CALCULATING FINAL MESH SIZE BEFORE MESH COMPLETION

Richard D. Lowrey, D.Sc., PE
Lockheed Astronautic Division
0/59-22, B/553
1111 Lockheed Way
Sunnyvale, Ca., 94088-3504

ABSTRACT

With the advent of truly automatic 3D solid finite element meshing on solid models, very complex meshes can be created in minutes to a few hours. However, due to CPU costs and core limitations, the size of the mesh must of course be well controlled. As soon as a mesh is started, a report of the predicted final mesh should be made to the user. Each subsequent step of the mesh process should issue an updated report to ensure continued compliance with the desired mesh goal. Manual checks can be performed as well when non-automatic software is used.

INTRODUCTION

The mesh designer needs to calculate the final number of grid points, elements, bandwidth, stiffness matrix size, and analysis execution cost prior to generating the mesh so that time and cpu are not wasted creating finite element meshes that will exceed the computer limits, analysis program limits, and/or the analysis budget restraints. The mesh size parameters can be computed exactly in the later mesh design stages, and approximately in the earlier stages.

The exact size of the quadratic or higher element-order surface mesh for a shell or solid can be computed from the linear surface mesh using the Euler formula for simple polygonal surfaces and closed-surface polyhedra. Adjustments for the number of holes in the surface or solid are made. The Gauss-Bonnet theorem shows that the Euler formula is actually applicable to arbitrary surfaced polyhedra (1) as well. The Euler formula is extended here-in to account for the number of solid elements, edges, and nodes inside the solid for solid finite element mesh predictions.

Equations for all-/hexa, penta, or tetra/hedra meshes are presented. Also, the bandwidth can be predicted accurately. Hence, the cpu cost to execute the analysis can be estimated. Holes in the model can be thru or non-thru holes; or tangent touching, tangent intersecting, partially intersecting, etc., without inducing error in the grid point calculation.

Solid element mesh size can be accurately calculated for various solid model types from the mesh one lays out on the model surface. The model types presented are: compact, thin, and long. The meshing technique is the only variable not accounted for.

Surface mesh size can be calculated reasonably well from the edge grid discretization sketched for the proposed surface mesh. Hence, the final quadratic mesh data can be predicted as early as the edge grid point discretization stage.

Shell, plane stress/strain, axisymmetric, etc. surface mesh calculations from edge grid points to surface mesh work well also. Equations for computing quadratic and higher order element mesh size from the linear surface mesh are exact.

Estimates of the quality of the bandwidth resulting from a bandwidth minimization can also be made. The user can then determine if additional bandwidth iterations should be performed.

Examples of several complex models are given to illustrate the versatility of the algorithm.

TRIANGULAR AND QUADRILATERAL 2D/3D SURFACE MESHES

The Euler characteristic is stated as: the number of vertices (V) plus faces (F) less edges (E) is invariant. Exactly two polygons meet at every edge. A simple polyhedron can be collapsed to a sphere (or point), i.e., no holes exist. Euler's formulae for simple polygons and polyhedra are applicable to even a mix of 3 to 'n' sided polygons, and are given by:

$$E = V + F - 1 \quad \text{(polygonal surface)}$$
$$E = V + F - 2 \quad \text{(surface of polyhedra solid)}$$

A surface mesh can be created for a shell, plane stress, plane strain, or axisymmetric model. Each finite element is a face (F), and there is one node on the surface at each finite element vertex (V).

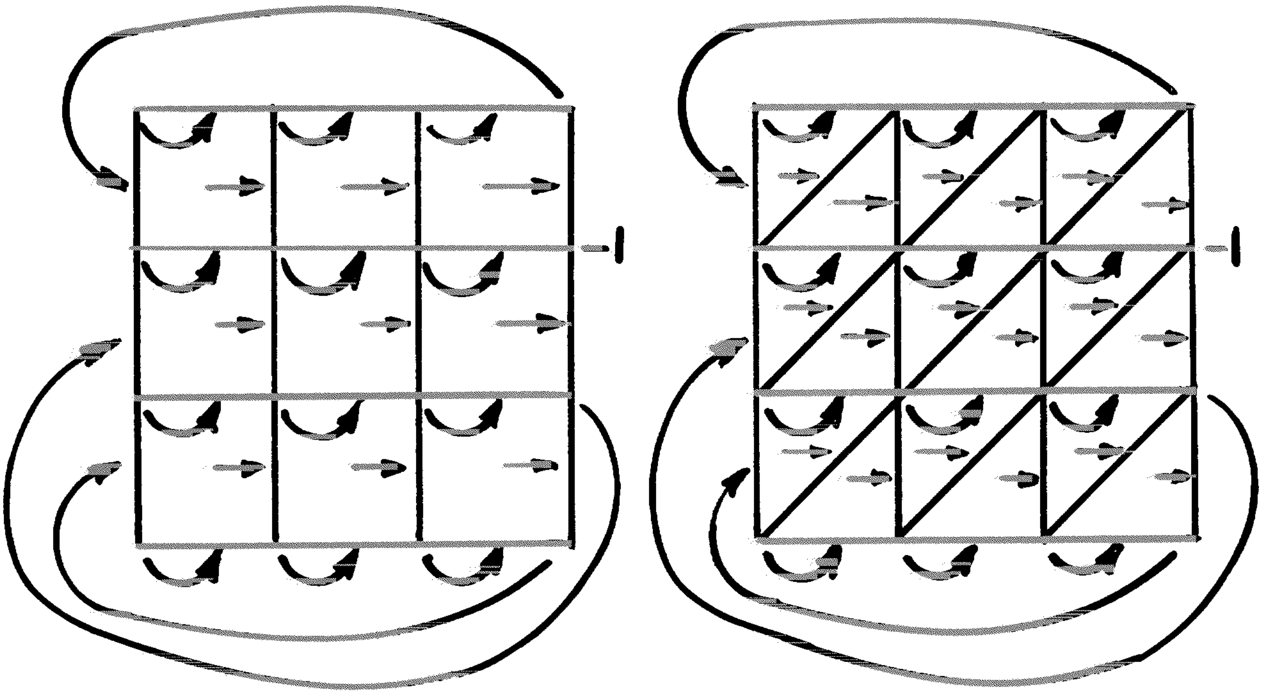
The quadratic element mesh will be formed from the linear surface mesh by adding one node (QN) on each edge (E), and none elsewhere. And the cubic mesh nodes (CN) formed by adding two nodes per edge, and so on for quartic (QRN) and higher order elements. This is stated as:

