Improved Eigensolution Reanalysis Procedures in Structural Dynamics

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ABSTRACT

An improvement to eigensolution reanalysis procedures has been developed in this paper. This new method is based on the well known Bubnov-Galerkin procedure with a set of truncated normal modes and the associated residual static modes as global approximation functions. The formulation has been implemented in DMAP for MSC/NASTRAN version 65. Numerical results showed that the inclusion of residual static modes drastically improves the solution accuracy with a minor increase in computational cost over the assumed mode method. Additionally, an iterative scheme has been introduced to further improve the reanalysis results.

INTRODUCTION

During the design evaluation of an aerospace vehicle, structural dynamic analyses have to be performed for various configurations. If the results showed unsatisfactory structural performance the design would be modified and the analyses repeated. These design analyses could be very time consuming and expensive. To circumvent the computational burden, efficient reanalysis techniques were developed to generate responses of a modified structure from those of a baseline design. State-of-the-art review of reanalysis techniques can be found in references 1 and 2.

For eigensolution reanalysis, the assumed mode method has been found to be quite effective for structures with moderate modification. This method has been applied to structural dynamic optimization [3,4] and has also been implemented in DMAP [5]. While it is very easy to implement, the assumed mode method loses its accuracy when the design modification alters the mode shapes significantly. In a recent paper, Noor and Whitworth [6] proposed new procedures for reanalysis of large structural systems. For eigensolution reanalysis, their approach uses eigenvector derivatives in addition to eigenvectors in the global approximation vector. Numerical results indicated this method is very efficient and accurate. While eigenvector derivatives can be computed by Nelson's method [7], its implementation is not so straight forward. In our opinion, this is a drawback of Noor's approach.

INTRODUCTION (Continued)

In this paper, Noor's method is modified so that static modes are used in lieu of eigenvector derivatives. Since the static modes are generated by solving static analysis problems of the original system, they can be generated efficiently. The solution procedures have been implemented as a set of DMAP alters to solution 63 (normal modes) of MSC/NASTRAN Version 65. Besides simple numerical examples, the new method has also been applied to a detailed structural model (30,000 degrees of freedom). The results showed not only excellent results in eigenvalues, but also good results when the modal data computed by reanalysis are used in forced response analysis.

EIGENSOLUTION REANALYSIS FORMULATIONS

The Bubnov-Galerkin formulation in reanalysis is based on the assumption that the eigenvectors of the modified system can be expressed as linear combinations of a set of global approximation vectors. That is, let

$$\{u\} = [T]\{q\} \tag{1}$$

Then the eigenproblem for the modified system can be expressed as

$$[T]^T([K] + [\Delta K])[T]\{q\} = \lambda[T]^T([M] + [\Delta M])[T]\{q\} \tag{2}$$

If N vectors are used in equation (1), equation (2) is an N^{th} order eigenproblem, from which we can solve for N-pairs of eigensolutions λ_i and q_i. The complete mode shape, in terms of physical coordinates, can then be computed from equation (1).

In the assumed mode reanalysis, [3,5], the matrix $[T]$ contains a set of truncated modes of the original system. In Noor's method [6], the matrix $[T]$ contains a set of modes as well as their path derivatives. In our proposed approach, $[T]$ contains eigenvectors and residual static modes. That is

$$[T] = [\phi\ \tilde{\psi}] \tag{3}$$

where $\phi \equiv$ truncated modal matrix of the original structure

$\tilde{\psi} \equiv$ residual static modes

The computation of the residual static modes is discussed in the next section.

COMPUTATION OF RESIDUAL STATIC MODES

Equation (2) may be written as

$$([K] + [\Delta K])\{\phi_l\} = \lambda_l([M] + [\Delta M])\{\phi_l\} \tag{4}$$

or

$$[K]\{\psi_l\} = \lambda_l[\Delta M]\{\phi_l\} - [\Delta K]\{\phi_l\} \tag{5}$$

where ψ_l is defined as the static mode due to the loading vector $\lambda_l[\Delta M]\{\phi_l\} - [\Delta K]\{\phi_l\}$. The static mode can be decomposed into two parts by the following equation:

