assumptions. The behavioral assumptions are needed so that the physical laws can be mathematically manipulated to yield a useful set of expressions. Each behavioral assumption introduces approximation, and associated error.

The primary purpose of idealization error control for a mathematical model is to qualify the behavioral assumptions used to derive that model. The application of analytically-based procedures is difficult since these require the existence of mathematical models to measure against. The exceptions to this would be where a related set of hierarchical mathematical models would be available to measure against, or at least compare to each other. However, it must be realized that this approach requires the existence of a high level mathematical model that is known (assumed) to be correct.

One idealization control method that can be applied at this level is the comparison of the results to known physical limits. Two simple examples of this type are checking the stress field from a linear elastic analysis against a yield criterion, and comparison of the temperature field from a thermal analysis excluding phase change with the melting temperatures of the constituents. In some classes of analyses such straight forward checks are not as simple. For example, it is often not possible to examine the results of a fluid flow analysis to see if the exclusion of a specific consideration would strongly affect the solution. In such cases, experience (either from experimental comparisons or trial and error) may be useful in providing guidance as to reasonable limits for the application of specific mathematical models. Finally, specifically designed experiments to measure selected critical parameters must often be devised to help test the appropriateness of a mathematical model.

Different levels of mathematical models are often used in the design process to evaluate an aspect of a design. The airframe structural analysis example presented earlier is one such example where several mathematical models are used. Often early models are used to efficiently give overall estimates of specific quantities. Later analyses use more accurate mathematical models that require more computational effort and detailed design information. Still, other mathematical models include physical behaviors not represented in earlier models. Control and coordination of such series of analyses must focus on the result desired from each analysis, the accuracy of result desired, the state of the design, and the analysis information available from previous analyses.

Domain. There is no additional error introduced into the analysis if the domain is complete with respect to the mathematical model being solved. The domain used in most analyses is typically not complete in that it may be limited to a portion of the total domain, with boundary conditions applied along the boundary introduced by the eliminated portion of the domain. If the portion of the domain eliminated is symmetric to that analyzed, and the proper essential boundary conditions are applied, there is no error introduced into the analysis. It is also common to eliminate geometric details; this also introduces errors into the solution.

Methods to control idealization due to domain simplifications depend upon how the geometric simplification effects the solution behavior.

The first category of domain simplifications that must be identified are those that alter the form of the solution behavior. For example, approximating a smooth boundary with a faceted one can change the exact values of many quantities to a given mathematical model from smooth to singular. Depending on the goal of the analysis such approximations may yield the solution useless [1], or simply indicate that the values to given solution parameters are not meaningful in specific areas [22], [29]. The determination of when these circumstances will arise requires an analytic understanding of the basic solution behavior [1], [2], [28]. The basic knowledge of such situations must be used to guide the design engineer on the application of analysis procedures. It may also be possible to use the specifics of the analytic solution behavior to provide estimates for the singular solution parameters.

A number of techniques are available to estimate and control geometric approximation errors when the approximations do not alter the smoothness of the solution with respect to the parameters of interest. A priori error estimates have been derived for specific classes of geometric approximation [26]. Such estimates can be used as the basis of the development of a posteriori error estimates for these approximations. Sensitivity analysis can be used to determine how important geometric variations may be on solution results. The various techniques for sensitivity analysis from shape optimization [6] provide a set of tools for this situation. Analytically-based results to idealized situations can also provide a useful set of techniques.

Sensitivity analysis methods can also be useful to estimate the influence of replacing a portion of a domain by a given set of boundary conditions that do not strictly adhere to a given symmetry situation. Sensitivity measures to small changes in the boundary conditions approximating the behavior of the eliminated portion of the domain can provide useful information in the control of errors due to this type of idealization.

Dimensional reductions and associated mathematical model alterations. It is often possible to selectively introduce specific behavioral assumptions over portions of the domain which allow the simplification of the mathematical model by reducing the physical dimensionality of that portion of the domain. The eliminated physical dimensions are accounted for by the introduction of specific parameters. The techniques available to control dimensional reductions vary greatly in sophistication and cost of application.

Given a starting mathematical model, the approximation errors introduced by its dimensional reduction can be controlled by analytically-based procedures if the dimensional reduction can be represented as a convergent sequence. One example that has been considered is the dimensional reduction associated with the analysis of plate like domains [31]. The key to this procedure is to not state a specific assumption on the 'through the thickness' behavior, but to discretize it with a convergence expansion.

Several of the other idealization control techniques are also useful with dimensional reductions. The various techniques can be used alone or in combination with others.

An example where test results are combined with dimensional reduction is the analysis of structural connections [30]. In this analysis the material the connection bears against is replaced by a set of nonlinear springs where the stiffness parameters of the springs are determined by matching the connection analysis behavior to that of a physical connection that was tested.

Material properties. The mathematical model as well as dimensional reduction and associated mathematical model alterations fix the framework of the material model. However, within that framework there is still the specification of the parameters.

The specification and control of material properties must ultimately rely on the input from physical tests of components designed to measure the parameters required in the selected form of constitutive relationship.

Often the form of mathematical model used is dictated by the form of the material constitutive model required to capture the needed physical behavior. In many cases analytic analyses indicate the form of constitutive relationship that is meaningful. For example, the theory of plasticity has been strongly influenced by the desire to employ constitutive forms that demonstrate specific mathematical properties.

Sensitivity analysis and statistical analysis have a strong influence on determining the variations of solution parameters based on expected variations in material parameters.

Boundary and initial conditions. The mathematical model as well as dimensional reduction and associated mathematical model alterations specify the boundary and initial conditions that are part of the current analysis. The boundary and initial conditions which must be specified are often difficult to abstract from the physical situation being analyzed. In addition, the values of many of them, particularly natural boundary conditions, are probabilistic in nature.

As with the domain approximations, the methods to control idealization due to boundary and initial condition approximations depend upon how they effect the solution.

