Analysis of Coupled Natural Frequencies of Thin-Walled Beams with Open Cross Sections Using MSC/NASTRAN

By

M. Fouad Ahmad[1]
Civil Engineering and Mechanics, U. of Wisconsin-Milwaukee,

Carl W. Guile
The MacNeal-Schwendler Corporation

Abstract

Analysis of vibration modes and natural frequencies of thin-walled beams with open cross sections is performed for studying the validity of the cross section contour indeformability assumption. The thin-walled beam basic assumptions and governing differential equations are presented first. Numerical examples have been solved using a finite element program for the general analysis of thin-walled beams as well as MSC/NASTRAN. Shell element models of these beams were developed to check the results of the thin-walled beam analysis. For the first few mode shapes, the differences between the results obtained using thin-walled beam models and those of the shell models are insignificant. However, this is not the case for higher modes where the values of natural frequencies obtained using thin-walled beam models differ from those obtained using shell models. The mode shapes obtained using shell models show that the beams at higher modes behave like true shells where cross sectional deformation is observed. The study recommends that for cases where higher vibration modes are important, three dimensional shell models should be developed.

Introduction

The behavior of thin-walled beams has, for many years, been a topic for investigation by research workers in the field of applied mechanics. Attention to thin-walled members accompanied the manufacturing of airplanes. The requirements of weight-saving necessitated the wide use of such members and the analysis of such members became an important, practical problem. Structural elements fitting the general description of thin-walled beams appear in many forms, sizes, and materials. Cold-formed or hot-rolled, welded or riveted metallic beams, columns, girders, and elements of frames are examples of thin-walled members.

The general free vibration equations of thin-walled beams were derived by Vlasov [9]. Gjelsvik [7] used the theory of cylindrical shells to formulate the static equilibrium equations, and Ahmad [1] extended his formulation to include the dynamic effect. Gere [5] considered the free torsional vibration of double symmetric sections with emphasis on different boundary conditions. Gere and Lin [6] classified the coupling into

[1]currently a visiting research scientist, The National Center for Supercomputing Applications, U of Illinois, Urbana-Champaign

two types, double and triple. The former occurs when the section has one axis of symmetry, while the latter is associated with no axes of symmetry.

Basic Assumptions

A thin-walled beam consists of a finite number of long narrow straight or curved plates rigidly connected along their longer sides. It is essential to state some of the assumptions normally associated with the behavior and shape of thin-walled beams. These assumptions form the basis of the engineering theory of these beams as presented by Vlasov [9] and Gjelsvik [7]. and they are:

1- Regardless of the loading system, the original shape of the projected cross-section is unaltered during deformation.

2- Because of the bending and torsional flexibility of the thin-walled open sections, the relative effect of shearing strain along the middle surface of the plates is extremely small and can be neglected.

3- The basic assumptions of the cylindrical shell theory are applicable for each plate element. In other words, the following assumptions from the shell theory can be drawn:

a) The thickness of the plate is small compared to its width and its radius of curvature.

b) Plane sections through the plate elements and normal to the middle surface remain plane after deformation.

The first assumption of indeformability of the contour basically differentiates the thin-walled beam problem from that of true shells for which a deformation, caused by a local load, appears to be essential. This assumption was adopted by all researchers in this area. According to Gjelsvk [7], it is necessary to stipulate that the beam is stiffened transversely by bulkheads which are completely rigid in their plane and perfectly flexible out of their plane. Further discussion of this point will be given later.

The second assumption was stated in this form by Vlasov [9] and is referred to as the Vlasov's assumption. This assumption can be understood upon recalling the engineering theory of beams. In the engineering theory of beams the case of pure bending, which is an exact case, was studied first. Then the conclusions of this case were used in deriving the Jourawski's formula for the shear stresses in beams. A similar approach can be followed here by studying the case of pure torsion of thin-walled beams, which is also an exact case, and using the conclusions of this case for the cases of flexure bending and non-uniform torsion. One of the conclusions for the case of pure torsion of thin-walled open sections is that the shearing stresses and strains of the middle surface are zero (see Ref. [8] pp. 273 and 287). This approach is also adopted in the case of closed section [7], since the shear strain for these sections is assumed to have the same distribution in the contour direction as it does in the case of pure torsion.

