OPTIMUM DESIGN OF SPACECRAFT STRUCTURES USING MSC/NASTRAN

R.A. Oldham and F.H. Chu
Mechanical Analysis
General Electric/Astro Space Division
Princeton, NJ

In the design and analysis of the complicated structures that exist in the aerospace industry, there is, without exception, the necessity for structural optimzation with respect to weight. In addition, there will almost always be constraints placed on this optimization such as stress restrictions, or, significant to the scope of this paper, frequency requirements.

What follows is an account of the application of an optimization technique which employs MSC/NASTRAN to the stiffness design of a typical spacecraft built at General Electric/Astro Space Division (GE/ASD). It is hoped that the results of this technique, which was developed at GE/ASD, may eventually be compared to those generated by the application of MSC/NASTRAN's optimization capability for normal modes analysis when it becomes operative in Version 66A.

Background

Due to the mathematical complexity of the finite element models used to simulate aerospace structures, weight optimization with frequency constraints is most aptly achieved through an iterative process involving dynamic analysis with some finite element code such as MSC/NASTRAN, an optimizing technique applied to critical parameters called design variables, and finally a dynamic re-analysis to determine whether or not the frequency constraints have been met.

According to the MSC/NASTRAN Handbook for Structural Optimization [1], the optimization problem may be posed as follows:

$$\text{Find } \mathbf{X} = (x_1, x_2, x_3, \ldots, x_n)$$
$$\text{Such that } F(\mathbf{X}) \text{ becomes a minimum and}$$
$$G_j(\mathbf{X}) \leq 0 \qquad j = 1,2,3,\ldots m$$
$$x_i^L \leq x_i \leq x_i^U \qquad i = 1,2,3,\ldots n \tag{1}$$

Where **X** is a vector containing design variables, F(**X**) is an objective function (in this case, weight as a function of the design variables), Gj(**X**) is a set of constraints (here, a pre-selected frequency), "m" is the total number of constraints to be applied, x_i^U and x_i^L are upper and lower bounds placed on the design variables, and "n" is the total number of design variables. A design variable is the parameter of a structural member (such as the area of a beam or the thickness of a plate) that has a profound effect on the formulation of the objective or the constraints. It is important to realize that the selection of design variables is neither simple nor trivial, requiring experience and insight into the behavior of the structure in question. A more rigorous explanation of optimization principle can be found in Reference 1.

Method

Available to the designer or analyst are many methods of optimization, each with their own virtues and vices, depending on the type of problem it is called upon to solve. At GE/ASD, one such method under scrutiny is that which was developed by Wang and Chu [2]. Here, a set of optimality criteria has been derived among design variables by the Lagrange Multiplier technique. When applied to the output of an MSC/NASTRAN normal modes analysis, eigenvalue and weight sensitivities as well as redesign equations are generated as follows:

$$C_i = \frac{\partial W}{\partial x_i} = \frac{\rho_i}{x_i} \times \frac{\sum_{e=1}^{ne} (ESE)_e}{\sum_{e=1}^{ne} (ESED)_e} \tag{2}$$

$$D_i = \frac{\partial \lambda_k}{\partial x_i} = \frac{2\beta_i}{M_k x_i} \times \sum_{e=1}^{ne} (ESE)_e \tag{3}$$

$$\mu = \frac{\Delta\lambda + (1+\alpha)\sum_{i=1}^{n} D_i x_i}{(1-\alpha)\sum_{i=1}^{n} \frac{D_i x_i}{C_i}} \tag{4}$$

$$x'_i = x_i\left[\alpha + \mu(1-\alpha)\frac{D_i}{C_i}\right] \tag{5}$$

Equation (2) represents the weight sensitivity with respect to the ith design variable , equation (3), the eigenvalue sensitivity with respect to the ith design variable, equation (4), the Lagrange Multiplier, and equation (5) represents the actual redesign formula.

