Flutter Simulation for Bridges

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1. Preface

Since the overall stiffness in long span bridge like suspension bridges and cable-stayed bridges is relatively small compared with medium and small span bridges, it is especially important to evaluate such aerodynamically unstable phenomena as vortex shedding*1), flutter*2) and so on. In the current design procedure for long span bridges, the evaluation of these phenomena is carried out in wind tunnel tests.

In the non-streamlined cross sections like bridge girders which are mainly subjected to wind force, the aerodynamic characteristics are considerably complex due to the separation of air flow or the generation of vortices. Therefore it is difficult to determine the unsteady aerodynamic force analytically. On the contrary, in streamlined sections like aircraft wings ,the unsteady aerodynamic force can be relatively easily determined based on the potential theory. The methods for analyzing aircraft flutter based on aeroelastic theory have been applied in practical use[1]

Scanlan et al[2]. showed the formulation of the unsteady aerodynamic force acting on non-streamlined cross sections by using the aerodynamic coefficients determined by wind tunnel tests as a function of reduced frequency. Since the aerodynamic coefficient includes the effect of separation of air flow and vortex shedding, it can be used to properly evaluate the aerodynamic behavior of bluff bodies like the cross section of bridge girders.

In this work, the method of flutter a nalysis for actual bridges was investigated by

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*1) Oscillation induced by alternate vortices in wake

*2) Self-excited vibration caused by the negative damping of aerodynamic force

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applying the experimental formulation to three-dimensional finite element analysis. The effectiveness was evaluated by performing the flutter analysis for the model of the Tacoma Narrows Bridge.

MSC / NASTRAN has several capabilities of aeroelastic analysis based on the potential theory. The analysis in this work was performed by using a newly developed DMAP, which introduced the aerodynamic coefficients into the aerodynamic force matrix.

2. Theory of flutter analysis

2.1 Equation of motion[3) 4)]

The flutter phenomena of bridges are a typical example of aeroelastic phenomena, and occur by interaction of three forces, that is, aerodynamic force, elastic force and inertia force.

The equation of motion for a structural system involving aerodynamic force $\{L\}$ becomes :

$$[M]\{\ddot{u}\} + [K]\{u\} = \{L\} \qquad (1)$$

where, $[M]$, $[K]$ are mass and stiffness matrices, respectively.

$\{u\}$, $\{\ddot{u}\}$ are displacement and acceleration vectors, respectively.

$\{L\}$ is aerodynamic force vector.

The section is assumed to oscillate harmonically in the air flow, representing by the following equation.

$$\{u\} = \{u^*\}\, e^{\iota\omega\tau} \qquad (2)$$

where, $\{u^*\}$ is displacement amplitude vector (complex number).

ω is circular frequency of harmonic oscillation.

The aerodynamic force acting on the oscillating section expresses generally as follows.

$$\{L\} = q\,[Q(k)]\,\{u^*\}\, e^{\iota\omega\tau} \qquad (3)$$

where, q is dynamic pressure ($= 1/2\ \rho v^2$)

ρ is mass density of air.

v is wind velocity.

k is reduced frequency defined by the following equation.

$$k = \frac{B\omega}{v} \tag{4}$$

B is representative length, the width of girder in case of bridges.

Q(k) is aerodynamic force matrix (complex number).

When Equations (2) and (3) are substituted into Equation (1), the following equation of motion is derived.

$$(-\omega^2[M] + [K] - q[Q(k)])\{u^*\} = 0 \tag{5}$$

2.2 Calculation for flutter velocity[1) 3) 4)]

Flutter is a self-excited oscillatory instability of a body suspended in air stream, and flutter velocity is one on the border of instability. This can be determined by using Equation (5). In Equation (5), the artificial structure damping g is introduced for developing the equation.

$$(-\omega^2[M] + (1+ig)[K] - q[Q(k)])\{u^*\} = 0 \tag{6}$$

the form of Equation (6) is changed into the following equation.

$$\{-([M] + \frac{\rho}{2}(\frac{B}{k})^2[Q(k)]) + (\frac{1+ig}{\omega^2})[K]\}\{u^*\} = 0 \tag{7}$$