$$\{\psi_l\} = \Sigma C_{il}\{\phi_i\} + \{\tilde{\psi}_l\} \tag{6}$$

where $\Sigma C_{il}\{\phi_i\}$ represents the contribution of the kept eigenvectors and $\{\tilde{\psi}_l\}$ is the contribution from the truncated higher modes (residual) as defined by equation (7):

$$\{\tilde{\psi}_l\} = \Sigma C_{il}\{\phi_i^o\} \tag{7}$$

where $\{\phi_i^o\}$ are the truncated higher modes. To compute $\{\tilde{\psi}_l\}$, equation (6) is multiplied by $\{\phi_j\}^T[M]$ to get

$$\{\phi_j\}^T[M]\{\psi_l\} = \Sigma C_{il}\{\phi_j\}^T[M]\{\phi_i\} + \{\phi_j\}^T[M]\{\tilde{\psi}_l\} \tag{8}$$

The last term of equation (8) is zero due to the orthogonality of the normal modes, the equation thus becomes:

$$C_{jl} = \{\phi_j\}^T[M]\{\psi_l\} \tag{9}$$

Note that in the above formulation orthonormal modes are used. Therefore the residual static mode can be written as

$$\{\tilde{\psi}_l\} = \{\psi_l\} - \Sigma C_{il}\{\phi_l\} \tag{10}$$

These residual static mode correction factors are appended to the original eigenvectors and used to modalize the new mass and stiffness matrices. From these, the new eigenvalues and hence eigenvectors are extracted. An iterative scheme can then be introduced to perform this process a user specified number of times, generally converging to the exact solution.

IMPLEMENTATION IN MSC/NASTRAN

For implementation into MSC/NASTRAN, DMAP alters are added to the database generating run to decompose the a-set stiffness matrix and save it along with the rest of the information. The decomposed stiffness matrix is used to calculate the static modes through gaussian elimination. Static mode DMAP alters are also added to existing reanalysis DMAP inside of the normal mode solution 63 sequence. The process is self contained and requires no pre- or post-processing with FORTRAN programs.

NUMERICAL EXAMPLES

The new formulation has been applied to the following three test cases. These include a simple cantilever beam, a cantilever flat plate, and a large scale structural model (30,000 degrees of freedom).

Example 1. Simple Cantilevered Beam

Figure 1 shows a three element cantilever beam modeled as a plane frame. The system has nine degrees of freedom. The original beam has a uniform cross-section. The modification involves the reduction of I_1 of the second beam element to 0.25 of its original value. The reanalysis results are shown in Table 1. It can be seen by the value of q^* that the residual static mode participates heavily in each mode thus insuring accuracy. For this simple example, the assumed mode method produces more than a 12% error even when 2 modes are used in the base run, while the inclusion of residual mode corrections drastically improves the accuracy of the reanalysis results.

Example 2. Cantilevered Flat Plate

Figure 2 shows a flat plate of titanium alloy modeled by 150 CQUAD4 elements. Ten modes were extracted from the original model. Seven modification cases were studied. They are listed below:

Case I - Thickness of two elements in the middle of the plate reduced to 0.05"

Case II - Thickness of one full row of elements reduced to 0.05", thus putting a kink in the mode shape (worst case)

Case III - Thickness of random elements increased to 1.0"

Case IV - Elements deleted randomly

Case V - Density increased from 0.161 to 0.25

Case VI - Five lb. CONM2's added to the plate's tip

Case VII - Combination of Case II and Case VI

It should be noted that Case I through Case IV are for stiffness variation, Cases V and VI involve mass modification while Case VII involves both stiffness and mass modification. The results are summarized in Tables 2 through 8.

From these results, it is apparent that when the modification is moderate or localized, as in Cases I, III, IV, VI, the error in the assumed mode method is also moderate, and the use of static correction always improves the solution accuracy. For Case II, since there is a drastic change in the design, the results of the assumed mode method involve gross error. The use of static mode correction improves the situation somewhat. Further improvement can be achieved by iteration.

Since a uniform density increase(Case V) does not change the mode shape, the assumed mode method predicted exact eigenvalues for the modified system. In Case VI, tip masses were added. The results of the assumed mode method showed large error for modes 4, 8, and 9 while the inclusion of static modes virtually eliminates these errors. Case VII is a combination of Case II and Case VI. The results of this case show a similar trend to Case II.

