The CTETRA(10) Element

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INTRODUCTION

Several commercial solid modeling software packages are capable of automatically generating tetrahedron elements within arbitrary solid three-dimensional shapes. This "free meshing" of arbitrary shapes can significantly reduce project turnaround time when compared with conventional "mapped" meshing techniques. Figure 1a is a solid model of an automotive rocker arm which was modelled independently by two individuals. The process of creating this solid model and generating a CTETRA mesh, Fig 1b, required 3 man-days. An analogous CHEXA[1] mesh created using conventional map-meshing techniques (i.e. the user creates the geometry as an assembly of hexahedral shapes, or hyperpatches) required 15 man-days. References [1][2][3][4] typically describe ratios of approximately 2/1 in favor of free meshing. The motivation for using the CTETRA element lies in the fact that it can dramatically reduce turnaround time when used in conjunction with free meshing.

However, there seems to be a consensus among finite element modelers that the CTETRA is either less accurate or less efficient than the CHEXA. Indeed, reference [5] claims that a tenfold compute penalty exists for the use of the parabolic CTETRA compared with a linear CHEXA. The two following benchmark problems establish the relative accuracy of the CTETRA(10) versus CHEXA(8) elements and determine effects of element distortion, particularly aspect ratio.

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[1]The tet model contained 2525 CTETRA(10) elements and 4541 nodes while the hex model contained 707 CHEXA(20) elements and 4447 nodes.

THE JOHN ROBINSON CANTILEVER BEAM TEST

This test was chosen because of its intuitive appeal, simplicity and history. Results for a variety of codes appear in several issues of Finite Element News [6][7][8][9][10][11][12]. The test determines the effect of aspect ratio upon accuracy. It is severe in the sense that only a single hexahedron element is used to model a structure. The test has been around sufficiently long that some users suspect hexahedron elements are optimized for this test.
We have extended this test as described in the following. Originally this test called for aspect ratios up to a value of 8/1. This was not high enough to degrade accuracy, so we continued to double aspect ratios until the results became obviously bad or until the value exceeded 8000/1. We also modified the test to include tetrahedra by filling the hexahedral shape (hyperpatch) with tetrahedra. Because a minimum of five tetrahedron elements are required to fill a hexahedral space, this test is presumed to be biased in favor of the CHEXA. Inasmuch as each of the five tetrahedra may have different aspect ratios, we found it convenient to use the aspect ratio of the parent hyperpatch as the independent variable. This is quite close to the average ratio of long to short sides for the member tetrahedra, except for very low aspect ratio values. Also note that the aspect ratio of a hexahedron is independent of angular measures of element distortion, such as skew and taper. This is not true for tetrahedra. High aspect ratio is always accompanied by severe angular distortion in a tetrahedron. For that reason, one might anticipate poorer performance from a tet under high aspect ratio.

Inspection of the loads and boundary conditions in Figure 2a reveals that the "fixed" end of the cantilever beam is not completely fixed but can expand or contract in that plane. That is, the beam is loaded by two equal and opposite moments, and just enough degrees of freedom are fixed to prevent rigid body motion. This formulation of the problem allows for comparison with results from Theory of Elasticity and includes the small displacement due

to Poisson ratio effects. Results in Figures 2 through 4 show that it is precisely this displacement which first goes awry for each element. In part, this is explained by the fact that this displacement is very small compared to the others. Therefore percentage-wise its error is high.
Figures 2a and 2b show virtually no error in the vertical displacement (v) and the extensional displacement (u) with which the bending stress is associated out to aspect ratio 4000/1. The lateral displacement of the tet model is in error at the observed corner, but is not in significant error at the opposite lower corner. Remember that the tet model is necessarily asymmetrical. The hex model has only 8 nodes X 3 degrees of freedom(dof)/node - 6 constrained dof = 18 dof. The tet model has 26 X 3 - 6 = 72 dof, so that one should conclude that indeed the hex model is more efficient for this test as expected.

If the test is modified as shown in Figure 3 to consist of two CHEXAs end to end or two sets of 5 CTETRAs end to end, the CHEXA again outperforms the CTETRA. Figure 4 shows the performance of the two elements when an internal plane is warped approximately 9 degrees. In that case the performance of the CHEXA(8) is severely degraded by the warping, while the CTETRA(10) performance is virtually unaffected. Although not shown here, other tests indicate that similar degradation occurs for much less warp.

THE ST. VENANT TORSION PROBLEM

Most benchmark problems in the literature deal with bending or extensional behavior. We felt it desirable to test torsion and to find a geometry where the tetrahedron would be at no special disadvantage. The torsion of a uniform triangular prism appeared to be a suitable candidate. Solutions are available in several books on Theory of Elasticity [13][14], and the stress distribution is non-trivial, Figure 5. Ignoring the possibility for cyclic symmetry, a minimum of three CHEXA elements or three CTETRA elements is required to fill the geometry, Figures 6a & 6b. In contrast to the Robinson problem where a known set of loads were applied, we specified tangential displacements at the two ends of the prism and calculated the resulting torque. This was facilitated by the use of a rigid body (RBE2) element, such that the torque was calculated as a (generalized) force of single point constraint at the independent node. This approach incidentally is far easier than attempting to calculate equivalent forces at the nodes of higher order elements.