Hereupon, by writing as $\lambda = (1 + ig)/\omega^2$, the complex eigenvalues λ_j ($j = 1, 2, \ldots n$: order of mode) are determined for a arbitrary values of k. The circular frequency ω_j and the damping g_j are derived from the real part and the imaginary part of λ_j as follows.

$$\omega_j = 1/\sqrt{\mathrm{Re}(\lambda_j)} \tag{8}$$

$$g_j = \mathrm{Im}(\lambda_j)/\mathrm{Re}(\lambda_j) \tag{9}$$

In this case, the relation of the damping g_j with the critical damping ratio h_j and the logarithmic damping rate ρ_j is as follows.

$$h_j = \frac{1}{2} g_j \tag{10}$$

$$\rho_j = \pi g_j / \sqrt{1 - \frac{g_j^2}{4}} \tag{11}$$

By using equation (4), the corresponding velocity can be determined as follows.

$$v_j = \frac{B\omega_j}{k} \tag{12}$$

The v_j and g_j are determined for the various values of k, and these can be plotted as shown in Fig. 1. The velocities at the intersections of these curves with the horizontal line show each flutter velocity.

2.3 Application to bridges

The aerodynamic force matrix Q(k) can be obtained relatively easily by the potential theory for streamlined bodies like aircraft wings. However, in bluff bodies like the cross section of bridge girders, the flow is considerably complex due to the separation of air flow or the generation of vortices. Therefore, it is difficult to determine Q(k) by numerical analysis.

Scanlan et al. formulated the unsteady aerodynamic force acting on a bluff body using the aerodynamic coefficients determined by wind tunnel tests as a function of reduced frequency. These aerodynamic coefficients involve the effect of separation of air flow and vortex shedding. Therefore, they can be used to properly evaluate the aerodynamic behavior of bluff bodies like the cross section of bridge girders. The unsteady aerodynamic force acting on the cross section of a bridge girder which oscillates both vertically and torsionally in uniform flow shown in Fig. 2 can be expressed as the following equation.

$$L_h = \frac{1}{2}\rho v^2 (2B)[kH_1^*(k)\frac{h}{v} + kH_2^*(k)\frac{B\alpha}{v} + k^2 H_3^*(k)\alpha] \tag{13}$$

$$M_\alpha = \frac{1}{2}\rho v^2 (2B^2)[kA_1^*(k)\frac{h}{v} + kA_2^*(k)\frac{B\alpha}{v} + k^2 A_3^*(k)\alpha] \tag{14}$$

where, L_h is aerodynamic lift.

M_α is aerodynamic moment

h is deflection (vertical displacement),

α is twist (angular displacement),

$H_l^*(k)$, $A_l^*(k)$ are aerodynamic coefficients determined by wind tunnel tests. These are functions of reduced frequency k.

The aerodynamic coefficients H_i^* and A_i^* obtained for some typical cross sections of bridge girders are shown in Fig. 3 . These aerodynamic coefficients involve the effect of excitation phenomena due to vortices at the experimental stage . Accordingly, when the aerodynamic force matrix Q (k) is made by using them, the flutter analysis taking the separation of air flow and vortex shedding into account is feasible .

3. Flutter analysis in MSC / NASTRAN

The MSC / NASTRAN has the following analytical capabilities in aeroelastic analysis .

(1) Static Aeroelastic Response (Sol. 21)

(2) Superelement Aerodynamic Flutter (Sol. 75)

(3) Superelement aeroelastic Response (Sol. 76)

The aeroelastic analysis is based on finite element method, and the finite aerodynamic elements which generate the aerodynamic force are used in combination with structural elements. Individual aerodynamic element has a strip or box form, and a wing geometry is modeled by combining these .

In this analysis, The CAERO4 based on the strip theory was used for the aerodynamic element. On the CAERO4, a rigid body deformation is assumed in wing chord direction , and the coupling of aerodynamic forces among individual strips in span direction is not considered . However it has a good accuracy for slender wings. And the bridge girder can be regarded as a kind of slender winds. The CAERO4 forms the aerodynamic force matrix Q (k) according to the strip theory based on the potential theory. So it cannot be applied directly to the flutter simulation for bridge. For this reason, the new DMAP which introduced the experimental aerodynamic coefficients H_i^* and A_i^* to Q (k) was developed and used .

