Dynamic Optimization Applied to Engine Structure

by

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1. INTRODUCTION

With the remarkable advances in the field of electronic computers, it has become possible to carry out vibration analysis of large structure using the FEM. Shifting the natural frequency of a vibration mode that creates problems by some effective structural change is very important in engine designing.

Till now, experience of the designer and analyzer has played a major role in making the structural changes. However, with structures becoming more and more complex and an increased necessity for taking action against any arbitrary vibration modes that may create problems, a situation has arisen where experience can no longer deal with the problem effectively.

This time, the author employed the super element method for carrying out vibration analysis of the cylinder body, bearing caps, crank shaft and flywheel systems, and shifted a given natural frequency to the designated frequency range efficiently with the use of the dynamic optimization program developed by Isuzu Motors Ltd., using the sensitivity of eigenvalues of the residual structure (cylinder body + bearing caps in this case).

2. THEORETICAL EQUATIONS

a. Super Element Method

We will carry out the calculation for the structural system shown in Fig. 1 in two steps; that of sub system and of main system.

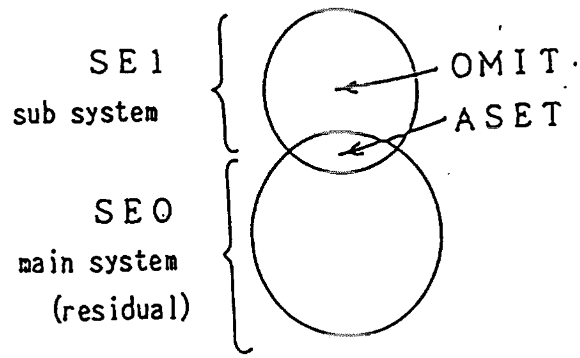

Fig1. STRUCTURAL SYSTEM

①. Calculation for Sub System

Let us carry out the eigenvalue analysis for determining the transformation matrix [T] that is used for reduction.
The use of suffixes depends on MSC/NASTRAN USER'S MANUAL.

$$\begin{Bmatrix} ut \\ uq \\ uo \end{Bmatrix} = \begin{bmatrix} [I] & [0] \\ [0] & [I] \\ [G] & [\phi] \end{bmatrix} \cdot \begin{Bmatrix} ut \\ uq \end{Bmatrix} \qquad (1)$$

$$= \begin{bmatrix} & T & \end{bmatrix} \cdot \begin{Bmatrix} ut \\ uq \end{Bmatrix} \qquad (2)$$

$$([Kvv] - \lambda [Mvv]) \{\phi vy\} = \{0\} \qquad (3)$$

$$[Kyy] = [\phi vy]^{t} \cdot [Kvv] \cdot [\phi vy] \qquad (4)$$

$$[Myy] = [\phi vy]^{t} \cdot [Mvv] \cdot [\phi vy] \qquad (5)$$

$$([Kxx] - \lambda [Mxx]) \{\phi xz\} = \{0\} \qquad (6)$$

$$[\phi wz] = [Gwx] \cdot [\phi xz] \qquad (7)$$

$$[\phi yz] = \begin{bmatrix} \phi xz \\ \phi wz \end{bmatrix} \qquad (8)$$

$$[\phi] = [\phi vy] \cdot [\phi yz] \quad (9)$$

Let us reduce the [Kff], [Mff] of the sub system into the main system.

$$[\tilde{K}aa]^{SE1} = [T]^t \cdot [Kff]^{SE1} \cdot [T] \quad (10)$$

$$[\tilde{M}aa]^{SE1} = [T]^t \cdot [Mff]^{SE1} \cdot [T] \quad (11)$$

②. Calculation for Main system

Let us carry out eigenvalue calculation in main system (residual structure)

$$[Kgg] = [Kjj] + [\tilde{K}aa]^{SE1} \quad (12)$$

$$[Mgg] = [Mjj] + [\tilde{M}aa]^{SE1} \quad (13)$$

If we use dynamic reduction, we get

$$[Kgg] \rightarrow [Kaa]$$

$$[Mgg] \rightarrow [Maa]$$

$$([Kaa] - \lambda [Maa]) \{\phi a\} = \{0\} \quad (14)$$

where, λ : eigenvalue

$\{\phi a\}$: eigenvector

b. Dynamic Optimization Method

①. Sensitivity of Natural Frequency

$$([K] - \lambda[M])\{\phi\} = \{0\} \qquad (15)$$

where, $\lambda = (2\pi f)^2$

$$([K] - f^2 \cdot 4\pi^2[M])\{\phi\} = \{0\} \qquad (16)$$

Here, if we take new a [M] for $4\pi^2$[M], and take i ($i = 1 \sim n$) as the harmonic number of natural vibration, we get

$$([K] - f_i^2[M])\{\phi_i\} = \{0\} \qquad (17)$$

If we differentiate the above equation by the design variable (plate thickness) tj and rearrange it, we get

$$\frac{\partial f_i}{\partial t_j} = \frac{\{\phi_i\}^t\left(\frac{\partial [K]}{\partial t_j} - f_i^2 \frac{\partial [M]}{\partial t_j}\right)\{\phi_i\}}{2 f_i \{\phi_i\}^t [M] \{\phi_i\}} \qquad (18)$$

