Evaluation of Direct Model Modification Methods via MSC/NASTRAN DMAP Procedures

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Abstract

Various dynamic model modification methods have been developed in the last two decades. They are used to correct the analytical models using modal test data. However, both advantages and disadvantages exist for each method. In other words, none of them are suitable for all problems. In this paper, some of the direct modification methods are examined to show the accuracy and efficiency of the methods. Also, further analyses are made after the modification, through the use of MSC/NASTRAN DMAP, to demonstrate the effects of incomplete model modification on frequency response functions and the transient responses. Accordingly, some discussions and application comments are given.

Introduction

The experiences in realistic modal analysis show that an analytical model usually does not initially produce dynamic characteristics that concur with modal test results. Thus, in many cases, appropriate adjustment or modification methods should be employed to improve the analytical model using mode data obtained from a ground vibration test. The validation of the analytical model is then based on a good correlation between analytical and test measured data. When dealing with simple structures, the model adjustment can be accomplished by trial and error approach which depends strongly upon the individual's experience and intuition. While with increasing complexity of the structural system, this task might become very difficult. During the past two decades, a number of systematic procedures have been developed by various means to improve the test/analysis correlation. However, none of them has received general acceptance due to certain shortcomings. The selection of a specific method in model modification depends on the type of problem encountered.

In the development of a new structural system, schedule and budget constraints are always very important considerations of the project. Since many design parameters are still unknown in the design stage, the engineer usually has to presume some of the design variables to a reasonable degree of accuracy. After the initialization, the reanalyses are invoked. From this point of view, it seems that emphasis should be laid on simplicity and efficiency rather than accuracy, especially when large problems are to be solved.

To meet the simplicity and efficiency requirements, direct model modification methods have been developed, through which mass and stiffness matrices can be directly improved by using test measured eigendata. Generally, the physical significance of the identified parameters may not be well preserved or physically unrealistic coupling will occur in the mass and stiffness matrices. Nevertheless, the direct modification procedure has proved to be a cost effective and mathemetically convenient method since no iterations are required.

In this paper, two of the direct modification methods are studied and examined by using DMAP capabilities of MSC/NASTRAN version 65. In addition, the methods are applied to the frequency response and transient response analyses by means of DMAP alters in the NASTRAN rigid format 30 and 31. The effects of the incompleteness of the correcting mode data are shown, and some of the engineering application notes are discussed.

Modification Techniques

In this section, direct model modification methods developed by Berman and Chen will be introduced. The theory of Berman's method and Chen's method are described fully in Ref.1 ~ 6 and Ref.7 ~ 9, respectively. Here, for convenience, an outline of the concepts and formulations used in each method are summarized.

1. Berman's method This method can also be termed as constraint optimization method. The constraint used in the mass matrix correction is the orthogonality relationship

$$\Phi^T M \Phi - I = 0 \tag{1}$$

where Φ is a rectangular matrix made up of an incomplete set of mass-normalized modes. There are infinite number of solutions for mass matrix to satisfy Eq.(1). Berman requested that a set of minimum changes in the analytical matrices which force the eigensolutions to agree with the correct (or test) mode data is to be found. Thus, one of the solutions is sought to minimize the norm

$$\epsilon = \| M_A^{-1/2} \, \Delta M \, M_A^{-1/2} \| \tag{2}$$

where

$$\Delta M = M - M_A \tag{3}$$

M_A represents the mass matrix used in the analytical model which is to be modified, and ΔM represents the changes in mass matrix required to satisfy Eq.(1). Using method of Lagrangian multipliers, a function Ψ may be written

$$\Psi = \epsilon + \sum_{i=1}^{m}\sum_{j=1}^{m} \lambda_{ij}(\Phi^T M \Phi - I)_{ij} \tag{4}$$

this function is minimized to solve for λ_{ij} which yields a matrix M that minimizes ϵ and satisfies Eq.(1). The result is of the form [3,6]

$$\Delta M = M_A \Phi M_0^{-1}(I - M_0)M_0^{-1}\Phi^T M_A \tag{5}$$

where $M_0 = \Phi^T M_A \Phi$. Then Eq.(5) is substituted into Eq.(3) to obtain the correct mass matrix.

In a manner similar to that used in mass matrix modification, the correct stiffness matrix may be carried out by the constraint equations

$$K\Phi - M\Phi\omega^2 = 0 \tag{6}$$

$$\Phi^T K \Phi - \omega^2 = 0 \tag{7}$$

$$K - K^T = 0 \tag{8}$$

and the norm

$$\epsilon = \| M^{-1/2}\, \Delta K\, M^{-1/2} \| \tag{9}$$

and the Lagrangian function

$$\Psi = \epsilon + \sum_{i=1}^{n}\sum_{j=1}^{m} \alpha_{ij}(K\Phi - M\Phi\omega^2)_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{m} \beta_{ij}(\Phi^T K \Phi - \omega^2)_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij}(K - K^T)_{ij} \tag{10}$$

Following the correct mass matrix derivation results in

$$K = K_A + (\Delta + \Delta^T) \tag{11}$$

where

$$\Delta = \frac{1}{2} M\Phi(\Phi^T K_A \Phi + \omega^2)\Phi^T M - K_A \Phi\Phi^T M \tag{12}$$

Thus the corrected stiffness matrix is obtained from Eq.(11) in combination with Eq.(12).

2. Chen's method Chen assume that the correct mass and stiffness matrices are close to those of the analytical quantities. Thus they can be expressed as

$$M = M_A + \varepsilon M_1 \tag{13}$$

$$K = K_A + \varepsilon K_1 \tag{14}$$

where εM_1 and εK_1 are considered to be small quantities. According to the perturbation theory, the solutions to the eigenvalue problem associated with M and K can also be expressed as

$$\Phi = \Phi_A + \varepsilon \Phi_1 \tag{15}$$

$$\omega = \omega_A + \varepsilon \omega_1 \tag{16}$$

Substituting Eq.(13)~(16) into Eq.(1) and (7), and neglecting the terms $O(\varepsilon^2)$, then using the definition

$$\Phi_A^T M_A \Phi_A = I \tag{17}$$

$$\Phi_A^T K_A \Phi_A = \omega_A^2 \tag{18}$$

the following equations can be obtained by neglecting the commom factor ε

$$\Phi_A^T M_A \Phi_1 + \Phi_1^T M_A \Phi_A = -\Phi_A^T M_1 \Phi_A \tag{19}$$

$$\Phi_A^T K_A \Phi_1 + \Phi_1^T K_A \Phi_A - 2\omega_A \omega_1 = -\Phi_A^T K_1 \Phi_A \tag{20}$$