Warping of cross sections was, of course, permitted.Unfortunately, solutions based on these coarse meshes did not perform sufficiently well for further study. The CHEXA model showed an error in torque of 39% while the CTETRA model showed an error of 56%. We therefore increased the CHEXA model to 12 elements and the CTETRA model to 15 elements, using 3 identical hyperpatches in each case. These hyperpatches were arranged the same as the elements of Fig 6a. The CHEXA model contains 111 dof while the CTETRA model contains 159 dof. The performance of the two elements is remarkably constant across a wide range of element aspect ratios, Figure 7. The tet model shows consistently better torque prediction. Comparing the peak shear stresses at the midside of a plane to the elasticity solution, both elements, (after averaging the stresses from all elements which share a node) show very low error. It is less than 1% for tet and less than 3% for hex elements.

THE PATCH TEST

The "patch test" is a test of element validity, which ensures convergence to the correct answer as the fineness of the mesh is indefinitely increased. An excellent discussion of this test, and some exceptions to the rule, are discussed in [15]. Table 1 is based on information from MSC and shows the status of the CHEXA and CTETRA in versions 65 and 66. The CTETRA did not pass the patch test in V65 for the case of curved edges, but it does meet this test in V66 without exception.

	Version 65	Version 66
CHEXA	Passes	Passes
CTETRA	Fails	Passes

Table 1. Patch Test Performance

CONCLUSIONS

Two elements, the CHEXA(8) and the CTETRA(10), were tested in MSC/NASTRAN version 66 using a traditional bending test and a "new" torsion test. Both elements performed well, even in situations of extremely high aspect ratio (8192/1 was an arbitrary cutoff point).

For regular geometries, especially those composed of straight surfaces and right angles, the CHEXA(8) will consistently outperform the CTETRA(10) even though the tet model has more degrees of freedom. For the situation of irregular geometry, and especially of warped surfaces, the CTETRA(10) can out-perform the CHEXA(8). It is precisely in this complicated geometrical situation where "free meshing" is likely to be employed. We conclude therefore that solid modeling, free meshing, and the use of quadratic tetrahedra should be considered by users of MSC/NASTRAN.

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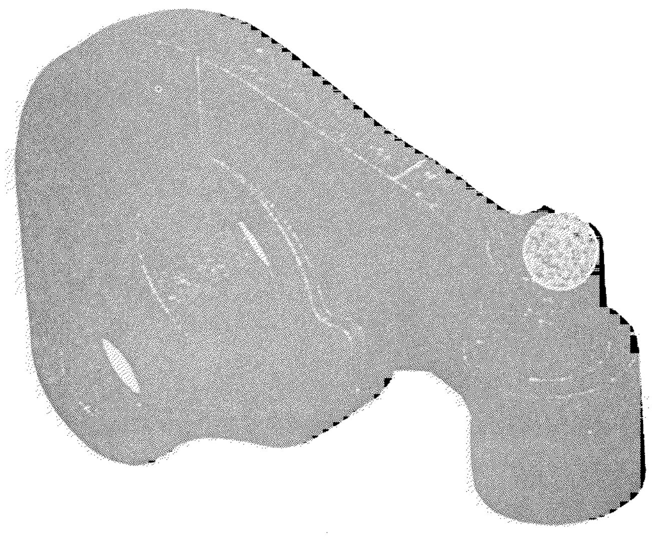