The subscript i represents quantities associated with any one design variable (x_i). Therefore, for each design variable chosen, there is an associated weight sensitivity (C_i), eigenvalue sensitivity (D_i), and *weight* density (ρ_i). Also associated with each design variable is a stiffness factor, β_i, which is determined by the type of finite element the design variable is related to. Wang and Chu [2] define this parameter as follows:

$\beta_i = 1$ for truss elements

$\beta_i = 2$ for beam elements

$\beta_i = 3$ for plate elements

The acronyms ESE and ESED represent **E**lemental **S**train **E**nergy and **E**lemental **S**train **E**nergy **D**ensity, respectively. These are related to design variables by virtue of the fact that each design variable is typically associated with a unique PID (property id) and every PID is referenced by a group of elements, each of which generates a value of strain energy and strain energy density when subjected to a normal modes analysis. The term

$$\sum_{e=1}^{ne} (ESE(D))_e ,$$

therefore, is actually the summation of strain energy (or strain energy density) over each element that references a unique PID ("ne" being the number of elements referencing the PID associated with the design variable, x_i). As MSC/NASTRAN only produces strain energies and strain energy densities with respect to each element, a separate routine must be developed , capable of grouping all elements that reference a common PID and summing their energy values. One may now be able to see why the selection of design variables requires some foresight as they are ultimately intertwined with the creation of the finite element model. It would behoove the analyst or designer to create his or her model with possible design variables in mind, providing for the proper and efficient definition of elements and their related properties.

Typically, the fundamental frequency (first mode) of a structure is of primary concern in the initial stages of design. This in mind, the subscript k in equation (3) will most likely take the value of 1, redefining the equation as "the eigenvalue sensitivity of the *first* mode with respect to the ith design variable". Consequently, the generalized mass quantity (M_k) becomes the generalized mass of the first mode, or M_1. If the MSC/NASTRAN analysis were to be allowed to default to a generalized mass normalized eigenvector, the following would hold:

$$M_1 = 1$$

The variable μ in equation (4) is a Lagrange multiplier developed by Wang and Chu [2] and used to relate the objective function (weight) with the constraints (frequency). The term $\Delta\lambda$ is the difference in the eigenvalue of the structure before applying the optimization routine and the desired eigenvalue or frequency constraint. Of course, $\lambda = (2\pi f)^2$. The term α in equations (4) and (5) is a relaxation factor used in the formulation of the Lagrange multiplier. Usually,

$$\alpha = 0.5$$

The x'_i term found in equation (5) is the *optimized* value of the design variable (x_i) after the objective function reaches an extremum and the constraint requirements have been met.

Benchmark

In the process of testing a new technique, a well behaved and fully explored example should be used as a benchmark. Such a benchmark is given by Wang [3] in the closed form solution of the design of a one-dimensional vibration system for minimum weight and specified fundamental frequency. Specifically, this problem deals with a cantilevered , variable cross-section beam with a mass attached to the free end and restrained to longitudinal motion (see Fig. 1). Wang sets the frequency constraint to 410 Hz and proceeds to analytically develop the following cross-sectional areas for each step in the beam:

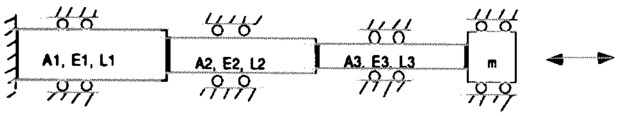
A1 = 12.33 sq. in

A2 = 9.28 sq. in

A3 = 5.60 sq. in

Theoretically, these values provide a permutation of the structure shown in Figure 1 which has a fundamental longitudinal frequency of 410 Hz at the lightest possible total weight (including the 38.64 lb tip mass) of 120.27 pounds. This same configuration, however, develops a fundamental longitudinal frequency of 405 Hz when subjected to a MSC/NASTRAN normal modes analysis.

The solution of this optimization problem is repeated, using the iterative method discussed above. Simply put, it is desired to find the cross-sectional area of a beam such as described in Figure 1 so that the structural weight is minimized and the fundamental longitudinal frequency is 405 Hz. The objective function, therefore, is the weight, the constraint is the first mode frequency, and the design variables are the cross-sectional areas of each step in the beam.



$$E_1 = E_2 = E_3 = 10.3\text{e}6 \text{ psi}$$

$$\rho_1 = \rho_2 = \rho_3 = .1 \text{ lb/in}^3$$

$$L_1 = L_2 = L_3 = 30 \text{ in}$$

$$m = .1 \text{ lb-s}^2/\text{in}$$

$$\mu = .33$$

Figure 1

Benchmark Example

The first step is to assume initial values for the three areas in question. This is easily done by allowing the beam to have a uniform cross section and theoretically solving for the cross-sectional area required for a first mode frequency of 405 Hz using formulas which can be found in a work such as Blevins [4]. This value is approximately 11 sq. in.

