

WHAT STRUCTURAL ENGINEERS SHOULD KNOW ABOUT MSC/EMAS

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ABSTRACT

Electromagnetic forces, as described by electric and magnetic fields, are important to the design of many mechanical structures. MSC/EMAS is a new program which solves general electromagnetic field problems using the finite element method. The unique formulation of MSC/EMAS produces a matrix equation that is identical to the matrix equation solved in structural analysis, e.g., by MSC/NASTRAN. Because the equations are identical, the advanced solution methods and numerical algorithms used in structural analysis are also applicable to field analyses. Thus, finite element technology from MSC/NASTRAN was used extensively to develop MSC/EMAS. The MSC/EMAS formulation also establishes an analogy between field analysis and structures, which is useful to structural engineers.

INTRODUCTION

Electromagnetic (E&M) forces are an important consideration in the design of many electromechanical devices, including large magnets, motors, solenoids, magnetic levitation systems, and E&M launchers. Such forces are usually described by electric and magnetic fields. With the release of MSC/EMAS, MSC's new general purpose E&M program, fields are analyzed using finite element techniques similar to those used in structural analysis. Thus, it is now much easier for structural engineers to calculate the E&M forces present in electromechanical devices since they are already familiar with the analysis methods.

The purpose of these paper is to describe the similarities between field analysis and structural analysis and to show how existing technology has been exploited in the development of MSC/EMAS. The unique formulation of MSC/EMAS results in a general matrix equation,

$$[M]\{\ddot{u}\} + [B]\{\dot{u}\} + [K]\{u\} = \{P\}, \quad (1)$$

that is **identical** to the equation solved by MSC/NASTRAN. In structures, the unknown degrees of freedom $\{u\}$ represent physical displacements and rotations, while the matrices $[M]$, $[B]$ and $[K]$ represent the mass, damping and stiffness properties of the model. In field analysis, the meaning of these quantities is entirely different. However, since the equations are the same, the advanced matrix methods and numerical algorithms developed for MSC/NASTRAN have been used, often with little change, in MSC/EMAS. Many of these methods, well known to structural analysts, are applied to field analysis for the first time. Equation (1) also establishes a complete analogy between field analysis and structures, one which is helpful to both electrical and structural engineers.

Many structural engineers are unfamiliar with the basic concepts of field analysis. This paper begins with a brief review of field theory and shows how finite element methods are applied to field analysis. The derivation of the matrix equation is described, followed by a thorough discussion of the fields/structures analogy. Finally, the important new features of MSC/EMAS and the electromagnetic features added to MSC's graphics pre/postprocessor, MSC/XL, are summarized.

FIELDS AND FORCES

Electromagnetic field analysis is a branch of mechanics that deals with the motion of charged objects under the influence of electromagnetic forces. Rather than deal directly with charge motion, we usually focus on the forces that mediate their interactions. These forces are represented by electric and magnetic *fields*. The origin of these fields is described in terms of charges and currents. Dynamic field interactions (even in vacuum) must also be taken into account. Once the forces are determined, the underlying charge motion, or *current*, is treated as a recovery quantity.

Electric fields are mathematical shorthand for the forces acting between *stationary* charged objects. Given some distribution of positive and negative electrical charge, a small positive "test charge" is placed at some location, and the force acting on the charge is measured. The ratio of the force to the size of the test charge is called the *electric field* \vec{E} . Figure (1) shows a mapping of the electric field around a single positive charge q . Note that electric field lines originate on positive charges (sources) and terminate on negative charges (sinks) because like charges repel and unlike charges attract.

