THE SURFACE OF ULTIMATE STRENGTH OF THIN-WALLED BEAMS

by

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ABSTRACT

A MSC/NASTRAN pre- and post-processor has been developed in CANADAIR [1] to analyse thin-walled beam-sections in elastic and plastic range. The plastic module is presented in this paper.

The graphic input and the output file of the surface of ultimate strength is created and displayed in CADAM [3].

The coordinates of a point on the surface define three ultimate allowable loads P, M1, M2 and the position of the plastic neutral axis. Any loadcase may be represented by a 3-D point in the same space.

The surface would shrink if shear stresses were taken into account but the corresponding point on the shrunken surface would define six ultimate allowable loads: P, M1, M2, Q1, Q2, T.

INTRODUCTION

The idea of a family of interactive curves creating a surface of ultimate strength is not new [2] but it couldn´t be practically accomplished without powerful computer graphics.

The surface of ultimate strength developed in CADAM [1], [3] is a convenient tool in analysis of load sensitivity and plastic margins of safety in thin-walled beams.

Two types of surfaces have been developed:
- Surface (a) of ultimate normal strength;
- Surface (b) of ultimate normal strength reduced due to shear.

(a): The three coordinates of any point on surface (a) are defined by three ultimate allowable loads related to normal stresses:
- Endload "P";
- Bending moment in plane 1 "M1";
- Bending moment in plane 2 "M2".

The acting loads P(L), M1(L), M2(L) define the three coordinates of point "L". The line $\overline{OL}$ from the origin "O" to point "L" extended to the surface (a) defines the point "A" of intersection and the ultimate allowable loads: P(A), M1(A), M2(A) as coordinates of the point "A".

The normal stresses are defined for any material of the beam as ultimate tensile or compressive allowables.

The plastic margin of safety may be analysed for any load case using the same surface (a).

(b): The surface (b) is created when both shear and normal stresses are taken into account. The surface (b) is created in the same way as surface (a) but the allowable normal stresses are reduced to meet Von Mises criterion for combined shear and normal stresses.

The shear stresses may be caused by:
- Shear force in plane 1 "Q1";
- Shear force in plane 2 "Q2";
- Torque "T".

Point "B" on intersection of the line $\overline{OA}$ with the surface (b) will define all six ultimate allowable loads: P(B), M1(B), M2(B), Q1(B), Q2(B), T(B), but the margin of safety may be calculated for one loadcase only since the surface (b) depends on loads Q1(L), Q2(L) and T(L).

SURFACE (a) OF ULTIMATE NORMAL STRENGTH

The three coordinates: P(A), M1(A), M2(A) of the point "A" on the surface depend on the position of the neutral axis (Fig.1) and may be calculated from formulas (1),(2) and (3):

$$P(A) = \sum_{i=1}^{n} [(Fb)(ti)(si)] \qquad \text{.......... (1)}$$

$$M1(A) = \sum_{i=1}^{n} [(Fb)(ti)(si)(yi)] \qquad \text{.......... (2)}$$

$$M2(A) = \sum_{i=1}^{n} [(Fb)(ti)(si)(zi)] \qquad \text{.......... (3)}$$

The allowable stresses (Fb) are equal to tensile allowable (Fst) on one side of the axis but they are equal to compressive allowable (Fcc) on the other side of the neutral axis.

There are two possible definitions of allowable stresses on both sides of the neutral axis and each of them will be related to one of the two parts of the surface: lower and upper.

-For the upper surface:

$$Fb = Fst \quad \text{......} \quad \text{if } [(zi)\sin(\alpha)-(yi)\cos(\alpha)+d]>0 \quad \text{.......... (4)}$$

$$Fb = -Fcc \quad \text{......} \quad \text{if } [(zi)\sin(\alpha)-(yi)\cos(\alpha)+d]<0 \quad \text{.......... (5)}$$

-For the lower surface:

$$Fb = Fst \quad \text{......} \quad \text{if } [(zi)\sin(\alpha)-(yi)\cos(\alpha)+d]<0 \quad \text{.......... (6)}$$

$$Fb = -Fcc \quad \text{......} \quad \text{if } [(zi)\sin(\alpha)-(yi)\cos(\alpha)+d]>0 \quad \text{.......... (7)}$$

This way two points may be defined for every position of the neutral axis: one point on the upper surface and the second on the lower one. The expressions "upper" and "lower" are conventional and are used to distinguish both parts of the surface rather than to describe their localization.

The position of the neutral axis is defined by the angle "α" and the distance "d" from centroid as shown on Fig. 1.

The coordinates P, M1 and M2 of every point on the surface (a) should be scaled to get units of length in any direction.

It is convenient to group points into splines for better visual presentation. Dashed lines on Fig.2 represent splines of the upper surface (a) of the beam section shown on Fig.3. All input data are described in [1]. Every spline on the surface (a) corresponds to one angle "α" of the neutral axis and every point on a spline corresponds to one distance "d".

The angle "α" should vary from 0 to 180 degree and the distance "d" should vary from one extreme fibre to the other.

This approach has been developed for thin-walled beam sections, but it may be also used to create a surface of ultimate strength for solid beam sections.

The coordinates of the point "L" which represents the applied loads should be scaled in the same way as coordinates of points on the surface (a).

The normal load ratio R(a) and plastic margin of safety M.S.(a) may be calculated from formulas (8) and (9):

$$R(a) = \overline{OL}/\overline{OA} = P(L)/P(A) = M1(L)/M1(A) = M2(L)/M2(A) \quad (8)$$

$$M.S.(a) = [1/R(a)]-1 \quad (9)$$

Once the surface (a) is created one can not only calculate plastic margin of safety for any loadcase but also analyse the load sensitivity of the beam. We can cut the surface with plane M2=0 and get the M1 versus P interactive curve as shown on Figs.2 and 6 and analyse the load sensitivity of the beam. We can learn from this interactive curve that maximum allowable bending moment M1(max) doesn't correspond to P=0 as it was assumed by Cozzone [5]. The M2 versus P and M2 versus M1 interactive curves are even more irregular.