In the derivation presented by Gjelsvik [7] for static equilibrium and Ahmad [1] for dynamic case, t he assumption of zero middle surface shear strains is made, the case of pure torsion is avoided, and an approach based on the bending theory of cylindrical shells is adopted. That is also the reason behind the existence of the third assumption. The use of this approach will allow the St. Venant torque and the influence of non-uniform distribution of longitudinal stresses, which is responsible for the secondary warping effect, to arise naturally. The effect of this secondary warping on the dynamic response of thin-walled beams was presented by Ahmad [2] in a previous study.

Governing Differential Equations

One of the differences between the elementary beam theory and the thin-walled beam theory appears when examining the generalized displacements. These generalized displacements will be referred to a set of right handed coordinate systems (x, y, z), where the z-axis is taken parallel to the straight longer side of the plates and is considered the beam axis, while the $x\&y$ axes are the principal axes of the cross sections. In the elementary beam theory there are six generalized displacement components, namely; the displacements components (u, v, and w), the slopes ($\frac{\partial u}{\partial z}$ and $\frac{\partial v}{\partial z}$), and the angle of twist (ϕ). In the thin-walled beam theory, a seventh generalized displacement component exists in addition to the previous six. This component is the rate of change of the angle of twist with respect to the beam axis, i.e., $\frac{\partial \phi}{\partial z}$. This generalized displacement is defined in a MSC/NASTRAN data deck through additional scalar points at the ends of the beam element.

Since thin-walled beams have seven generalized displacement components, there will also be seven equilibrium equations. These equilibrium equations in terms of the beam stress resultants are [1] :

$$\frac{\partial N}{\partial z} - N_i + \mathcal{N} = 0 \tag{1}$$

$$\frac{\partial V_x}{\partial z} - V_{xi} + \mathcal{P}_x = 0 \tag{2}$$

$$\frac{\partial V_y}{\partial z} - V_{yi} + \mathcal{P}_y = 0 \tag{3}$$

$$\frac{\partial T}{\partial z} - T_i + \mathcal{T} = 0 \tag{4}$$

$$\frac{\partial M_x}{\partial z} - V_y - M_{xi} + \mathcal{M}_x = 0 \tag{5}$$

$$\frac{\partial M_y}{\partial z} + V_x - M_{yi} + \mathcal{M}_y = 0 \tag{6}$$

$$\frac{\partial M_\omega}{\partial z} + T_\omega - M_{\omega i} + \mathcal{M}_\omega = 0 \tag{7}$$

The first equation is the axial force (N) equilibrium equation. In this equation the term N_i is axial inertia force and the term $\mathcal{N}$ is an external axial forcing function. The second and third equations are the transverse shearing forces ($V_x\&V_y$) equations, where $V_{xi}\&V_{yi}$ are the the inertia forces and $\mathcal{P}_x\&\mathcal{P}_y$ are the external transverse forcing functions. The fourth equation is the torsional equilibrium equation about the shear center axis where the terms $T_i \& \mathcal{T}$ are the inertia torque and external torsional functions, respectively. Similarly, the fifth and sixth equations are the moment equilibrium equations about the principal x and y axis of the cross section. $M_{xi} \& M_{yi}$ are the rotary inertia moments and $\mathcal{M}_x \& \mathcal{M}_y$ are external moment functions. The seventh equation represents the major difference between the elementary beam theory and the thin-walled one. It is due to the out of plane warping of the cross section. M_ω is the bimoment and $M_{\omega i} \& \mathcal{M}_\omega$ are the bimoment rotary inertia and external bimoment function, respectively. Finally, T_ω is the warping torque and the relation between this torque and the total torque (T) is given by:

$$T = T_s + T_\omega \tag{8}$$

where T_s is the St. Venant torque.