②. Structural Change with Pseudo Least Square Method

By using the sensitivity of the above equation, we can estimate linearly, as in the following equation, the natural frequency that is obtained after the change in the design variable tj.

$$[Z] \cdot \{\Delta t\} = \{\Delta f\} \qquad (19)$$

where, [Z] : sensitivity matrix

$$Z_{ij} = \frac{\partial f_i}{\partial t_j} \qquad (i = 1 \sim n,\ j = 1 \sim m)$$

$\{\Delta t\}$: increment vector of design variable
$\{\Delta f\}$: increment vector of natural frequency

$$\begin{bmatrix} \frac{\partial f_1}{\partial t_1} & \frac{\partial f_1}{\partial t_2} & \frac{\partial f_1}{\partial t_3} & \sim & \frac{\partial f_1}{\partial t_j} & \sim & \frac{\partial f_1}{\partial t_m} \\ \frac{\partial f_2}{\partial t_1} & \frac{\partial f_2}{\partial t_2} & \frac{\partial f_2}{\partial t_3} & \sim & \frac{\partial f_2}{\partial t_j} & \sim & \frac{\partial f_2}{\partial t_m} \\ \wr & \wr & \wr & & \wr & & \wr \\ \frac{\partial f_i}{\partial t_1} & \frac{\partial f_i}{\partial t_2} & \frac{\partial f_i}{\partial t_3} & \sim & \frac{\partial f_i}{\partial t_j} & \sim & \frac{\partial f_i}{\partial t_m} \\ \wr & \wr & \wr & & \wr & & \wr \\ \frac{\partial f_n}{\partial t_1} & \frac{\partial f_n}{\partial t_2} & \frac{\partial f_n}{\partial t_3} & \sim & \frac{\partial f_n}{\partial t_j} & \sim & \frac{\partial f_n}{\partial t_m} \end{bmatrix} \begin{Bmatrix} \Delta t_1 \\ \Delta t_2 \\ \wr \\ \Delta t_j \\ \wr \\ \Delta t_m \end{Bmatrix} = \begin{Bmatrix} \Delta f_1 \\ \Delta f_2 \\ \wr \\ \Delta f_i \\ \wr \\ \Delta f_n \end{Bmatrix} \tag{20}$$

If the sensitivity matrix [Z], and the amount of variation of natural frequency $\{\Delta f\}$ are given, we can get the increment vector of design variable $\{\Delta t\}$ from equation (19).
Equation (19) is a simultaneous equation with more variables than the number of equations, and hence has innumerable solutions $\{\Delta t\}$. However, if we put the condition that $\{\Delta t\}$ is normed minimum, then immediately the solution narrows to just one.

In short, if we take $\{\ell\}$ as the Lagrange's multiplier vector and H as the Lagrange's function, we get

$$H(\{\Delta t\}, \{\ell\}) = \frac{1}{2}\{\Delta t\}^t\{\Delta t\} - ([Z]\{\Delta t\} - \{\Delta f\})^t\{\ell\} \tag{21}$$

We need only find out the $\{\Delta t\}$ that will reduce the Lagrange's function

H to minimum under the conditions of equation (19).

$$\frac{\partial H}{\partial \{\Delta t\}} = 0 \tag{22}$$

From the equation (22) , we get

$$\{\Delta t\} = [Z]^t \cdot \{\ell\} \tag{23}$$

If we substitute the equation (23) in equation (19), we get

$$[Z] \cdot [Z]^t \cdot \{\ell\} = \{\Delta f\} \tag{24}$$

$$\{\ell\} = ([Z]\cdot[Z]^t)^{-1}\{\Delta f\} \tag{25}$$

By substituting equation (25) in equation (23), we get

$$\{\Delta t\} = [Z]^t \cdot ([Z]\cdot[Z]^t)^{-1}\{\Delta f\} \tag{26}$$

This equation (26) is the well-known formula used in the pseudo least square method. However large the number of elements might be, in carrying out the calculation of equation (26), $[Z]\cdot[Z]^t$ will be reduced to the matrix size of the number of natural frequency and calculation of its inverse matrix can be carried out in a brief period of time.

3. CALCULATION FLOW

The calculation consists of five steps, coming in the following sequential order.

①. Initial Run

②. Sub System Calculation Run

③. Main System Calculation Run

④. Sensitivity Calculation Run

⑤. Optimization Run

The flowchart of calculation has been shown in Fig. 2.