Some of these irregularities in 2-D interactive curves were analysed in [6].

The beam may be also analysed by NASTRAN [4] in the plastic range using Solution 66 and PBCOMP cards but every loadcase needs a separate run and interactive curves are not available in MSC/NASTRAN.

THE SURFACE (b) OF ULTIMATE NORMAL STRENGTH REDUCED DUE TO SHEAR

The ultimate allowable normal stresses "Fb" should be reduced to "fb" and the ultimate allowable shear stresses "Fs" should be reduced to "fs" if normal and shear stresses are acting simultaneously on the beam section. This normal and shear stress interaction is defined in MIL-HDBK-5 [2] as follows:

$$[fs/Fs]^2 + [fb/Fb]^2 = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

If $(Fb)^2 = 3(Fs)^2$ the formula (10) would be identical with the Von Mises criterion (11).

$$(fb)^2 + 3*(fs)^2 = (Fb)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The load ratio "R" is defined by formula (12):

$$R = \overline{OL}/\overline{OB} = P(L)/P(B) = M1(L)/M1(B) = M2(L)/M2(B) = T(L)/T(B) =$$

$$= Q1(L)/Q1(B) = Q2(L)/Q2(B) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The same load ratio "R" may also define the relation between acting shear stresses "fsa" and allowable shear stresses "fs" as shown in formulas (13):

$$fsa = (R)*(fs) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The ultimate allowable normal stresses reduced due to shear "fb" may be evaluated from formulas (10) and (13) as follows:

$$fb = Fb \{ 1 - [fsa/(R*Fs)]^2 \}^{0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The three coordinates: P(B), M1(B), M2(B) of the point "B" on surface (b) may be calculated from formulas (1), (2), (3) but the ultimate allowable normal stress "Fb" should be replaced by the reduced allowable normal stress "fb".

The load ratio "R" is defined by the coordinates of points "L" and "B" in formula (12) but to calculate the coordinates of the point "B" we need to know the value of the load ratio first.

This problem may be solved by iterations. In the first step we can assume arbitrary R=1 and calculate the allowable stresses from formula (14) and create the first approach of surface (b). Now we can calculate the second load ratio from formula (12).

The iterations should be continued until the differences between the value of load ratio "R" calculated in the previous step and those in the present one are satisfactorily small. The load ratio "R" represents all six loads as shown in the formula (12).

The corresponding plastic margin of safety M.S.(b) may be calculated from formula (15):

M.S.(b) = [1/R]-1 .... (15)

The difference between the surface (a) and (b) depends on the level of shear stresses. Both surfaces are compared on Figs.6,7 and 8. The loads are shown on Fig.3, the corresponding shear stress graph on Fig.4 and the printed report sheet on Fig.5.

Figs.6,7 and 8 also show the "P" versus "M1", "P versus "M2" and "M1" versus "M2" interactive curves respectively.

The surfaces (a) and (b) are analysed using a post-processor called "PLASTIC",which has been developed in CADAM but it can be translated into any other 3-D graphic system like MSC/XL,PATRAN, CATIA or CAEDS(SUPERTAB).

MSC/NASTRAN PRE- AND POST-PROCESSORS DEVELOPED IN CADAM

The MSC/NASTRAN pre- and post-processors [1] are written in FORTRAN but the input and output files are stored in CADAM. An example of the input file is shown on Fig.3.

The pre-processor calculates section properties in MSC/NASTRAN format as PBAR or PBEAM cards. A complimentary file stored in CADAM contains four graphs:

-Shear flow due to unitary vertical load "Q1=1",
-Shear flow due to unitary horizontal load "Q2=1",
-Warping due to unitary twist angle "ϕ'=1",
-Shear flow due to unitary warping torsion "TW=1".

The section properties and the coordinates of the shear centre are also printed on the report form.

This MSC/NASTRAN pre-processor has been used in CANADAIR since 1985.

There are also two MSC/NASTRAN post-processors:

-"ELASTIC" and
-"PLASTIC".

The post-processor ELASTIC calculates normal, shear and Von Mises stresses and stores them as three graphs in the CADAM output file. The formulas for stresses have been evaluated by Timoshenko [7].

The complimentary report sheet contains the stress ratio, margin of safety and indicates the place of the maximum stress ratio.

Eight loads may be considered: P,M1,M2,Q1,Q2,T,TW,BM. All but one may be found in MSC/NASTRAN output. The eighth one: "BM" is the bi-moment named also "M_ω" by Meyer [8].

The post-processor PLASTIC calculates the surface(a) or (b) and stores it in CADAM. The complimentary FORTRAN output prints scale factors for the surface, the ultimate allowable loads, the position of plastic neutral axis, load ratio and the plastic margin of safety, which is also repeated on the report sheet as shown on Fig.5.

Both post-processors have been used in CANADAIR since 1986.

FOLLOW-UP

The post-processor PLASTIC has been used in the analysis of single beam sections, mostly for crash load conditions and for some ultimate load conditions.

This post-processor could also analyse more complex beam structures like a section of wing or fuselage where the advantage of using the surface of ultimate strength would increase even more.

The complex beam sections of a wing or a fuselage could be also analysed in the plastic range in case of discrete sources damage or damage tolerance conditions where plastic stress distribution is acceptable [9].

An example of fuselage section with rotorburst damage is shown on Fig.9 and the correspondent interactive curves M1 versus M2 are shown on Fig.10.

Future development of the post-processor PLASTIC will be oriented to enhance the analysis of complex thin-walled beam structures.

Another future development may be oriented towards solid beam sections.

CONCLUSIONS

The MSC/NASTRAN post-processor PLASTIC developed in CANADAIR analyses the surface (a) of ultimate normal strength and the surface (b) of ultimate normal strength reduced due to shear. It has proved to be a convenient tool in the analysis of load sensitivity and plastic margins of safety in thin-walled beams.

Future development will be oriented towards complex beam structures like a whole fuselage or wing. Another future development may be oriented towards solid beam sections.