The first category of simplifications that must be identified are those that alter the form of the solution behavior. For example, approximating a distributed load or boundary condition with a shape variation can introduce singularities into the solution. These singularities can range from analytic singularities, to non-analytic singularities for which the solution is meaningless [1], [30]. These situations would be handled with the same types of techniques as domain simplification.

The same techniques as available for domain simplification are available when the approximations do not alter the smoothness of the solution with respect to the parameters of interest.

Correlation of analysis and test results can also be effectively used to specify the appropriate boundary conditions to use in an analysis [14].

Discretization. Discretization consists of representing the continuous field by one written in terms of a specific number of parameters. This reduction of the solution test space introduces additional approximations into the analysis process. The common generalized discretization techniques used include finite difference, finite elements, finite volumes and boundary elements.

The mathematical basis of the discretization process makes it possible to control the discretization errors through the application of a posteriori error estimation techniques [3], [11]. Over the past several years these techniques have been developed and combined with adaptive techniques that automatically control discretization errors through the successive enrichment of the discretization.

In the long term, it is expected that most discretization errors will be controlled by adaptive analysis techniques. At this time good adaptive procedures to control all discretization errors do not exist, and even when they do, the overall efficiency of the process may be improved by combining it with other techniques. For example, rules based on an analytic understanding of the solution behavior, and/or previous experience can provide a priori information for the development of an initial discretization. Such a capability is needed when there are no adaptive procedures available to control the discretization errors. They can also be useful in conjunction with adaptive techniques to improve the computational efficiency of the analysis process [13], [20].

MODELING SYSTEM SUPPORTING IDEALIZATION CONTROL

The ability to apply idealization control requires a design modeling system which can house various levels of analysis idealization with design methodologies and engineering analysis tools. Figure 1 shows the overall framework of an engineering modeling system for mechanical objects that is specifically structured to support the idealizations used in engineering modeling and analysis [17], [24].

This system framework is consistent with the architectures being considered to support design modeling systems [8], [9], [21]. It does not represent an entire design system since it includes only the model representations and analysis tools. As indicated by Dixon, et al. [8] the analysis tools only provide the data needed for evaluation in design. However, it is important to point out that the model representations determined most appropriate for the support of analysis idealization processes contain both a functional and geometric model. This level of representation is also important in design modeling systems to support the application of knowledge-based techniques [21] and feature modeling [9]. The concept of features has a direct relationship to the analysis idealization process

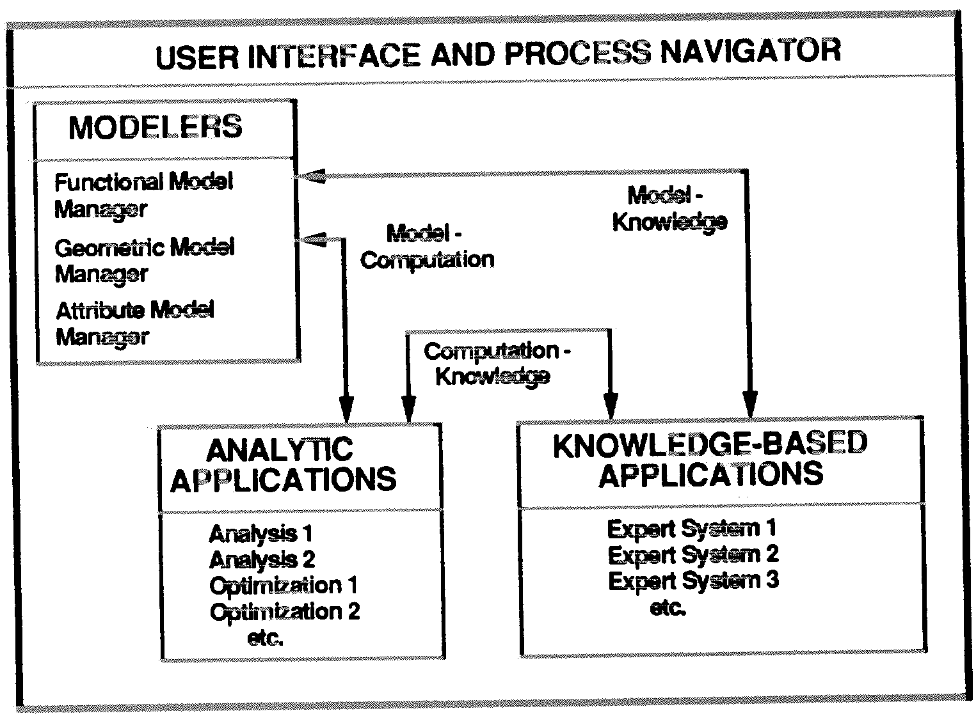


Figure 1. Modeling system to support idealizations

in that the analysis features [15] are defined for the purpose of associating idealization information with some portion of the model.

The heart of the system is the representation of the object being designed and the modelers that support that representation. To support the functions necessary in the design evolution of an artifact, its representation is housed in linked functional, geometric and attribute model structures which are controlled by the appropriate modelers. The other operational components of the modeling system are the applications. The applications include analysis procedures to answer performance questions, algorithms to alter the design based on analysis results, and procedures to plan the manufacturing processes, etc. Applications are separated into two groups based on the technology underlying their implementation, not on the functions addressed. The first group are analytic applications. The vast majority of the applications in this group are numerically based analysis and optimization procedures. The second group are knowledge-based applications. Knowledge-based applications are assumed here to operate from codified heuristics placed in rule sets.

The task of analysis idealization control falls to the process navigator. The process navigator interacts with the models, applications, and databases to track the various activities that have been performed and to guide the application of those that are requested. By tracking the idealizations used and analyses performed to the current point in the design, the process navigator provides the designer with 1) guidance as to the next steps in the process, 2) feedback as to the appropriateness of performing the next task request, and 3) directions to the applications appropriate to perform the requested task.