In modal flutter analysis method, three methods (K, KE and PK) can be selected . The PK method which can obtain V-g curves, complex eigenvalues and flutter deformation modes ,was used in this analysis, .

4. Example of analysis

In order to evaluate the effectiveness of the analytical procedure, the flutter analysis was performed for an actual bridge. The bridge is the Tacoma Narrows Bridge which is famous as one broken by flutter. This bridge was a suspension bridge having the span of 853m, but in 1940, the wind at a velocity of only 19 m/sec caused violent oscillation of the bridge girder and it fell in finally[5)6)].

The analytical model was a three-dimensional frame model complied with the main properties of the actual bridge as shown in Fig. 4. The structural damping was assumed to be 1% according to the standard value[7)] in suspension bridges. In flutter analysis, the deformation modes which should be taken into account are deflection and twist of a bridge girder. Therefore, the degrees of freedom in out-of-plane direction and bridge axis direction of bridge girder were fixed in the model. The types of finite element used were rod elements for cable and hanger, and bar element for all the other parts. The real mode shapes under the condition of no aerodynamic force are shown in Fig. 5. These real eigenvalues and eigen mode almost coincide with the values measured in the actual bridge.

The type of aerodynamic element used for the analysis is CAERO4 which has a chord length coincided with the girder width of 11.88m. These elements were connected to the whole length of the bridge girder. The air flow was assumed to be horizontal uniform flow in the direction of right angles to the bridge axis (X direction in Fig. 4), and air density was 1.25×10^{-4} ton·sec^2/m^4.

Flutter analysis was carried out on two cases using the aerodynamic coefficient of a flat plate and that of the Tacoma Narrows Bridge. On the case of using the aerodynamic coefficient of a flat plate, the developed DMAP was not used. The range of wind velocity for the analysis was taken from 3 m/sec to 30 m/sec.

On the case of using the aerodynamic coefficient of flat plate, the relation between wind velocity V and the aerodynamic damping g (V-g curves) in each eigen mode is shown in Fig. 6. As for every mode, the aerodynamic damping g increases with wind velocity, and it shows that aerodynamic force acted in the direction of restricting oscillation. The air damping is positive in the range of wind velocity for the analysis. The velocity when flutter occurs is over 30 m/sec for all modes. These behavior doesn't correspond with the actual behavior. On CAERO4, the aerodynamic force matrix is formed according to the potential theory, and the effect of vortex shedding is not taken into account. This is considered to be the main cause for the disagreement.

The aerodynamic coefficients of the Tacoma Narrows Bridge shown in Fig. 3 were obtained by two-dimensional wind tunnel test, and it involves the effect of the oscillating force due to vortices. The V-g curves in each eigen mode are shown in Fig. 7, and the aerodynamic unstable behavior arises remarkably in the region of the wind velocity above 10 m / sec. In the first order mode (primary deflection), aerodynamic damping increases initially, but begins to decrease above the wind velocity of 6 m / sec, and exceeds the structural damping of 1% at 12 m/sec, thus flutter occurred. In all modes, flutter occurs in the range up to the wind velocity of 30 m / sec. The flutter velocities V_f increase with higher order modes for each deflection and twist mode. These flutter velocities V_f correspond with the range of wind velocity when aerodynamic unstable phenomena occurred in the actual bridge. For example, the wind velocity just before the bridge fell was 19 m / sec, and the actual mode of oscillation corresponds with the sixth order mode in this analysis, and the analytical flutter velocity V_f is 16 m / sec.

5. Postscript

It was shown that the complex aeroelastic behavior of actual bridges can be simulated properly by using the aerodynamic coefficients obtained by wind tunnel test.

In order to obtain the aerodynamic coefficients accurately, it is necessary to carry out special wind tunnel tests such as forced oscillation. Such aerodynamic coefficients have been rarely obtained in actual bridges. Therefore it may be difficult to directly apply this method to the evaluation of flutter for actual bridges in most cases. However, there is the possibility to considerably improve the accuracy of prediction for flutter by using the aerodynamic coefficients of the similar cross-sectional form of bridge girder, compared with the conventional design method [7].