This configuration is then run in MSC/NASTRAN under a SOLUTION 3 for normal modes analysis, requesting the output of elemental strain energy data. The elemental strain energy data is then summed over each PID associated with the design variables. Finally, this data is used in equations (2) through (5) to develop the first iteration of optimized design variables.

These values then replace the original design variables and the process repeats until the resulting parameters converge to the optimum condition. As can be seen in Table 1 and Figure 2, this problem converges after approximately four to five iterations and develops a 123.5 lb system with a fundamental longitudinal frequency of 405 Hz.

Iterat	Freq	Weight	Area1	Area 2	Area 3	ESED 1	ESED 2	ESED 3
0	401.750	137.600	11.0000	11.0000	11.0000	4643.75	3401.96	1608.30
1	417.930	132.590	12.6270	10.7220	7.9680	3670.19	3728.25	3590.19
2	410.620	126.860	11.8570	10.1430	7.4060	3720.63	3763.47	3868.72
3	406.720	124.450	11.4560	9.8540	7.2930	3783.48	3805.07	3839.31
4	405.480	123.730	11.3270	9.7710	7.2640	3808.67	3814.60	3822.55
5	405.140	123.530	11.2920	9.7480	7.2550	3815.28	3817.19	3819.06
6	405.040	123.470	11.2820	9.7420	7.2520	3817.14	3817.76	3818.52

Table 1

Optimization Results by Iteration

(Benchmark Example)

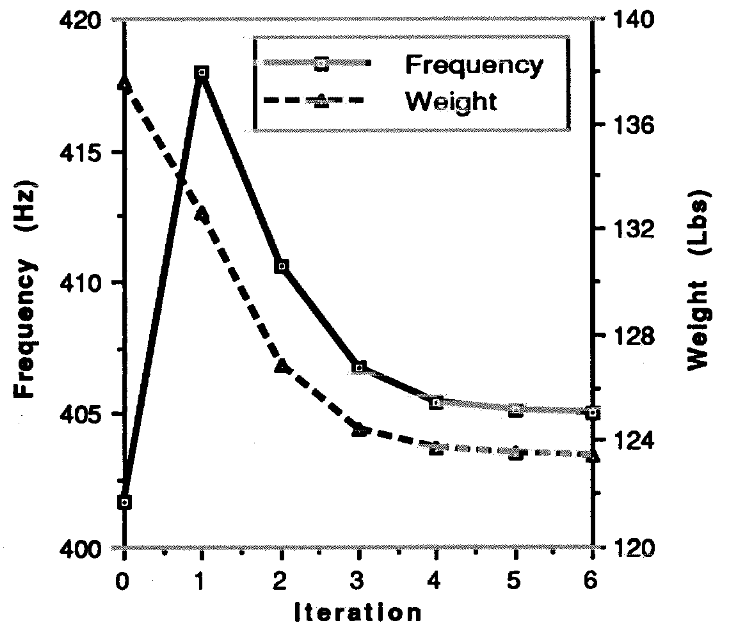


Figure 2

Optimization Results

Frequency and Weight vs Iteration

(Benchmark Example)

At the end of six iterations , the system shows a 10.3% reduction in overall weight, a 14.3% reduction in structural weight, and a .82% *increase* in the fundamental natural frequency. Interestingly, the weight of 123.5 pounds is only 2.7% greater than the value of 120.27 pounds as found in the closed form solution by Wang.

Practical Application

The above example is suitable as a test, as the small size and directional restrictions eliminate the many variables that can effect the outcome of an optimization problem with frequency constraints. The critical test of new techniques, however, does not involve well behaved systems, but systems which represent actual structures -- structures such as a typical communications satellite designed and built at GE/ASD.

Such a satellite is shown in Figure 3. It is constructed of four equipment panels which are connected to a center cylinder through the use of what are known as bulkheads. Towers and reflectors necessary for the satellite's operation reside topside, while below, a conical adapter is used to interface the spacecraft with the booster vehicle which will send it into orbit.

Obviously, the spacecraft must be designed to be as light in weight as possible and, in order to make it safe for launch, it must meet certain frequency requirements as necessitated by the vendor of the booster vehicle. Typically, the first mode of the spacecraft is paramount and will therefore be the concern of this example.

