Curved Shell Analysis
Some Problems and Solutions

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1. Introduction

This study of the behavior of curved shell analysis will concentrate on aspects of equilibrium. In fact the balance between membrane forces and bending forces plays a dominant role in curved shells.

Simple curved beam models will serve to illustrate some principal problems, discussing element design and correct loads in relation to modelling strategy. Cylindrical shells are easily related to the concepts of curved beam modelling. For doubly curved shells the discussions apply in principle, but the formal treatment is considerably more involved, and will not be treated in this paper.

In most circumstances membrane action is many orders stiffer than bending. Hence deviations from equilibrium result in wrong distributions between membrane & plate bending forces, and can lead to substantial errors in deformations.

Membrane Locking occurs when more load is carried by membrane forces than theoretically correct.

Excessive Flexibility occurs when excessive shell bending is caused by equilibrium errors.

Many practical curved shell problems are transition between membrane and plate stress status. The solution of these problems demands careful detail design of curved elements including surface pressures, where practical cases are dominated by equilibrium between pressure and membrane stresses.

Two different modelling principles can be distinguished:

(1) Faceted modelling using planar elements (or straight BARS,BEAMS). This is the only reliable method with current panel elements

(2) Curved modelling, using curved elements, currently has limited success with circular curved BEAM elements which are force equilibrium function elements. However the MSC elements CURVE, CBEND do not include lateral pressure.

FE results will be presented using beam elements in MSC/pal 2 *.

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* MSC/pal 2 is a registered trademark of the MacNeal-Schwendler Corporation, Los Angeles, California.

2. Membrane Stress Theory

Membrane theory will shed some light on shell analysis problems. It investigates equilibrium conditions when only membrane stresses occur, without transverse shears and bending. It is treated extensively by Fluegge in Ref.1.

The case of general curvature in two directions is rather complex, but the special case of a cylindrical surface is very instructive and applicable to the simple explorations of this paper, probably including ruled surfaces.

The expression for the hoop load is well known from a circular cylinder, but is generally valid for variable pressure and radius:

$$N_\Phi = p \cdot R \qquad (1)$$

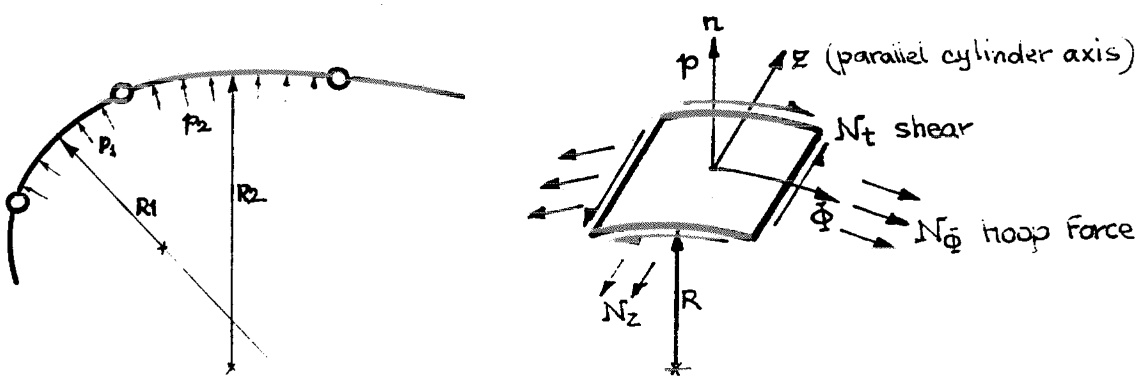

Figure 1. Membrane Forces on a Cylindrical Surface

If the product $p \cdot R$, and consequently the hoop load, is not constant, a tangential membrane shear force is needed to maintain equilibrium.

$$N_t = -(z/R) \cdot dN_\Phi / d\Phi \qquad (2)$$

Variation of this shear results in normal stresses in the z direction:

$$N_z = 0.5 \cdot z^2 \cdot dN_t/ds + N_z(o) \qquad (3)$$

The membrane shear is selfbalanced around a closed section (resultant = zero). It increases linearly in the axial direction z of the cylinder until able to cancel on a bending resistant frame or bulkhead, which interrupts the membrane status. In the absence of discrete frames, the bending stiffness of the shell must take over. Since it is usually fairly flexible, large displacements occur, and thus any imbalance between membrane and bending in shell elements leads to errors mentioned previously.

3. Principles of Facet and Curved Modelling

The examples in this section were briefly discussed by R.S. Lahey and this author in references 2 & 3. The principles were used in a statement by Professor Clough in 1971: " Inconsistent refinements of analysis often result in larger errors than no refinement". Experiences tell that this principle bears repeated elaboration from time to time.