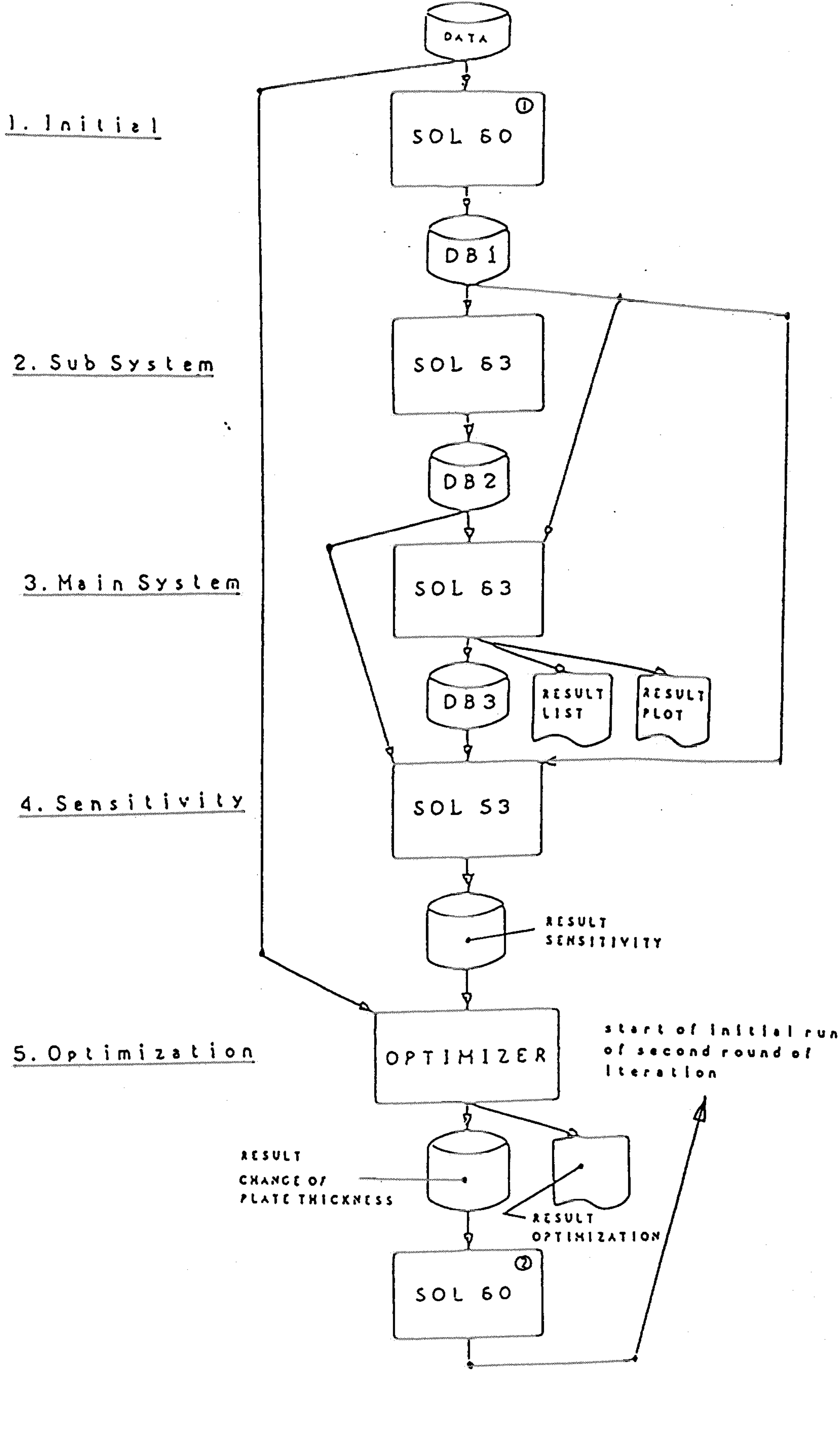

Fig2. CALCULATION FLOW

4. APPLICATION TO ENGINE STRUCTURE

The calculation model used here consisted of cylinder body, bearing caps, crank shaft and flywheel systems.

We took up the torsional mode of the low frequency side, that constituted one of the principal vibration modes in the engine structure, and tried to apply this method for making structural change to 600 Hz, which was about 10% higher than the present frequency of 548 Hz. We assumed the coupling of each part to be rigid coupling using RBE2.