Careful scrutiny of the structure reveals that only the center cylinder (26.67 lbs) plays a significant role in the determination of the frequency of the first bending mode. This is an *extremely* crucial observation as it aids in the selection of the design variables which will be modified during the optimization process (remember, a design variable is the parameter of a structural member that has a profound effect on the formulation of the objective or the constraints).

Keeping the importance of the center cylinder in mind, a finite element model of the spacecraft is constructed. It should be noted that in the initial stages of design, particularly when dealing with frequency constraints, the best finite element model is not necessarily a complex collection of beams, plates, and hexes, but one which will simply and accurately depict the physical nature of the structure. Such a model is shown in Figure 4. The center cylinder of the communications satellite is represented by the long vertical bar element, centrally located in the model. The remainder of the

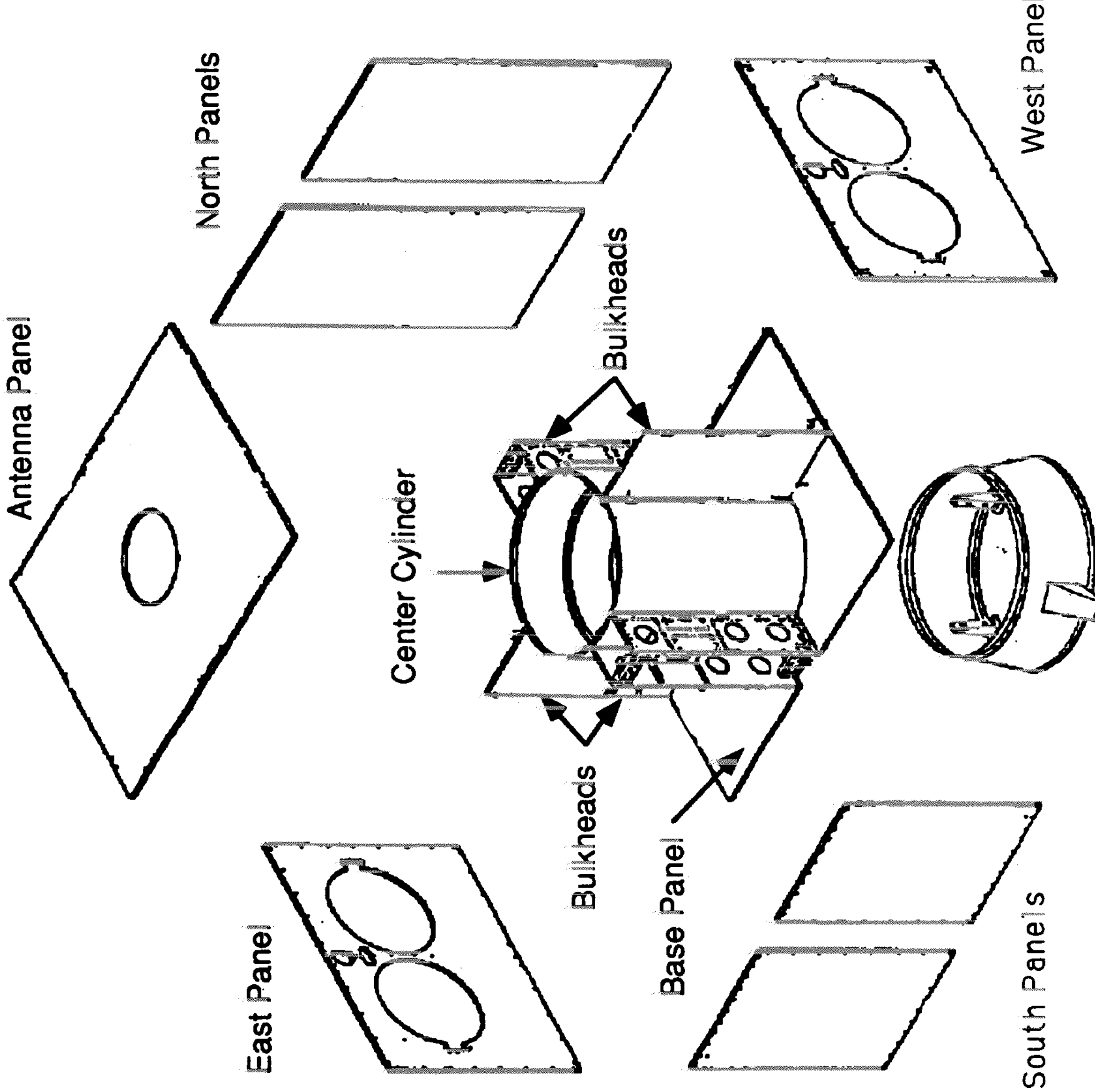


Figure 3

Typical Communications Satellite