Equilibrium between membrane forces and bending forces on straight or planar elements is no problem. On curved surface elements it is critical to their design, including membrane forces in equilibrium with lateral pressure.

Faceted modelling of a curved beam

Faceted modeling uses only planar elements. This approach is currently the most successful. Incorrect hoop forces can be avoided by correct determination of nodal forces from lateral pressure for irregular mesh distance and variable radius, discussed in section 4.

Line or surface pressure load input for an irregular mesh will lead to inconsistent results.

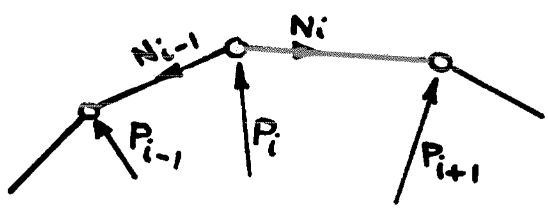

Figure 2. Hoop Force Equilibrium on Faceted Curved Beam

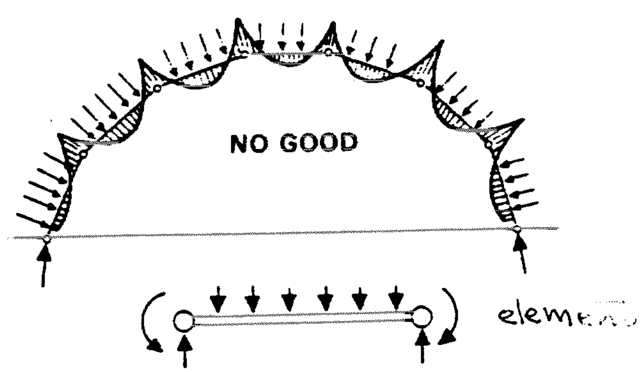

Figure 3. End Moments on Faceted Modelling.

Consistent end-moments are not applicable

Pressure loads on a curved structure must not use end-moments, neither for faceted nor curved modelling. They are inconsistent with the membrane equilibrium status and result in an unreal bending perturbation. For a truly straight beam structure end-moments improve results, as is well known.

Curved Elements and Radial Nodal Forces

For true concentrated forces, curved elements can give better results than faceted representation.

For Pressure nodal loads, however, hoop forces in curved elements joining a node cannot establish equilibrium. Transverse shear and bending shown in figure 3 is the result. The curved beam in MSC/NASTRAN & MSC/pal satisfy internal hoopload-bending equilibrium, but do not include the pressure load term.

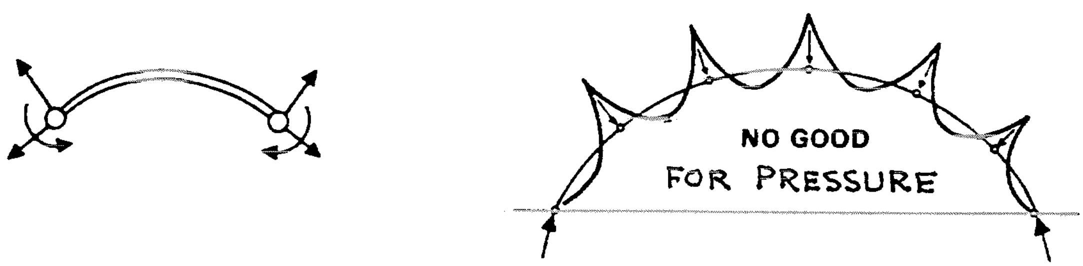

Figure 4. Concentrated Radial Loads on Curved Beams.

Curved Element Including Pressure

The hoop (membrane) forces due to pressure must be imbedded in the design of a curved element. This leads to terms of a similar nature than thermal expansion loads. The hoop forces of joining elements must be in equilibrium at the node, thus curved elements should be able to join tangentially at the nodes, for non-circular sections this will be difficult to achieve.

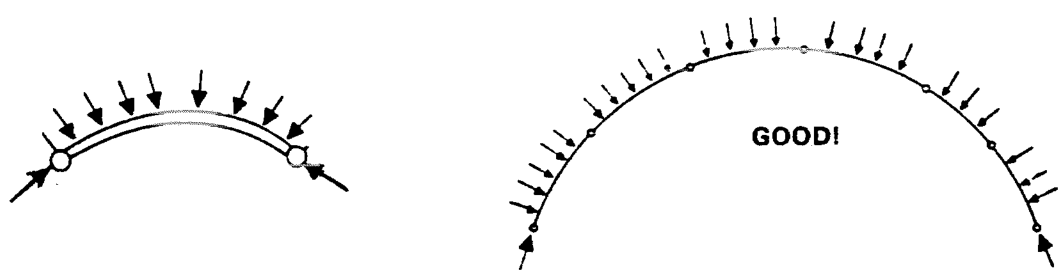

Figure 5. Hoop Force Equilibrium on Curved Beam Model.