Besides, wind tunnel tests on three-dimensional model are rarely carried out. However it is easy to perform three-dimensinal analysis on this method. It will help to understand the complex phenomena in actual bridges.

The authors hope that this method is applied to the evaluation of wind endurance of actual bridges in the purpose of supplementing the conventional evaluation method.

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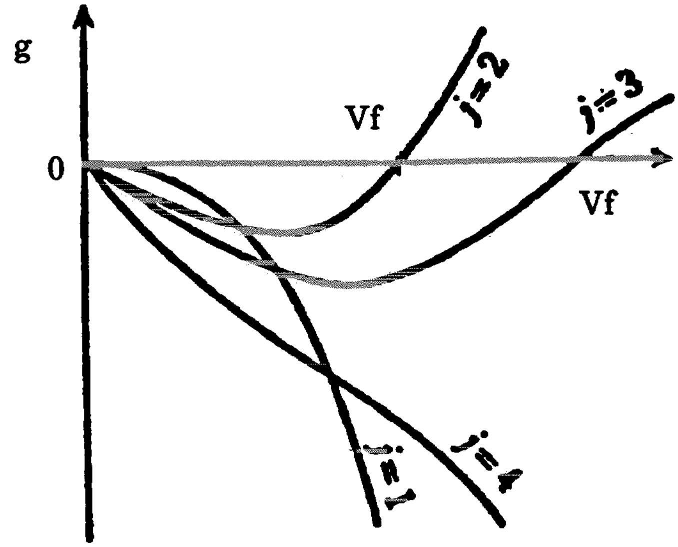


Fig. 1. V−g Curves

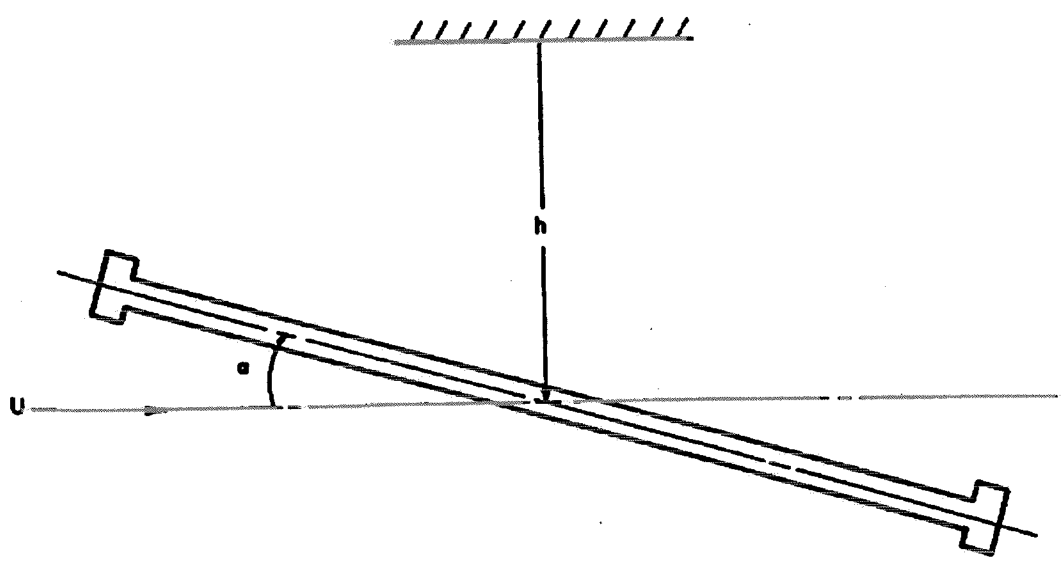


FIG. 2 Degrees of Freedom of Section of a Line-like Structure[4)]