Substitute Eq.(15) and (16) into Eq.(19) and (20) to eliminate Φ_1 and ω_1, one obtains

$$\Phi_A^T M_1 \Phi_A = 2I - \Phi_A^T M_A \Phi - \Phi^T M_A \Phi_A \tag{21}$$

$$\Phi_A^T K_1 \Phi_A = 2\omega\omega_A - \Phi_A^T K_A \Phi - \Phi^T K_A \Phi_A \tag{22}$$

Pre- and postmultiplying the righthand side of Eq.(21) and (22) by $\Phi_A^T M_A \Phi_A$ which is equal to unity and cancelling Φ_A^T and Φ_A, one obtains

$$\varepsilon M_1 = M_A \Phi_A (2I - \Phi_A^T M_A \Phi - \Phi^T M_A \Phi_A) \Phi_A^T M_A \tag{23}$$

$$\varepsilon K_1 = M_A \Phi_A (2\omega\omega_A - \Phi_A^T K_A \Phi - \Phi^T K_A \Phi_A) \Phi_A^T M_A \tag{24}$$

Then Eq.(23) and (24) can be substituted into Eq.(13) and (14), respectively, to obtain the correct mass and stiffness matrices.

DMAP Implementation

DMAP (Direct Matrix Abstraction Program) is the Data Block-oriented language used by MSC/NASTRAN to solve engineering problems. It allows the user of MSC/NASTRAN access to the matrix routines which would support the structural solution. By utilizing the strong capability of DMAP, the aforementioned model modification methods can be conveniently implemented to be undertaken by MSC/NASTRAN.

Fig.1 and Fig.2 show the DMAP procedures established for generating the direct model modification by using Berman's and Chen's methods, respectively. In the programs, the mass and stiffness matrices to be modified are read from a binary file with unit number 51. On the other hand, the 'correct' or the test measured eigenvectors and eigenvalues are requested to be pre-written on files with unit number 52 and 72, respectively. The modification procedures include : mass and stiffness matrices modification, orthogonality checks, and eigensolution reanalyses by READ module. The rules to construct the data

```
ID Modification, Berman
TIME 20
$
BEGIN $
INPUTT4 /MXX,KXX,PHIX,,/3/51/-1 $
$ MXX、KXX、PHIX represent analytical M、K、Φ
INPUTT4 /PHIE,,,,/3/52/-1 $
INPUTT2 /LAMA,,,,/-1/72 $
$
$ Generate the mass matrix modification
$
PARAML  MXX//TRAILER/2/V,N,SIZE  $
MATGEN  ,/UNITY/1/V,N,SIZE  $
SMPYAD  PHIE,MXX,PHIE,,,/MAS/3////1////6 $
SOLVE   MAS,/MASINV/ $
ADD     MAS,UNITY/MMID/(-1.0,0.0)/(1.0,0.0) $
SMPYAD  MXX,PHIE,MASINV,MMID,MASINV,/MPMIM/5 $
SMPYAD  MPMIM,PHIE,MXX,,,/MDELT/3/////1  $
ADD     MXX,MDELT/MNEW/  $
$
SMPYAD  PHIE,MNEW,PHIE,,,/MCHK/3////1////6  $
MATPRN  MAS,MCHK//  $
$
$ Generate the stiffness matrix modification
$
LAMX,   ,LAMA/LMAT/-1 $
MATMOD  LMAT,,,,,/OMEGA2,/28 $
SMPYAD  PHIE,KXX,PHIE,,,OMEGA2/KMID/3////1 $
SMPYAD  KXX,PHIE,PHIE,MNEW,,/KPPM/4/-1/////1 $
ADD     MNEW,/MHALF/(0.5,0.0)  $
SMPYAD  MHALF,PHIE,KMID,PHIE,MNEW,KPPM/DELTA/5///////1 $
TRNSP   DELTA/DELTT  $
ADD5    DELTA,DELTT,KXX,,/KNEW/ $
$
SMPYAD  PHIE,KXX,PHIE,,,/KCHKO/3////1////6  $
SMPYAD  PHIE,KNEW,PHIE,,,/KCHK/3////1////6  $
MATPRN  KCHKO,KCHK// $
$
$ Eigensolutions reanalysis
$
READ    KNEW,MNEW,,,DYNAMICS,,CASECC/LAMA,
        PHINEW,MI,OEIGS/MODES/S,N,NEIGS/1  $
OFP     LAMA,OEIGS// $
MATPRN  PHINEW// $
END  $
$
CEND
TITLE=Model Modification Using Berman's Method
METHOD=50
BEGIN BULK
EIGR,50,MGIV,0.,800.,,,,,+EIG   1
+EIG   1,MASS
ENDDATA
```

Fig.1 DMAP module package for model modification using Berman's method

```
ID Modification, Chen
TIME 20
BEGIN $
INPUTT4 /MXX,KXX,PHIX,,/3/51/-1 $
$ MXX、KXX、PHIX represent analytical M、K、Φ
INPUTT4 /PHIE,,,,/3/52/-1 $
INPUTT2 /LAMAO,,,,/-1/71 $
INPUTT2 /LAMAE,,,,/-1/72 $
$
$ Generate the mass matrix modification
SMPYAD  PHIX,MXX,PHIE,,,/POMOP/3/-1///1 $
SMPYAD  PHIE,MXX,PHIX,,,/PMOPO/3/-1///1 $
PARAML  MXX//TRAILER/2/V,N,SIZE  $
MATGEN  ,/UNITY/1/V,N,SIZE  $
ADD5    UNITY,POMOP,PMOPO,,/PAREN1/(2.0,0.0) $
SMPYAD  MXX,PHIX,PAREN1,PHIX,MXX,/EPM/5///////1 $
ADD     MXX,EPM/MNEW/ $
SMPYAD  PHIE,MXX,PHIE,,,/MCHKO/3////1 $
SMPYAD  PHIE,MNEW,PHIE,,,/MCHK/3////1 $
MATPRN  MCHKO,MCHK// $
$
$ Generate the stiffness matrix modification
LAMX,   ,LAMAE/LMATE/-1 $
MATMOD  LMATE,,,,,/OMEGAE2,/28 $
MATMOD  LMATE,,,,,/LAMAWE,/1/2 $
MATMOD  LAMAWE,,,,,/OMEGAE,/28 $
LAMX,   ,LAMAO/LMATO/-1 $
MATMOD  LMATO,,,,,/OMEGAO2,/28 $
MATMOD  LMATO,,,,,/LAMAWO,/1/2 $
MATMOD  LAMAWO,,,,,/OMEGAO,/28 $
SMPYAD  PHIX,KXX,PHIE,,,/POKOP/3/-1///1 $
SMPYAD  PHIE,KXX,PHIX,,,/PKOPO/3/-1///1 $
MPYAD   OMEGAO,OMEGAE,/WOWE/////0  $
ADD5    WOWE,POKOP,PKOPO,,/PAREN2/(2.0,0.0) $
SMPYAD  MXX,PHIX,PAREN2,PHIX,MXX,/EPK/5///////1 $
ADD     KXX,EPK/KNEW $
SMPYAD  PHIE,KXX,PHIE,,,/KCHKO/3////1////6 $
SMPYAD  PHIE,KNEW,PHIE,,,/KCHK/3////1////6 $
MATPRN  KCHKO,KCHK// $
$
$ Eigensolution reanalysis
READ    KNEW,MNEW,,,DYNAMICS,,CASECC/LAMA,
        PHINEW,MI,OEIGS/MODES/S,N,NEIGS/1  $
OFP     LAMA,OEIGS// $
MATPRN  PHINEW// $
END  $
$
CEND
TITLE=Model Modification Using Chen's Method
METHOD=50
BEGIN BULK
EIGR,50,MGIV,0.,800.,,,,,+EIG   1
+EIG   1,MASS
ENDDATA
```