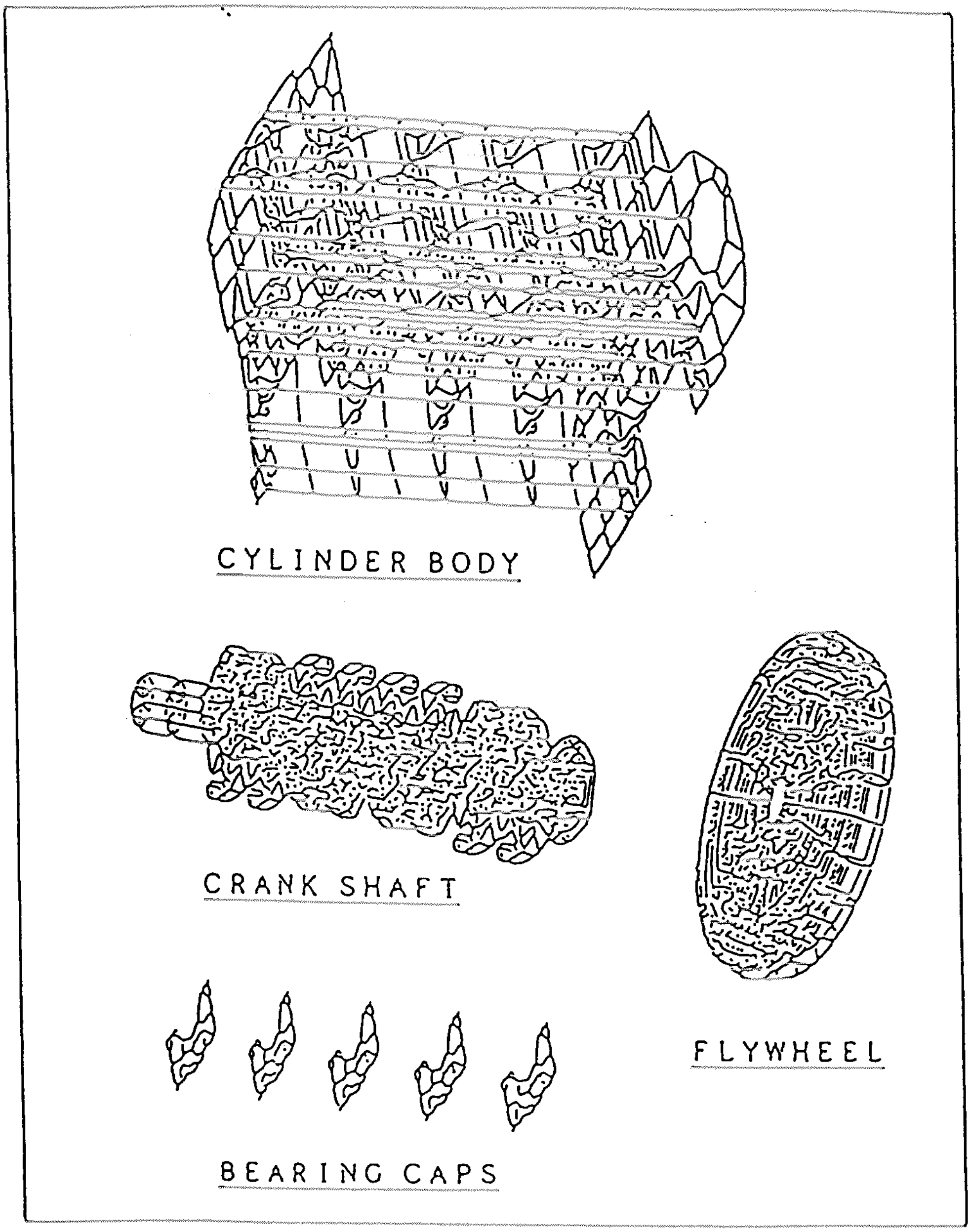

Fig3. PARTS OF ENGINE STRUCTURE

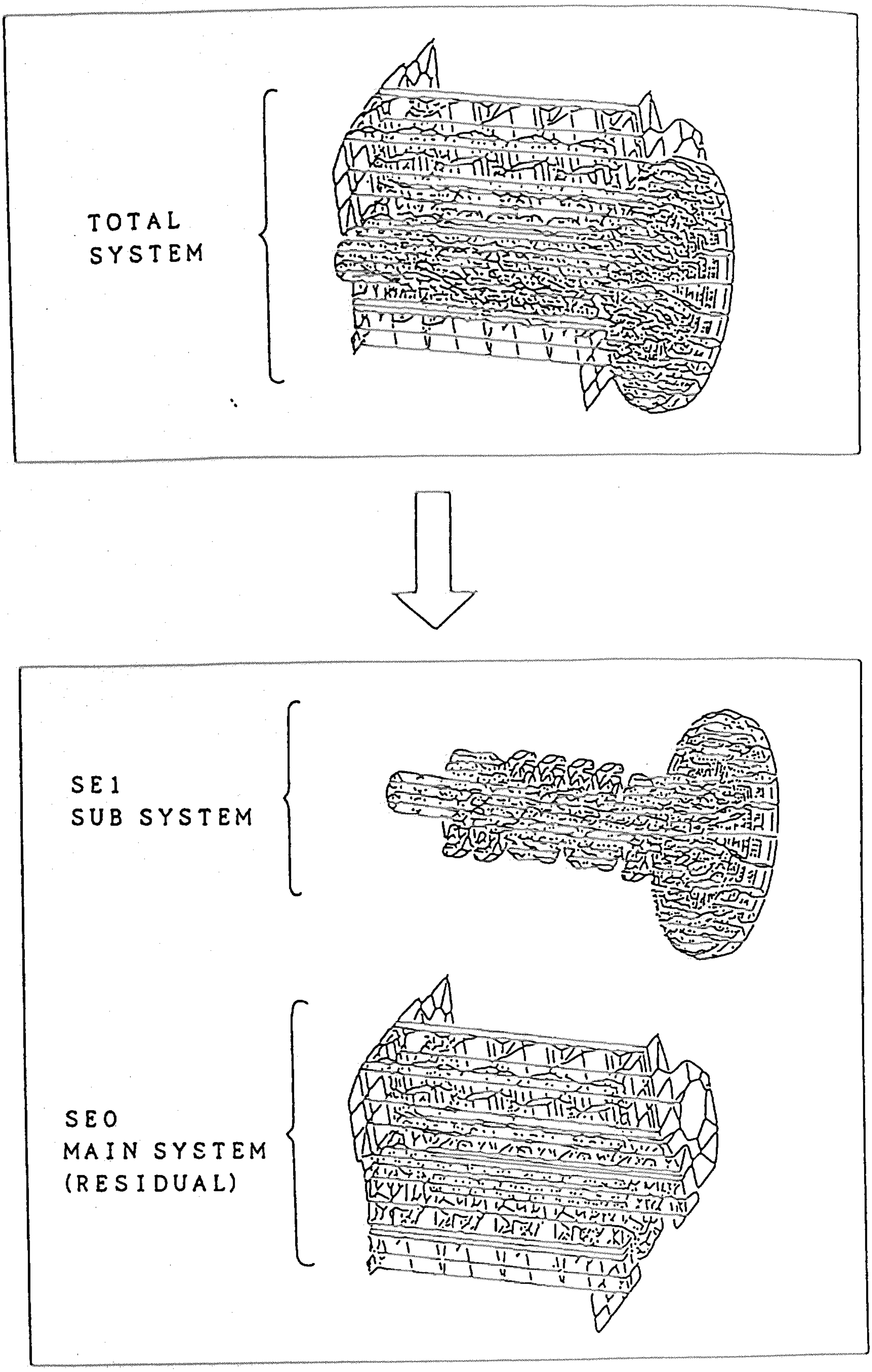

Fig4. SUPER ELEMENT CALCULATION MODEL

5. RESULT OF CALCULATION

Fig. 5 shows the appearance of optimization achieved when the torsional mode at 548 Hz was shifted to 600 Hz by our method. Repetition was carried out for 7 times.

Fig. 6 shows the cylinder body and the bearing caps that were the residual structures for which we wanted to make design change. Figs. 7 ~ 9 show the locations of design change (plate thickness) identified by the method of optimization.
The original plate thickness was 4mm.

Here, Fig. 7 shows the locations of design change involving plates of thickness 4.5 mm and above. Fig. 8 shows the locations of design change involving plates of thickness 5.0 mm and above, and Fig. 9 shows the locations of design change involving plates of thickness 6.0 mm and above.

We came to know from the results of above calculation that the areas that made big contribution in improving the natural frequency of torsional mode were