The post-processor PLASTIC has been developed in CADAM but it can be translated into other 3-D graphic systems such as MSC/XL,PATRAN, CATIA or CAEDS (SUPERTAB).

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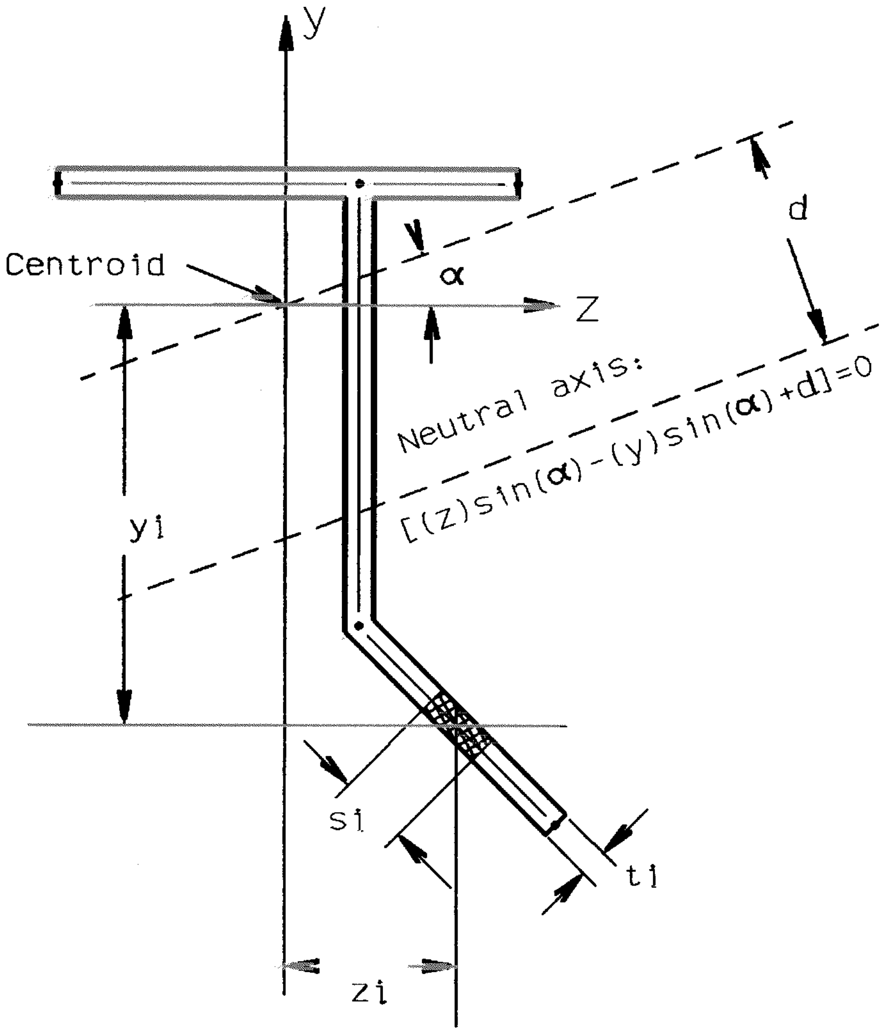
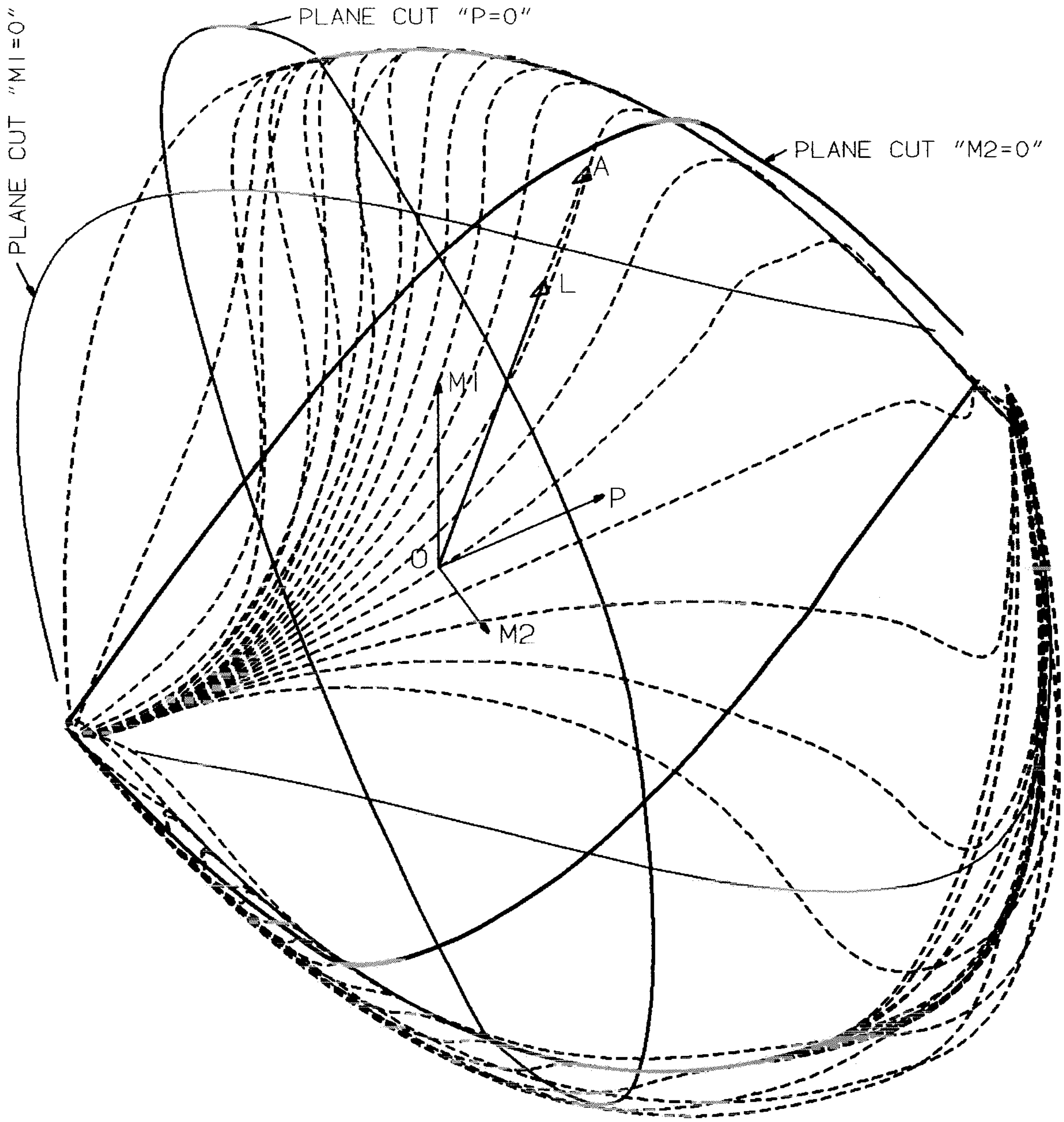