Fig.2 DMAP module package for model modification using Chen's method files and the format associated with them are described in Ref.10, which also provides the descriptions of the functional modules used in the program.

In the case of frequency response and transient response analyses, the DMAP program should be inserted into the solution sequence of SOL 30 and SOL 31 at the point that eigensolution is carried out. This requires an ALTER statement followed by the DMAP instructions which are used to generate the model modification. It is of the form

```
ALTER 429 $
INPUTT4 /PHIE..../3/52/-1 $
INPUTT2 /LAMA..../-1/72 $
..........followed by DMAP package
```

where the sequence number 429 is the same for both frequency response and transient response analyses. The DMAP packages following the ALTER statement are the same as those shown in Fig.1 and 2, except that the input of the analytical M , K , Φ are unnecessary and the eigenvectors output in the READ module must be named PHIX. With the model modification DMAP package added to the Executive Control Deck of the frequency or transient response analyses, one has to construct the original structural model only to a reasonable degree of accuracy, then simply go through the SOL 30 or SOL 31 with the provided alter. The resultant responses will thus match the dynamic characteristics with those obtained from the mode data in files with unit number 52 and 72.

Illustrative Examples

A simple cantilever beam is first used to illustrate the modification procedures. Fig.3(a) shows the original analytical model which is to be modified. Fig.3(b) shows a 'target model' which will produce the 'correct' mode data. Another target model shown in Fig.3(c) introduces larger difference with the original model and requires a larger modification.

With the first target model, i.e. the one which has smaller difference with the original model, the constructed DMAP procedures are employed. Table.1 shows the modified mass matrices by Berman's and Chen's methods as well as the original and target values. It can be seen that very small off-diagonal terms are produced which are negligible. The stiffness matrices before and after adjustment are listed in Table.2. Unlike the adjusted mass matrices, considerable unrealistic coupling can be observed. However, the eigenvalues reproduced by the modified models, shown in Table.3, are very close to that produced by target model. Note that in Berman's method, exact n eigenvalues are obtained when

n modes are used in the modification.

We then use the model shown in Fig.3(c) as the target model. The adjusted mass matrices in comparison with that of the target model are shown in Table.4, and the eigenvalue comparison is shown in Table.5. It is found that more considerable errors are induced by using Chen's method. However, it still holds that n modes produce n exact eigenvalues in Berman's method.

Secondly, a cantilever rectangular plate shown in Fig.4(a) is employed to generate test runs of MSC/NASTRAN SOL 30 and 31 with application of

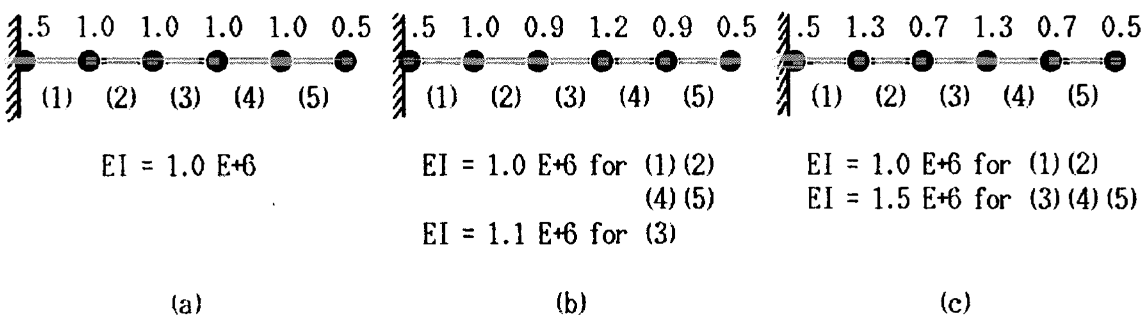


Fig.3 Illustrative cantilever beam model
(a) Original model to be modified
(b) Target model with small changes
(c) Target model with large changes

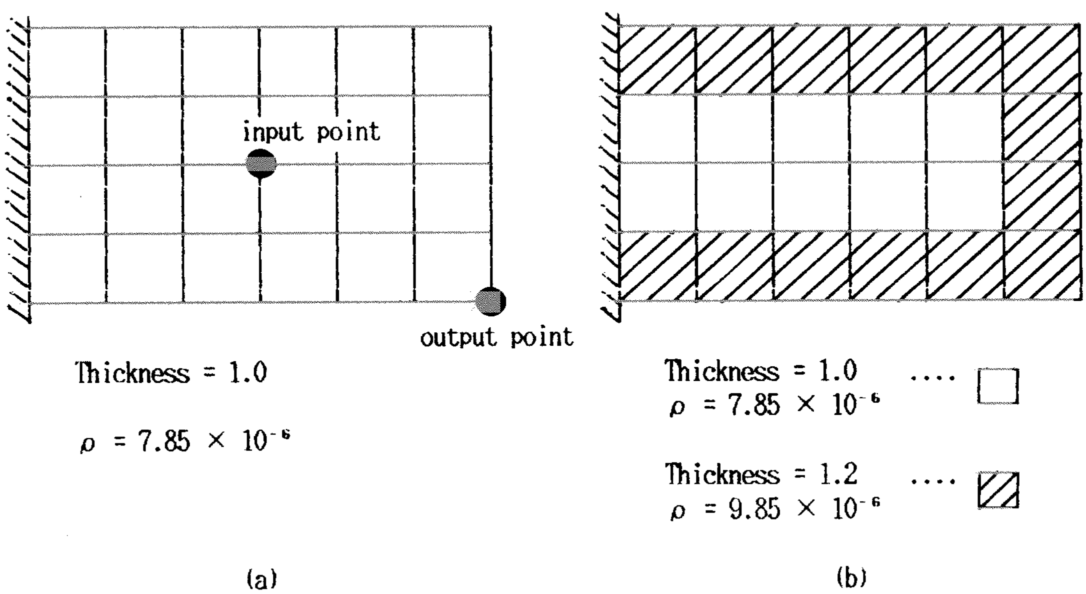
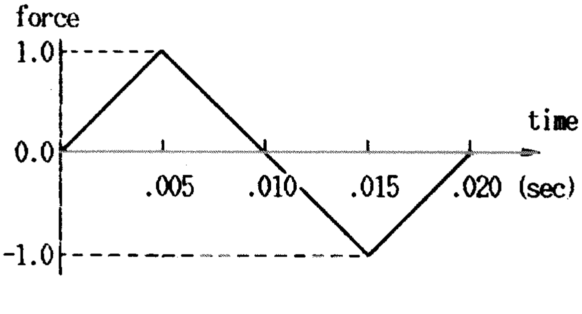