Exploded View

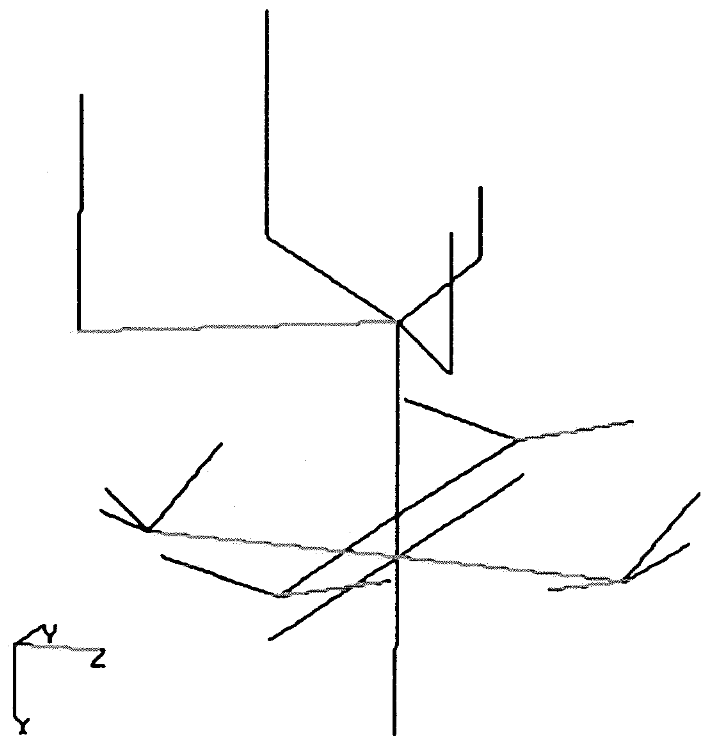


Figure 4
Typical Communications Satellite
Stick and Spring Model
(Spacecraft Example)

spacecraft (panels, bulkheads, towers, reflectors, and adapter), which are considered unimportant for this optimization formulation, are simulated by a combination of bars, springs, and generalized elements. It can be seen in Figure 5 that the actual center cylinder is to consist of three sections, each separated by a latitudinal ring. In the finite element model, therefore, each section is represented by a unique bar element with properties directly related to the cross section of the center cylinder, which is a very thin annulus. During the initial modeling, these section properties are not chosen arbitrarily, but, as in the benchmark example, are developed and tuned to create a model that will predict the primary modes of the spacecraft. Since these properties do not necessarily represent an optimal condition, they are fine tuned with the optimization technique.

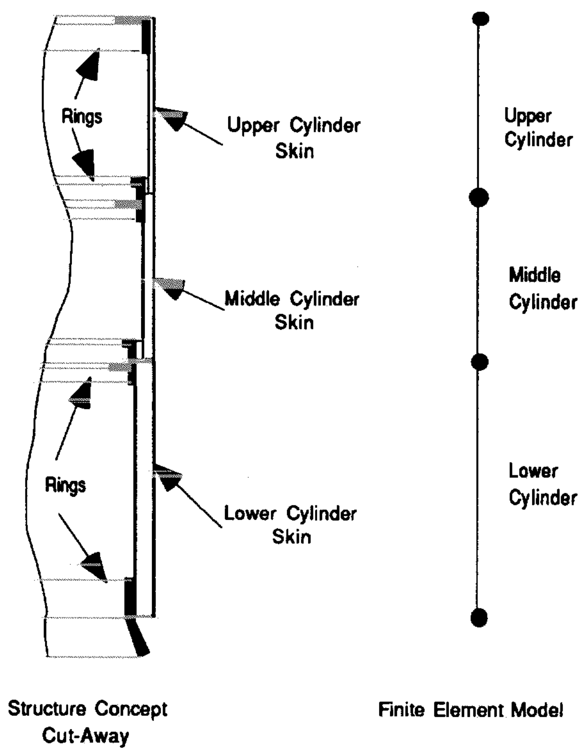


Figure 5

Structure Concept vs Finite Element Model

of Center Cylinder

(Spacecraft Example)