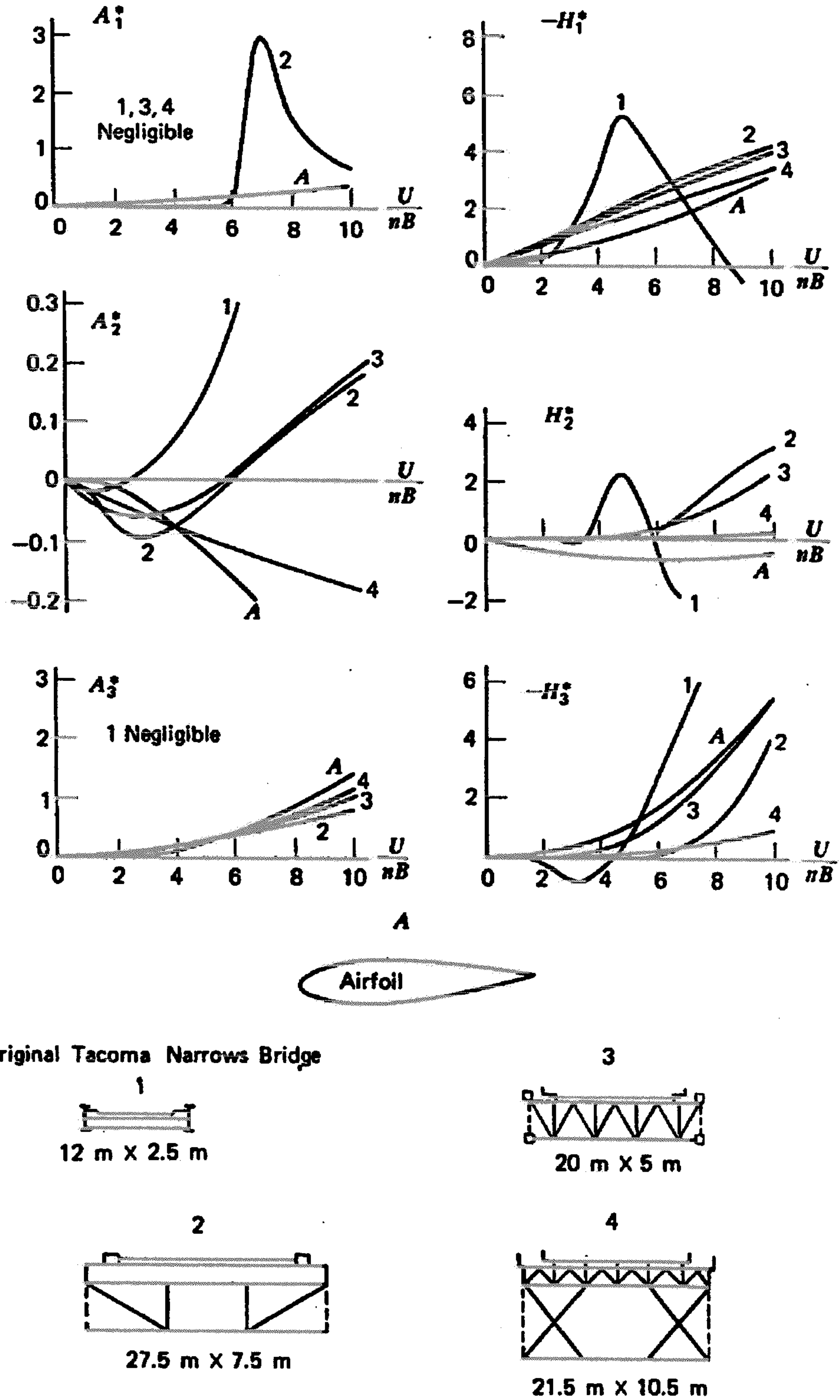
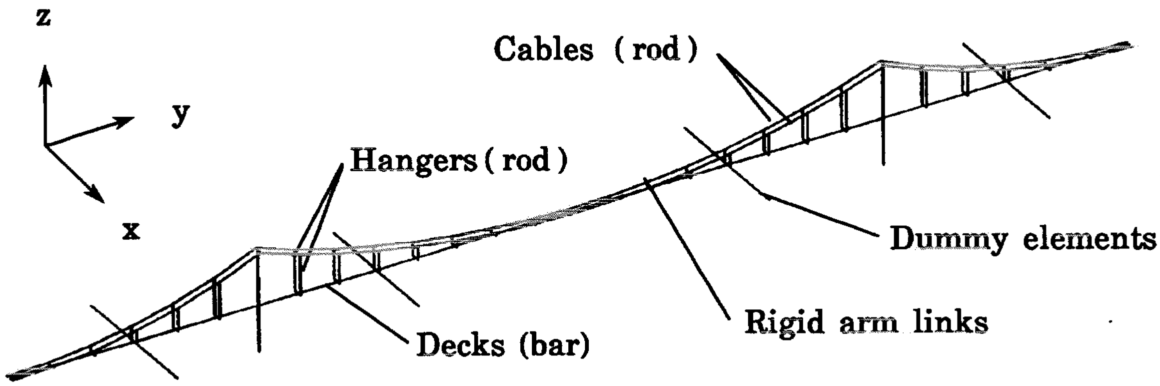