Magnetic fields represent a different type of force acting between *moving* charges. Given some current distribution, a small "test current" is placed at some location, and the resulting force produced by magnetic fields is measured. This magnetic force is proportional to the size of both the test current and the underlying magnetic field, and it acts in a direction that is perpendicular to both. Figure (1) shows a mapping of the magnetic field near a narrow current (flowing out of

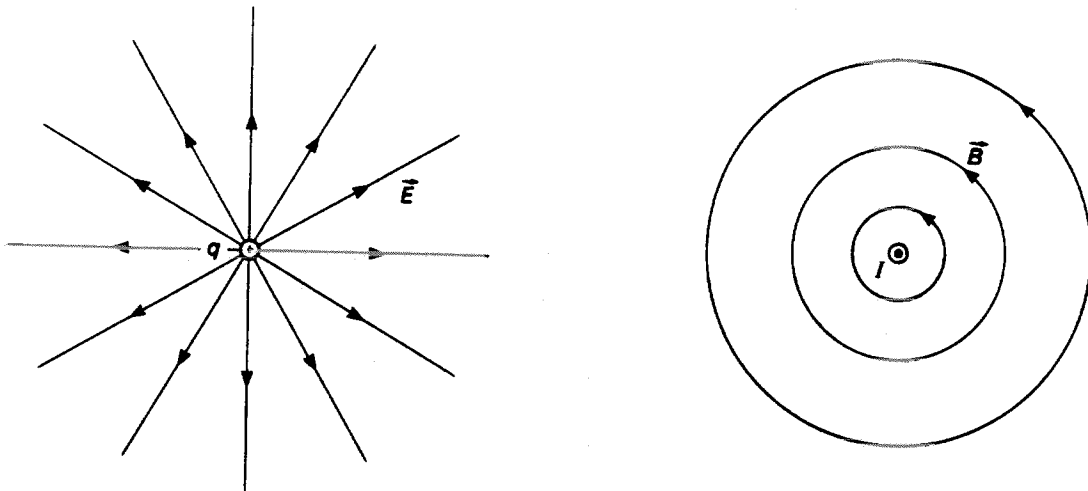


Figure 1: Electrical charges are the sources of electric fields. Electric fields \vec{E} originate on positive charges q and terminate on negative charges. Electrical currents produce magnetic fields \vec{B} . Magnetic fields have no sources or sinks, but instead circulate around currents.

the page). Magnetic fields have no sources or sinks; rather, they circulate around currents in a sense given by the right-hand rule.

Dynamic fields interact in two important ways. First, Faraday's Induction Law states that a time-varying magnetic field *induces* a circulating electric field. This idea must seem rather abstract at first. Here are two force vectors interacting (even in vacuum), seemingly independent of their sources. Nevertheless, induction is an everyday phenomenon, and is responsible for the operation of many simple, practical devices. The second dynamic interaction is the reciprocal of induction; time-varying electric fields act as current sources (called *displacement currents*¹) for circulating magnetic fields. Taken together, induction and the displacement current account for the propagation of electromagnetic waves (radio, light waves, etc.) through vacuum, independent of physical charges and currents.

¹The term "displacement current" is historical and has nothing to do with physical movement.

MATERIAL PROPERTIES

On a microscopic scale, on the scale of atoms and molecules, E&M material behavior can be very complicated. As in mechanics, we adopt a *macroscopic* view in which complicated details are replaced by more simple models of average behavior. It is useful to distinguish between mobile charge, which is free to move through conducting bodies (e.g., electrons in metals) and bound charge, which moves only within microscopic regions (e.g., electrons bound in atomic orbits). There are two basic material models for mobile charge:

- **Charge Continuity** - Charge is conserved. Current flowing into a volume must be accounted for by a charge increase within the volume.
- **Ohm's Law** - Charge moves under the influence of electric fields. The amount of current at any point is proportional to the electric field present and to the electrical conductivity σ of the material.

For bound charge, which cannot produce current in a macroscopic sense, there are also two basic models:

- **The Dielectric Response** - When subjected to an electric field, bound charges produce electric *dipoles*, and a net electric dipole moment per unit volume \vec{P} . The vector, $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, represents forces due only to free charges ρ_{free} , while \vec{E} represents forces due to all charges, including exposed electric dipoles.
- **The Magnetic Response** - Bound charges subjected to a magnetic field produce magnetic dipoles, and a net magnetic dipole moment per unit volume \vec{M} (also called the *magnetization*). The new force vector, $\vec{H} = \nu_0 \vec{B} - \vec{M}$, represents forces from free currents \vec{J}_{cond} , while \vec{B} represents forces from all currents, including exposed magnetic dipoles.