Demonstration Problem Circular Arc Models with MSC/pal

Closed Ring Arc of 90° double symmetry, four beam elements.

Area A = 1.0*0.1 = 0.1; Inertia I = 0.833333-04; Module S = 1.667-03;
Radius = 10.0; Pressure = 100.0 ; Constant hoop force.

	Hoop Force	Hoop Stress	Bending Stress	Radial Displ.
Theory:	1000.0	10 000.0	0.0	1.0-02
Tests: Arc-pressure nodal loads from table below.				
A. Equal Facets :	1000.0	10 000.0	0.66-05	1.0-02
B. Biased Facets:	1000.0	10 000.0	1.73-05	1.0-02
C. Curved, Equal:	980.8	9 807.9	7.68+04	4.69-02
D. Curved Beam with built in pressure not currently available.				

Equal Angle Nodal loads, equ 6 & 7:

Node	Angle	Fx	Fy
1	0.0	1.9509032201E+02	0.0000000000E+00
2	22.5	3.6047991100E+02	1.4931566810E+02
3	45.0	2.7589937928E+02	2.7589937928E+02
4	67.5	1.4931566810E+02	3.6047991100E+02
5	90.0	0.0	1.9509032201E+02

Biased Angle Nodal loads, equ 6 & 7:

Node	Angle	Fx	Fy
1	0.0	8.7155742747E+01	0.0000000000E+00
2	10.0	2.5486440058E+02	5.6502077306E+01
3	30.0	3.0076746636E+02	1.7364817767E+02
4	50.0	2.9690501110E+02	4.2402429979E+02
5	90.0	0.0	3.4202014332E+02

Test B1. LINE Pressure Loads on biased faceted model.
Results show the incorrect, non-constant, hoop forces given by equation (15):
N1 = 996.2; N2 = N3 = 984.8; N4 = 939.7;
Radial displacements at node 1 =.956-02; node 5 =.976-02

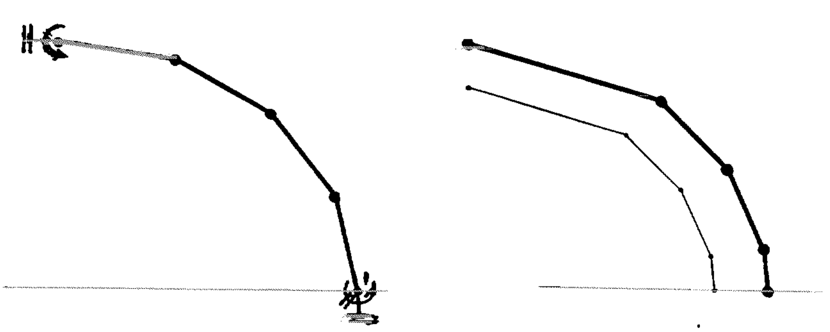

Figure 6. Equal Faceted and Biased Faceted Quarter Arc Beam Model.

4. Pressure Nodal Loads For Cylindrical Vessels

A general procedure is given to determine pressure nodal loads for faceted cylindrical surfaces with
Variable mesh distance, curvature radius, and surface pressure.