① Rear half of the lower deck

② Central part of the upper deck

③ Side walls coupled to the above areas

④ Bulkhead area

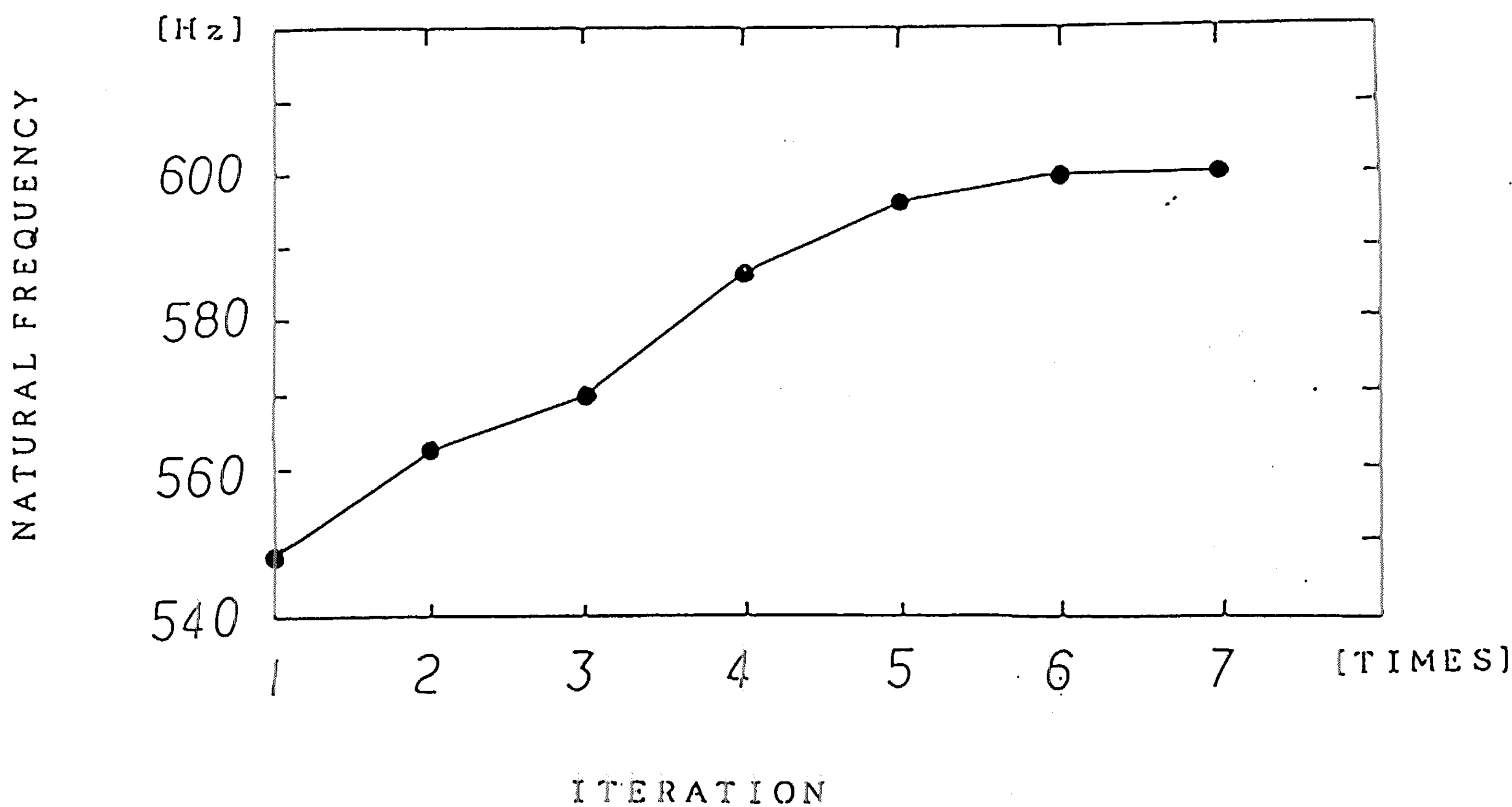

Fig5. OPTIMIZATION OF TORSIONAL MODE

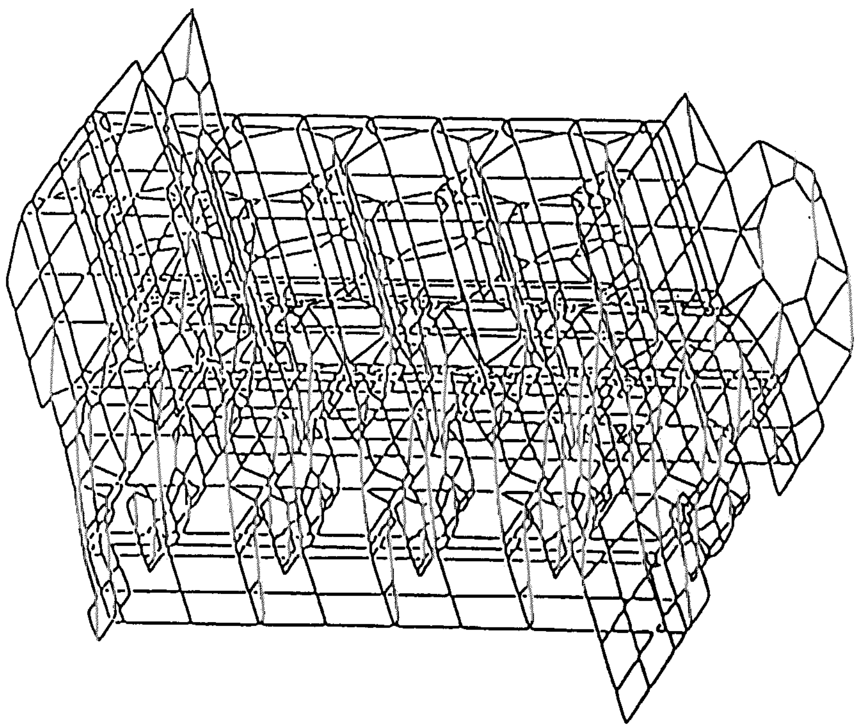

Fig6. CYLINDER BODY · BEARING CAPS STRUCTURE

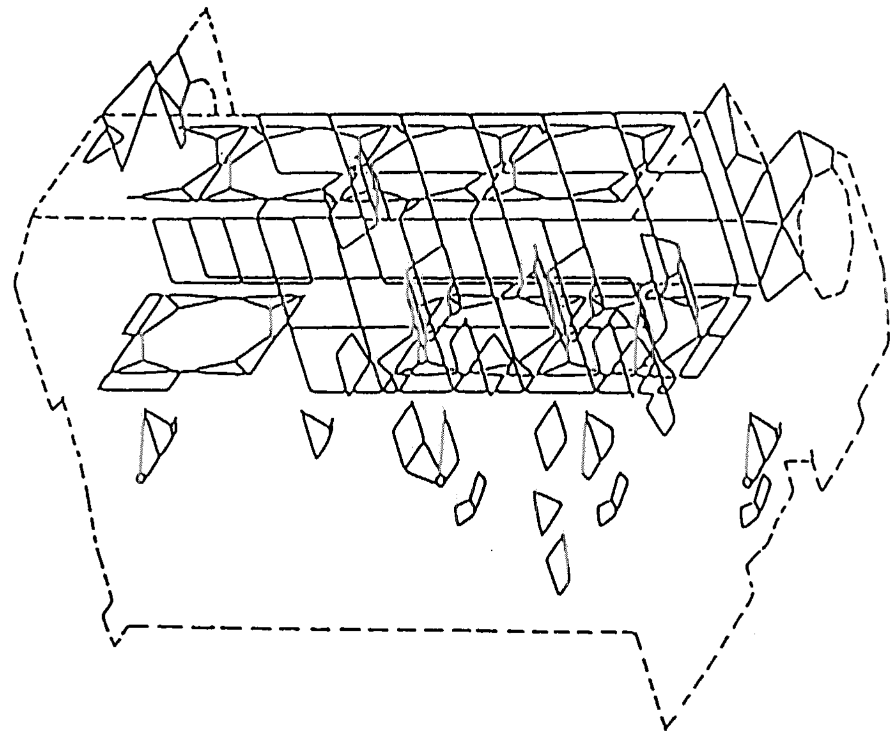

Fig7. LOCATION OF CHANGE OF PLATE THICKNESS 4.5mm AND ABOVE

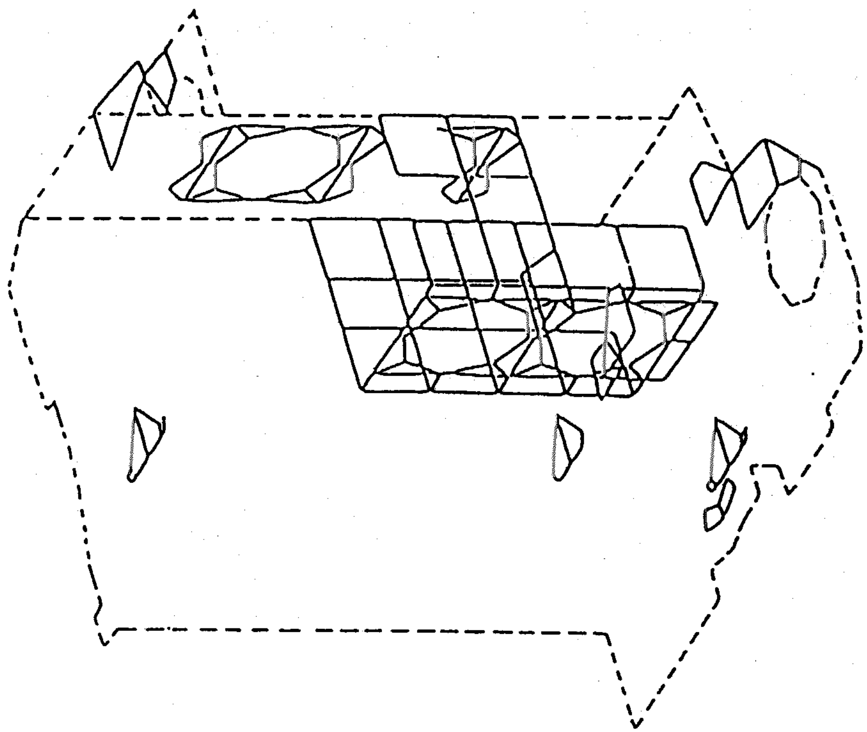

Fig 8. LOCATION OF CHANGE OF PLATE THICKNESS 5.0mm AND ABOVE

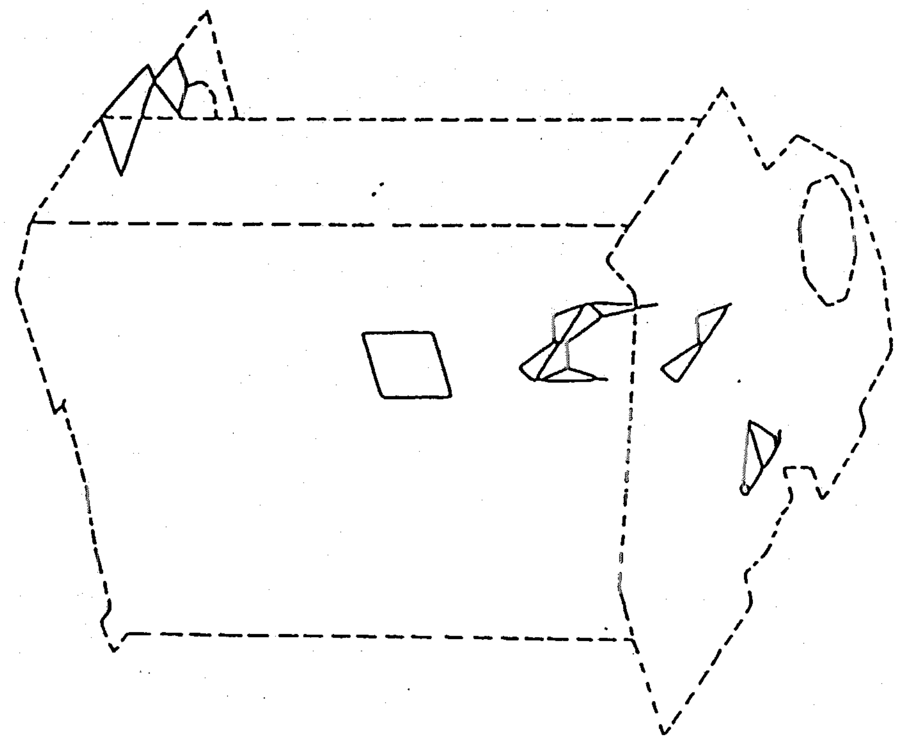

Fig 9. LOCATION OF CHANGE OF PLATE THICKNESS 6.0mm AND ABOVE

Table 1. CPU TIME IN EACH RUN

NO	RUN	SOLVER	CPU (SEC)
1	INITIAL RUN	SOL 60	8·4
2	SUB SYSTEM RUN	SOL 63	352
3	MAIN SYSTEM RUN	SOL 63	407
4	SENSITIVITY RUN	SOL 53	20
5	OPTIMIZATION RUN	OPTIMIZER	1·2

Table 2. CPU TIME OF MAIN MODULES IN MAIN SYSTEM RUN

DHAP NO	MODULE	OPERATION	CPU (SEC)
335 - 337	SEMA	$\overset{\sim SE1}{[K_{aa}]} + [K_{jj}]$	12