FIG.1.CROSS-SECTION OF THE BEAM



- SCALE FACTOR "P" 1 LBS = .0004877 IN
- SCALE FACTOR "M1" 1 LBS*IN = .0005281 IN
- SCALE FACTOR "M2" 1 LBS*IN = .0025104 IN

FIG.2. SURFACE OF ULTIMATE NORMAL STRENGTH

```
M1=  12000.          EID=100
M2=  120.            LID=00002
Q1=  1800.
Q2=  18.
P= 3600.
T= 0.0000000
TW= 0.000000
LF= 1.0
BM= 0.000000
SC1= 1.0
FACT= 10.0
MATERIAL 1 =AL.7075-T6-QQ250/13
FST1= 73000.
FCC1= 46390.
FSS1= 14000.
MATERIAL 2 =AL.7075-T6-QQ250/13
FST2= 73000.
FCC2= 48980.
FSS2= 28000.
MATERIAL 3 =AL.2024-T62-QQ250/5
FST3= 60000.
FCC3= 48980.
FSS3= 28000.
RIVET A=MS20426-AD5
QALL,A= 610.
PREPARED= Z.BOROWIEC
CHECKED=
APPROVED=
MODEL= GENERAL
PAGE= FIG.5
REPORT= MSC USERS CONF.
DATE= MARCH  1990
SUBJECT= PLASTIC STRESS DISTRIBUTION
TITLE= UPPER SURFACE OF ULTIMATE STRENGTH
SUBTITLE 1 = ALL LOADS APPLIED
```

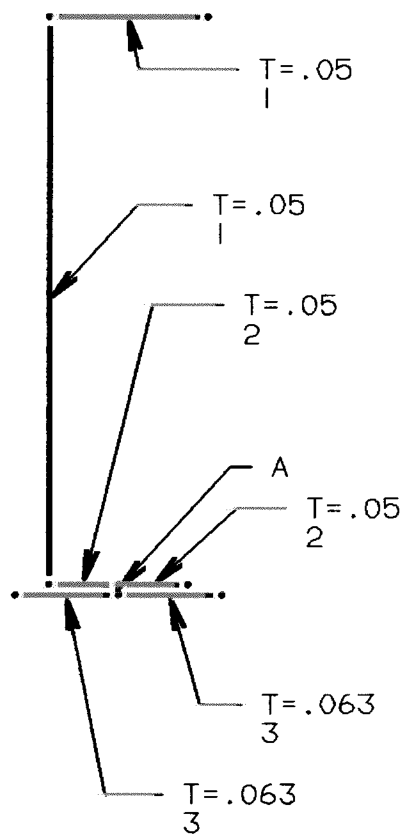
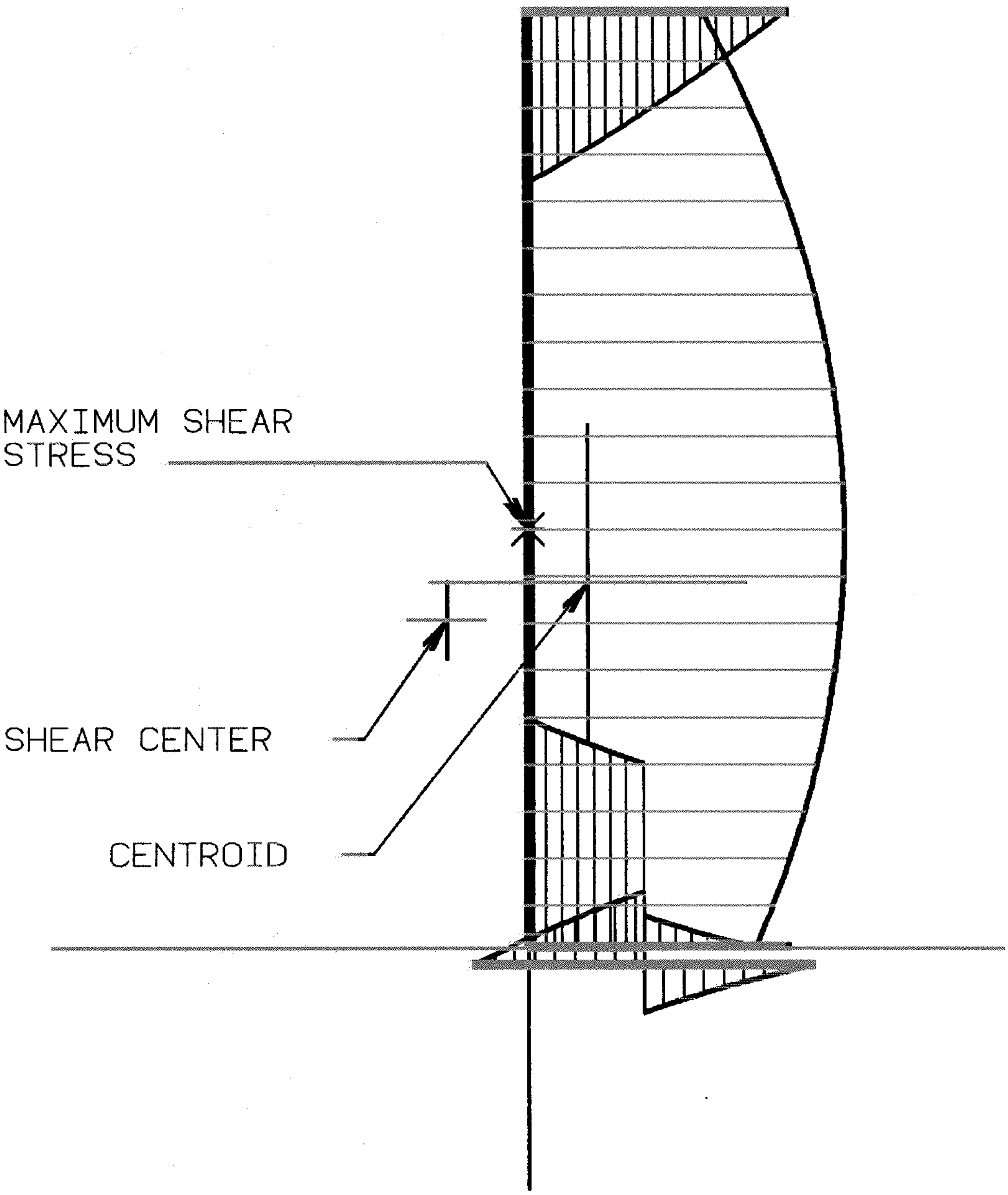