Taken together, these four models constitute a fairly general description of material behavior. In some cases (e.g., in steel) \vec{M} is a nonlinear function of \vec{B} . Such cases are treated in MSC/EMAS in a manner analogous to nonlinear elasticity.

MAXWELL'S EQUATIONS

The basic principles of electromagnetics are combined with material models into a single set of four coupled, partial differential equations, known as *Maxwell's Equations*:

$$\nabla \cdot \vec{D} = \rho_{free} \quad (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\nabla \cdot \vec{B} = 0 \quad (4)$$

$$\nabla \times \vec{H} = \vec{J}_{cond} + \frac{\partial \vec{D}}{\partial t} \quad (5)$$

Together these equations describe both the sources of E&M fields and their dynamic interactions. For more than 100 years these equations have been used to analyze the behavior of virtually all electromagnetic devices, from something as simple as a high-voltage insulator to the most complex digital computers. MSC/EMAS solves these equations in their **complete and general** form.

THE FORMULATION

Rather than solve directly for \vec{E} and \vec{B} , MSC/EMAS makes use of *electromagnetic potentials*. Some programs solve directly for the six unknowns associated with \vec{E} and \vec{B} . However, only four unknowns are actually required because \vec{E} and \vec{B} are related through Maxwell's equations. Further, additional interface conditions (analogous to force equilibrium conditions) on the tangent and normal field components must be applied, potentially on every face of every finite element. The situation is analogous to the choice between the force method and the displacement method in structural analysis. Just as in MSC/NASTRAN, MSC/EMAS uses the "displacement method" based on electromagnetic potentials.

Potential functions are chosen to automatically satisfy two of the four Maxwell equations. Since $\nabla \cdot \vec{B} = 0$, \vec{B} must be the curl of some vector. Call it \vec{A} , the *vector potential*. Faraday's induction law, Eq. (3), can also be automatically satisfied by taking $\vec{E} = -\dot{\vec{A}}$. To account for static electric fields, the gradient of a scalar function is added to \vec{E} to produce the final result:

$$\vec{B} = \nabla \times \vec{A} \quad (6)$$

$$\vec{E} = -\nabla\psi - \dot{\vec{A}} \quad (7)$$

These new potential functions \vec{A} and ψ are the "displacement" functions used in MSC/EMAS.

The physical meaning of \vec{A} and ψ deserves comment. Commonly called the *electrostatic voltage*, $\phi = \psi$ just represents, for example, the voltage between the two terminals of a battery. Some think of \vec{A} as simply a mathematical convenience. Others argue that it represents a single underlying

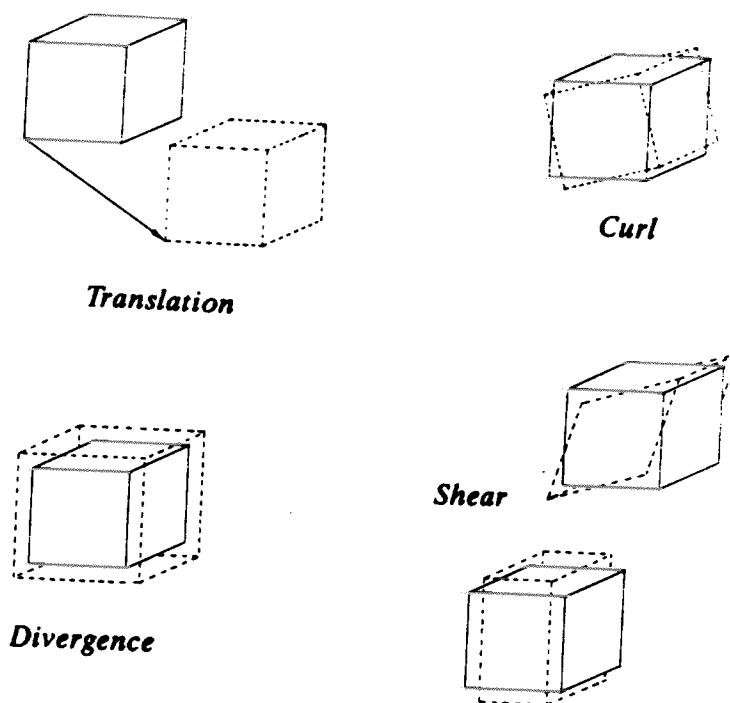


Figure 2: When the vector potential \vec{A} is interpreted as physical displacement, then the twelve independent zero and first-order “motions” may be interpreted as translation, rotation (curl), expansion (divergence) and shear. The only motion that produces electromagnetic energy is rotation; all others represent singular motions analogous to the rigid body modes of mechanical objects.