Functionally the process navigator consists of three components: the request interpretor, the analysis strategist, and the process monitor. At the most basic level, the request interpretor is responsible for accepting a request to perform an operation, to determine if the basic information and capabilities required to perform the task exist, and to invoke the strategist to carry out the request. The strategist is responsible for formulating and controlling the process steps required to perform the request. The process monitor is responsible for maintaining information about the status of the design and the tasks that have been performed previously.

Given an analysis request and the current state of the design, the analysis strategist must be able to apply the various levels of idealization error control on each source of idealization error to produce the most reliable solution possible within the given cost constraints. An analysis strategist must employ feedback procedures to exercise the various levels of idealization control. Figure 2 gives an indication of the various steps the analysis strategist must perform. The integration of the analysis strategist into an engineering design system must address the issue of dealing effectively with incomplete design specifications.

The ability of the analysis strategist to interact with the design process is aided by the process monitor maintaining the appropriate information about the state of the design and the analyses performed to the current point in the design process. One tool critical to the functioning of the process monitor is the analysis goal graph [32] and its interactions with the information in the design system.

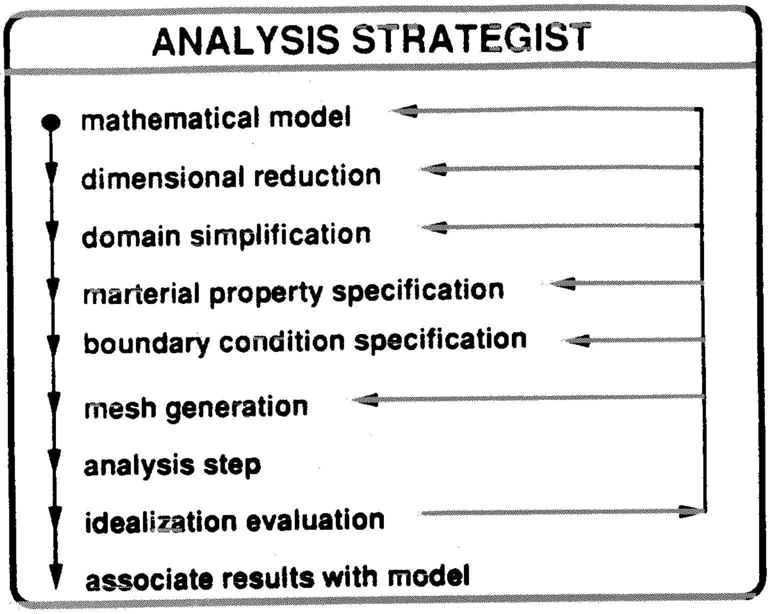


Figure 2. Analysis strategist

CONTROLLING DIMENSIONAL REDUCTIONS IN AIRFRAME ANALYSIS

The majority of the dimensional reduction idealization control procedures for structural analysis of airframes employ rules [10], [12]. To support the process of designing an airframe idealization control techniques must support a variety of structural analyses [19] including:

1. Simple beam analysis to determine overall load distribution.
2. Internal load distributions based on one-dimensional frame and truss members to determine member section property requirements.
3. Overall static stress analysis representing main members as combinations of one- and two-dimensional entities employing various levels of idealization rules.
4. Overall vibration analysis using appropriate sets of member stiffness and mass property idealization.
5. Component fatigue and failure analyses using appropriate local idealizations and boundary conditions obtained from the previous analyses.

Current efforts on the development of an analysis idealization procedure are focused on the application of knowledge-based idealizations associated with the structural analysis of airframes for internal load distributions [7], [17], [24], [32]. The prototype system is being implemented primarily within the framework of the KEE environment [16] which is used to house the functional model, house the dimensional reduction idealization rules, support the process navigator, provide the inferencing techniques, and provide the user interface. The geometric information is provided by a geometric modeling system. (Currently a simple set of geometry types are supported.) The finite element models ultimately generated will be analyzed by MSC/NASTRAN [18].

The process of performing the dimensional reductions appropriate to the analysis begins by invoking the request interpretor and indicating a structural analysis is to be performed. An updated analysis goal graph for the class of analysis requested is presented to the user to guided the selection of the specific analysis task. The user is then presented with the major airframe components and selects the one to be operated on. At that time a high level functional view of that component (Fig. 3) is made available for the user as well as a version of the analysis strategist specialized to the analysis type under consideration. After employing some graphics tools to specify the particular members of interest the user specifies the information needed to support the internal loads analysis. The specification of this information can be by applying the dimensional reduction idealization rules to the individual members, or the direct definition of the idealization structural properties needed for the analysis. In either case the result will be a specification of the structural idealization and properties needed to support the analysis. For example, the internal loads structural idealization of a frame would typically result in an idealization consisting of truss members for the flanges and a shear panel for the web (Fig. 4). This structural idealization is then discretized into finite elements for analysis by MSC/NASTRAN.