$$QN = V + E$$
$$CN = V + 2E$$
$$QRN = V + 3E$$

The linear-to-quadratic element surface mesh evolution is shown graphically in Figure 1 where an arrow points from each vertex to an edge, and an arrow points from each face element centroid to an edge. Additionally, one node must be added for each hole (H) as discussed below, resulting in:

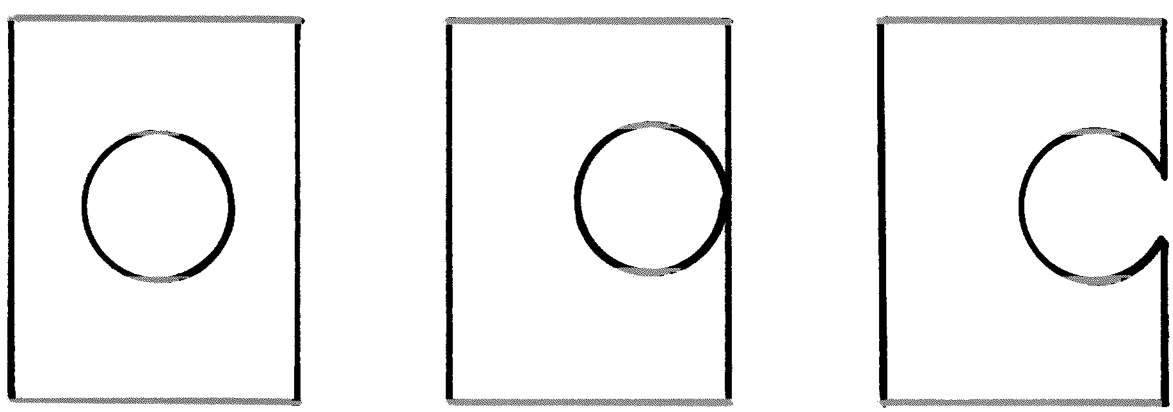
$$E = V + F + H - 1$$

A hole (H) in a quadrilateral element mesh reduces face elements (F) by one, but the number of edges (E) is unchanged. Hence, the added (H) corrects for the decrease in (F). Similarly, a triangular element mesh with the same quadrilateral hole (H) reduces the face elements (F) by two, and the edges (E) by one. Hence, the same (H) corrects for the decreases in (F) and (E). Substituting E into the equations for QN, CN, QRN yields:



Quadrilateral elements   Triangular elements

FIGURE 1. 2D Surface Elements: E = V +F +H -1



HOLE   HOLE   NOT A HOLE

FIGURE 2. SURFACE hole topology

$$QN = 2V + F + H - 1$$
$$CN = 3V + 2F + 2H - 2$$
$$QRN = 4V + 3F + 3H - 3$$

SURFACE HOLE TOPOLOGY

Topological holes in the surface mesh are shown in Figure 2. The hole must be enclosed by the outer edges of the surface mesh for it to be a topological hole. It may even be tangent to the outside edge. But if it breaks thru the outer edge, it is only an edge deviation, not a topological hole in the surface.

EDGE MESH

The edge mesh is simply the discretization the mesh designer sketches out on the edges of the model. There will be approximately 'a' elements in one direction, and 'b' elements in the other. The directions of 'a' and 'b' need not be orthogonal. The number of nodes for a quadratic or higher element mesh can be approximated from the actual number of nodes used on the edges per the equations provided in TABLE I.

TABLE I : FINAL NODES IN TRIANGULAR AND QUADRILATERAL SURFACE ELEMENT MESHES

MESH STAGE	NODES IN FINAL MESH QUADRATIC (QN)	CUBIC (CN)	QUARTIC (QRN)	NODE ERR. (%)
EDGE	EN*2ab/(a+b)	EN*3ab/(a+b)	EN*4ab/(a+b)	+/-20.
LIN.SRF	2V +F +H -1	3V +2F +2H -2	4V +3F +3H -3	0.