Example 3. Large Scale Aircraft Structural Model

Shown in Figure 3 is a line drawing of a portion of a finite element model of an aircraft structural component. This model has 30,000 degrees of freedom. It was analyzed with and without the addition of ballast weight. Using the model with ballast weight as the original model, the eigensolution of the structure without ballast weight was predicted by the proposed method. The results of Table 9 show that reanalysis predicts the natural frequency of the first ten modes almost exactly. This problem was solved on a CRAY X-MP. Table 10 shows that using the proposed method, the cpu time is reduced by a factor of 1.9. As a hard test for accuracy, the ten modes computed from the proposed method are used in a frequency response analysis to get a RMS stress distribution. These results are virtually the same as the solution computed by using ten modes from the exact analysis as shown on Figure 4. Assumed mode reanalysis has produced approximately a 30% error on similar models.

CONCLUDING REMARKS

In this paper, we have demonstrated that the inclusion of residual static modes can improve the eigensolution reanalysis accuracy. Furthermore, in the aircraft vertical tail model, the accuracy of forced response analysis using modes computed by the proposed method has also been demonstrated. The method has been automated in DMAP alters. It appears that the proposed method can be used to perform multiple parametric studies accurately and efficiently. More studies are required to establish the range of applicability of the proposed method.

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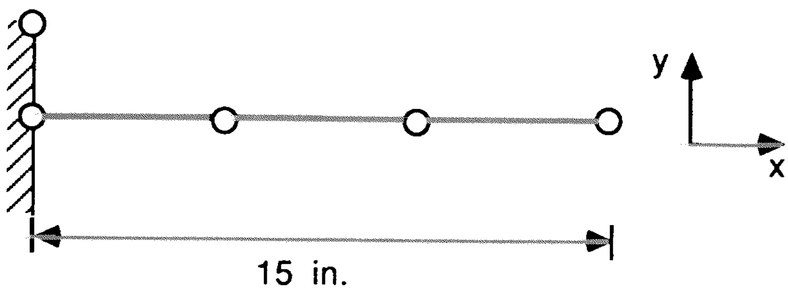
Figure 1 A Simple Cantilever Beam



Titanium Alloy
Material Properties
E= 16.0E6 psi
Density= 0.161 lb/in**3
Area= 1.0 in**2
Inertia= 0.083 in**4