The equilibrium equations can be reduced to four equations by introducing the relations between the stress resultants as well as the inertia forces and the displacement components and their spatial and time derivatives. These four equations are [1] :

$$EA\frac{\partial w^2}{\partial^2 z} - \rho A\frac{\partial^2 w}{\partial t^2} + \mathcal{N} = 0 \tag{9}$$

$$EI_{yy}\frac{\partial^4 u}{\partial z^4} - \rho I_{yy}\frac{\partial^4 u}{\partial^2 z \partial^2 t} + \rho A[\frac{\partial^2 u}{\partial t^2} + Y_p\frac{\partial^2 \phi}{\partial t^2}] - \mathcal{P}_x + \frac{\partial \mathcal{M}_y}{\partial z} = 0 \tag{10}$$

$$EI_{xx}\frac{\partial^4 v}{\partial z^4} - \rho I_{xx}\frac{\partial^4 v}{\partial^2 z \partial^2 t} + \rho A[\frac{\partial^2 v}{\partial t^2} - X_p\frac{\partial^2 \phi}{\partial t^2}] - \mathcal{P}_y - \frac{\partial \mathcal{M}_x}{\partial z} = 0 \tag{11}$$

$$EI_{\omega\omega}\frac{\partial^4 \phi}{\partial z^4} - GJ\frac{\partial^2 \phi}{\partial z^2} - \rho I_{\omega\omega}\frac{\partial^4 \omega}{\partial^2 z \partial^2 t} + \rho[A(-X_p\frac{\partial^2 v}{\partial t^2} + Y_p\frac{\partial^2 u}{\partial t^2}) + I_p\frac{\partial^2 \phi}{\partial t^2}] - \mathcal{T} + \frac{\partial \mathcal{M}_\omega}{\partial z} = 0 \tag{12}$$

where E is the modulus of elasticity, G is the shear modulus, ρ is mass density of the beam material, A is the cross sectional area, I_{xx} is the principal moment of inertia about the x-axis. I_{yy} is the principal moment of inertia about the y-axis, $I_{\omega\omega}$ is the principal second sectorial moment of inertia or warping constant, J is the St. Venant torsional constant, $X_p \& Y_p$ are the coordinates of the shear center, and I_p is the polar moment of inertia about the shear center axis which is defined as:

$$I_p = I_{xx} + I_{yy} + A(X_p^2 + Y_p^2) \tag{13}$$

Equation [9] determines the longitudinal vibrations of the beam and it is totally independent from the other three. The second term in equations [10] and [11] as well as the third term in equation [12] are the rotary inertia terms. These terms have small effect on the beam vibrations and are usually neglected. For the case when $X_p \& Y_p$ are equal to zero, i.e., the centroid coincides with the shear center, equations [10], [11], and [12] become uncoupled and can be solved seperately.

Natural Frequencies and Mode Shapes Analysis

The finite element stiffness and mass matrices formulation can be found in Ref. [3]. A finite element program for the general analysis of thin-walled beams (TINWAL)

Mode Number	Mode Type According to Thin-Walled Beam Analysis	TINWAL 10 Elements	MSC-NASTRAN		
			CBEAM	CQUAD4	
			10 Elements	450 Elements	1800 Elements
1	Minor Axis Bending	5.767	5.741	5.787	5.732
2	Torsional	10.94	10.89	10.23	10.32
3	Major Axis Bending	25.76	25.64	25.84	25.82
4	Minor Axis Bending	36.14	35.58	35.49	35.15
5	Torsional	48.07	47.28	45.78	45.99
6	Minor Axis Bending	101.2	98.62	86.09	86.36
7	Torsional	122.4	119.1	93.56	100.9
8	Major Axis Bending	164.1	158.9	108.3	116.3
9	Minor Axis Bending	198.5	191.2	117.3	117.6
10	Torsional	234.2	225.2	119.6	123.2

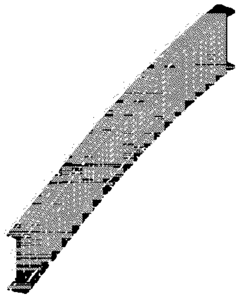
Table 1: Natural Frequencies of a 15 ft. W18X50 Cantilever

has been developed using this formulation. The organization of the program follows the procedures outlined in Ref. [4] for developing general purpose finite element programs. The eigen-value analysis used in this program is based on the subspace iteration method. MSC/NASTRAN is used for verifying the results obtained by TINWAL and studying the effect of modeling these beams using shell elements. In order to avoid missing any eigen-values, the enhanced inverse power method and the modified Householder's method with and without dynamic reduction were used in the MSC/NASTRAN analyses. These different methods gave very close results.