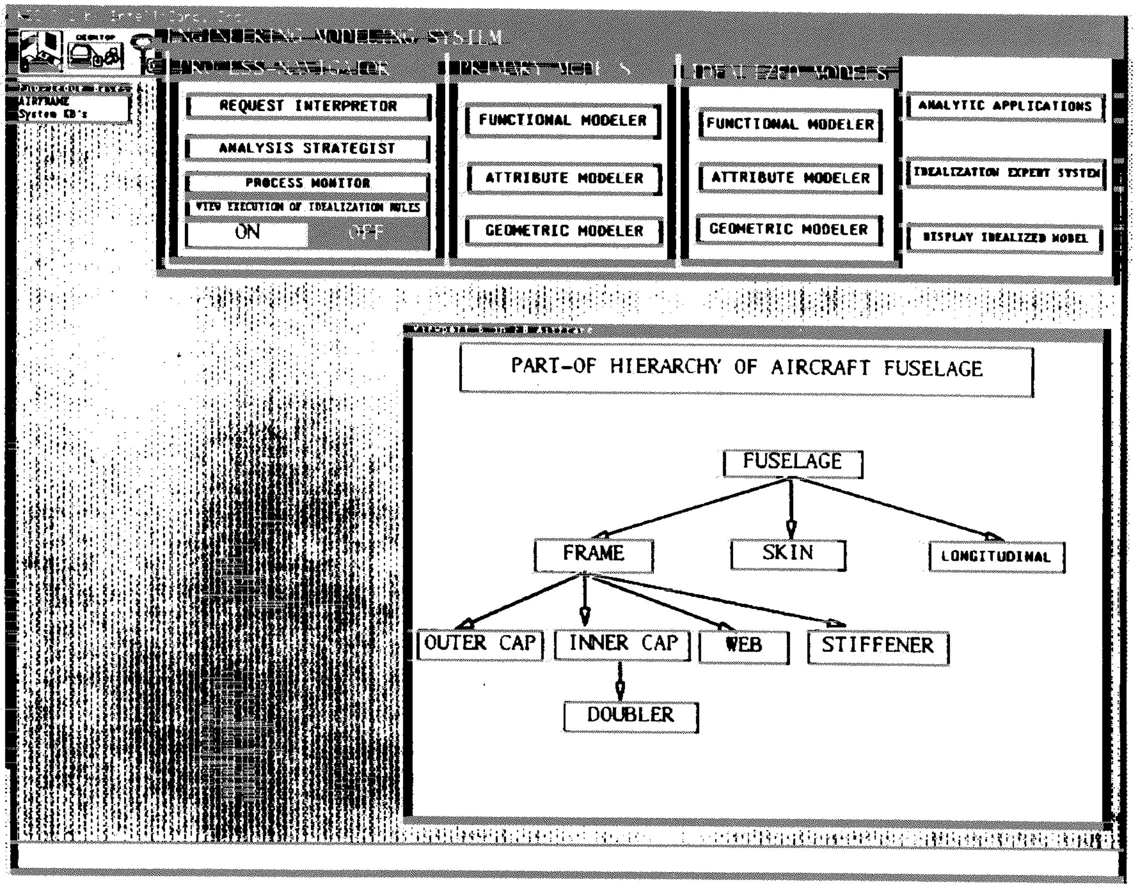


Figure 3. Functional model for portion of airframe

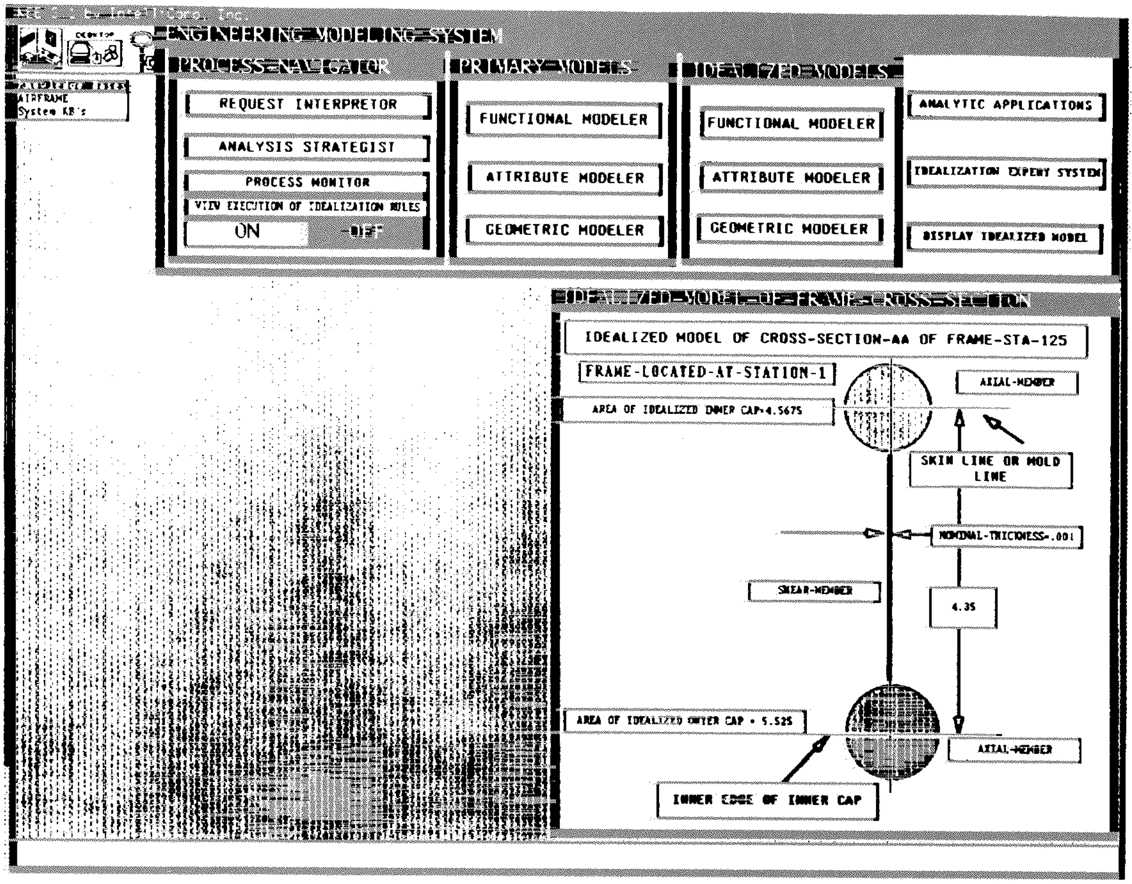


Figure 4. Dimensional reduction idealization for frame member

AUTOMATED ADAPTIVE ANALYSIS TO CONTROL MESH DISCRETIZATION ERRORS

The combination of automatic mesh generation and adaptive analysis techniques allows for the reliable control of the idealization errors associated with the mesh discretization in each norm for which accurate a posteriori error estimates exist. The initial acceptance of these tools by the engineering community requires that they operate in conjunction with the commercial finite element analysis tools they currently use. Procedures meeting these requirements have been developed for h-refinement in both two and three dimensions and for hp-refinement in two dimensions.