Results of the Unmodified Structure

Mode	Frequency (Hz)
1	133.55
2	753.90

Figure 2 A Cantilever Plate Model

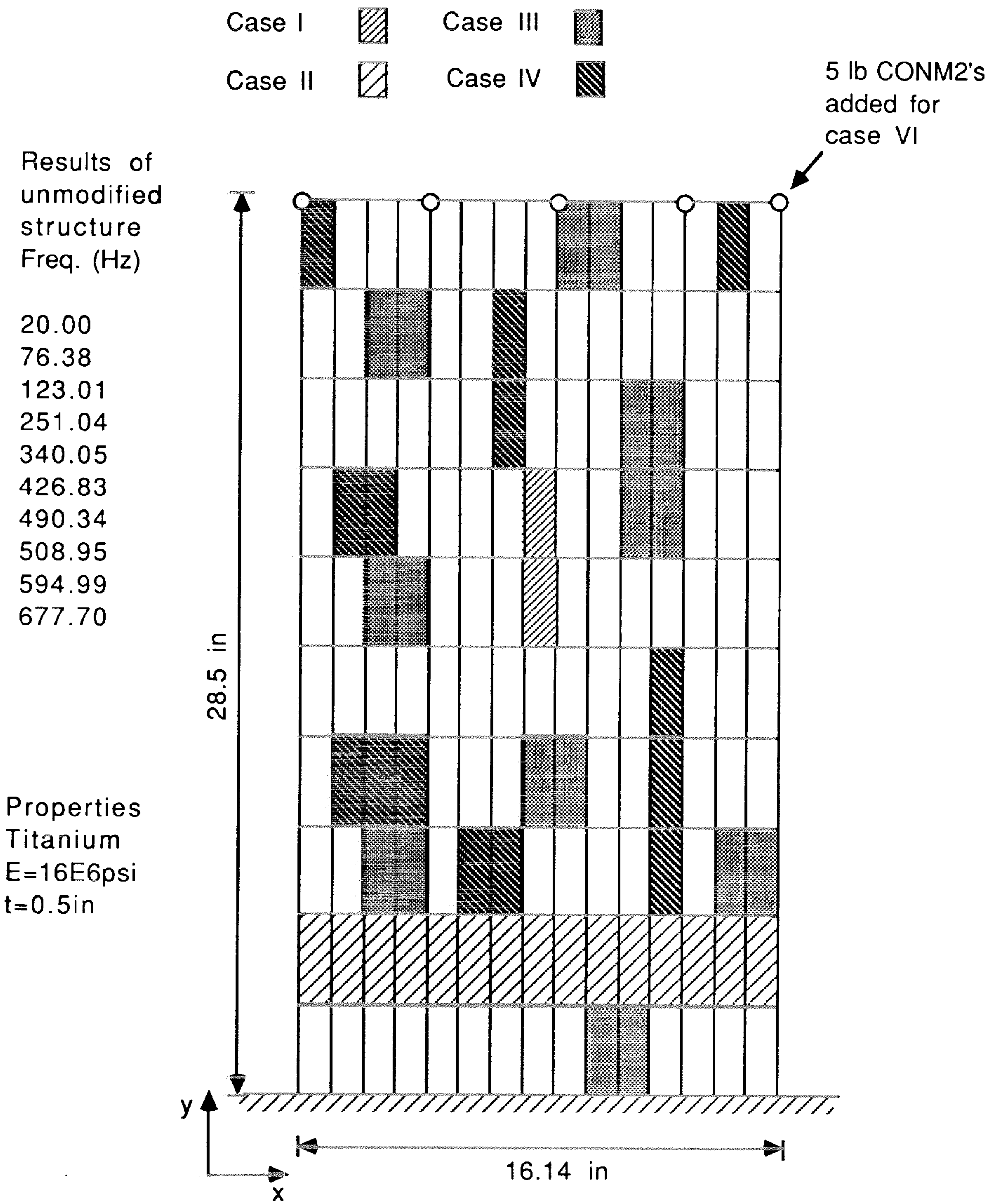


Figure 3 Finite Element Mesh of Part of Aircraft Structural Component

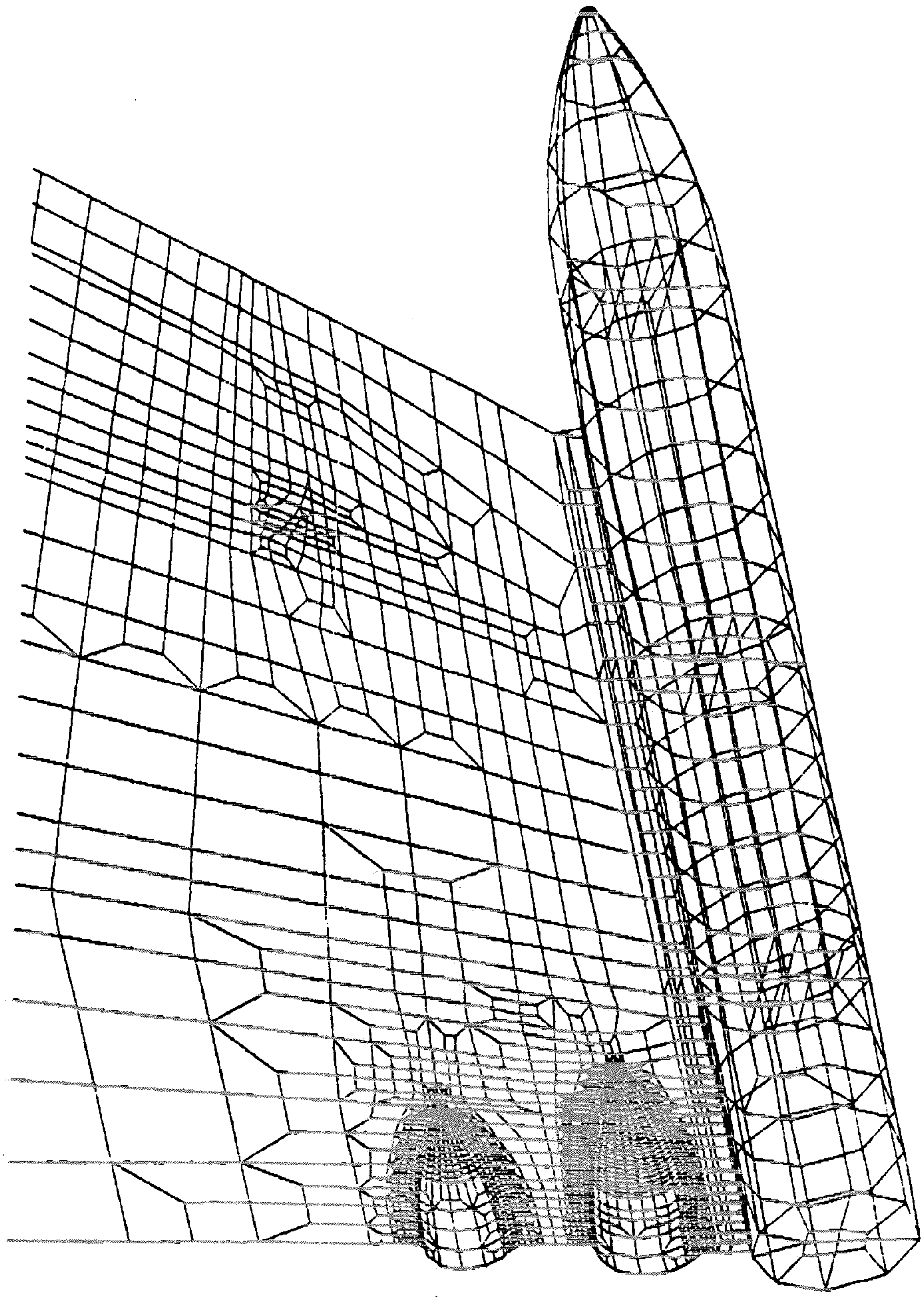


Figure 2 A Cantilever Plate Model

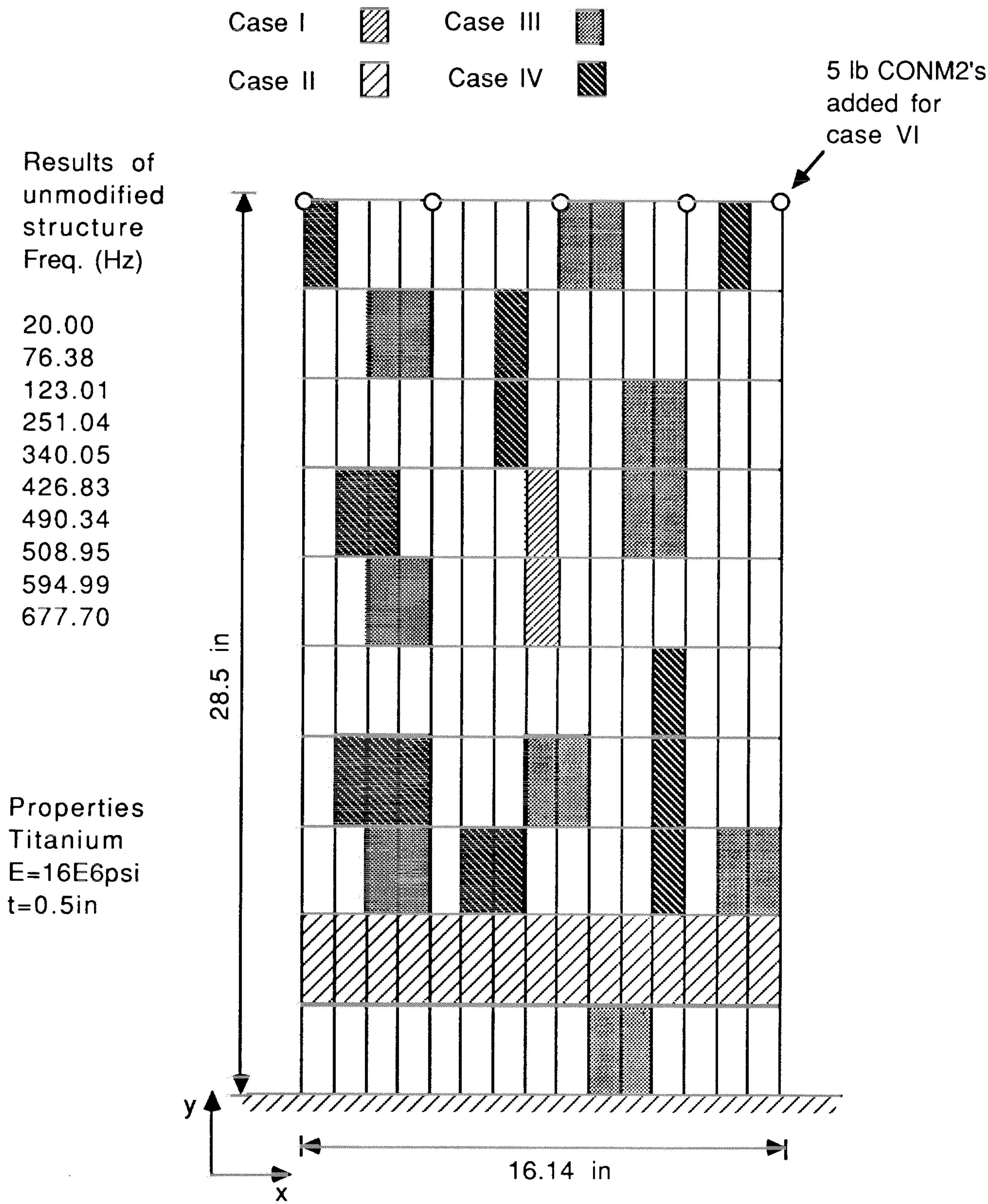


Figure 4 Percent Stress Change Comparison

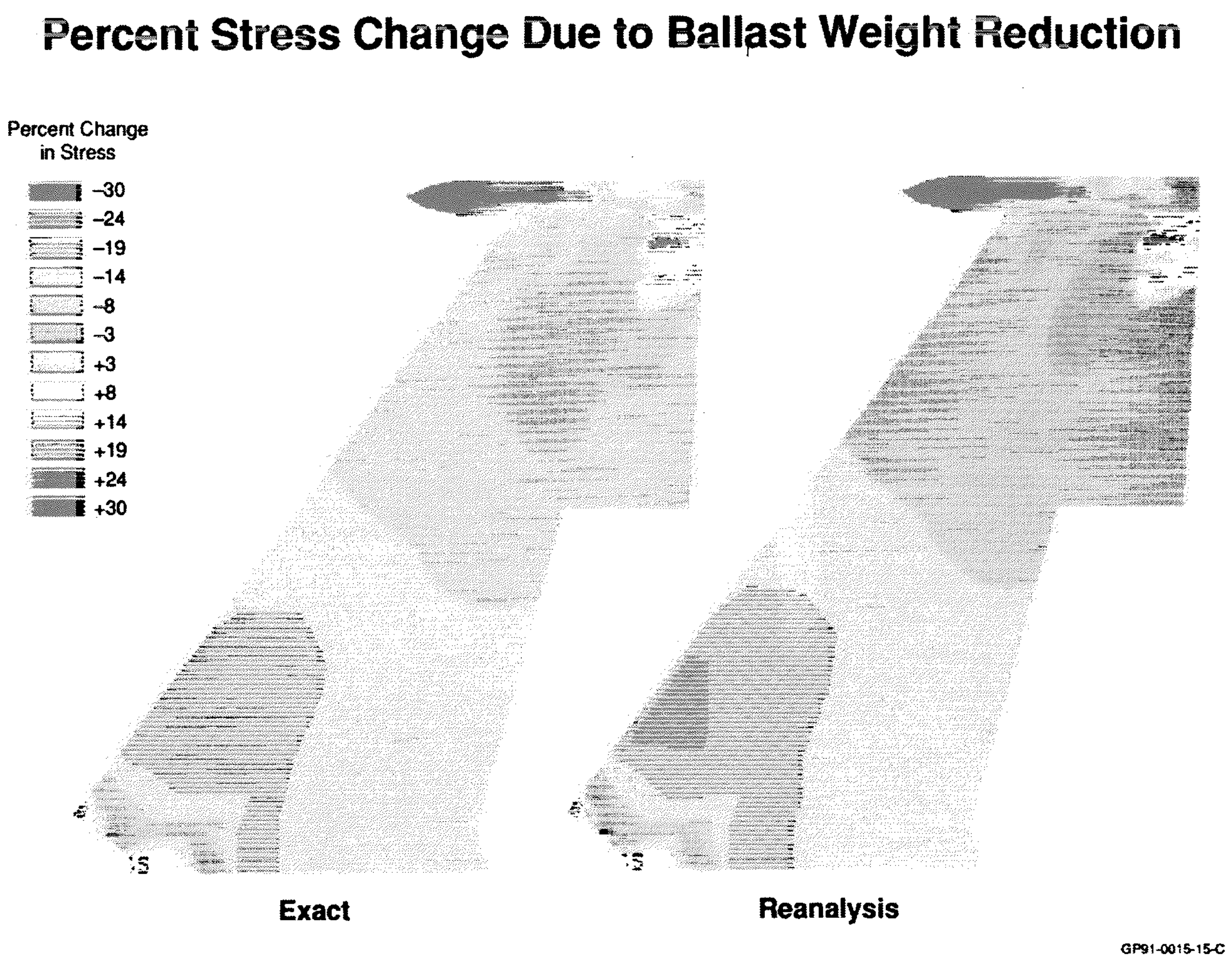


Table 1 Reanalysis Results

	Assumed Mode Method			Residual Static Mode Correction		
	Freq. (Hz)	q^*	Error	Freq. (Hz)	q^*	Error