FIG.3.EXAMPLE OF INPUT DATA FILE



EID=100 LID=00002

FIG.4.SHEAR STRESS GRAPH

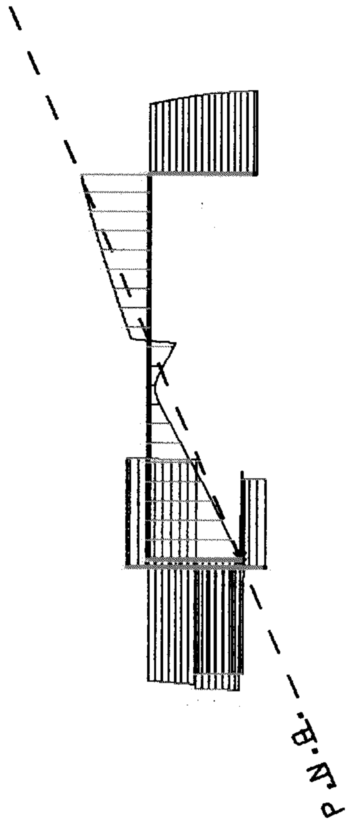
UPPER SURFACE OF ULTIMATE STRENGTH

ALL LOADS APPLIED

EID=100 LID=00002

M1= 12000.
M2= 120.
Q1= 1800.
Q2= 18.
P= 3600.
T= 0.
TW= 0.
BM= 0.
LF= 1.0
SC1= 1.0
FACT= 10.
FST1= 73000.
FCC1= 46390.
FSS1= 14000.
FST2= 73000.
FCC2= 48980.
FSS2= 28000.
FST3= 60000.
FCC3= 48980.
FSS3= 28000.
FST4= 0.
FCC4= 0.
FSS4= 0.