entity that is responsible for both electric and magnetic effects. In any event, we can think of \vec{A} and ϕ simply as mathematical “displacement” functions that are used to solve for electromagnetic fields.

The potential function \vec{A} is not uniquely determined by Maxwell’s equations. For the moment, think of \vec{A} as representing physical displacement (see Fig. (2)). Just as in any vector field, there are twelve independent zero and first order motions associated with \vec{A} :

- three translations,
- three rotations (curl),
- one volume expansion (divergence) motion, and
- five independent shear motions.

$$\begin{aligned}
\delta w = & \int_{vol} dv \int_{t_0}^{t_1} dt \{ (\delta(\nabla\psi) + \delta\vec{A}) \cdot [\epsilon](\nabla\dot{\psi} + \dot{\vec{A}}) \leftarrow [M] \\
& - (\delta(\nabla\psi) + \delta\vec{A}) \cdot [\sigma](\nabla\dot{\psi} + \dot{\vec{A}}) \leftarrow [B] \\
& - \delta(\nabla \times \vec{A}) \cdot [\nu](\nabla \times \vec{A}) \leftarrow [K] \\
& - \alpha \delta(\nabla \cdot \vec{A}) \cdot (\nabla \cdot \vec{A}) \leftarrow \text{penalty } [K] \\
& - \delta\psi(\rho_o - \nabla \cdot \vec{P}_o) \leftarrow \text{load } \psi - \text{charge} \\
& + \delta\vec{A} \cdot (\nabla \times \vec{M}_o) \leftarrow \text{load } \vec{A} - \text{current} \\
& + \int_{surf} ds \int_{t_0}^{t_1} dt \{ \delta\vec{A} \cdot ((\vec{H} + \vec{M}_o)^* \times \hat{n}) \leftarrow \text{B. C. } \vec{A} \\
& - \delta\psi(\hat{n} \cdot (\vec{J} + \vec{D})^*) \leftarrow \text{B. C. } \psi \\
& + \left[\int_{surf} ds \delta\psi(\hat{n} \cdot (\vec{D} - \vec{P}_o)^*) \right]_{t=t_0}^{t_1} \leftarrow \text{initial condition } \psi
\end{aligned}$$

Figure 3: The finite element formulation of electromagnetics used in MSC/EMAS is derived from this expression for virtual work, which is equivalent to Maxwell's Equations in their complete and general form. Various terms play different roles in the final matrix equation, as indicated.

In elasticity, no energy is associated with uniform translation and rotation. These motions are said to be *singular* and must be eliminated by constraints. In electromagnetics, translation, divergence and shear are **all singular**; only curl is determined by Maxwell's equations. Translation and shear motions are global (like the translation and rotation *rigid body motions* in mechanics), and are eliminated using a total of eight constraints. Divergence motions are local and must be eliminated by an artificial penalty energy in each element.

In order to use finite elements, a variational principle equivalent to Maxwell's equations must first be derived in terms of the potential functions \vec{A} and ψ . Without a detailed derivation, the appropriate expression for virtual work is shown in Fig. (3). A total of nine terms are shown, along with labels which describe their ultimate role in the finite element matrix equation Eq. (1). Each term is described briefly as follows:

1. The work done by electric fields, $\delta\vec{E} \cdot \vec{D}$. Eventually forms the dielectric (or mass) matrix $[M]$.
2. The work done by Ohmic losses. Eventually forms the conduction (or damping) matrix $[B]$.