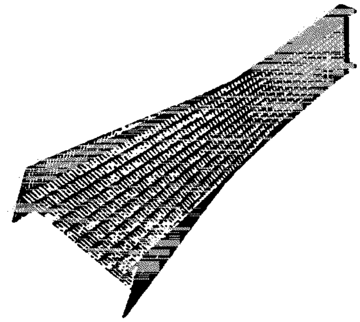
Two different cantilever beams were analyzed to compare the results obtained using thin-walled beam elements against the resullts obtained using shells. The first case represents an uncoupled case where a symmetrical I-beam is analyzed. As for the second case an antisymmetric channel is used where torsion couples with flexural bending about the two principal axes of the cross section.

For the case of no coupling, a 15 ft. W18X50 steel cantilever beam was analyzed. The values of the first ten natural frequencies obtained by different analyses are given in table 1. The table also identifies the mode type according to thin-walled beam analysis. The differences between the values of the frequencies obtained ranges between 1% to 5% for the first five natural modes. However, this is not the case for the next five modes where major differences exist between the beam and shell models. This fact can also be observed in Fig. [1] where the first ten modes of the 1800 CQUAD4 elements model are shown. The first five modes can be identified as thin-walled beam modes, while the next five modes can be classified as shell modes. In other words the first assumption of the contour indeformability seems to be true for only the first few modes.

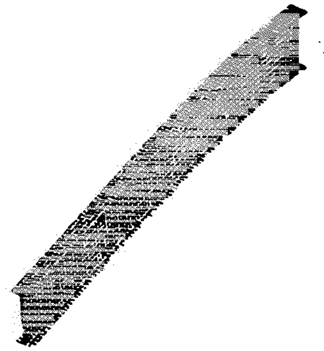
For the case of coupling, a 100 in. steel cantilever is considered. The cross section of this cantilever is an antisymmetric channel with 2 in. top flange, 4 in. bottom flange, and 10 in. web. The thickness of the flanges and the web is 0.5 in. The values of the



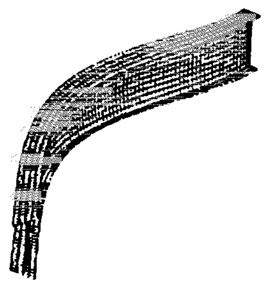
[1]



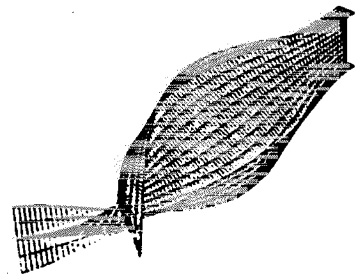
[2]



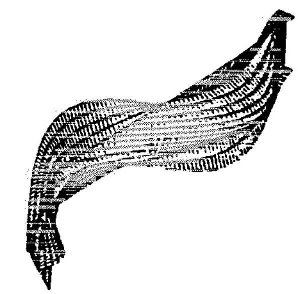
[3]



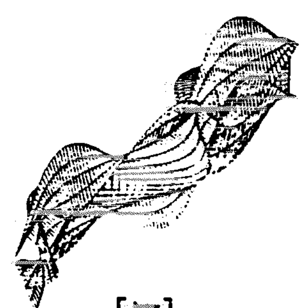
[4]



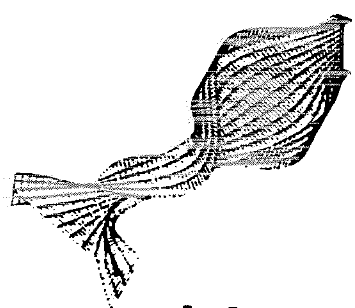
[5]



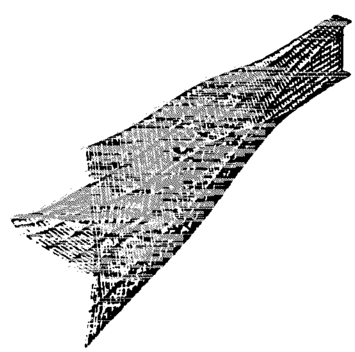
[6]



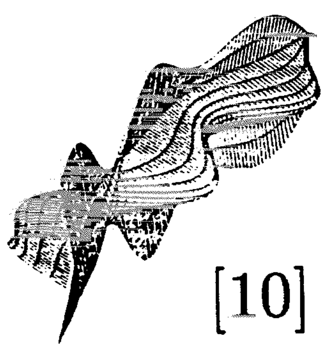
[7]