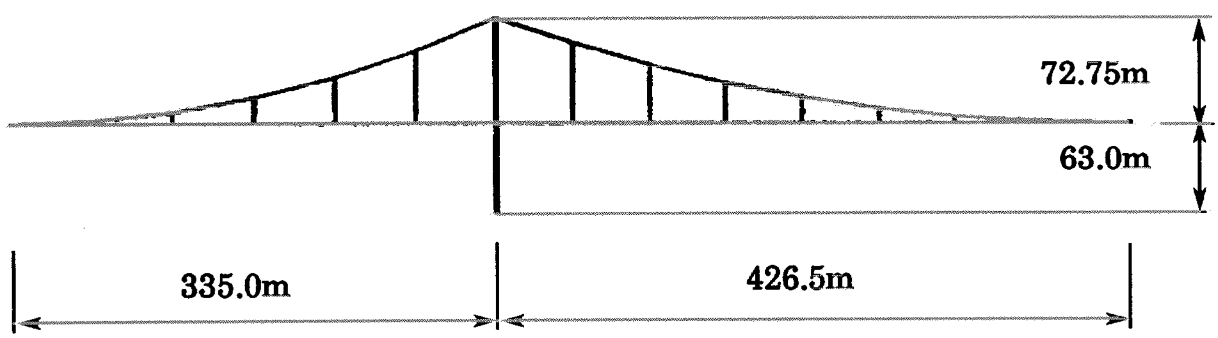