Where (a,b) are the number of elements in the model (x,y) coordinates.
And, V = NUMBER OF SURFACE ELEMENT VERTICES = LINEAR SURFACE NODES
E = NUMBER OF EDGES
F = FACE ELEMENTS
EN = EDGE NODES
QN = QUADRATIC NODES
CN = CUBIC NODES
QRN = QUARTIC NODES

SOLID MODEL TYPES

Gauss-Bonnet can be used to calculate the number of edges (E), face elements (F), and vertices (V) on the surface of any arbitrary closed surfaced solid. The finite elements can be thought of as polyhedra joined to form such a arbitrary surfaced solid. The Euler characteristic is extended here-in to include an accounting of the internal and external (on surface) edges of the solid (Es), internal and external vertices (Vs), and solid elements (S). The internal nodes and edges are (Vs -V) and (Es -E).

The model types used here-in are COMPACT, THIN, and LONG. The definition of each used here-in is given in terms of solid type structures in three dimensional space using entirely either hexahedron of tetrahedron solid elements. There are generally thickened regions of the structure which require that 3D solid elements be used to accurately determine displacements and stresses. Solid elements coupled to shell and beam elements or 2D/3D solid transition elements are not accounted for here.

COMPACT models have their three principal dimensions of similar magnitude. A true 3D solid structure is normally analysed with solid elements as no other element type is appropriate. COMPACT model type can always be used.

THIN models have two principal dimensions of similar size, and the third is significantly less than the other two dimensions by about one order of magnitude. A single element thru the thickness is assumed predominate.

LONG models have one principal dimension much longer than the other two dimensions by about one order of magnitude. A one by one element cross-section is assumed to be predominate.

There are other model types which occur and may be approximated by these equations as well. Several are mentioned below.

VERY LONG models have one principal dimension more than one order of magnitude greater than the other two principal dimensions. The equations provided for LONG models cover this model type adequately. The bandwidth as a percentage of the nodes will be proportionately less.

VERY THIN model. The equations provided for THIN models cover this model type. However, modeling with solid elements into the shell regions and using shell elements stopping at the larger of 4 thicknesses or 3 radii away from the solid regions of the structure with coupled degrees of freedom are frequently needed from a cost stand point.

THIN LONG models have three principal dimensions which are each an order of magnitude different. The equations provided for THIN models yield accurate results for this model type.

SOLID MODEL MESHES

The solid element mesh has a node at every solid element vertex (Vs). The quadratic mesh is created by adding one node to each solid element edge (Es). The cubic mesh is created by adding two nodes to each element edge (Es). And like wise for each higher order element. This is stated as:

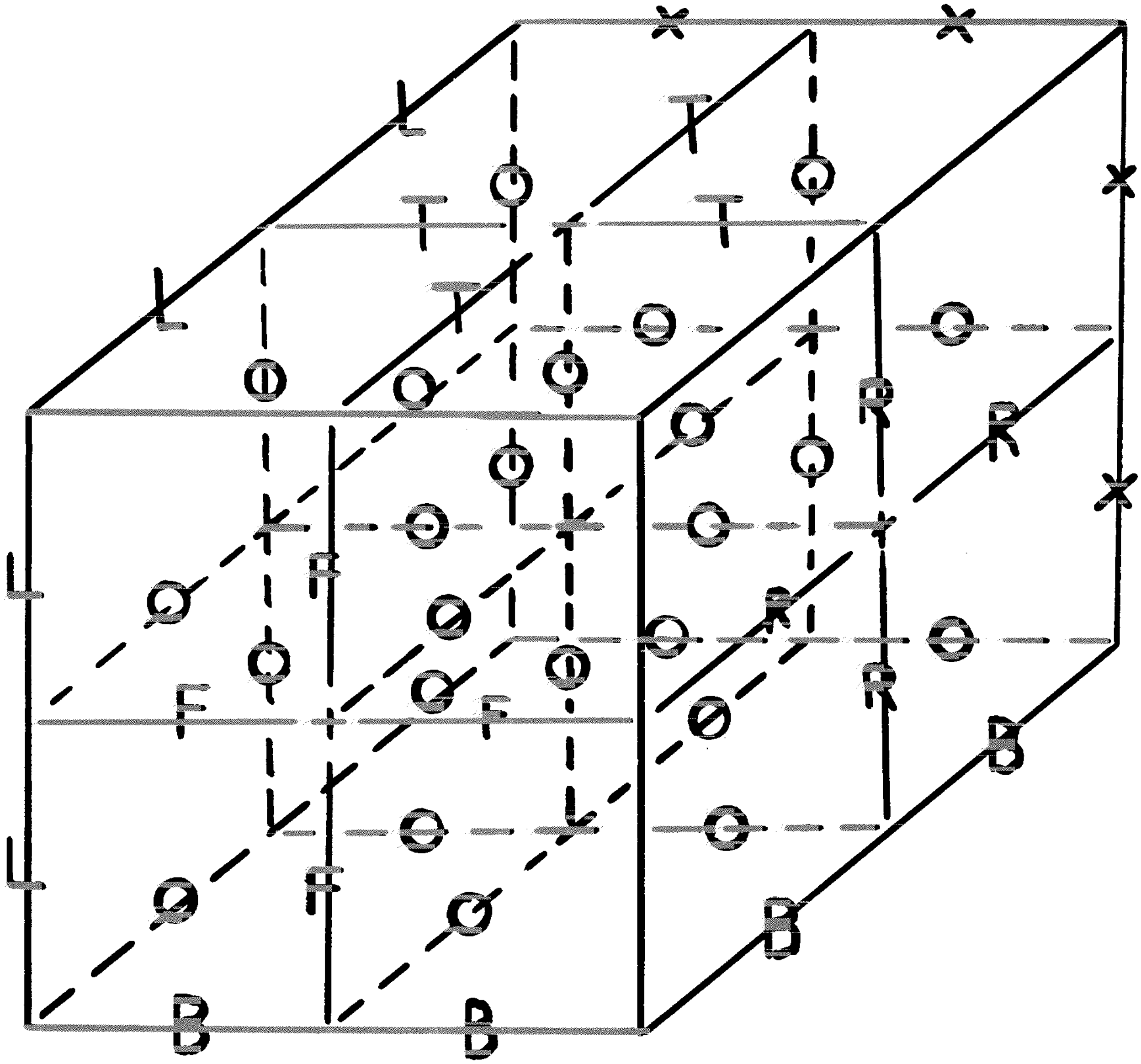
$$QN = Vs + Es$$
$$CN = Vs + 2Es$$

The equations derived for hexahedron and tetrahedron solid meshes differ.

HEXAHEDRON ELEMENTS

The number of edges (Es) in the linear hexahedron mesh is equal to the number of face elements (F) on the exterior of the solid plus 3 times the number of solid hexahedron elements (S) plus 1 nodes for each hole (H) in the solid plus the principal surface element discretization counts a +b +c. This is shown graphically in Figure 3, and can be stated as:

$$Es = F + 3S + H + (a + b + c)$$



KEY:

O	3 edges per element at left lower rear corner
X	1 edge per rear surface
F	1 edge per Front surface
L	1 edge per Left surface
R	1 edge per Right surface
B	1 edge per Bottom surface
T	1 edge per Top surface

FIGURE 3. Hexahedron Elements: $Es = F + 3*S + H + (a + b + c)$

The equations for number of quadratic (QN) and cubic (CN) hexahedron mesh nodes are stated below. And (QN) is also given in TABLE II.

$$QN = Vs + F + 3S + H + (a + b + c)$$
$$CN = Vs + 2*F + 6S + 2H + 2(a + b + c)$$

The term (a +b +c) is usually in the range of 3 to 50, so knowing the exact value for a highly variable mesh is not too important except for meshes near the node limit. Assuming a value of EN/4 will usually be conservative.

As was shown for surface meshes earlier, the holes (H) adjust for the change in the number of volume elements (S) and surface elements (F). Holes in solids are harder to count since the holes can differ in shape, location, and orientation.

When the model has extensive detail, and consequently, a lot of edges, then the edge node (EN) to quadratic node (QN) prediction can be high.

The number of nodes on the surface in the quadratic mesh (QN surface) is calculated from the relationship found for closed solid surfaces; and the number of interior nodes QN(interior) are stated as:

$$QN(surface) = 2V + F + H - 2$$
$$QN(interior) = QN - QN(surface)$$

The compactness of the solid can be evaluated as the percentage of interior nodes to total nodes. A thin solid will have a very low percentage of nodes internal, while a compact solid can have over half the nodes internal.

TABLE II: NODES IN QUADRATIC HEXAHEDRON ELEMENT MESHES

MESH STAGE	NUMBER OF QUADRATIC NODES (QN) SOLID MODEL TYPE: COMPACT	THIN	LONG	NODE ERR. (%)
EDGE	EN*1.5abc/(a+b+c)	EN*[2ab/(a+b) +1]	3.5EN	30.
SURFACE	V*[2.7abc/(ab+bc+ca) +1.5]	V*4ab/[(a+1)(b+1)]	3V	10.
LINEAR	<------------ Vs +F +3S +H +(a +b +c)		------------>	0.

NOTE: V*4ab/[(a+1)(b+1)] < 4V

PENTAHEDRON ELEMENTS

Where the pentahedra are created such that two pentahedra can be grouped to fill a hexahedron space throughout the solid, and consequently, there are an even number of solid elements, S, then the number of edges in the solid (Es), number of quadratic nodes (QN) in the solid and on the surface are given by:

$$Es = F + 3S/2 + H + (a + b + c)$$

$$QN = Vs + F + 3S/2 + H + (a + b + c)$$

$$QN(surface) = 2V + F + H - 2$$

TETRAHEDRON ELEMENTS

The number of edges (Es) in the linear tetrahedron mesh is equal to the number of surface nodes (V) plus the number of solid tetrahedron elements (S) plus the number of linear nodes (Vs) plus 3 nodes for each hole (H) in the solid less 3 nodes. The number of quadratic (QN), cubic (CN), and quartic (QRN) tetrahedron nodes are stated below. And (QN) is given in TABLE III.

$$QN = 2Vs + V + S + 3(H - 1)$$
$$CN = 3Vs + 2V + 2S + 6(H - 1)$$
$$QRN = 4Vs + 3V + 3S + 9(H - 1)$$

Again, the holes (H) adjust for the change in the number of elements (S) and edges (E).