368 - 370	SEMA	$\overset{\sim SE1}{[M_{aa}]} + [M_{jj}]$	11
558	DYCNTRL	DETERMINING OF λs	84
563	DCMP	DECOMPOSITION OF $[A_{vv}]$	83
566	DYNREDU	SOLVING OF $([K_{vv}] - \lambda[M_{vv}])\{\phi_{vy}\}=\{0\}$	133
630	SMPYAD	$\{\phi_{vy}\}^t[K_{vv}]\{\phi_{vy}\}$	20
645	SMPYAD	$[MLAA]$	12
801	READ	SOLVING OF $([K_{xx}] - \lambda[M_{xx}])\{\phi_{xz}\}=\{0\}$	1

6. CONCLUSION

The super element and dynamic optimization methods were applied here to analyze the engine structure. The following information was obtained.

① The areas that made large contribution in improving the natural frequency of torsional mode in the engine structure were:

a. rear half of the lower deck,
b. central part of the upper deck,
c. side walls coupled to the above areas, and
d. bulkhead area

② Considering the CPU time of the computer, the method employed here took most of the main system (residual system).
Dynamic reduction proved to be effective when the scale of the main system was large.

③ Since the method employed here required many repetitious calculations until the target value was achieved, the super element method using a data base proved to be very effective when considering the CPU time.

Till now, the designer and the analyzer had to rely on experience to a great extent in evolving action against the vibration mode of complicated structure. The present study revealed that such action could be evolved more efficiently by employing the method used in this study.

7. ACKNOWLEDGMENT

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8. REFERENCES

1. A.Nagamatsu, "Moodo kaiseki", 1985, Baifukan, Japan

2. M.Ohkuma, P.Sok-chu, A.Nagamatsu and M.Nanpei, "A Technique of Structural Dynamic Modification (Use of Mathematical Pseudo Inverse Method and Substructure Synthesis Method)" 1989, ASME/JSME PVP CONFERENCE, Vol. 179, P155

3. M.Iwahara, M.Ohkuma and A.Nagamatsu, "Optimal Design Method for Dynamic Behavior of Structure" 1989, ASME/JSME PVP CONFERENCE, Vol. 179, P187