3. The work done by magnetic fields, $\delta \vec{B} \cdot \vec{H}$. Eventually forms the reluctivity (or stiffness) matrix.
4. The artificial penalty energy for suppressing divergence motions. Included in $[K]$.
5. Loads on ψ due to charges (in various forms). Eventually forms part of the load vector $\{P\}$.
6. Loads on \vec{A} due to currents (in various forms). Eventually forms part of the load vector $\{P\}$.
7. Work done on \vec{A} at the model surface by boundary conditions. Either constrain the tangent components of \vec{A} or apply a load by specifying the value of \vec{H} at the surface.
8. Work done on ψ at the model surface by boundary conditions. Either constrain ψ or apply a load by specifying the current at the surface.
9. Work done on ψ at the model surface as part of the calculation of electrostatic initial conditions. Either constraint ψ or apply a load representing the value of \vec{D} at the surface.

The next step is to apply the finite element approximation to the variation principle shown in Fig. (3). The volume integral is converted to a sum of integrals over smaller finite element volumes. Within each element, potential functions are given a simple, low-order polynomial dependence. The solution is represented by parameters, or *degrees of freedom*, which are simply the values of the potential functions at specified locations, called *grid points*. Matrices, DOF vectors and loads may then be constructed for each element in the model.

Matrices and vectors are then assembled into a single large system of algebraic equations in the form:

$$[M]\{\ddot{u}\} + [B]\{\dot{u}\} + [K]\{u\} = \{P\} \quad (8)$$

The \vec{A} -partition of Eq. (8) represent Ampere's law, Eq. (5), which describes the sources of magnetic fields in terms of currents. The ψ -partition does not represent the remaining Maxwell equation, Eq. (2), but rather its first time derivative, an expression of charge continuity. Electrostatics is treated separately as an initial condition.

THE ANALOGY WITH STRUCTURAL MECHANICS

The form of Eq. (8) establishes an analogy with other physical systems, including structural mechanics and circuit analysis. Of particular interest here is the structures analogy which is summarized in Fig. (4). As can be seen, the potential functions \vec{A} and ψ are analogous to the displacement vector \vec{u} . When a suitable spatial operator is applied to \vec{u} , the result is strain.

Likewise, when a different spatial operator (the curl) is applied to \vec{u} in electromagnetics, the result is the magnetic field \vec{B} . When strain is multiplied by stiffness, the result is stress. In electromagnetics, the quantity analogous to stress is the magnetic field strength \vec{H} . In a similar manner, it is clear that \vec{E} is analogous to velocity, while \vec{D} is analogous to momentum.² Loads in the form of currents and charges are generally analogous to mechanical forces, but the specific form of these loads is quite varied. The energy stored in electric fields $\vec{E} \cdot \vec{D}$ is analogous to the kinetic energy of the system. Similarly, the energy stored in electric fields $\vec{B} \cdot \vec{H}$ is analogous to strain energy. Finally, the energy dissipated through Ohmic heating $\vec{J} \cdot \vec{E}$ is directly analogous to friction heating.

Finite element electromagnetic field models often have a different character than structural models. In structures, energy is present only in restricted regions, i.e., within the part being modeled. One could model regions outside the part, but mass and stiffness is zero outside; so the energy in these regions is zero. In principle, electromagnetic fields extend out to infinite distances. Even vacuum has an effective mass and stiffness, and so can store electromagnetic energy. Thus, the user is obliged to model all space—a daunting task. Fortunately, various methods can be used to truncate the mesh (asymptotic behavior, metallic boundaries, infinite boundaries) and still obtain accurate solutions. Nevertheless, E&M models tend to be large and three dimensional.

MSC/EMAS FEATURES

Much of the technology in MSC/NASTRAN has been adapted for use in MSC/EMAS. The input records have been designed to look like electromagnetics to the user. Still, approximately two-thirds of the 103 different BULK DATA records used in MSC/EMAS are exactly the same as in MSC/NASTRAN. Of course, the elements have been changed to reflect electromagnetic behavior, and are quite different from the structures versions. The solution methods, DMAP sequences, and numerical algorithms are all basically unchanged. Data recovery, however, is quite different. E&M has a large number of important data recovery quantities, many more than in structures. A limited number of basic output quantities are computed within MSC/EMAS itself; the rest are computed using the postprocessor in MSC/XL.