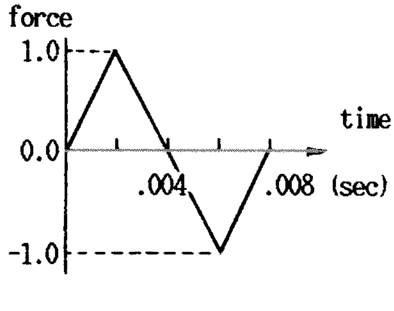
Fig.4 Rectangular cantilever plate (a) Original model (b) Target model

Berman's method. In SOL 30, i.e. the frequency response analysis, a white noise is input on the center point of the plate in the direction normal to the plate. It is remarkable that the natural frequencies of this typical plate may be separated into two groups: one in the lower frequency range (18 modes under 500Hz), and the other in the higher frequency range (other modes above 2KHz). Since the higher frequency modes are of the less importance in most of the structural responses, only the modes under 500Hz are used in the modal formulation. In the target model, a part of the original plate is thickened as shown by the hatched area in Fig.4(b). The calculated acceleration responses of the original model and the target model are plotted in Fig.5. Using ALTER 429 followed by model modification DMAP package, the responses of the modified model are calculated and shown in Fig.6. It can be seen that the model adjusted by 18 modes produces a good response function which matches perfectly with that of the target model under 500 Hz. On the other hand, a shift in peak frequency can be observed in the response function of the model adjusted by 7 modes. Nevertheless, the function curves under 75.5 Hz, the natural frequency of the 7th mode, still fits very well.

In SOL 31, the same cantilever plate is used again to calculate the time domain responses. In the analysis, a sawtooth-shaped forcing function shown in Fig.7(a) is introduced. The acceleration response time history of the original and target model are plotted in Fig.8 comparably. After modification, the response curves plotted in Fig.9 show a significant improvement, except the magnitude of higher frequency peaks. Furthermore, if we let the forcing function be compressed in the abscissa as shown in Fig.7(b), more errors will be induced, shown in Fig.10, especially when the integration time increases. However, if the attention is focused on the displacement responses, fairly valid results can be achieved as shown in Fig.11.



(a)



(b)

Fig.7 Forcing function used in the transient response analysis (SOL 31)
(a) a low frequency sawtooth (b) a high frequency sawtooth

Conclusions and Recommendations

According to the results of the above examples, it can be proved that both Berman's and Chen's methods produce reasonably accurate mass matrix when the original errors in the analytical model are small. Also, in such case, the unrealistic couplings in the modified stiffness matrix seem to have minor effects since they are an order of magnitude smaller than the real coupling terms. Generally, Berman's method is recommended when the analytical model is not so close to the correct one, in which case the Chen's method will induce larger errors due to the neglect of $O(\varepsilon^2)$ terms in the derivation. Alternatively, when very large structures are to be modified, Chen's method will show a better efficiency since only basic matrix operations such as additions, multiplications are needed.

In the application to the frequency response analysis, the number of modes to be used in the modification can be roughly determined by the frequency consideration. It is recommended to use as many as the number of modes included in the frequency range of interest. Similarly, the modal transient response analysis requires at least the same number of modes as that should be taken into consideration. Significant errors will occur when the frequency of the external load exceeds that of the highest mode in the incomplete set used in the modification. However, these errors are mainly 'shifts' in peak frequencies instead of differences in amplitude. Nevertheless, the displacement responses are much less sensitive to these errors since part of the local effects in the acceleration time history smooth away after integration twice.

Finally, we conclude that MSC/NASTRAN DMAP capabilities can be efficiently and conveniently applied to not only the model modification check runs but also the dynamic response analysis after the model modification.

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Table.1 Mass matrices comparison (small model changes)

	1	2	3	4	5	6	7	8	9	10
1(O)	1.000D+0									
1(C)	1.003D+0	-7.055D-5	1.354D-3	-2.793D-5	-1.859D-3	-3.626D-5	-3.241D-3	3.130D-6	-6.869D-4	-1.063D-5
1(C6)	1.004D+0	-2.286D-5	1.378D-3	4.442D-5	-1.708D-3	1.792D-5	-3.195D-3	6.145D-6	-6.880D-4	-1.749D-5
1(B)	1.000D+0									
1(B6)	1.000D+0	2.649D-7	1.892D-5	-1.430D-6	1.657D-4	-2.499D-8	4.215D-5	1.026D-6	6.041D-8	1.394D-7
1(T)	1.000D+0									
2(O)		1.000D-1								
2(C)		1.007D-1	4.533D-6	7.535D-4	5.384D-5	7.061D-4	-1.180D-5	4.871D-4	3.744D-5	1.470D-4
2(C6)		1.000D-1	8.847D-4	-1.565D-5	4.258D-4	2.303D-5	1.128D-4	-1.829D-5	4.557D-6	1.075D-5
2(B)		1.000D-1								
2(B6)		9.999D-2	8.419D-4	-3.489D-6	5.601D-4	5.110D-6	9.849D-5	3.748D-6	4.078D-8	-5.087D-7
2(T)		1.000D-1								
3(O)			1.000D+0							
3(C)			8.962D-1	6.437D-6	8.325D-3	1.143D-4	2.086D-3	1.633D-4	-4.592D-4	5.940D-5
3(C6)			8.965D-1	-2.199D-4	8.346D-3	-6.063D-4	1.980D-3	1.414D-5	-4.535D-4	1.004D-4
3(B)			9.000D-1							
3(B6)			9.002D-1	-1.889D-4	3.027D-5	-6.183D-4	-1.009D-4	-3.151D-5	1.523D-5	8.406D-5
3(T)			9.000D-1							
4(O)				1.000D-1						
4(C)				1.012D-1	8.216D-5	8.139D-4	-6.922D-5	-1.504D-4	-5.365D-5	-2.985D-4
4(C6)				1.000D-1	-1.675D-3	-3.529D-5	-3.934D-4	2.996D-5	-4.491D-6	-2.220D-5
4(B)				1.000D-1						
4(B6)				1.000D-1	-1.908D-3	-1.187D-6	-2.868D-4	-1.195D-5	-5.022D-7	-1.915D-7
4(T)				1.000D-1						
5(O)					1.000D+0					
5(C)					1.180D+0	2.410D-4	7.244D-3	-3.118D-5	-5.114D-4	9.145D-6
5(C6)					1.179D+0	3.201D-4	7.264D-3	1.128D-3	-4.811D-4	-1.150D-4
5(B)					1.200D+0					
5(B6)					1.199D+0	2.513D-4	5.833D-5	1.265D-3	2.569D-5	-1.800D-4
5(T)					1.200D+0					
6(O)						1.000D-1				
6(C)						1.008D-1	6.254D-5	1.272D-4	-9.638D-6	-8.300D-5
6(C6)						1.000D-1	9.257D-4	-5.415D-5	6.895D-6	3.589D-5
6(B)						1.000D-1				
6(B6)						9.999D-2	8.306D-4	7.829D-7	9.550D-7	6.318D-6
6(T)						1.000D-1				
7(O)							1.000D+0			
7(C)							8.972D-1	-1.108D-4	-7.393D-4	-2.034D-5
7(C6)							8.977D-1	1.199D-4	-8.320D-4	-7.411D-4
7(B)							9.000D-1			
7(B6)							9.003D-1	7.402D-5	-9.925D-5	-7.221D-4
7(T)							9.000D-1			
8(O)								1.000D-1		
8(C)								1.014D-1	1.439D-4	7.529D-4
8(C6)								1.001D-1	-1.136D-5	-3.415D-5
8(B)								1.000D-1		
8(B6)								1.000D-1	2.412D-7	-5.812D-7
8(T)								1.000D-1		
9(O)									5.000D-1	
9(C)									5.010D-1	8.949D-5
9(C6)									5.010D-1	-1.863D-5
9(B)									5.000D-1	
9(B6)									5.000D-1	-7.556D-7
9(T)									5.000D-1	
10(O)										5.000D-2
10(C)										5.058D-2
10(C6)										5.001D-2
10(B)										5.000D-2
10(B6)										4.999D-2
10(T)										5.000D-2