Figure 1a. Solid Model Of Rocker Arm

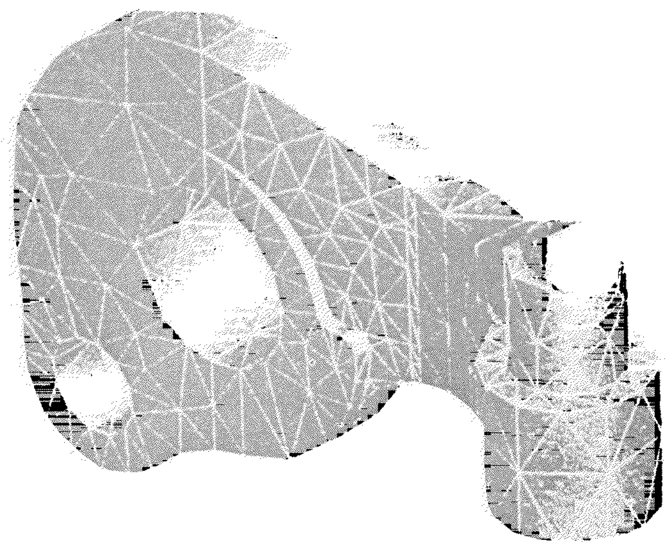


Figure 1b. Free Mesh Of Rocker Arm

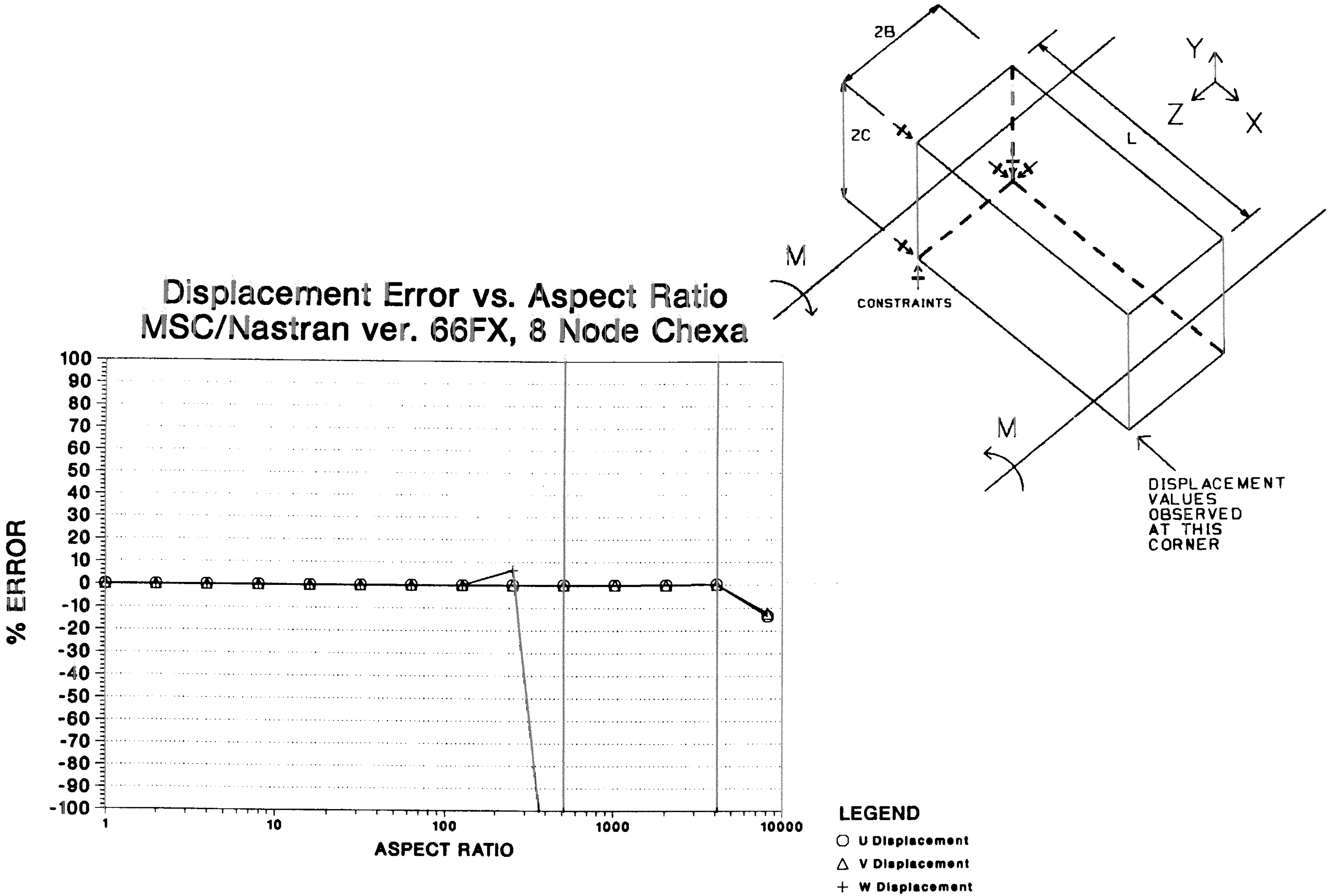