The number of nodes on the surface in the quadratic mesh is calculated from the relationship found for surfaces (and identical to the hexahedron element relationship). QN(surfaces) and QN(interior) are stated as:

$$QN(surface) = 2V + F + H - 2$$
$$QN(interior) = QN - QN(surface)$$

TABLE III: NODES IN QUADRATIC TETRAHEDRON ELEMENT MESHES

MESH STAGE	NUMBER OF QUADRATIC NODES SOLID MODEL TYPE COMPACT	THIN	LONG	NODE ERR. (%)
EDGE	EN*2.25abc/(a+b+c)	EN*[3ab/(a+b) +1.5]	5.3EN	30.
SURFACE	V*[4abc/(ab+bc+ca) +2]	V*6ab/[(a+1)(b+1)]	4.5V	10.
LINEAR	<----------- 2Vs +V +LE +3H -3	--------------->		0.

NOTE: V*6ab/[(a+1)(b+1)] < 6V

SOLID HOLE TOPOLOGY

The number of nodes produced is a function of the number of holes in the solid. Tetrahedron elements add 3 nodes per hole, while hexahedron elements add 1 node per hole.

For most solids, the number of holes is limited, and hole will have little effect on the final mesh size. However, solids such as manifolds and complex castings and forgings can have enough holes to significantly alter the final mesh size, and sometimes the size limitations of the analysis program or the CPU's CORE size will be exceeded.

MSC/NASTRAN up thru Version 65 had a degree of freedom limit of 2*32768-1, with 6 degrees of freedom per node. Consequently, the solid element mesh size was then limited to 10992 nodes, and counting holes was often very important.