1 mode base run	123.5	-1.0	18%	105.0	0.995 2.643	0%
2 mode base run	118.6	0.998 0.0579	13%	105.0	-0.995 -.0971 -2.57 -.00665	0%
	608.5	-0.0579 0.998	14%	534.3	-0.0964 0.994 -7.28 2.16	0%

q^* is the eigenvector for the reduced problem

Table 2 Results of Case I

Frequencies (Hz)

Mode	Exact	Assumed Mode	Static Correction 1 Iteration	Static Correction 5 Iterations	5 Iteration % Error
1	20.065	20.069	20.065	20.065	0.00
2	75.84	75.88	75.85	75.84	0.00
3	122.22	122.48	122.23	122.22	0.00
4	249.33	249.53	249.42	249.33	0.00
5	338.60	338.88	338.60	338.60	0.00
6	400.02	424.42	401.16	400.04	0.01
7	488.26	489.18	488.91	488.26	0.00
8	507.14	510.58	507.21	507.14	0.00
9	588.36	594.22	588.64	588.38	0.00
10	676.84	678.09	676.88	676.85	0.00

Table 3 Results of Case II

Frequencies (Hz)

Mode	Exact	Assumed Mode	Static Correction 1 Iteration	Static Correction 5 Iterations	5 Iteration % Error
1	1.2846	16.56	2.91	1.285	0.03
2	30.43	71.42	68.30	34.06	11.93
3	45.61	120.56	93.98	45.75	0.31
4	175.92	245.34	240.78	177.02	0.63
5	191.50	349.09	248.81	191.66	0.08
6	300.68	423.52	316.86	314.86	4.72
7	385.99	453.20	419.64	386.51	0.13
8	402.32	496.94	484.80	403.95	0.41
9	476.64	582.61	524.08	478.80	0.45
10	555.28	701.48	601.53	557.32	0.37

Table 4 Results of Case III

Frequencies (Hz)

Mode	Exact	Assumed Mode	Static Correction 1 Iteration	Static Correction 5 Iterations	5 Iteration % Error
1	22.20	25.17	22.53	22.21	0.05
2	87.64	96.38	89.59	87.82	0.21
3	136.84	150.82	138.81	136.98	0.10
4	288.21	319.27	296.63	289.26	0.36
5	388.00	440.13	398.31	388.86	0.22
6	466.65	503.49	474.77	467.72	0.23
7	495.27	551.36	495.35	495.35	0.02
8	550.09	635.61	569.96	552.09	0.36
9	685.08	798.29	706.06	690.27	0.76
10	749.24	868.68	770.25	750.54	0.17