[8]



[9]



[10]

Figure 1: Mode Shapes of a 15 ft. W18X50 Cantilever

Mode Number	TINWAL 10 Elements	MSC-NASTRAN		
		CBEAM	CQUAD4	
		10 Elements	280 Elements	1120 Elements
1	11.05	11.00	10.74	10.68
2	26.54	26.46	25.37	25.70
3	46.45	46.15	46.57	46.76
4	62.40	61.50	59.51	59.38
5	104.2	102.7	99.90	100.9
6	147.2	143.0	137.9	137.6
7	252.4	244.3	234.7	234.4
8	266.6	256.6	244.3	244.9
9	286.7	281.7	256.2	256.9
10	415.3	391.7	354.4	353.0

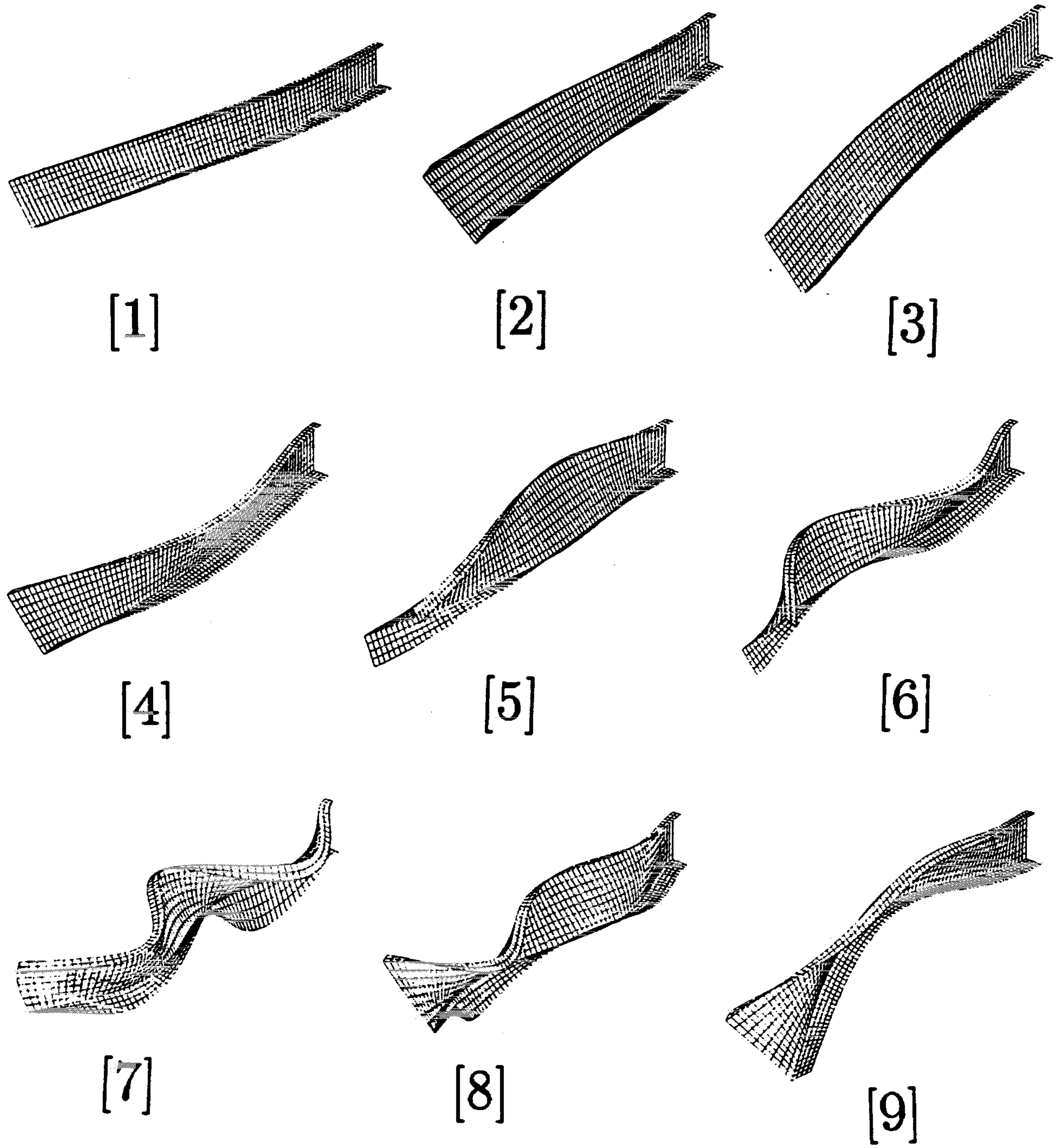
Table 2: Natural Frequencies of a 100 in. Antisymmetric Channel