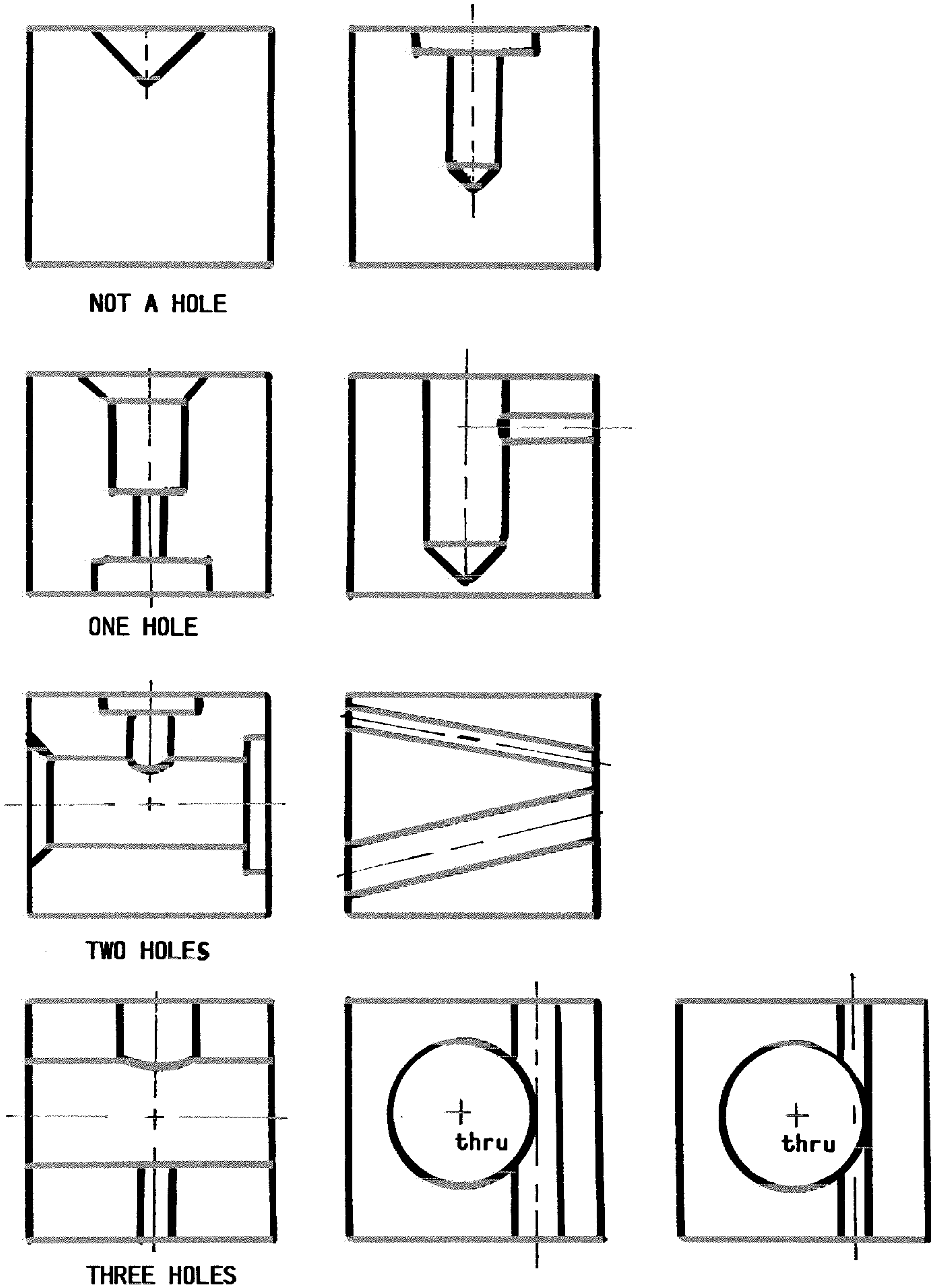


FIGURE 4. SOLID hole topology

RULES FOR COUNTING SOLID MODEL HOLES

A hole must extend between two surfaces (i.e., a distinct intersection at each end of the hole must occur) before it is a topologically solid hole. One or both of the surfaces the hole connects to may be other holes or the exterior of the solid. Examples of holes and the appropriate count are shown in Figure 4. The rules for counting holes are as follows.

NOT A HOLE:
A spotface, drill point, or any other surface depression.
A non-thru hole not intersecting any other hole.

ONE HOLE:
A thru hole not intersecting any other hole.

TWO HOLES
A non-thru hole intersecting a thru hole, regardless of diameters or amount of intersection.
Two thru holes which do not touch, or just touch tangent (with no intersection).

THREE HOLES
Two intersecting thru holes. Any intersection of the holes will do. The holes may be of different radii, intersect obliquely or only partially.

A hole may be of any complexity, but it is still topologically just one hole. For example, the hole may have large counterbores on both ends with a tool radius at the bottom of the counterbores, and a greatly enlarged bore or cavity inside the solid, and it is still just one hole.

NODAL BANDWIDTH (NBW) VS. MODEL TYPE AND MESH SIZE

MODEL TYPE and MESH SIZE both have an impact on the magnitude of the nodal bandwidth.

As MESH SIZE increases, the bandwidth only increases by the 2/3 power of the increase, given of course, that the mesh technique remains unchanged. For example, if the number of nodes (N) increases by a factor of 2, the bandwidth increases by a factor of (2)**2/3 = 1.5874.. And if the number of nodes increases by a factor of 8, the bandwidth increases be a factor of only 4.

Very small meshes can have a nodal bandwidth of 25% of the number of nodes. Very large meshes frequently have a bandwidth of less than 3% of the nodes.

Keep in mind of course, that the CPU cost is increasing by the number of nodes and bandwidth squared.

MODEL TYPE	TYPICAL NBW CALCULATION	IDEALIZED MODEL SHAPE
COMPACT	(N)**2/3	Uniform cube mesh
THIN	(N)**1/2	Uniform square mesh
LONG	(N)**1/2	Length/Depth = 10
	(N)**1/3	Length/Depth = 100

These are generally the upper limit of the bandwidth. Another banding technique should be considered if the bandwidth relative to the shape is out of proportion.

CPU COST CALCULATION

The CPU cost is a function of the number of nodes (N) and the nodal bandwidth (NBW) squared. Actual data of CPU cost verses NODES times NODAL BANDWIDTH**2 can be plotted on log-log graph paper. Two straight line segments will plot when a very large range of nodes is considered with the knee of the two lines reflecting the point at which the CPU cost increases due to a decrease in core efficiency of the analysis algorithm.

CPU costs in a given region can be compared simply by computing 'alpha1' for an actual analysis in this region, and then using the resulting equation below for the comparisons desired.

CPU = alpha1(NODES)(NBW)**2

Using degrees of freedom (DOF, DOFBW), analyses with different numbers of DOF per node can be compared in the same equation using:

CPU = alpha2(DOF)(DOFBW)**2

EXAMPLE - LOWER CONTROL ARM (Compact solid)

The lower control arm mesh is shown in Figure 5. A large mesh was expected. It is mostly thin with extensive thick regions, has fillets on all corners, chamfers on most edges, extensive pocketing, and 6 thru holes. Tetrahedron solid elements are used, and the planned mesh discretization (a,b,c) is (35, 20, 2). Hence, compact equations are used in the quadratic nodal predictions.

MESH STAGE	CURRENT # NODES	SURF/VOL ELEMENTS	BANDWIDTH (NODES, %)	QUAD. N**2/3	PREDICTED QUAD NODES	ERROR (%)
EDGE	232			547.8	12821	-1.5
SURFACE	1688	3123		609.5	15046	+15.6
LINEAR	3676	3965	171.8/4.67	553.4	13020	0.
QUAD	13020	3965	451.8/3.47			

Since the predicted quadratic nodes based on edge nodes is over 15000 nodes, the need to predict after each mesh stage is very real.