The automated h-refinement system [4] employs the finite quadtree and finite octree mesh generators for two and three dimensional domains respectively. The adaptive portion of the automated h-refinement system consists of error estimation and local remeshing. Two error estimation procedures are used. In the first, the error is calculated from the residuals of the primary solution variable [4]. The second error estimator employs stress projector techniques [33], [25]. The magnitudes of the elemental errors used to determine the number of levels the elements are to be refined [4]. Either local or global remeshing procedures are used to automatically update the mesh to the requested refinement levels [5].

The automated system components are constructed in a modular form with the communications between major modules being through files. The adaptive procedures require only standard solution output and a knowledge of element type to support adaptive analysis of both first and second order simplex elements (3– and 6–noded triangles, 4– and 10–noded tetrahedra). With this structure, the integration with MSC/NASTRAN [18] consists of creating a small program to re-format data.

Figure 5 shows a two-dimensional bracket analyzed by the finite quadtree based automated system. The initial and final meshes are shown in Figure 6. Figure 7 shows the meshes for a three-dimensional example automatically analyzed by the finite octree based procedure. In both examples quadratic elements are used since experience has indicated they require much less computation to obtain a given level of accuracy.

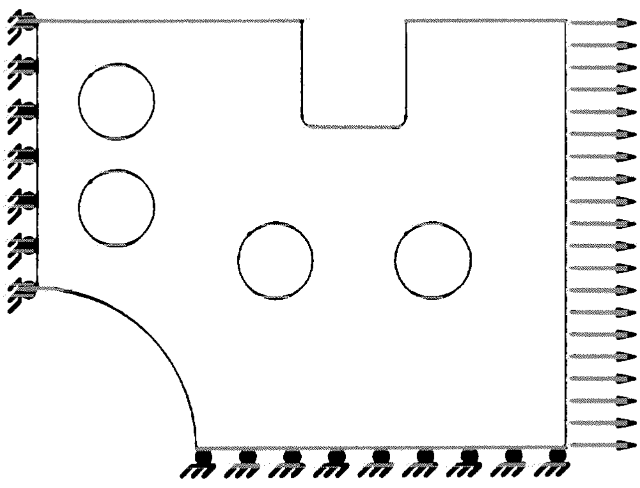
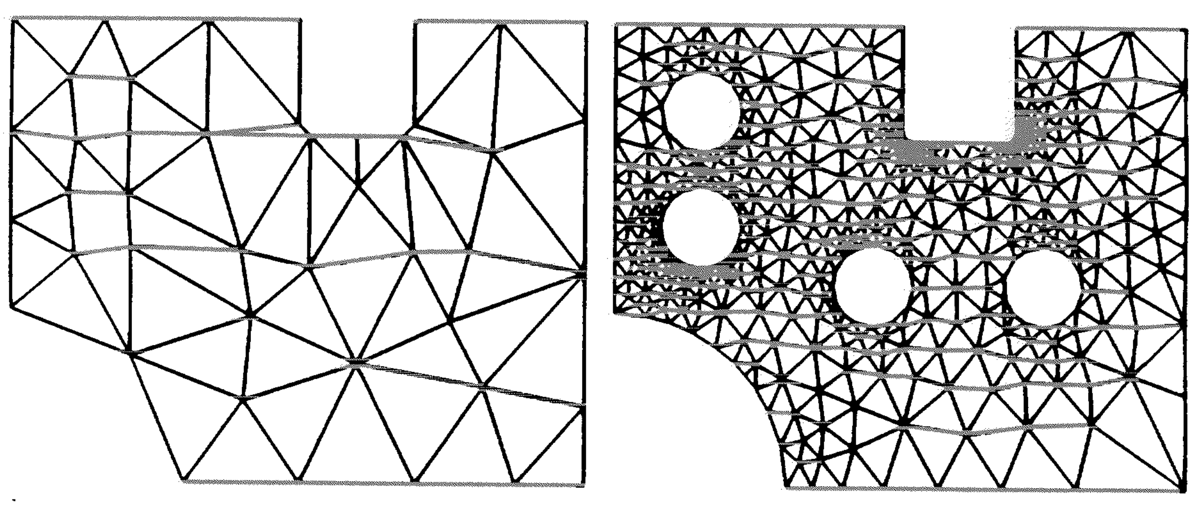


Figure 5. Problem definition



a)

b)

Figure 6. Two-dimensional adaptive h-refinement: a) initial mesh, b) final mesh

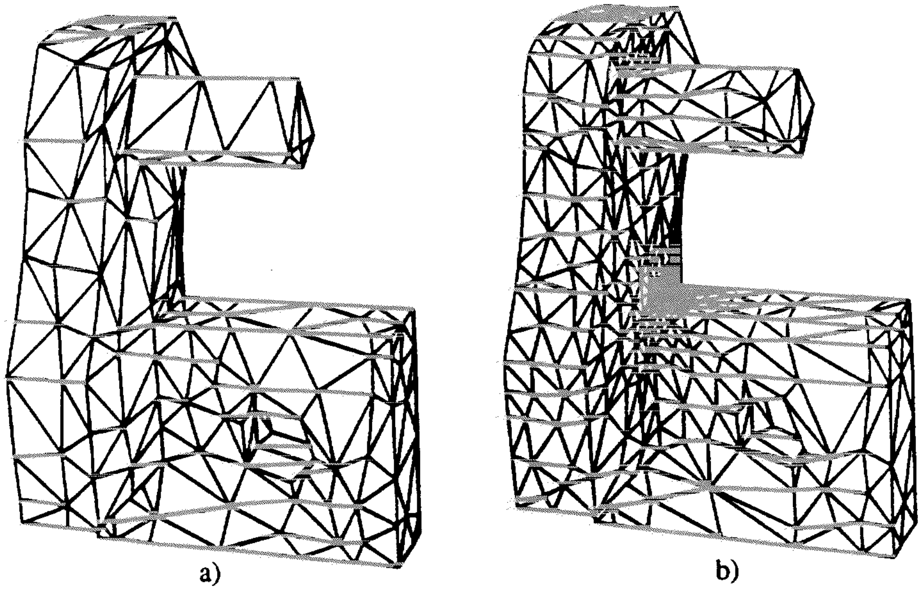


Figure 7. Three-dimensional adaptively h-refinement: a) initial mesh, b) final mesh