(symmetric)

(O) --- Original values
(C) --- Chen's method using complete modes
(C6) --- Chen's method using first 6 modes
(B) --- Berman's method using complete modes
(B6) --- Berman's method using first 6 modes
(T) --- Target values

Table.2 Stiffness matrices comparison (small changes)

	1	2	3	4	5	6	7	8	9	10
1(O)	2.400D+4	0.000D+0	-1.200D+4	-6.000D+4						
1(C)	2.409D+4	-4.965D+2	-1.201D+4	-6.078D+4	-5.751D+1	-5.863D+2	-3.859D+1	1.768D+1	1.687D+1	1.072D+2
1(C6)	2.422D+4	-5.340D+1	-1.229D+4	-5.989D+4	1.769D+2	-1.473D+2	-4.075D+1	1.724D+2	4.732D+0	-9.036D+1
1(B)	2.400D+4	1.328D-3	-1.200D+4	-6.000D+4	2.541D-4	6.742D-4	7.051D-5	7.352D-4	-5.878D-5	-7.833D-4
1(B6)	2.383D+4	3.916D+1	-1.213D+4	-5.748D+4	2.348D+2	1.482D+3	3.662D+1	-1.617D+2	-5.186D+0	8.924D+1
1(T)	2.400D+4	0.000D+0	-1.200D+4	-6.000D+4						
2(O)		8.000D+5	6.000D+4	2.000D+5						
2(C)		8.076D+5	6.001D+4	2.095D+5	2.707D+2	8.256D+3	3.433D+2	4.778D+3	3.203D+2	1.384D+3
2(C6)		8.003D+5	5.997D+4	1.993D+5	2.708D+2	8.802D+2	-2.241D+2	-1.029D+3	3.976D+1	5.395D+2
2(B)		8.000D+5	6.000D+4	2.000D+5	-2.244D-3	-2.744D-3	6.635D-4	8.000D-3	8.531D-5	-2.038D-3
2(B6)		7.997D+5	5.987D+4	1.997D+5	-1.105D+2	1.692D+3	2.235D+2	9.044D+2	-4.000D+1	-4.811D+2
2(T)		8.000D+5	6.000D+4	2.000D+5						
3(O)			2.400D+4	0.000D+0	-1.200D+4	-6.000D+4				
3(C)			2.513D+4	-5.705D+3	-1.313D+4	-6.564D+4	-2.602D+1	1.911D+2	3.119D+1	1.511D+2
3(C6)			2.447D+4	4.097D+1	-1.239D+4	-6.005D+4	1.523D+2	6.000D+1	-2.378D+1	-3.135D+1
3(B)			2.520D+4	-6.000D+3	-1.320D+4	-6.600D+4	3.429D-4	6.055D-4	-4.990D-5	1.096D-4
3(B6)			2.498D+4	-3.737D+3	-1.304D+4	-6.348D+4	1.794D+2	4.138D+2	-3.195D+1	-2.286D+2
3(T)			2.520D+4	-6.000D+3	-1.320D+4	-6.600D+4				
4(O)				8.000D+5	6.000D+4	2.000D+5				
4(C)				8.523D+5	6.651D+4	2.286D+5	6.727D+2	1.531D+3	-1.492D+2	-1.074D+3
4(C6)				8.012D+5	5.949D+4	1.983D+5	4.245D+2	1.958D+3	-7.539D+1	-1.026D+3
4(B)				8.400D+5	6.600D+4	2.200D+5	-2.246D-3	-6.779D-3	8.341D-4	7.979D-3
4(B6)				8.020D+5	6.260D+4	1.947D+5	-9.204D+2	1.030D+3	1.479D+2	-5.563D+2
4(T)				8.400D+5	6.600D+4	2.200D+5				
5(O)					2.400D+4	0.000D+0	-1.200D+4	-6.000D+4		
5(C)					2.525D+4	5.830D+3	-1.194D+4	-6.089D+4	-1.111D+2	-5.956D+2
5(C6)					2.454D+4	6.955D+2	-1.232D+4	-6.081D+4	5.868D+1	4.253D+2
5(B)					2.520D+4	6.000D+3	-1.200D+4	-6.000D+4	3.050D-4	1.346D-3
5(B6)					2.487D+4	4.302D+3	-1.201D+4	-5.963D+4	2.164D+0	-1.847D+2
5(T)					2.520D+4	6.000D+3	-1.200D+4	-6.000D+4		
6(O)						8.000D+5	6.000D+4	2.000D+5		
6(C)						8.482D+5	6.060D+4	2.051D+5	2.515D+2	1.005D+3
6(C6)						8.022D+5	5.942D+4	1.973D+5	1.031D+2	1.406D+3
6(B)						8.400D+5	6.000D+4	2.000D+5	-7.455D-4	-5.702D-3
6(B6)						8.103D+5	5.781D+4	1.946D+5	3.823D+2	2.884D+3
6(T)						8.400D+5	6.000D+4	2.000D+5		
7(O)							2.400D+4	0.000D+0	-1.200D+4	-6.000D+4
7(C)							2.416D+4	-7.111D+2	-1.214D+4	-6.072D+4
7(C6)							2.423D+4	6.781D+2	-1.204D+4	-6.035D+4
7(B)							2.400D+4	1.309D-3	-1.200D+4	-6.000D+4
7(B6)							2.378D+4	-7.264D+2	-1.196D+4	-5.962D+4
7(T)		(symmetric)					2.400D+4	0.000D+0	-1.200D+4	-6.000D+4
8(O)								8.000D+5	6.000D+4	2.000D+5
8(C)								8.151D+5	6.175D+4	2.088D+5
8(C6)								8.031D+5	5.988D+4	1.983D+5
8(B)								8.000D+5	6.000D+4	2.000D+5
8(B6)								7.970D+5	6.013D+4	2.015D+5
8(T)								8.000D+5	6.000D+4	2.000D+5
9(O)									1.200D+4	6.000D+4
9(C)									1.223D+4	6.119D+4
9(C6)									1.201D+4	6.006D+4
9(B)									1.200D+4	6.000D+4
9(B6)									1.199D+4	5.993D+4
9(T)									1.200D+4	6.000D+4
10(O)										4.000D+5
10(C)										4.063D+5
10(C6)										4.008D+5
10(B)										4.000D+5
10(B6)										3.991D+5
10(T)										4.000D+5