The actual bandwidth of 451.8 nodes is reasonably good because it is less than the predicted value of (13020)**2/3 = 553.4 nodes.

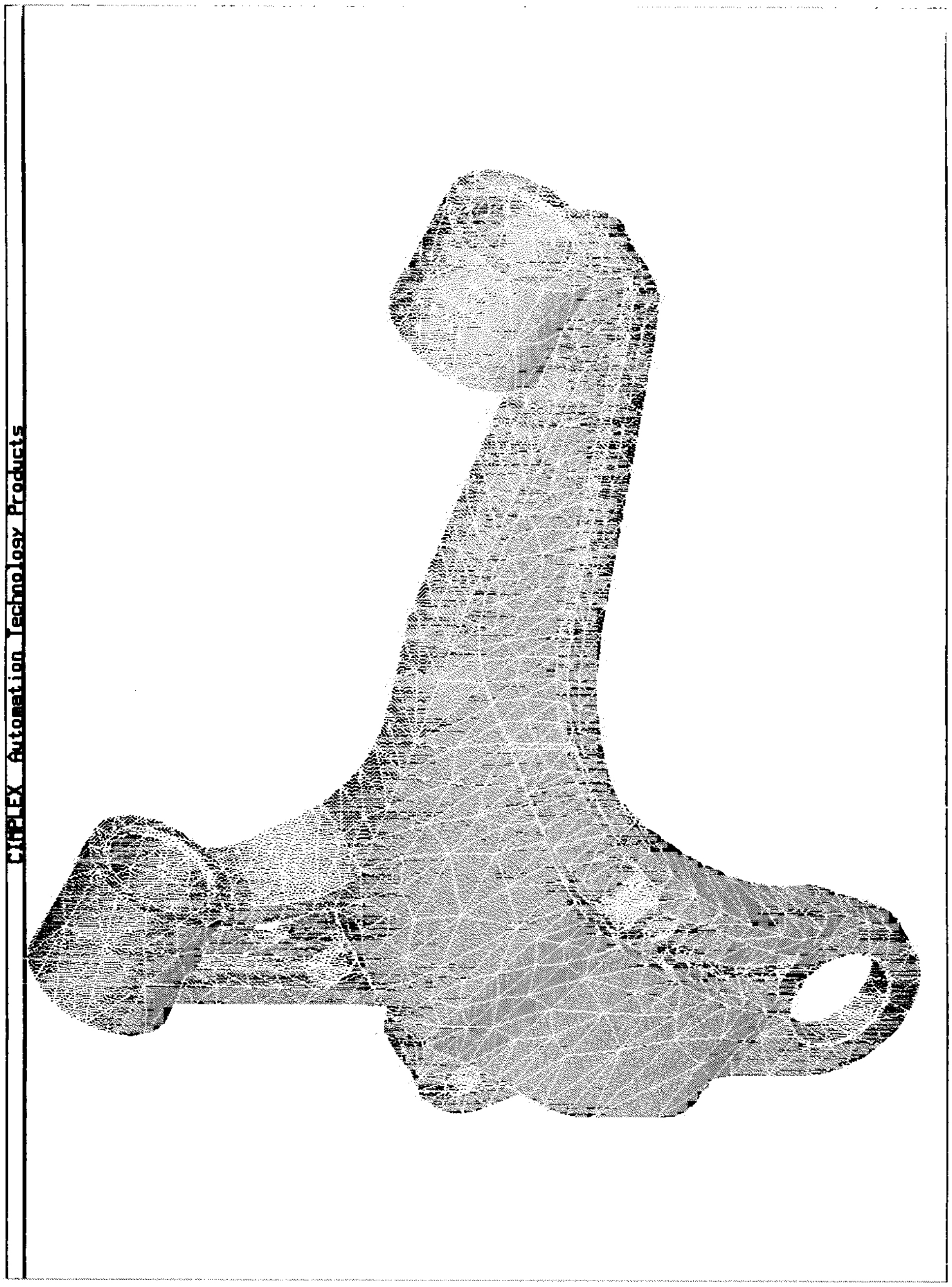


FIGURE 5. Tetrahedron Mesh on Lower Control Arm With Six Thru holes

The actual ratio of quadratic nodes to surface nodes is 8.9136 and gives a measure of the compactness of the solid. This ratio for thin solids with tetrahedron elements is always less than 6. And for long solids, it is less than 4.5.

If thin solid approximations were used, less than 10000 nodes would have been predicted. This is because much of the actual mesh has more than one element thru the thickness to accommodate the thickened regions.

EXAMPLE - PISTON (Compact solid)

The piston mesh is shown in Figure 6. It is mostly thin with extensive thick regions, has fillets in all corners, extensive detail, and 2 thru holes. Tetrahedron solid elements are used, and the planned mesh discretization (a,b,c) is (24, 8, 2). The thickness average assumes 60% at 1 element, 30% at 3 elements and 10% at 4 elements for an average of 2 elements. Hence, compact equations are used in the quadratic nodal predictions.