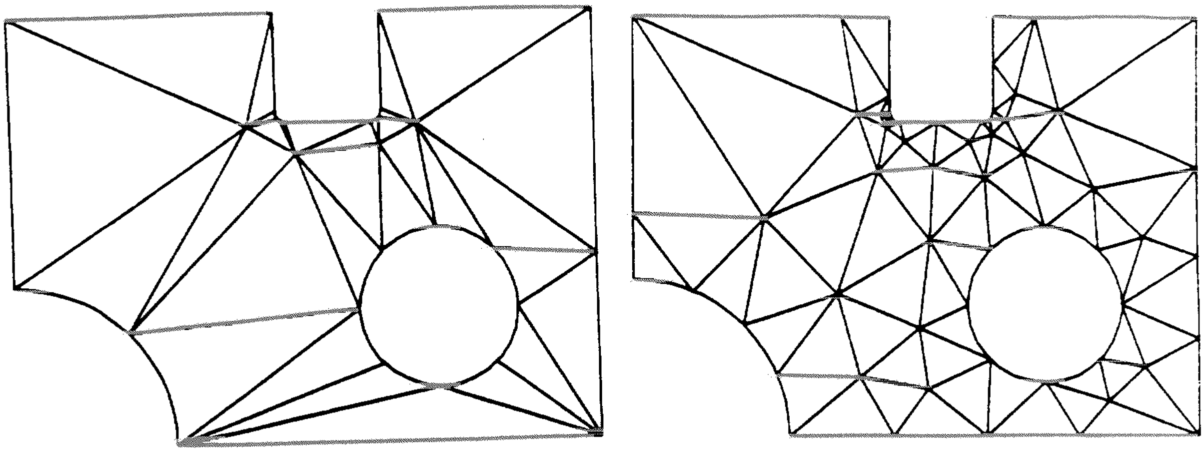
The automated hp-refinement system employs an element removal mesh generator designed to work in conjunction with the hp-correction indication procedures in the system [13]. The hp-refinement system was defined to minimize total computation cost by using analytically-based criteria tuned by numerical experimentation[13]. To minimize the computational effort in the modeling process, error prediction techniques are used to specify the best combination finite element discretization, h, and the polynomial order, p, needed to just achieve the required accuracy. The solution process is carried out using PROBE [27].

The starting mesh for a bracket analyzed by the hp-refinement system is shown in Figure 8a. Three preliminary analyses at levels $p = 2$, 3, and 4 are performed to determine information that is needed by the adaptive analysis procedures. Based on this feedback, the preliminary mesh is refined as shown in Figure 8b. Three additional analyses at levels $p= 2$, 3, and 4 are performed on the refined mesh. To achieve less than 2.0% error in the energy norm, the adaptive procedures predicted a polynomial level of $p = 4$ which did yield the required accuracy.

The combination of the element removal mesh generator and hp-adaptive procedures [13] with the PROBE analysis program [27] is very efficient computationally. The total CPU required for all aspects of the automated analysis was two minutes (117 seconds) on a four MIP (VAX 6320) computer. This CPU time includes all mesh generation, analysis steps, error estimation and mesh enrichment.

As an example of the importance of balancing the idealization errors in an analysis, consider again the stress analysis of the bracket shown in Figure 5. The purpose of performing the stress analysis is to determine if the bracket will perform its intended function under the action of the applied loads. Two performance requirements of concern are the ability of the bracket to support the load without failing, and to maintain the overall deformations within a specific limit.

Early in the design process, simplified analyses using basic strength of material formula coupled with stress concentration formula may be used. Alternatively, or in addition, the bracket may be



a) b)

Figure 8. Adaptive hp-refinement: a) preliminary mesh, b) final mesh

analyzed using a finite element solution procedure under the assumption that a more accurate solution will be obtained.

A common domain idealization that may be used in the finite element analysis is to simplify the representation of the small fillets by using sharp corners in their place. This idealization may be used because the analysis is being performed at a point in the design process before the fillets are designed, or it is assumed that the finite element analysis process will be simpler if they are not included in the domain analyzed.

This particular form of geometric simplification introduces a singularity in the solution of the stress field at the re-entrant corners. Therefore, the exact solution for the stresses in these corners is infinity.

It may be argued that since the finite element solution on a given mesh will always produce a finite value to the stresses, even in the case of a sharp corner, this simplification is adequate. If the goal of the analysis is to only predict the deformations of the bracket, this is an acceptable geometric simplification since its effect on overall deformations will be negligible. However, if the goal of the analysis is to obtain an estimate of the peak stresses for use in a fatigue prediction procedure, it is not adequate. This is because the value of the stress obtained in the vicinity of the corner will be solely a function of the mesh. No matter what mesh is used, making it just a little finer at the re-entrant corners will cause sizable increases in the stresses, leading to a different prediction of the fatigue life.