(O) --- Original values
(C) --- Chen's method using complete modes
(C6) --- Chen's method using first 6 modes
(B) --- Berman's method using complete modes
(B6) --- Berman's method using first 6 modes
(T) --- Target values

Table 3. Natural frequencies comparison (small model change)

MODE NO.	Original model	Target model small change	Berman's method use 10 modes	Berman's method use 6 modes	Chen's method use 10 modes	Chen's method use 6 modes
1	6.950156E-01	7.023570E-01	7.023570E-01	7.023570E-01	7.020784E-01	7.021313E-01
2	4.171632E+00	4.193796E+00	4.193796E+00	4.193796E+00	4.191044E+00	4.192067E+00
3	1.125137E+01	1.146739E+01	1.146739E+01	1.146739E+01	1.148567E+01	1.148807E+01
4	2.099506E+01	2.123248E+01	2.123248E+01	2.123248E+01	2.132823E+01	2.133129E+01
5	3.078613E+01	3.124122E+01	3.124122E+01	3.124122E+01	3.140933E+01	3.143068E+01
6	3.261097E+02	3.292055E+02	3.292055E+02	3.292055E+02	3.290384E+02	3.291866E+02
7	3.787458E+02	3.823973E+02	3.823973E+02	3.786815E+02	3.823048E+02	3.787458E+02
8	4.506643E+02	4.548506E+02	4.548506E+02	4.506157E+02	4.545477E+02	4.506643E+02
9	5.124052E+02	5.170541E+02	5.170541E+02	5.124047E+02	5.163932E+02	5.124052E+02
10	5.470512E+02	5.533481E+02	5.533481E+02	5.470436E+02	5.540402E+02	5.470512E+02

Table 4. Mass matrices comparison (large model changes)

	1	2	3	4	5	6	7	8	9	10
1(T)	1.300D+0									
1(B)	1.300D+0									
1(C)	1.334D+0	3.660D-3	9.420D-2	-5.810D-3	-2.932D-2	3.224D-3	4.332D-2	7.668D-3	-1.318D-2	-5.995D-3
2(T)		1.000D-1								
2(B)		1.000D-1								
2(C)		3.963D-1	4.187D-2	-3.320D-2	-1.396D-2	-1.399D-1	-8.523D-3	7.664D-2	4.432D-3	-3.968D-3
3(T)			7.000D-1							
3(B)			7.000D-1							
3(C)			7.409D-1	-1.703D-3	-8.118D-2	-2.072D-2	-3.554D-2	9.178D-3	5.995D-3	-6.399D-5
4(T)				1.000D-1						
4(B)				1.000D-1						
4(C)				2.052D-1	1.818D-2	-5.631D-2	-2.676D-2	-9.360D-2	7.768D-3	8.822D-2
5(T)					1.300D+0					
5(B)					1.300D+0					
5(C)					1.308D+0	-7.164D-3	6.331D-2	-1.811D-2	-3.548D-3	1.592D-2
6(T)						1.000D-1				
6(B)						1.000D-1				
6(C)						2.259D-1	2.515D-2	2.849D-2	-8.336D-3	-6.492D-2
7(T)							7.000D-1			
7(B)							7.000D-1			
7(C)							6.946D-1	2.329D-2	-1.977D-2	-2.486D-2
8(T)								1.000D-1		
8(B)								1.000D-1		
8(C)								1.945D-1	-6.012D-3	-7.510D-2
9(T)									5.000D-1	
9(B)									5.000D-1	
9(C)									5.066D-1	7.707D-3
10(T)										5.000D-2
10(B)										5.000D-2
10(C)										1.306D-1

(symmetric)

(T) --- Target values

(B) --- Berman's method using complete modes

(C) --- Chen's method using complete modes

Table 5. Natural frequencies comparison (large model change)

MODE NO.	Original model	Target model large change	Berman's method use 10 modes	Berman's method use 6 modes	Chen's method use 10 modes	Chen's method use 6 modes
1	6.950156E-01	7.407978E-01	7.407978E-01	7.407978E-01	7.397597E-01	7.401015E-01
2	4.171632E+00	4.743587E+00	4.743587E+00	4.743587E+00	4.681365E+00	4.686918E+00
3	1.125137E+01	1.239651E+01	1.239651E+01	1.239651E+01	1.190004E+01	1.190042E+01
4	2.099506E+01	2.425475E+01	2.425475E+01	2.425475E+01	2.513124E+01	2.520630E+01
5	3.078613E+01	3.603771E+01	3.603771E+01	3.603771E+01	3.788996E+01	3.799681E+01
6	3.261097E+02	3.812757E+02	3.812757E+02	3.707436E+02	3.679705E+02	3.697154E+02
7	3.787458E+02	4.278987E+02	4.278987E+02	3.812757E+02	3.968174E+02	3.787458E+02
8	4.506643E+02	4.987677E+02	4.987677E+02	4.487258E+02	4.607303E+02	4.506643E+02
9	5.124052E+02	5.918720E+02	5.918720E+02	5.122286E+02	5.814278E+02	5.124052E+02
10	5.470512E+02	6.653505E+02	6.653505E+02	5.470475E+02	6.504360E+02	5.470512E+02