Figure 2a. Robinson Test Using Single Chexa

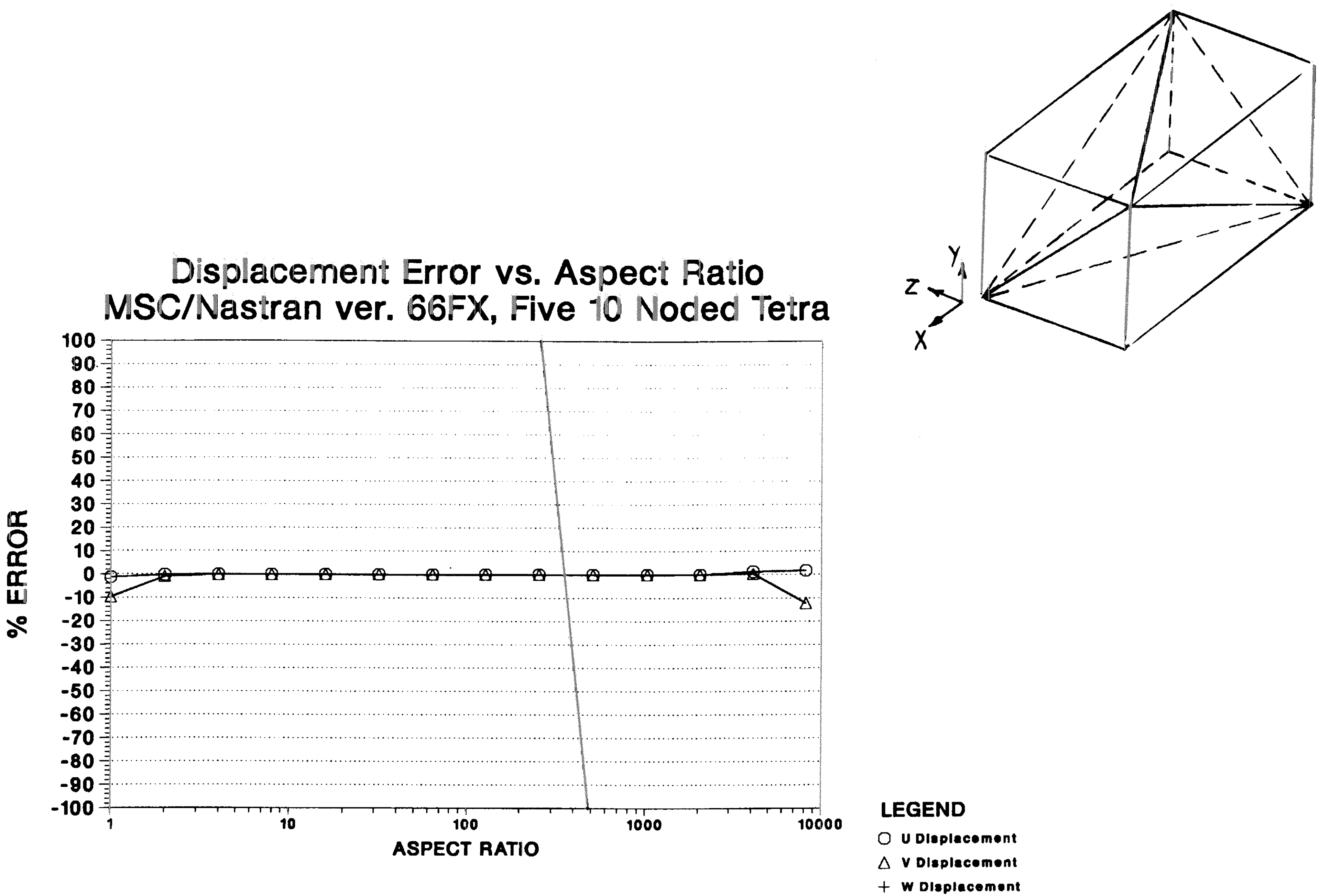
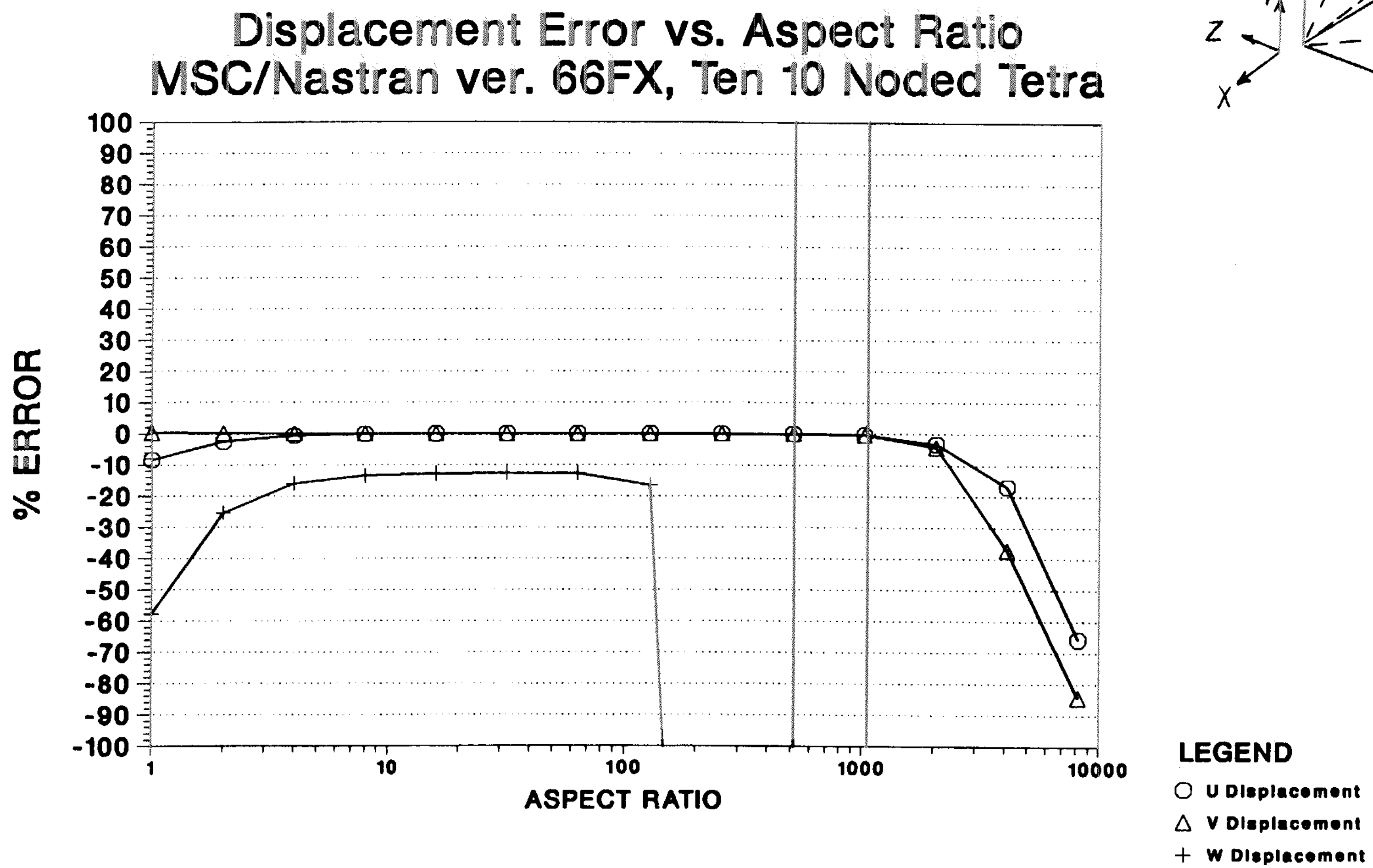