To demonstrate the importance of balancing the influence of idealizations on the solution obtained, the bracket is analyzed by both maintaining the proper geometry, and idealizing the fillets with sharp re-entrant corners. The adaptive analysis procedure employs analytically-based error estimation procedures to control the discretization error in total strain energy. Since the total strain energy in both cases is finite, the adaptive solution process converges. Figure 9 shows a close-up view of the meshes in the area of the fillets for both the exact and simplified geometry. An examination of the convergence of this stress shows that the value predicted in the mesh of Figure 9a has changed very little in the last step of the adaptive solution process, while the value predicted for the mesh in Figure

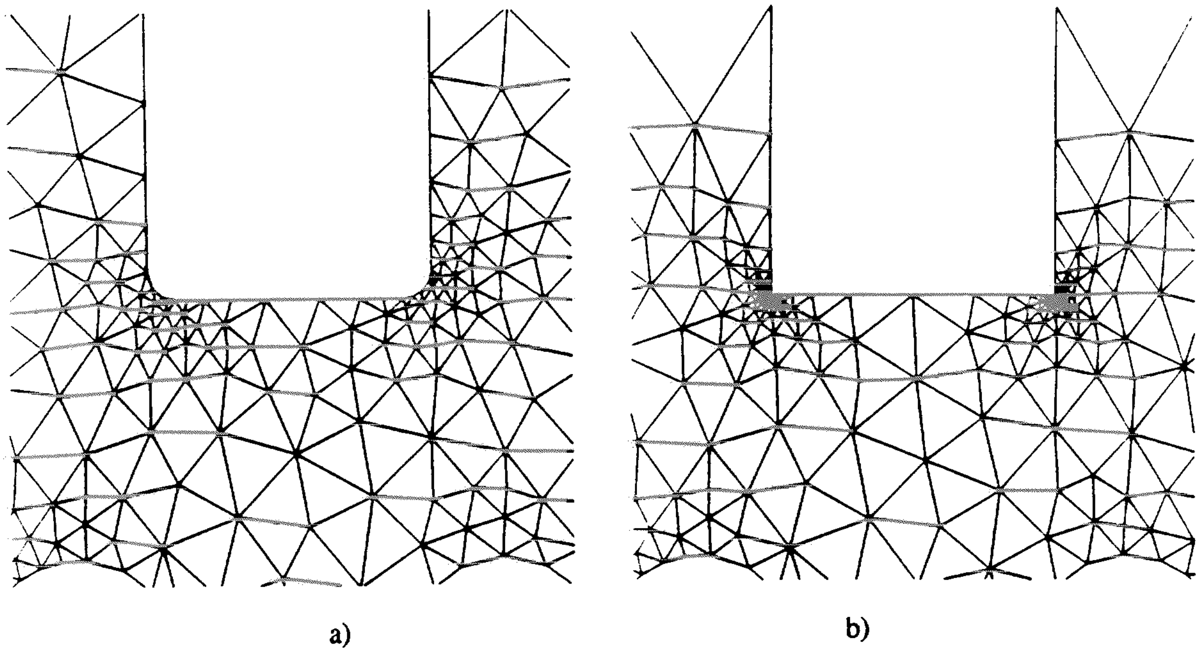


Figure 9. Close-up view of mesh at fillets: a) complete geometry, b) simplified geometry

9b continues to demonstrate large changes with each mesh refinement. This is not surprising since the stress field for the complete geometry converges to a finite value, while the stress fields at the re-entrant corners in the idealized case are infinite and will not converge.

It is also interesting to note that since the idealized geometry introduced stress singularities into the solution the idealized geometry required twice the computer time required for the automated analysis of the idealized geometry. Therefore, in this particular example, the solution cost was twice that needed to obtain a result which over-predicted a particular quantity by more than 350%.

A solution to the situation demonstrated by this example is to always use the complete geometry in any analysis. However, it is not possible to include geometric details that are not yet defined, even though it may be desirable to perform an analysis to determine the overall deformations. In addition, the number of small geometric details may be so large that their inclusion in an analysis will dominate the entire solution process and make it unacceptably expensive. The appropriate solution to the above issues is to provide the engineer with tools that are part of a design system that helps control the idealizations performed and indicates the limitations of analyses performed based on the idealizations used.

CLOSING REMARKS

The availability of advanced general purpose finite element analysis codes such as MSC/NASTRAN and adequate computational power at reasonable cost, has made it possible to perform accurate engineering analysis at almost every step of the engineering design process. However, the effective application of finite element analysis in the design process requires that techniques that control all analysis idealization steps be employed. This paper has outlined the various techniques that can be used to control engineering analysis idealizations, and demonstrated the application of a few of them on specific problems.

Since the techniques available to control engineering analysis idealizations vary in level of reliability and underlying technology, it is important that idealization control techniques be available within a general modeling system. A consideration of the requirements of such a system leads to the definition of a framework that is consistent with those being proposed to support design methodologies.

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REFERENCES

[1] I. Babuska. Uncertainties in engineering design: Mathematical theory and numerical experience. In J. E. Bennett and M. E. Botkin, editors, *The Optimum Shape: Automated Structural Design*, pages 171–197, Plenum Press, NY, 1986.

[2] I. Babuska. Adaptive mathematical modeling. In J. E. Flaherty, P. J. Paslow, M. S. Shephard, and J. D. Vasilakis, editors, *Adaptive Methods for Partial Differential Equations*, pages 1–14. SIAM, 1989.

[3] I. Babuska, O. C. Zienkiewicz, J. Gago, and d. A. Oliveria, editors. *Accuracy Estimates and Adaptive Refinements in Finite Element Computations*. John Wiley, Chichester, 1986.

[4] P. L. Baehmann. *Automated Finite Element Modeling and Simulation*. PhD thesis, Rensselaer Design Research Center, Rensselaer Polytechnic Institute, Troy, NY, May 1989.

[5] P. L. Baehmann and M. S. Shephard. Adaptive multiple level h-refinement in automated finite element analyses. *Eng. with Computers*, 5(3/4):235–247, 1989.

[6] J. A. Bennett and M. E. Botkin, editors. *The Optimum Shape: Automated Structural Design*. Plenum Press, New York, NY, 1986.