MESH STAGE	CURRENT # NODES	SURF/VOL ELEMENTS	BANDWIDTH (NODES, %)		QUAD N**2/3	PREDICTED QUAD NODES	ERROR (%)
EDGE	841				770.1	21371	+38.5
SURFACE	1819	4543			596.0	14552	-5.7
LINEAR	2400	8812	160.8	6.7	619.9	15434	0.
QUAD	15434	8812	788.7	5.1			

The actual ratio of quadratic nodes to surface nodes is 8.4849 and again indicates that this is a compact solid.

The actual bandwidth is 788.7, which is greater than (15434)**2/3 = 619.9 which is predicted. The connectivity of the mesh across the top of the piston may not allow a significantly lesser value.

SUMMARY

With these equations, the mesh designer can calculate the number of nodes, bandwidth, cpu time, etc. And hence, one can determine early in the mesh design process if the final quadratic or higher element order mesh will likely have too many grid points, too large of a bandwidth, too high of analysis cost, etc. Adjustments in the mesh design can then be made early in the mesh process to achieve the desired result in a more cost effective manner.

REFERENCE

1. 'Geometric Modeling', Michael E. Mortenson, John Wiley & Sons, 1985.

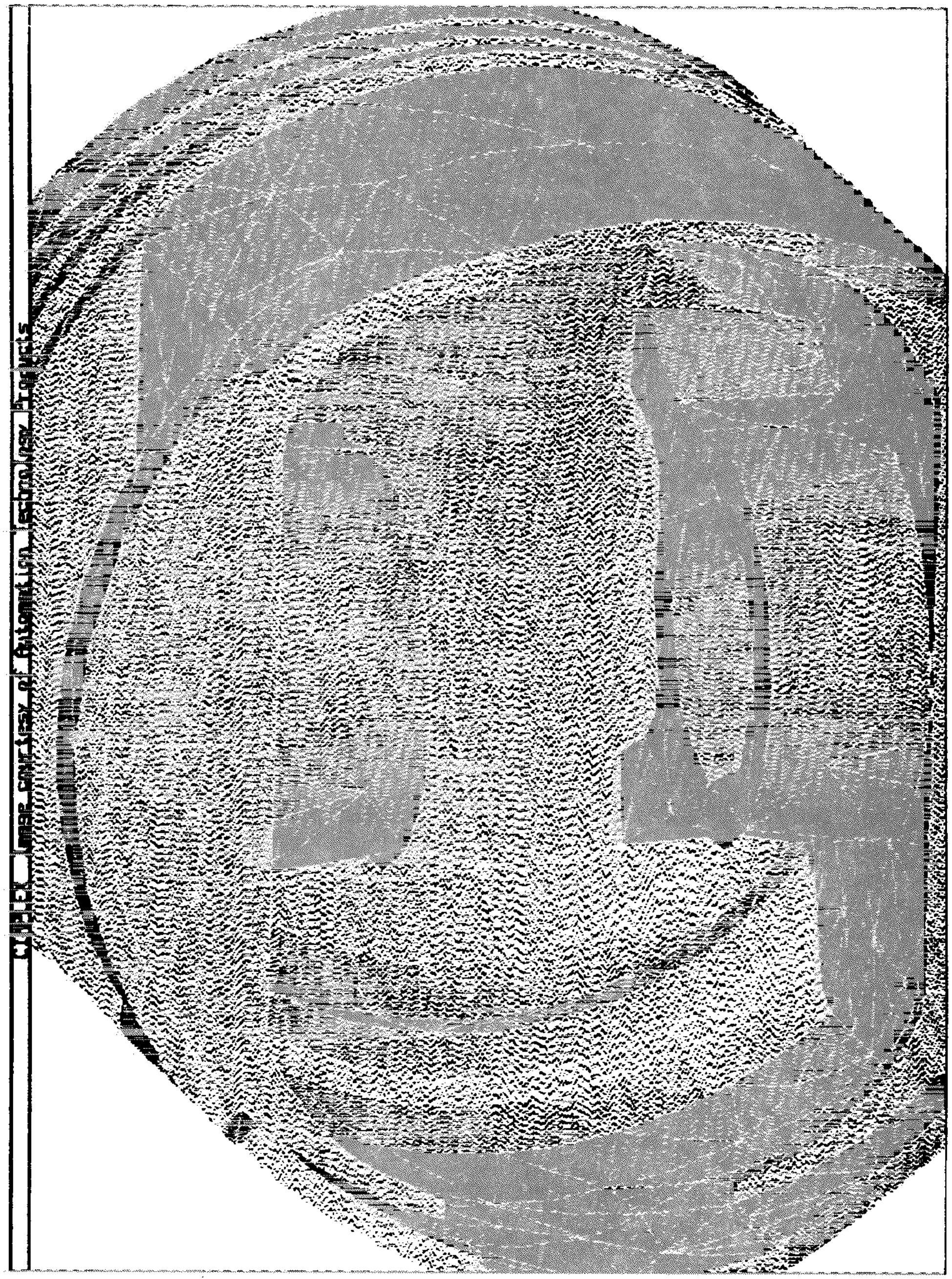


FIGURE 6. Tetrahedron Mesh on Piston With Two Thru Holes