Figure 2b. Robinson Test Using Five Tetra



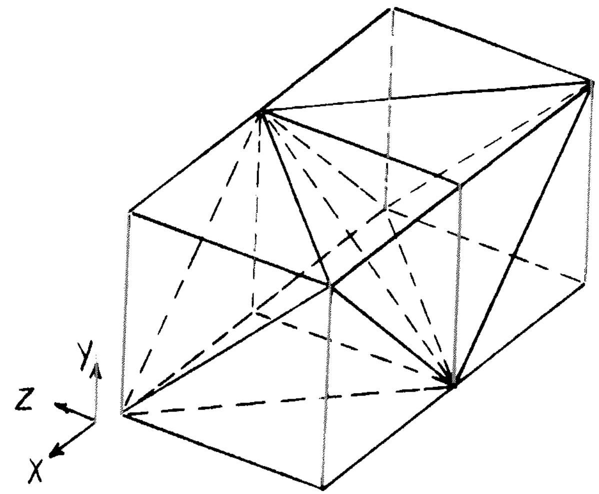
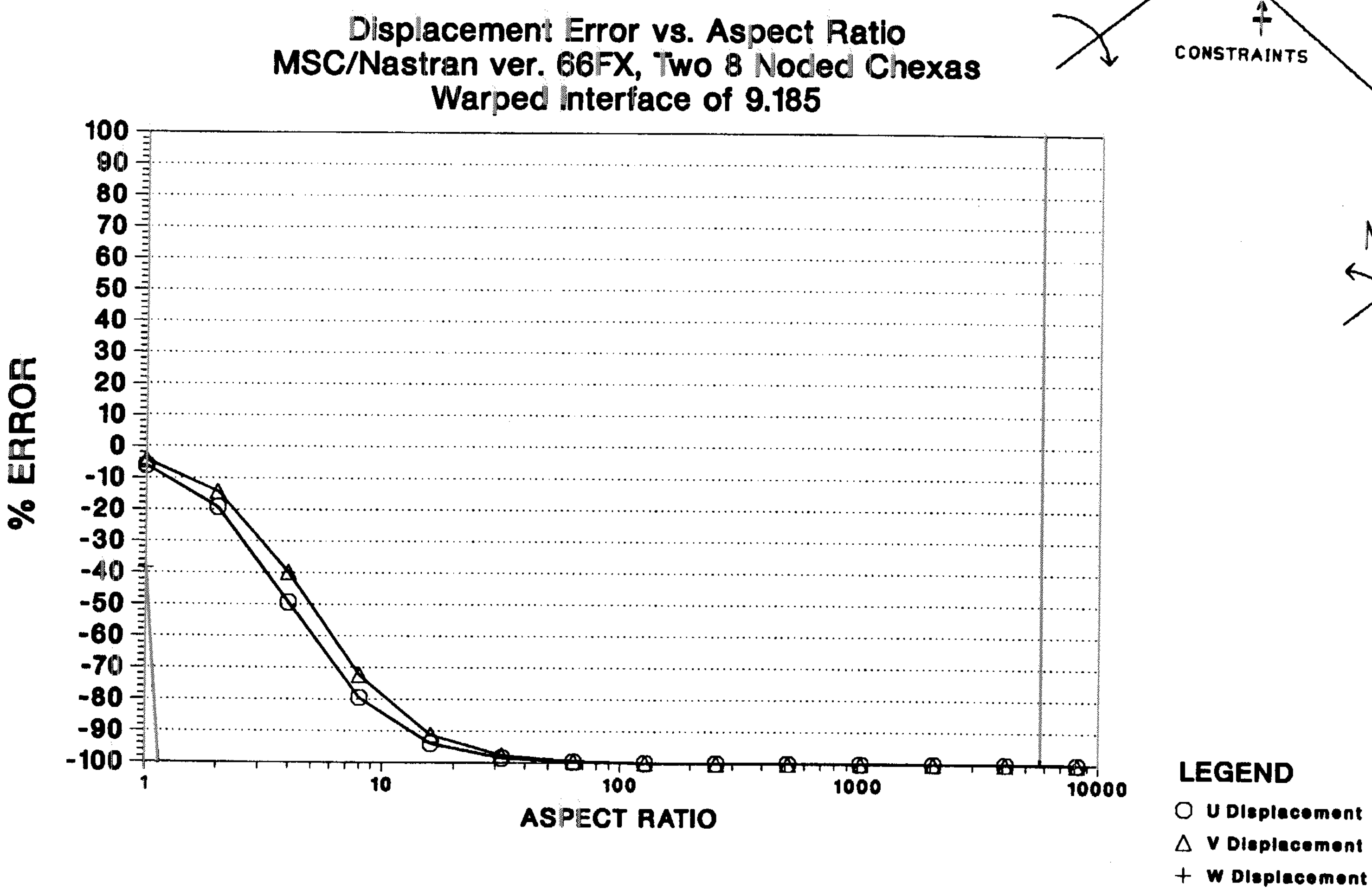