ULTIMATE NORMAL STRESSES
REDUCED DUE TO SHEAR



SCALE=0.68

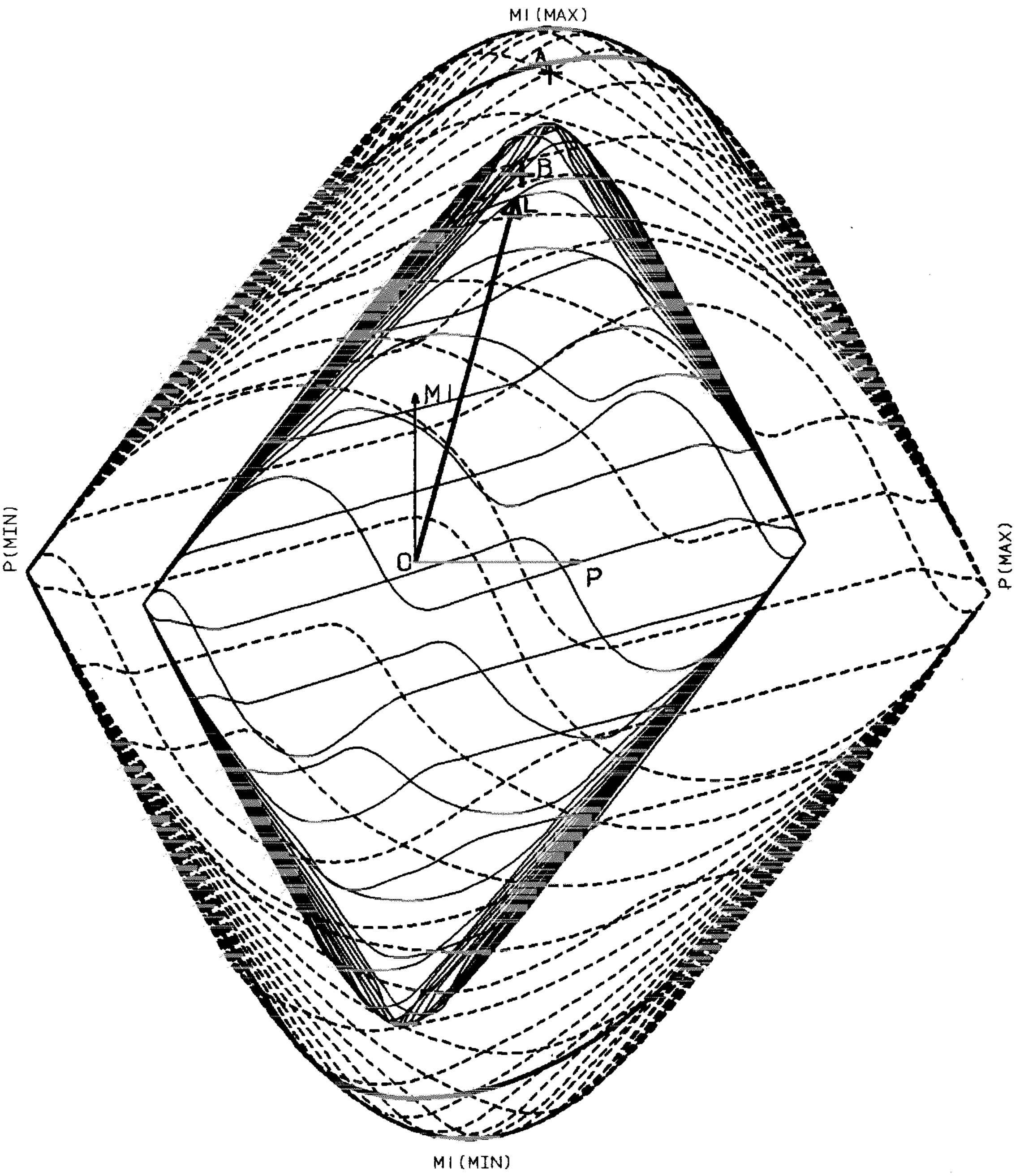
ALLOWABLE ULTIMATE LOADS

M1,ALL= 12163. LB*IN   M2,ALL= 122. LB*IN
Q1,ALL= 1824. LBS   Q2,ALL= 18. LBS
P,ALL= 3649. LBS   T,ALL= 0. LB*IN
TW,ALL= 0. LB*IN   BM,ALL= 0. LB*IN**2

(PLASTIC) LOAD RATIO = 0.987

MARGIN OF SAFETY (PLASTIC) M.S.= 0.01

FIG.5. PRINTED OUTPUT FILE



——— P VERSUS MI INTERACTION FOR M2=0 IN SURFACE (a)

- - - - - SURFACE OF ULTIMATE NORMAL STRENGTH (a)

——— SURFACE OF REDUCED NORMAL STRENGTH DUE TO SHEAR (b)