first ten natural frequencies are shown in table 2. All of these frequencies are coupled due to lack of symmetry. Although, significant differences in the frequency values obtained by shell and beam analyses are not obvious as in the previous uncoupled case, a jump in this difference from about 4% at the eighth mode to over 9% at the nineth can be detected. The mode shapes shown in Fig. [2] shows similar observations as for the case of no coupling.

Further analyses were conducted to study the effect of providing a form of rigid bulkhead. RBE2 elements were used, as outlined in the MCS/NASTRAN Application Manual, to represent two bulkheads at mid span and the tip of the cantilever. The results obtained were similar to those presented in both values and mode shapes. In addition, no significant differences have been obtained by providing additional two bulkheads at the quarter points of the span. According to Gjelsvik [7], the longer the bar and the thicker the wall, the less important the bulkheads. Furthermore they are more important in closed than in open cross sections.

Conclusions

It can be concluded from this study that the thin-walled beam theory provide adequate natural frequencies and mode shape results for the first few modes. For cases where higher modes are considered to be a major design factor, it is recommended to consider three dimensional shell models for verifying the thin-walled beam analysis. Since many different parameters such as the plates' thickness, cross sectional dimensions, and beam length play an important role in the beam vibrational characteristics, the need for more numerical experimentation in these areas is evident.



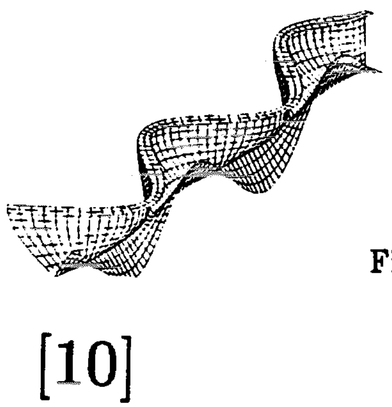


Figure 2: Mode Shapes of a 100 in Antisymmetric Channel

Acknowledgement

Part of the analysis in this study was performed using the CRAY X-MP/48 system at the National Center for Supercomputer Applications, University of Illinois at Urbana-Champaign. The authors are indebted to this Center for providing excellent technical support and convenient computing facilities.

References

[1] AHMAD, M. F., '*Complex Frequency Analysis of Damped Thin-Walled Beams with Open Cross Sections*,' Ph.D. Dissertation, the University of New Hampshire (1984).

[2] AHMAD, M. F., '*Secondary Warping Effect on the Dynamic Response of Thin-Walled Beams with Open Cross Sections*,' in STRUCENG, Structural Engineering, FEM & CAD, Technology Transfer Series, Institute for Industrial Technology Transfer (IITT-International), France, 1989.

[3] BARSOUM, R. S., '*A Finite Element Formulation for the General Stability Analysis of Thin-Walled Members*,' Ph.D. Dissertation, Cornell University (1970)

[4] BATHE, K. J., '*Finite Element Procedures in Engineering Analysis*,' Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1982.

[5] GERE, J. M., '*Torsional Vibrations of Beams of Thin-Walled Open Section*,' Journal of Applied Mechanics, pp. 381-387, December 1954.

[6] GERE, J. M., and LIN, Y. K., '*Coupled Vibrations of Thin-Walled Beams of Open Section*,' Journal of Applied Mechanics, pp. 373-378, September 1958.

[7] GJELSVIK, A., '*The Theory of Thin-Walled Bars*,' John Wiley & Sons, New York (1981).

[8] TIMOSHENKO, S. P., and GOODIER, J. N., '*Theory of Elasticity*,' McGraw-Hill Book Company, Inc., New York, 1953.

[9] VLASOV, V. Z., '*Thin-Walled Elastic Beams*,' Second Edition, Israel Program for Scientific Translations, Jerusalem (Translated from Russian) (1961).