Table (1) lists some of the important features in MSC/EMAS. It is interesting to see how many of these features have been brought over from the MSC/NASTRAN technology and yet are new in finite element electromagnetics (in **bold face type**).

One new feature, the use of scalar elements to model electrical circuits, deserves further comment. Years ago, structures were analyzed using analog computers. Two features were brought over from this work into the early design of NASTRAN. *Scalar elements* were included in NASTRAN

²The novel use of $\dot{\phi}$ rather than the traditional ϕ is critical here. If ϕ is used, the resulting matrix equations are not symmetric because the expression for \vec{E} contains mixed time derivatives. By using $\dot{\phi}$, \vec{E} represents a true velocity, and the matrices are sparse, banded, symmetric, and positive semi-definite, just like their structural counterparts.

s = suitable spatial operator

<i>symbol</i>	<i>electrodynamics</i>	<i>mechanics</i>
$\{u\}$	\vec{A} ψ	displacement rotation
$\{su\}$	\vec{B}	strain
$\{\dot{u}\}$	\vec{E}	velocity
$[M]$	dielectric matrix	mass matrix
$[B]$	conduction matrix	damping matrix
$[K]$	reluctivity matrix	stiffness matrix
$[k \text{ or } \nu]\{su\}$	\vec{H}	stress
$[m \text{ or } \epsilon]\{\dot{u}\}$	\vec{D}	momentum
$\{P\}$	currents	forces
$\{\dot{u}\}^T[M]\{\dot{u}\}$	electric energy	kinetic energy
$\{u\}^T[K]\{u\}$	magnetic energy	strain energy
$\{\dot{u}\}^T[B]\{\dot{u}\}$	Ohmic heating	friction heating

Figure 4: Because the matrix equations for electromagnetics and structural mechanics have the same form, an analogy is established. This table relates analogous quantities in the two systems.

ELEMENTS	isoparametric, optional midside nodes 1D, 2D and 3D mixed within the same model axisymmetric elements scalar elements
LOADS	seventeen different load types inhomogeneous Neumann boundary conditions dynamic loads generated from static load sets nonlinear loads
CONSTRAINTS	local coordinate systems unlimited MPC conditions direct matrix input transfer functions Lagrange multipliers
SOLUTION METHODS	DMAP electrostatics magnetostatics direct frequency response direct transient analysis modal real eigenvalue analysis complex eigenvalue analysis nonlinear magnetostatics (tangent) direct nonlinear transient analysis

Table 1: Many of the important features in MSC/EMAS are appearing in a commercial finite element electromagnetics product for the first time (**bold face**).

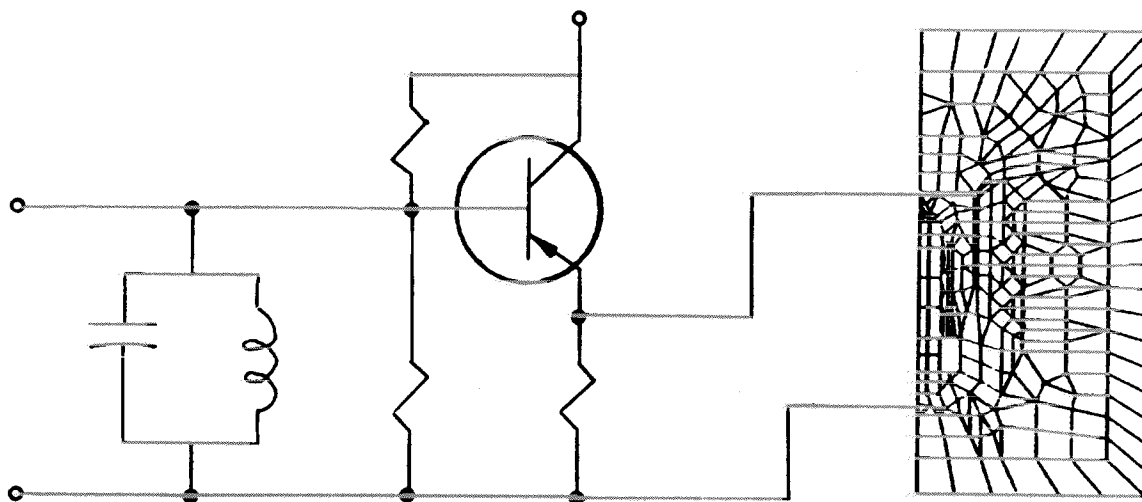


Figure 5: MSC/EMAS allows the user to combine finite element models of electromagnetic devices with scalar element models of electronic drive and control circuits. The two interacting systems are then analyzed as a single, combined model.