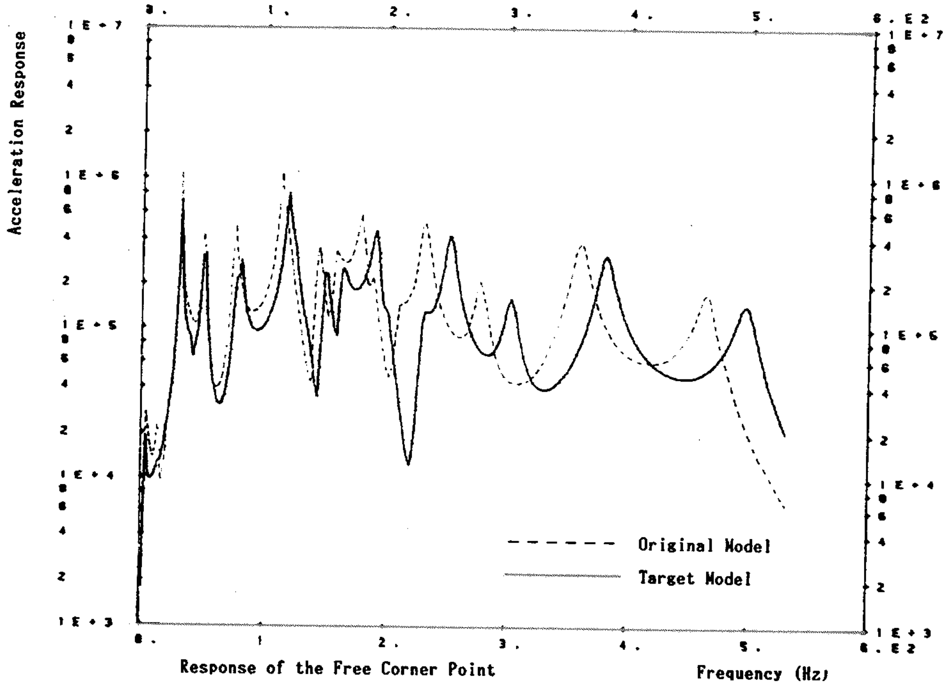

Fig.5 Frequency response of the original and target models excited by a white noise

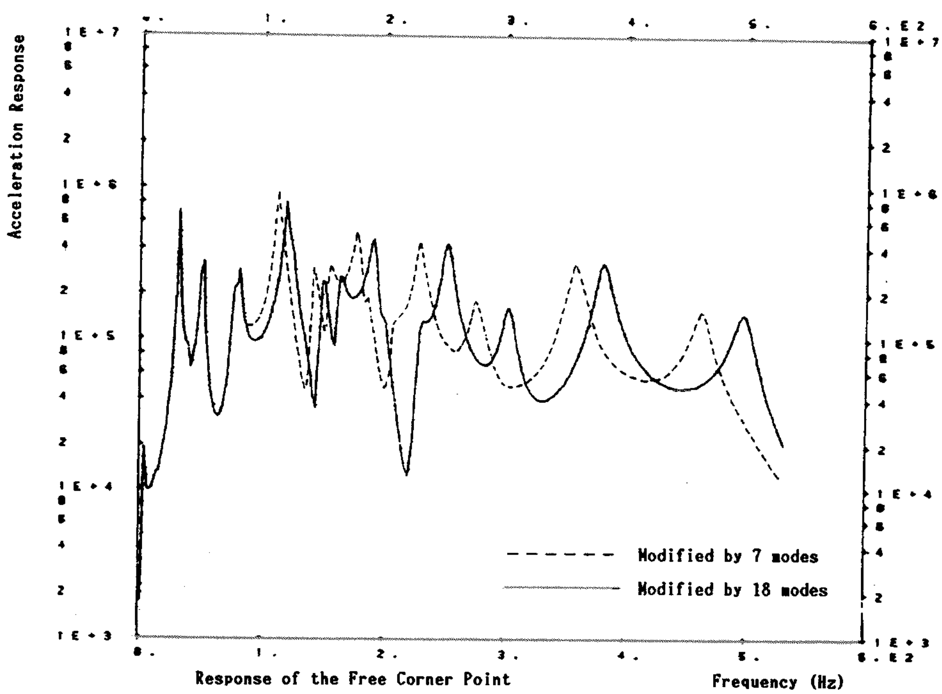

Fig.6 Frequency response of the modified models excited by a white noise

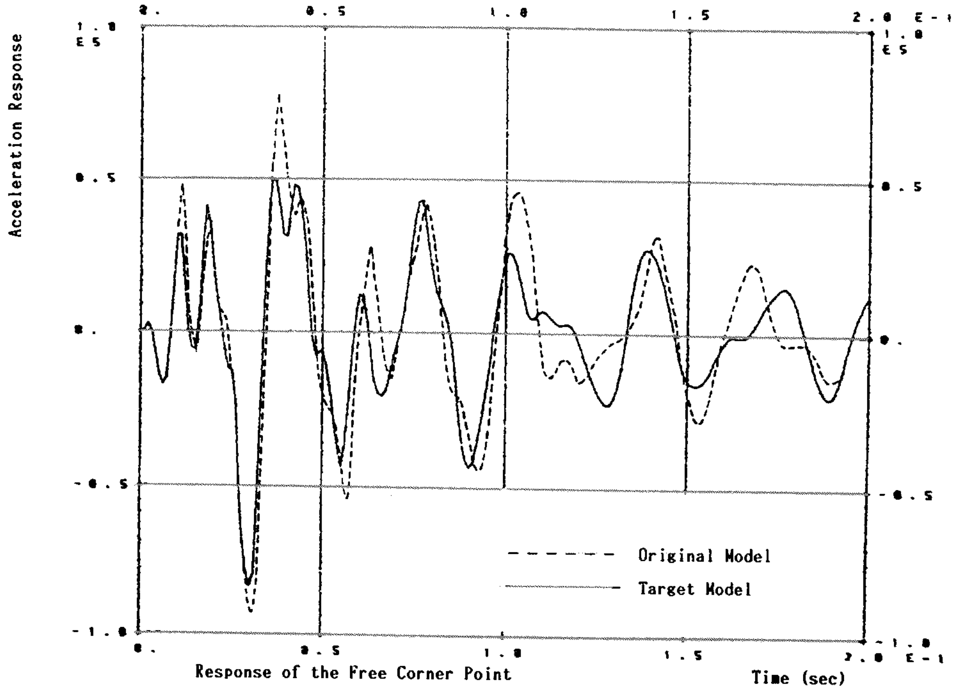

Fig.8 Transient response of the original and target models forced by a low frequency saw-tooth