Figure 3b. Robinson Test Using Ten Ctetra



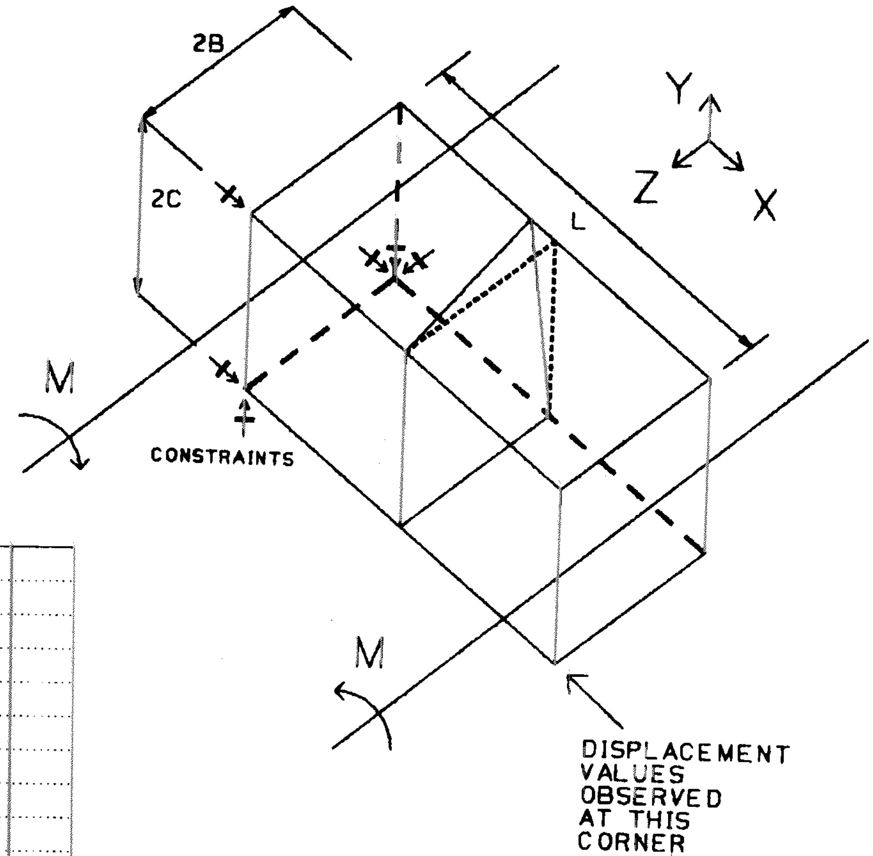
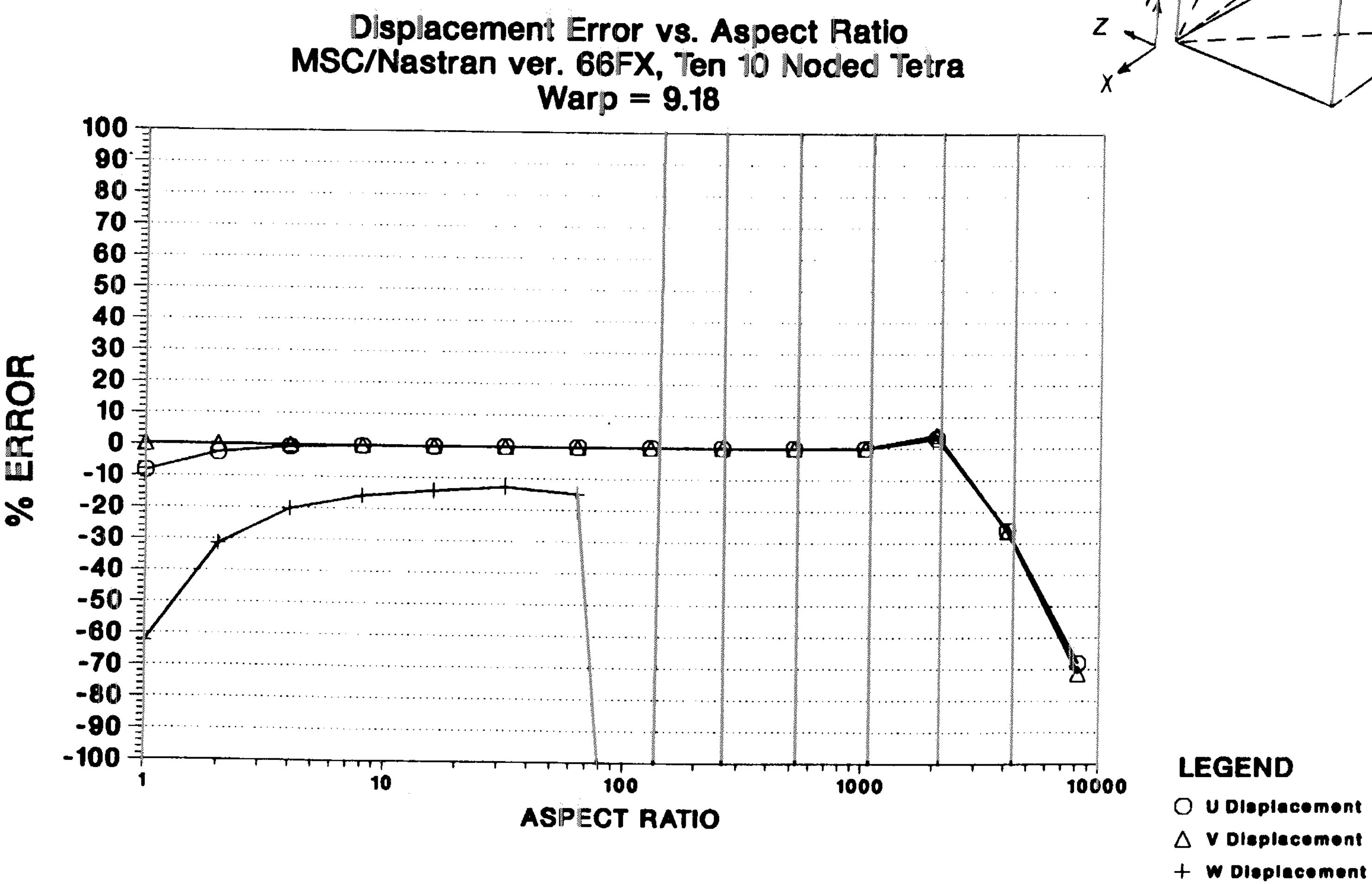