Table 5 Results of Case IV

Frequencies (Hz)

Mode	Exact	Assumed Mode	Static Correction 1 Iteration	Static Correction 5 Iterations	5 Iteration % Error
1	19.72	20.19	19.76	19.73	0.05
2	76.75	77.93	76.98	76.92	0.22
3	124.56	126.37	124.76	124.61	0.04
4	254.92	261.92	258.35	257.01	0.82
5	332.01	342.50	334.25	332.93	0.28
6	397.57	438.21	407.99	403.35	1.45
7	476.97	508.24	488.09	487.50	2.21
8	489.49	517.38	498.44	495.36	1.20
9	551.77	605.98	587.75	576.53	4.49
10	673.31	691.90	679.58	676.60	0.49

Table 6 Results of Case V

Frequencies (Hz)

Mode	Exact	Assumed Mode	Static Correction 1 Iteration	Static Correction 5 Iterations	5 Iteration % Error
1	16.05	16.05	16.05	16.05	0.00
2	61.29	61.29	61.29	61.29	0.00
3	98.72	98.72	98.72	98.72	0.00
4	201.46	201.46	201.46	201.46	0.00
5	272.88	272.88	272.88	272.88	0.00
6	342.53	342.53	342.53	342.53	0.00
7	393.50	393.50	393.50	393.50	0.00
8	408.43	408.43	408.43	408.43	0.00
9	477.48	477.48	477.48	477.48	0.00
10	543.85	543.85	543.85	543.85	0.00

Table 7 Results of Case VI

Frequencies (Hz)

Mode	Exact	Assumed Mode	Static Correction 1 Iteration	Static Correction 5 Iterations	1 Iteration % Error
1	10.32	10.32	10.32	10.32	0.00
2	38.51	38.62	38.51	38.51	0.00
3	94.29	94.36	94.29	94.29	0.00
4	162.75	172.22	162.75	162.75	0.00
5	175.83	177.33	175.83	175.83	0.00
6	274.53	280.21	274.53	274.53	0.00
7	293.91	294.68	293.91	293.91	0.00
8	345.51	401.86	345.71	345.71	0.06
9	403.16	484.97	403.26	403.26	0.02
10	504.41	511.90	504.74	504.73	0.06

Table 8 Results of Case VII

Frequencies (Hz)

Mode	Exact	Assumed Mode	Static Correction 1 Iteration	Static Correction 5 Iterations	5 Iteration % Error
1	0.70	8.69	1.60	0.70	0.00
2	19.04	36.86	35.66	20.71	8.77
3	39.22	90.79	79.08	40.01	2.01
4	111.54	169.86	162.51	113.05	1.35
5	150.07	172.43	164.96	150.77	0.47
6	162.75	249.21	235.18	162.84	0.06
7	168.66	301.65	256.88	170.65	1.18
8	300.52	402.81	362.81	301.34	0.27
9	345.71	481.74	406.10	345.95	0.07
10	418.26	501.88	479.50	447.03	6.88

Table 9 Large Scale A/C Structural Component Results

Frequencies (Hz)

With Ballast	W/o Ballast Exact	W/o Ballast Static Correction Reanalysis	% Error
8.67	8.62	8.62	0
12.66	12.70	12.69	-.08
16.09	16.12	16.11	-.06
21.84	21.85	21.85	0
25.08	25.14	25.14	0
32.26	31.09	31.09	0
34.73	38.29	38.29	0
38.63	38.54	38.54	0
40.47	39.75	39.75	0
41.81	41.31	41.36	0.12

Table 10 Comparison of Reanalysis Solution Times (CRAY)

Large Scale A/C Structural Component Model Solution

	Exact cpu (sec)	Reanalysis cpu (sec)	Reduction Factor
Vib soln	1305	514	2.5
Data Processing	339	339	0
Total	1644	853	1.9