Following the same procedure used to minimize the weight of the benchmark, the center cylinder of this model is optimized with an arbitrary frequency constraint of 21.71 Hz on the first mode, using the cross-sectional areas of each segment of the center cylinder as the design variables. Referring to Table 2 and Figure 6, it can be seen that after seven iterations, the final center cylinder weight is 20.08 pounds, representing a 25% decrease in structure weight, and the spacecraft suffers no change in the frequency of the first bending mode. Also, although the frequency constraint is not reached until the seventh iteration, there is no appreciable reduction in weight after the fifth iteration. Therefore, if a first bending mode frequency of 21.76 Hz is considered acceptable to the designer, the optimization process could have been terminated at that point.

Iterat.	Freq.	Weight	Upper Area	Middle Area	Lower Area	Upper ESED	Middle ESED	Lower ESED
0	21.71	26.67	6.767	8.825	14.21	3.886	10.03	11.16
1	22.63	25.58	4.652	8.682	14.75	9.084	9.410	9.055
2	22.14	22.38	4.061	7.695	12.86	11.02	11.05	10.96
3	21.92	21.16	3.842	7.290	12.14	11.88	11.87	11.84
4	21.81	20.60	3.745	7.103	11.82	12.28	12.28	12.27
5	21.76	20.28	3.700	7.017	11.67	12.48	12.48	12.47
6	21.74	20.22	3.679	6.976	11.60	12.57	12.57	12.57
7	21.71	20.08	3.667	6.953	11.56	12.62	12.61	12.63

Table 2

Optimization Results by Iteration

(Spacecraft Example)

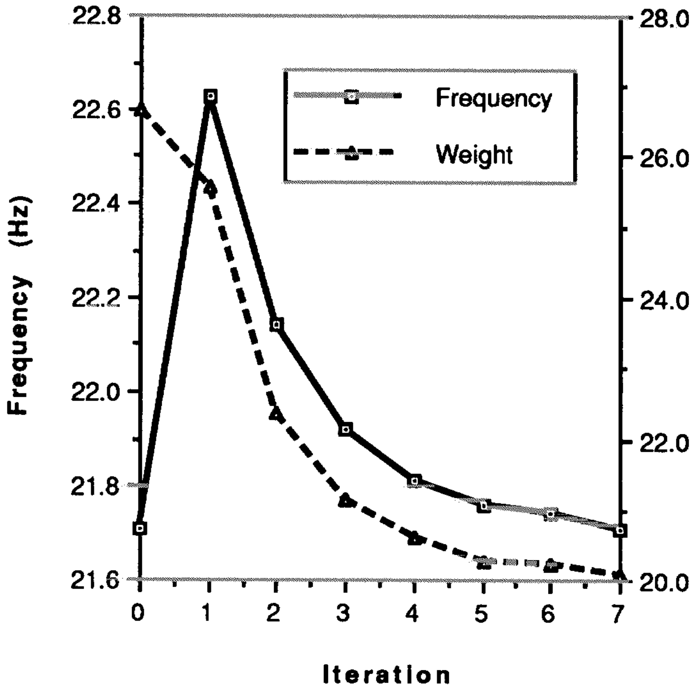


Figure 6

Optimization Results

Frequency and Weight vs Iteration

(Spacecraft Example)

Conclusion

In the above example, the structure of a typical spacecraft was optimized with respect to weight while constrained to a specific first mode frequency using an implementation of the optimality criterion introduced by Wang and Chu [2] which effectively utilizes the strain energy data produced by a MSC/NASTRAN SOLUTION 3 (normal modes analysis) analysis. After seven iterations, the weight of a section of the spacecraft significant in the formulation of the first mode frequency was reduced from 26.67 pounds to 20.08 pounds while maintaining a frequency of 21.71 Hz. That represents a 25% drop in weight.

Hence it has been shown that, when used intelligently, Wang and Chu's method, by virtue of its simplicity in use and programming, can be an efficient and effective tool in the minimal weight design of spacecraft structures with frequency constraints.

REFERENCES

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