Figure 4a. Robinson Test Using Two Chexas With Warped Interface



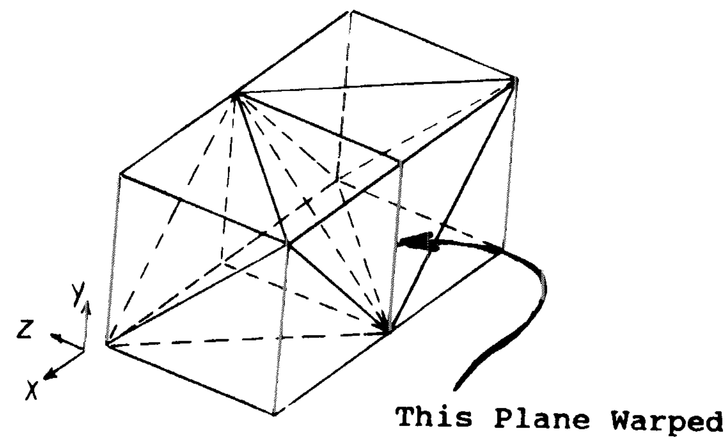


Figure 4b. Robinson Test Using Sets of Tetrahedra With Warped Interface

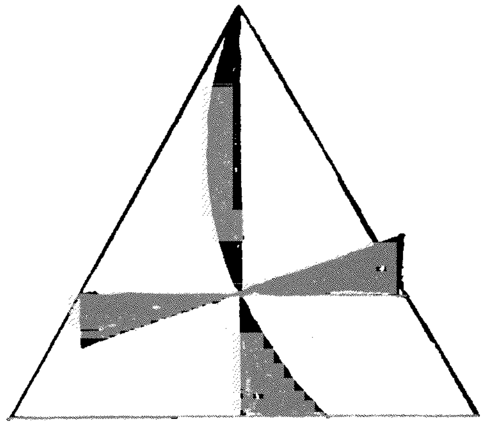


Figure 5. Shear Stress Distribution For Torsion

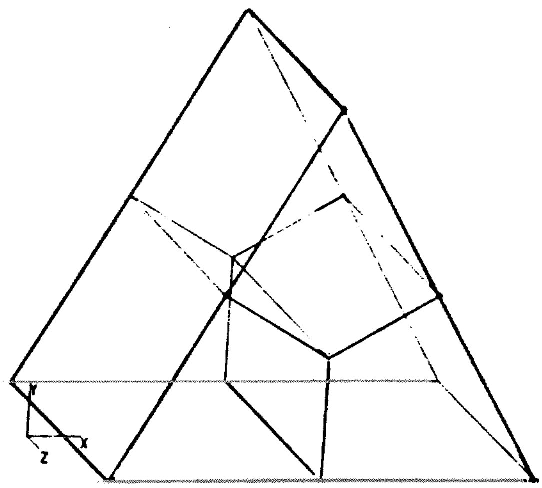