FIG.6.SURFACES (a) AND (b) PLANE P,MI

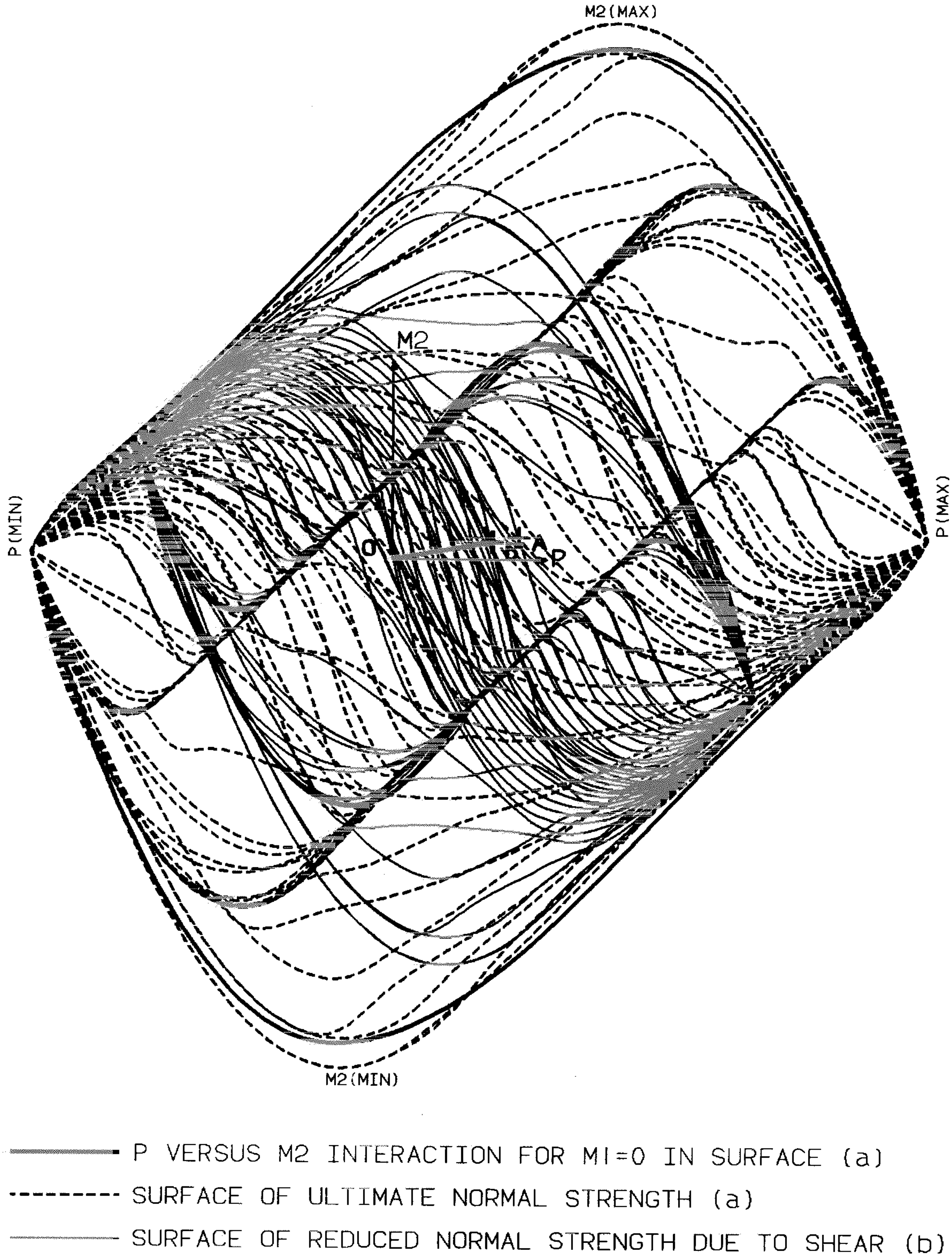


FIG.7.SURFACES (**a**) AND (**b**) PLANE P,M2

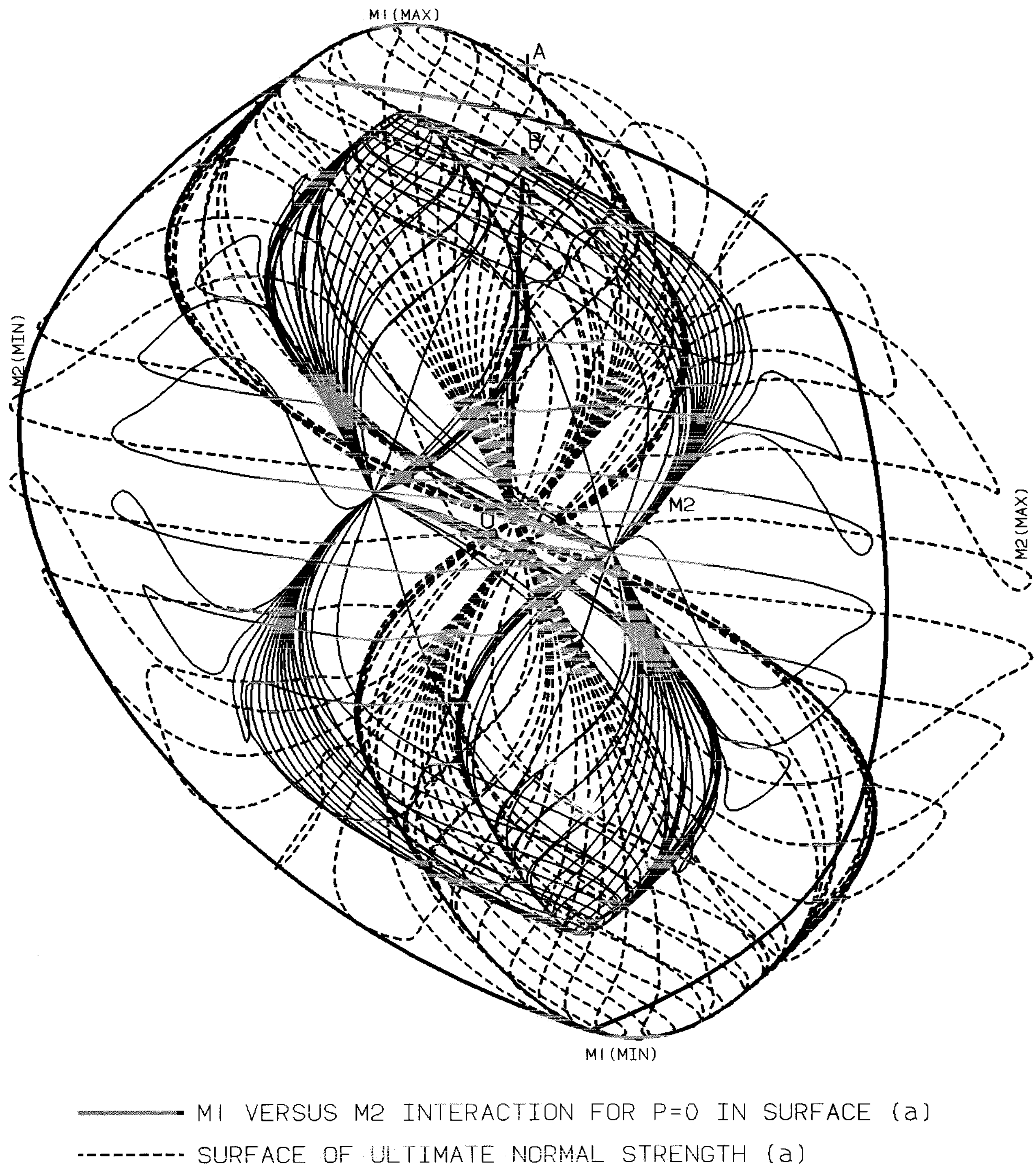


FIG.8.SURFACES (a) AND (b) PLANE M2,MI