to simulate the resistors, capacitors and inductors used in the earlier analog work to represent dampers, masses and springs. Likewise, *nonlinear loads* were incorporated to simulate the multipliers found in analog computers. It is amusing that these same features are now used in MSC/EMAS to model electronic drive circuits coupled with finite element models (see Fig. (5)). This feature is unique in electromagnetic field analysis.

MSC/XL ENHANCEMENTS FOR ELECTROMAGNETICS

Graphic pre/postprocessing is an important feature of any finite element modeling system, and is especially important in the realm of 3D electromagnetics. Version 1B of MSC/XL contains full electromagnetic model building and data analysis capabilities. Table (2) lists some of the electromagnetic features added to MSC/XL. The treatment of geometry, constraints and meshing is essentially the same, but many features have been added for greater convenience. Material properties, loads and data recovery have been changed to represent E&M quantities, so that MSC/XL

has the look and feel of electromagnetism. Of particular interest to field analysts are arrow plots of vector quantities and the results calculator which lets the user form new output quantities from existing quantities in the database.

THE FUTURE

The experienced MSC/NASTRAN user will notice that many important features have not been incorporated into MSC/EMAS. These include superelements, cyclic symmetry, modal dynamics, and design optimization. MSC has plans to include these more sophisticated methods within MSC/EMAS as time and client needs allow. Several capabilities peculiar to E&M are also planned. Some treatment of magnetic hysteresis in three-dimensions, either isotropic or anisotropic, is desirable. (Magnetic hysteresis is analogous to anisotropic nonlinear elasticity with yielding.) Modeling of open boundaries is also desirable. More elaborate material models—including magnetic recording media, semiconductors, and superconductors—are also possible. Because MSC/EMAS and MSC/NASTRAN share a common formulation and a common technology base, numerical enhancements will quickly spread to both products.

The prognosis for finite element field analysis, and MSC/EMAS in particular, is a bright one. Finite element analysis is successful in field analysis for the same reasons it is with structures; it is the only method available for solving complicated field problems involving “real world” geometries and material properties. Structural engineers can now extend their repertoire by first calculating electromagnetic forces, and then predicting their effects on mechanical structures. Fortunately, both analyses can now be done using the same basic methods.

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SUGGESTED READING

1. David Halliday and Robert Resnick, *Physics: Part II*, John Wiley and Sons, Inc. (New York, 1962).

USER INTERFACE	grouping improved calculator expressions multiple applications expanded MACROs titles
GEOMETRY	aligning curves/surfaces use of local coordinates
FINITE ELEMENTS	extruding Delauney-based solid meshing constraint generation by CS plane E&M elements E&M properties E&M loads MSC/EMAS Case and Exec
RESULTS PROCESSING	XY plots results calculator import/export summaries path plots E&M forces arrow plots

Table 2: Many features have been added to MSC/XL Version 1B to support electromagnetic field analysis.

2. John David Jackson, *Classical Electrodynamics*, John Wiley and Sons, Inc. (New York, 1962).
3. Julius Adams Stratton, *Electromagnetic Theory*, McGraw-Hill Book Company, Inc. (New York, 1941).
4. *What Every Engineer Should Know About Finite Element Analysis*, John R. Brauer (ed.), Marcel Dekker, Inc. (New York, 1988).
5. *MSC/EMAS User's Manual*, Bruce E. MacNeal (ed.), The MacNeal-Schwendler Corporation (Los Angeles, 1989).
6. *MSC/EMAS Applications Manual*, John R. Brauer (ed.), The MacNeal-Schwendler Corporation (Los Angeles, 1989).