Fig. 3. Coefficient H^*_i and A^*_i for Various Sections[4)]





Structural Properties

Cable : CSA $= 1.189 \times 10^{-1}$ m²

Horizontal component of

cable tension $= 5.69 \times 10^{3}$ t / cable

Deck : I(Vent) $= 1.54 \times 10^{-1}$ m⁴

Mass = 6.7 t / m

Width = 11.88 m

Hanger : CSA $= 1.574 \times 10^{-3}$ m²

Fig. 4. Flutter Analysis Indealization for Tacoma Narrows Bridge

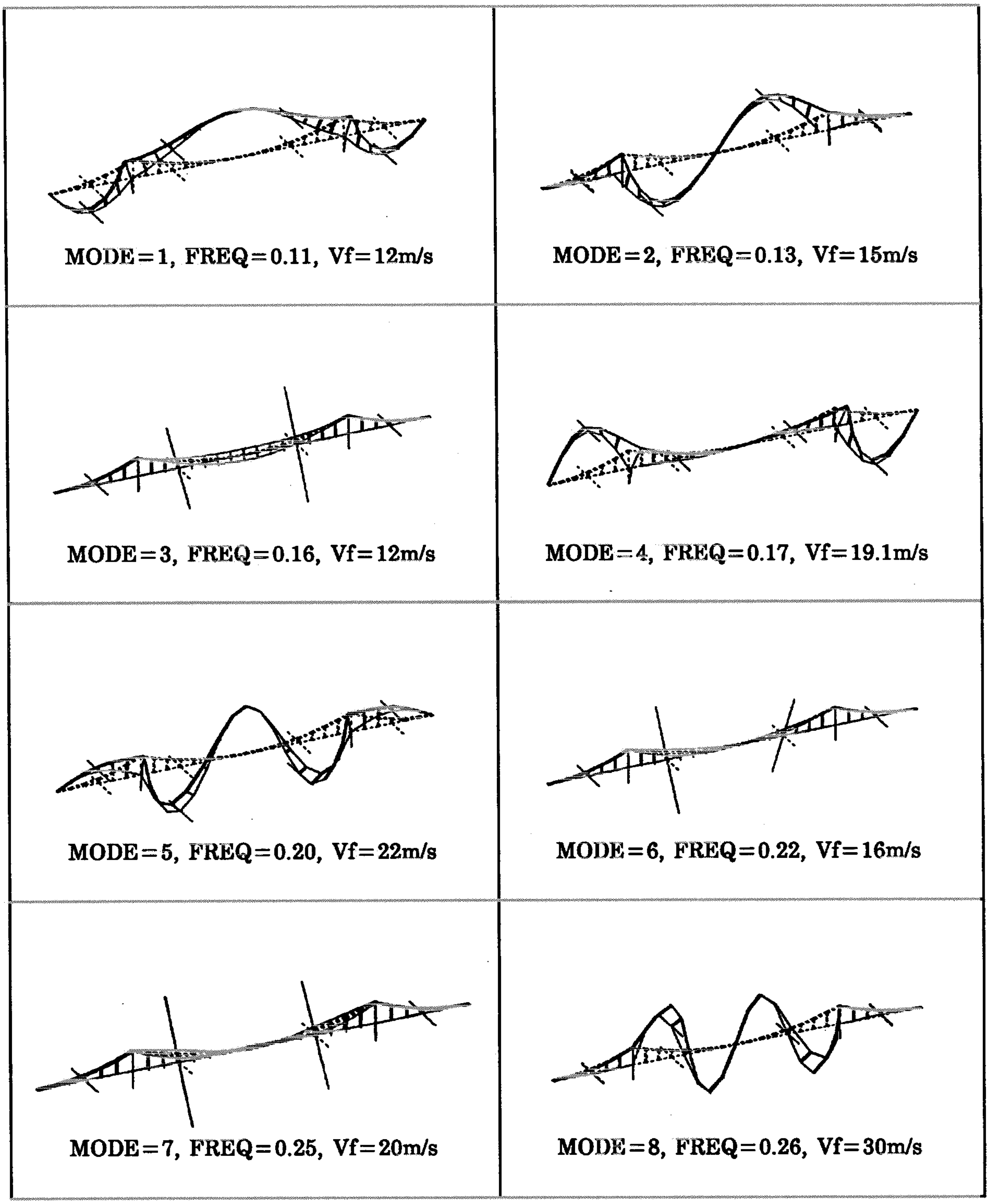


Fig. 5. Mode Shapes of Model for Tacoma Narrows Bridge in Vacuum Condition

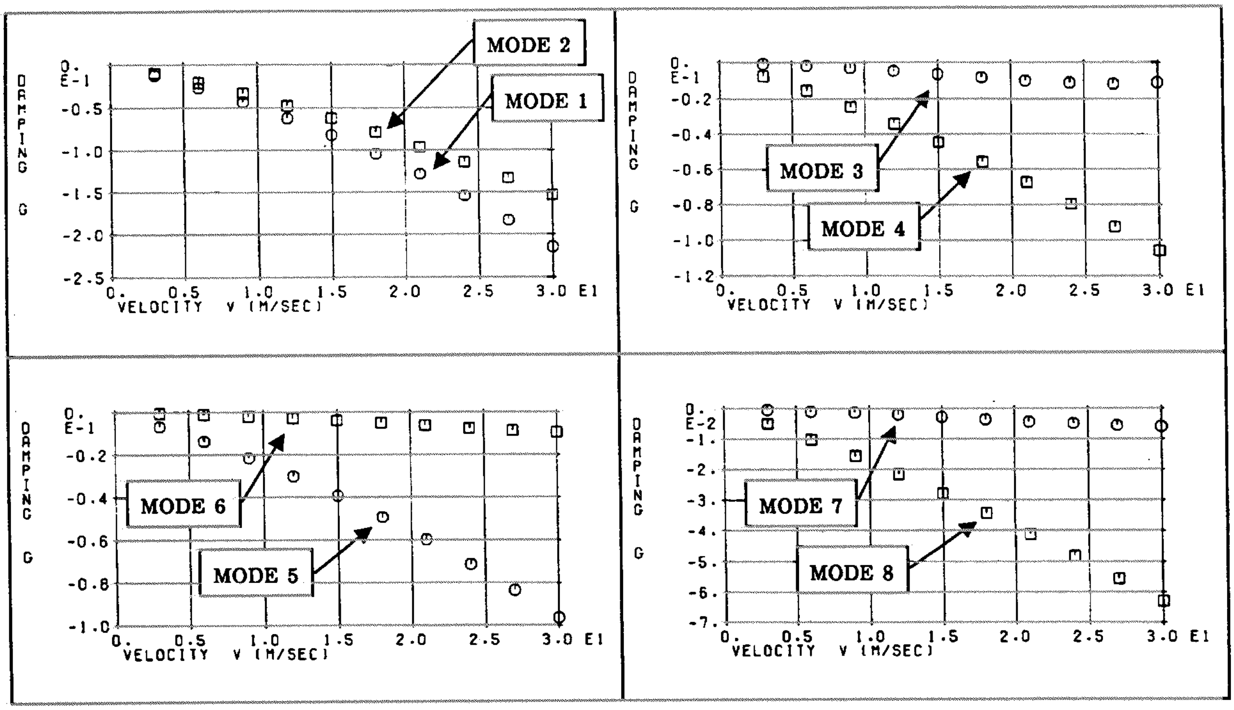


Fig. 6 V - g Curves for Model of Tacoma Narrows Bridge using Plane Section's Aero Coefficients

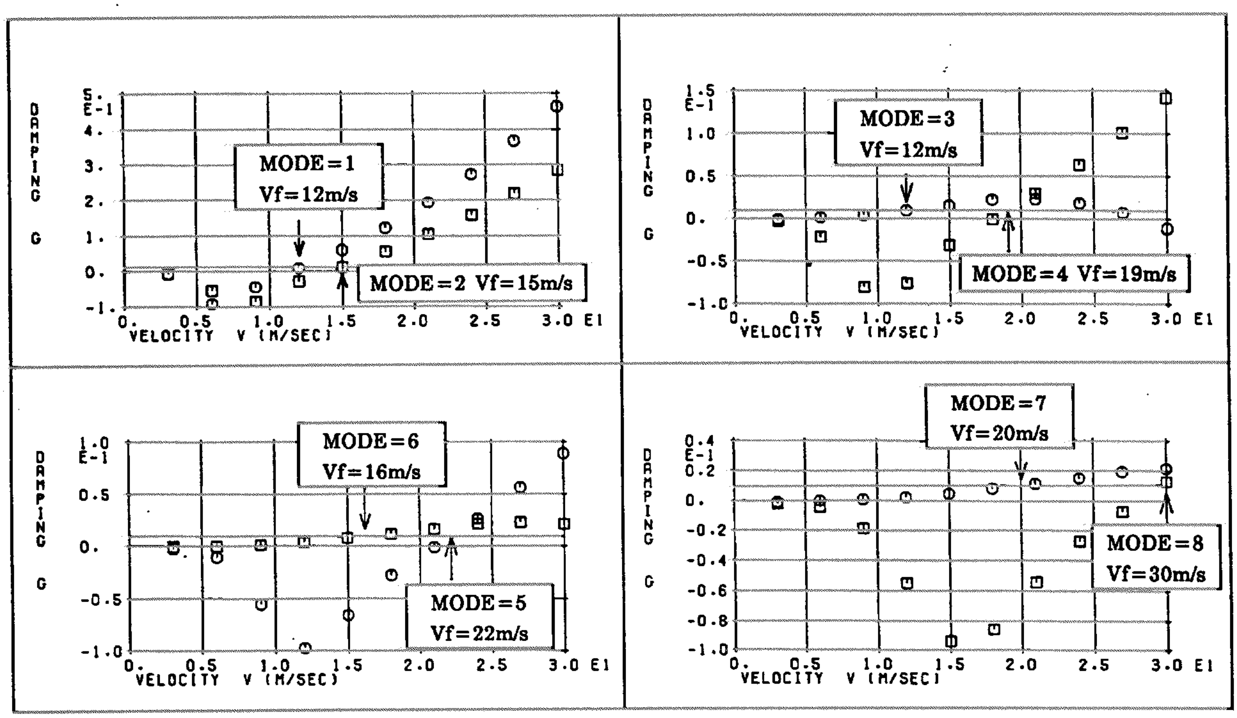


Fig. 7 V - g Curves for Model of Tacoma Narrows Bridge using Tacoma's Aero Coefficients