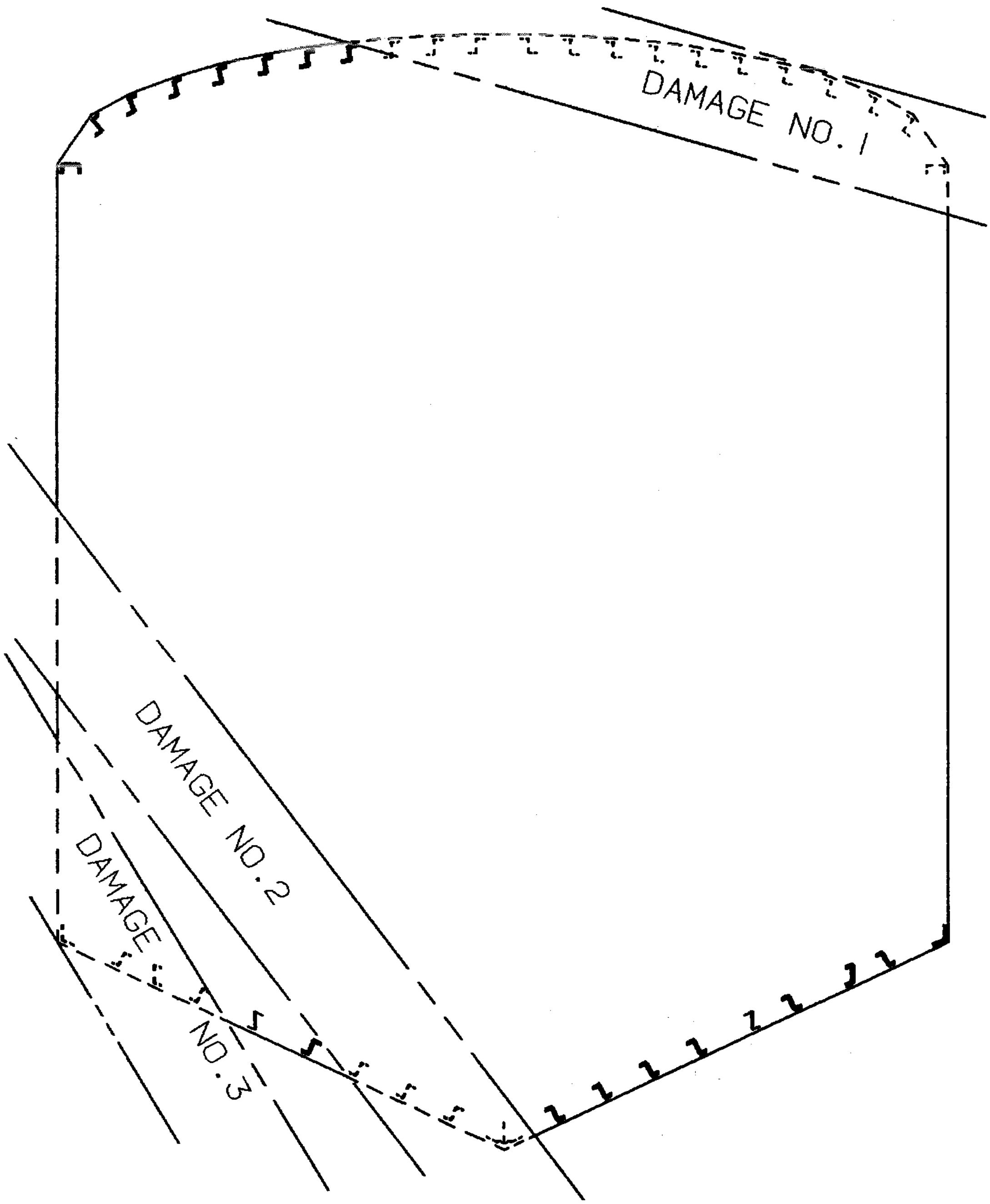


FIG.9.FUSELAGE DAMAGED BY ROTORBURST

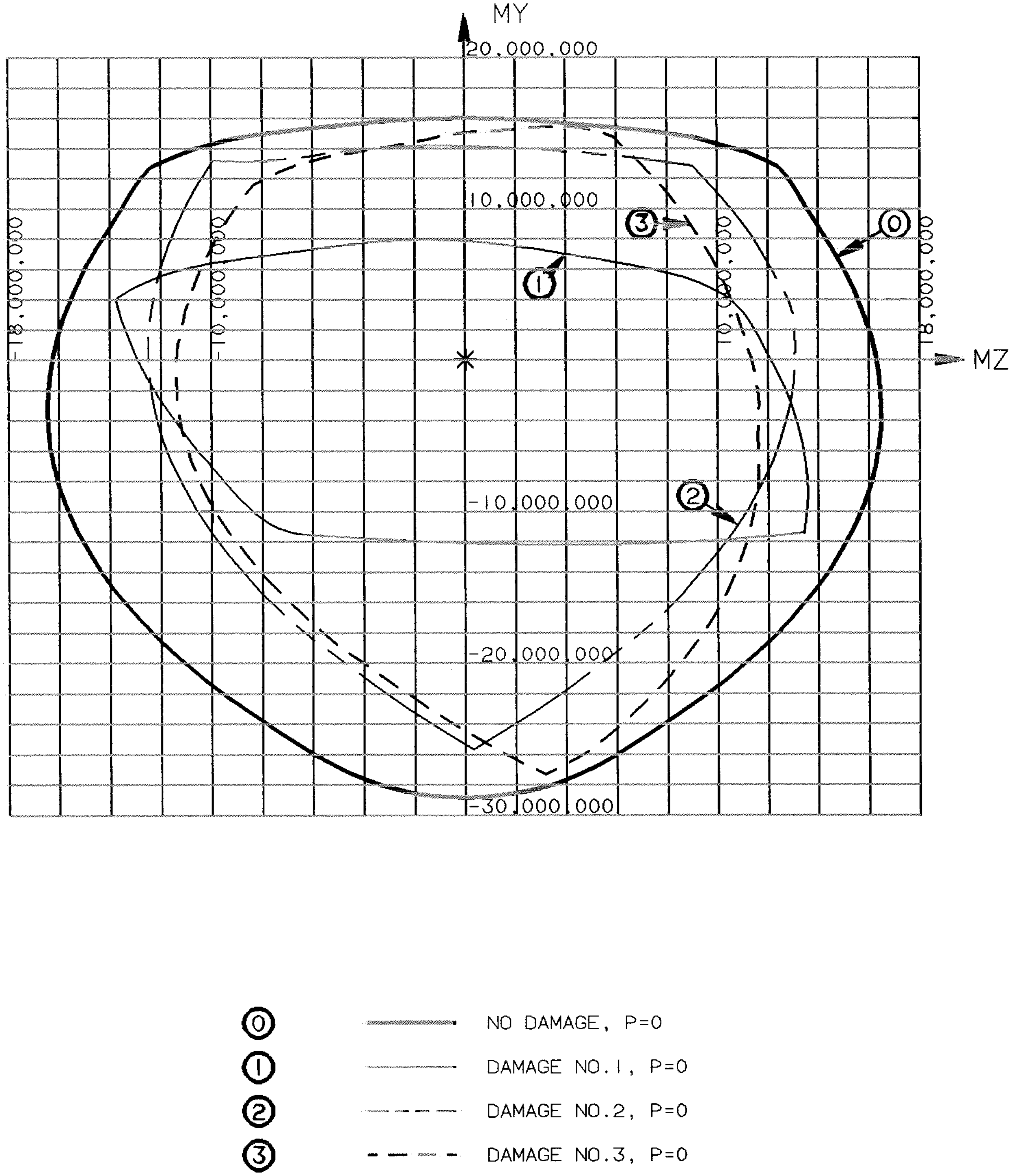


FIG.10. M1 VERSUS M2 INTERACTION FOR DIFFERENT DAMAGES