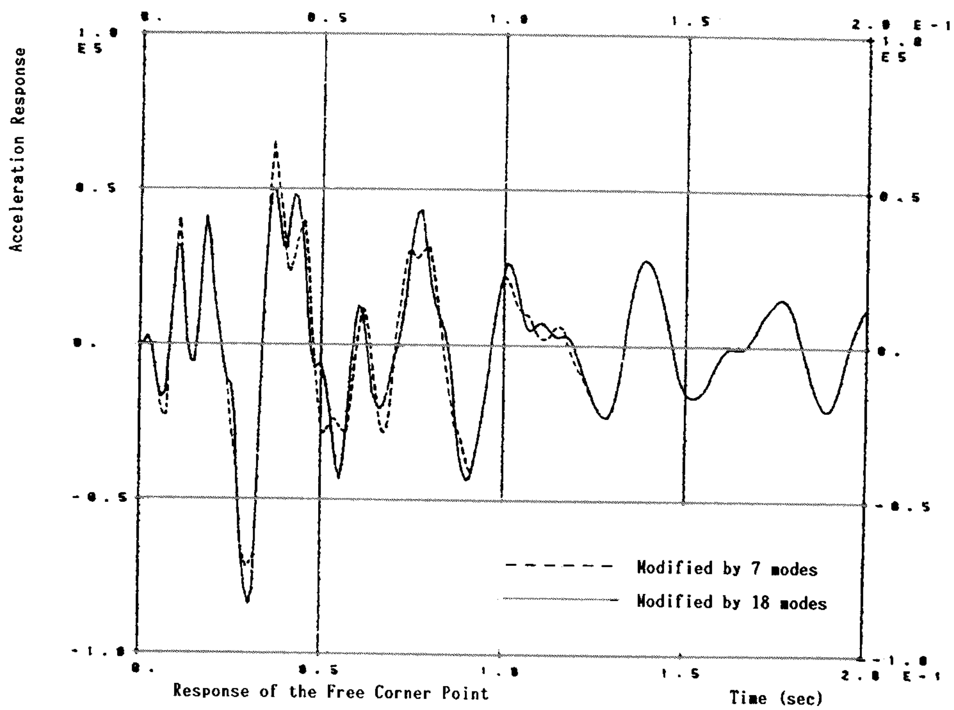

Fig.9 Transient response of the modified models forced by a low frequency saw-tooth

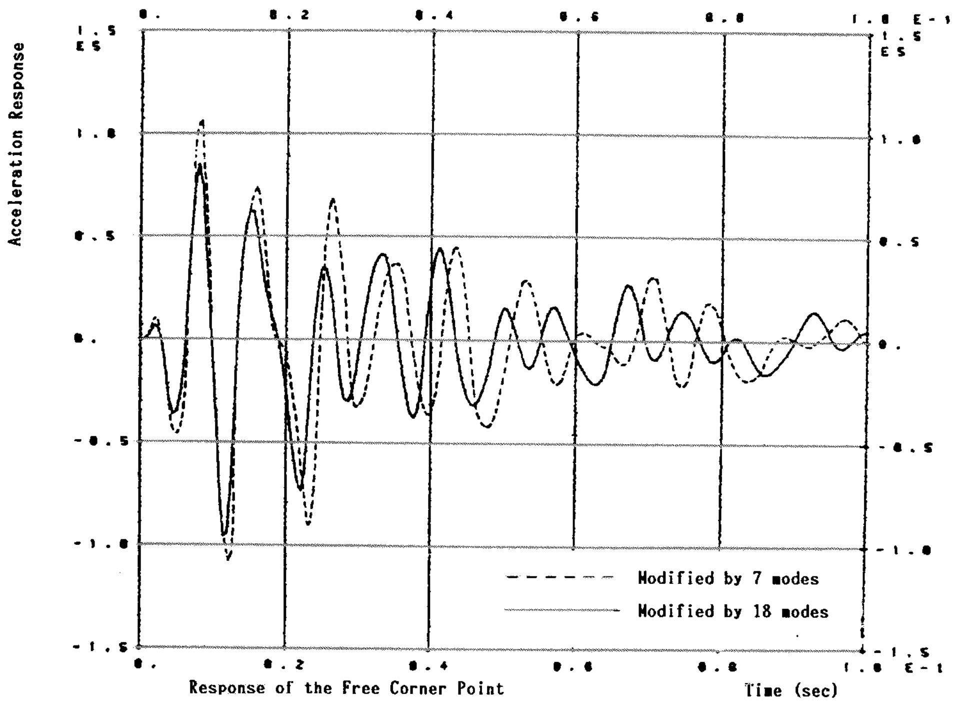

Fig.10 Acceleration response of the modified models forced by a high frequency saw-tooth

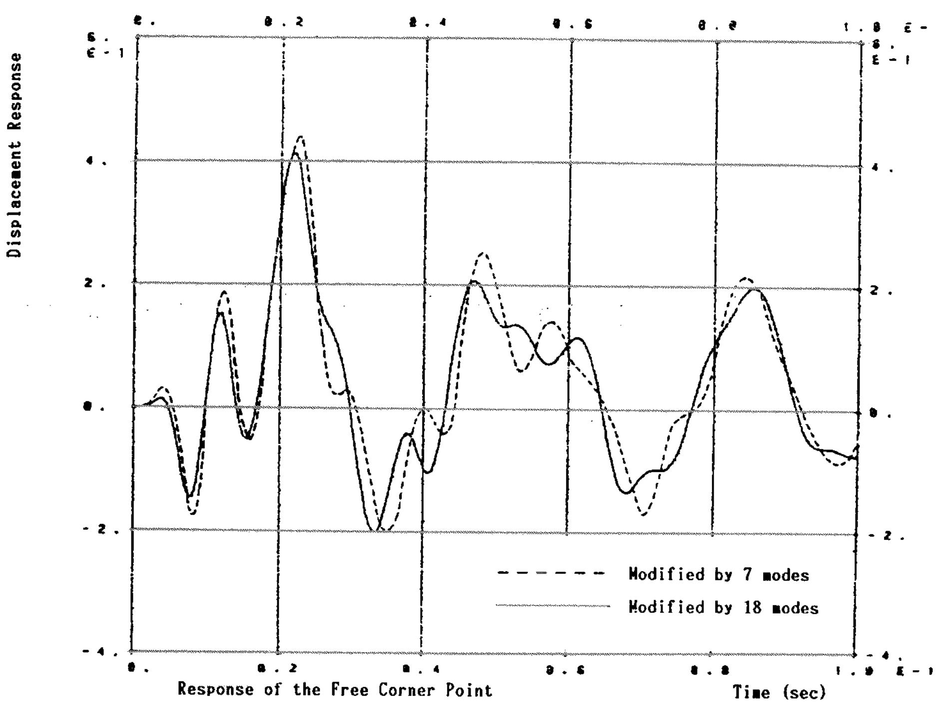

Fig.11 Displacement response of the modified models forced by a high frequency saw-tooth