[7] A. Budhiraja, R. Wentorf, M. S. Shephard, and E. V. Korngold. The functional model of an aircraft fuselage and idealization procedures for internal load distribution. Technical Report TR-89059, Rensselaer Polytechnic Institute, Rensselaer Design Research Center, Program for Automated Modeling, 1989.

[8] J. R. Dixon, M. J. Guenette, R. K. Irani, E. H. Nielsen, M. F. Orelup, and R. V. Welch. Computer-based models of design processes: The evaluation of designs for redesign. In *Preprints of NSF Engineering Design Research Conference*, pages 491–506, U. Massachusetts, Amherst, MA, 1989.

[9] J. R. Dixon, E. C. Libardi, and N. E. H. Unresolved research issues in development of design-with-features. In *Geometric Modeling for Product Engineering*, pages 183–196. North-Holland, Amsterdam, 1990.

[10] R. V. Dompka, M. C. Sciascia, D. R. Lindsay, and Y. T. Chung. Plan, formulate and discuss a nastran finite element vibrations model of the bell acap helicopter airframe. Technical report, NASA, Langley Research Center, Hampton, VA, 1989. NASA Contractor Report 181774.

[11] J. E. Flaherty, P. J. Paslow, M. S. Shephard, and J. D. Vasilakis, editors. *Adaptive Methods for Partial Differential Equations*. SIAM, Philadelphia, PA, 1989.

[12] R. Gabel, W. J. Kesack, and R. A. Reed. Planning, creating and documenting a NASTRAN finite element vibration model of a modem helicopter. NASA Contractors Report 165722, NASA Langley, Hampton, VA, 1981.

[13] M. K. Georges. *A Two-Dimensional Automatic Modeling System for the h-p Version of the Finite Element Method*. PhD thesis, Rensselaer Design Research Center, Rensselaer Polytechnic Institute, Troy, NY, 1989.

[14] H. R. Grooms and J. H. Johnson. Structural analysis of the space shuttle orbiter. In *Research in Structures and Dynamics - 1984*. NASA Scientific and Technical Info. Branch, 1984. NASA Conf. Publ. 2335.

[15] M. R. Henderson. Feature definition techniques analysis. In *Geometric Modeling for Product Engineering*, pages 321–335. North-Holland, Amsterdam, 1990.

[16] IntelliCorp, Inc. *KEEtutor: A Basic Course*, 1988.

[17] E. V. Korngold, M. S. Shephard, R. Wentorf, and D. L. Spooner. Architecture of a design system for engineering idealizations. In *Advances in Design Automation - 1989: Vol. 1, Computer Aided and Computational Design*, pages 259–265. ASME, 1989.

[18] The MacNeal-Schwendler Corporation, 815 Colorado Boulevard, Los Angles, CA. *MSC/NASTRAN User's Manual*, 1985.

[19] M. C. Y. NIU. *Airframe Structural Design*. Conmilit Press Ltd., Hong Kong, 1988.

[20] E. Rank and I. Babuska. An expert system for the optimal mesh design in the hp-version of the finite element method. *Int. J. Numer. Meth. Engng.*, 24:2087–2106, 1987.

[21] S. A. Safier and M. S. Fox. The role of archtiecture in computer-assisted design systems. In *Preprints of NSF Engineering Design Reaserch Conference*, pages 507–520, U. Massachusetts, Amherst, MA, 1989.

[22] M. S. Shephard. Idealization in engineering modeling and design. *Research on Engng. Design*, to appear:—, 1990.

[23] M. S. Shephard, P. L. Baehmann, M. K. Georges, and E. V. Korngold. Framework for the reliable generation and control of analysis idealizations. *Comp. Meth. Appl. Mech. Engng.*, –:–, 1990. in review.

[24] M. S. Shephard, E. V. Korngold, and R. Wentorf. Design systems supporting engineering idealizations. In *Geometric Modeling for Product Engineering*, pages 279–300. North-Holland, Amsterdam, 1990.

[25] M. S. Shephard, Q. Niu, and P. L. Baehmann. Some results using stress projectors for error indication and estimation. In J. E. Flaherty, P. J. Paslow, M. S. Shephard, and J. D. Vasilakis, editors, *Adaptive Methods for Partial Differential Equations*, pages 83–99. SIAM, 1989.

[26] G. Strang and G. J. Fix. *An Analysis of the Finite Element Method.* Prentice-Hall, Englewood Clifs, N.J., 1973.

[27] B. A. Szabo. *PROBE: Theoretical Manual.* NOETIC Technology Corporation, 7980 Clayton Road, Suite 205, St. Louis, MO 63117, 1985.

[28] B. A. Szabo. Estimation and control of error based on p convergence. In I. Babuska, O. C. Zienkiewicz, J. Gago, and de A. Oliveria, editors, *Accuracy Estimates and Adaptive Refinements in Finite Element Computations*, pages 61–78. John Wiley, Chichester, 1986.

[29] B. A. Szabo. Geometric idealizations in finite element computations. *Communications in Applied Numerical Methods*, 4(3):393–400, May-June 1988.

[30] B. A. Szabo. On errors of idealization in finite element analysis of structural connections. In J. E. Flaherty, P. J. Paslow, M. S. Shephard, and J. D. Vasilakis, editors, *Adaptive Methods for Partial Differential Equations*, pages 15–28. SIAM, 1989.

[31] M. Vogelius. On mathematical modeling - dimensional reduction. Technical report, Institute for Physical Science and Technology, University of Maryland, College Park, Maryland, 1980. Tech. Note BN-940.

[32] R. Wentorf, M. S. Shephard, and E. V. Korngold. The use of functional models to control engineering idealizations. In *Proc. of 1989 ASME International Computers in Engineering Conference.* ASME, 1989.

[33] O. C. Zienkiewicz and J. Z. Zhu. A simple error estimator and adaptive procedure for practical engineering analysis. *Int. J. Numer. Meth. Engng.*, 24:337–357, 1987.