Figure 6a. Element Or Hyperpatch Arrangement

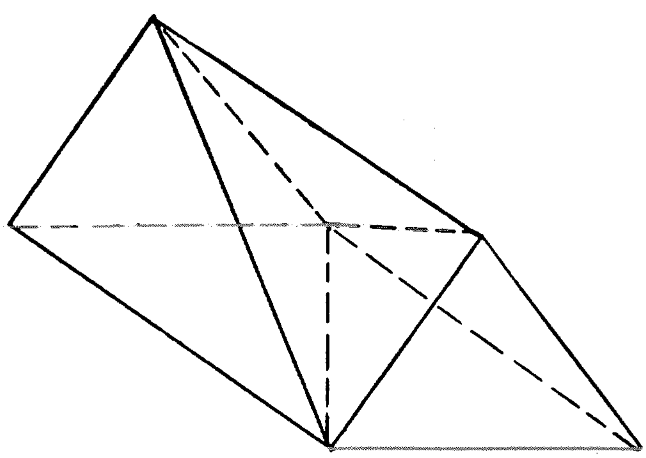


Figure 6b. Element Arrangement For Three Ctetra(10) Comprising Torsion Bar

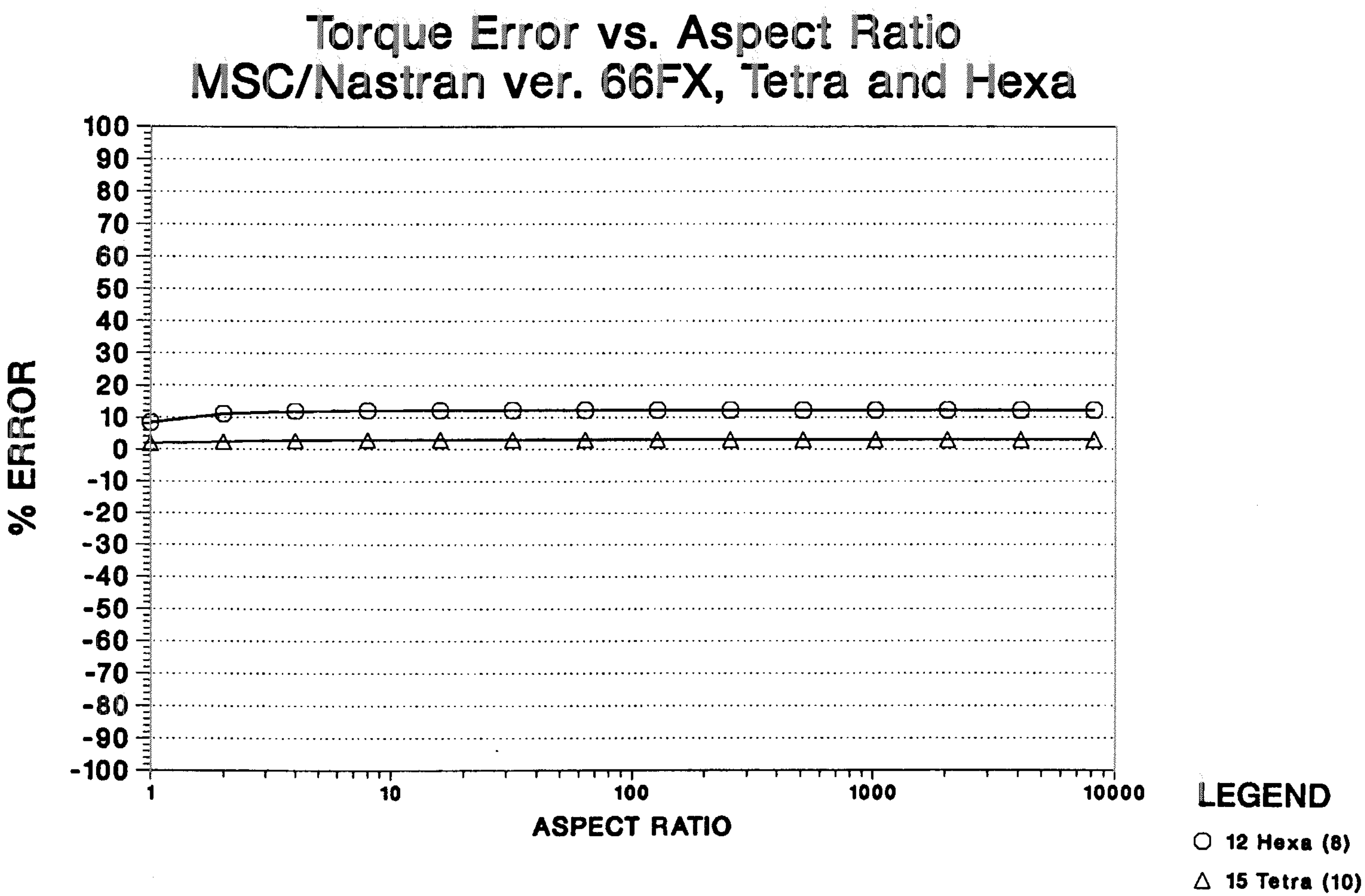


Figure 7. Twelve Chexa(8) vs. Fifteen Ctetra(10) With Enforced Rotation