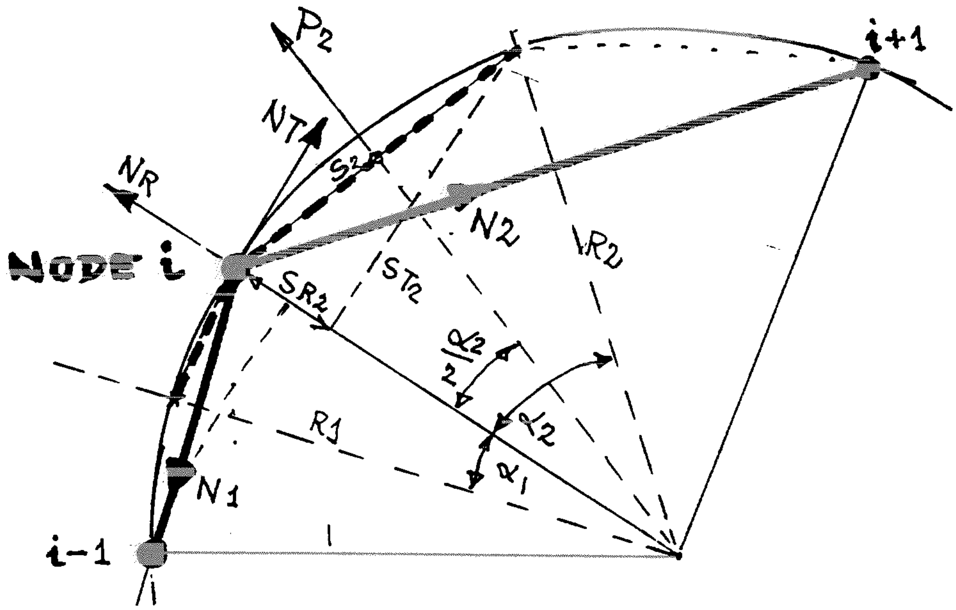

Figure 7. Arc-Pressure Loads at Node i.

The pressure is applied to the arc between midpoints adjacent to node i,
The load on node i results from the two adjacent branches:

Facet subscript 1: node i halfway to node i-1
Facet subscript 2: node i halfway to node i+1

- α Half-angle of mesh interval
- R Radius, variable between intervals
- p Pressure
- P Pressure nodal load at node i
- N Hoop load at node i
- r radial component
- t tangential component

The chord over the halfangle α is $S = 2 \cdot R \cdot \sin(\alpha/2)$

Its radial and tangential components are:

$$Sr = R(1-\cos\alpha) \tag{4}$$

$$St = R \cdot \sin\alpha \tag{5}$$

Arc Pressure load radial and tangential components at node i for variable pressure, radius and mesh angle:

$$Pir = p1 \cdot R1 \cdot \sin\alpha 1 + p2 \cdot R2 \cdot \sin\alpha 2 \quad (6)$$

$$Pit = - p1 \cdot R1 \cdot (1 - \cos\alpha 1) + p2 \cdot R2 \cdot (1 - \cos\alpha 2) \quad (7)$$

The adjacent hooploads are:
N1 (segment i,i-1) & N2 (segment i,i+1)
Theyr radial & tangential components are:

$$Nr = - N1 \cdot \sin\alpha 1 - N2 \cdot \sin\alpha 2 \quad (8)$$

$$Nt = - N1 \cdot \cos\alpha 1 + N2 \cdot \cos\alpha 2 \quad (9)$$

Equilibrium of radial components at node i:

$$Nr + Pr = 0 = (p1 \cdot R1 - N1) \cdot \sin\alpha 1 + (p2 \cdot R2 - N2) \cdot \sin\alpha 2 \quad (10)$$

Adjacent hoop loads are thus determined per membrane equation (1).

Tangential equilibrium at node i:

$$Nt + Pt = 0 = - N1 \cdot \cos\alpha 1 + N2 \cdot \cos\alpha 2 \quad (11)$$

$$- p1 \cdot R1 \cdot (1 - \cos\alpha 1) + p2 \cdot R2 \cdot (1 - \cos\alpha 2)$$

Both equilibrium conditions are satisfied for variable mesh-angle α:

When both R=const and p=const (pressurized circular cylinder)
Or if: $p \cdot R = const$ (equation 1).

If not, then shell bending and/or membrane shear will occur.

Limitation of Line & Surface Pressure Loads

The loads normal to the straight chord from node i to i+1:

$$P2 = p2 \cdot R2 \cdot 2 \cdot \sin\alpha 2 \quad (12)$$

At node i the radial component is:

$$P2R = 0.5 \cdot P2 \cdot \cos\alpha 2 = p2 \cdot R2 \cdot \sin\alpha 2 \cdot \cos\alpha 2 \quad (13)$$

The tangential component:

$$P2T = 0.5 \cdot P2 \cdot \sin\alpha 2 = p2 \cdot R2 \cdot \sin^2\alpha 2 \quad (14)$$

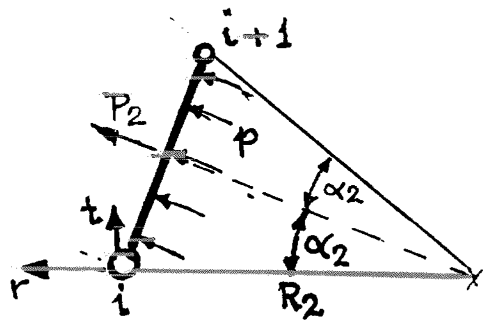

Figure 8. Line Pressure Components.

Equilibrium at node i with hoop force components from equations 8 & 9:

$$0 = \sin\alpha 1(p1 \cdot R1 \cdot \cos\alpha 1 - N1) + \sin\alpha 2(p2 \cdot R2 \cdot \cos\alpha 2 - N2) \qquad (15)$$

This leads to hoop forces below (Compare equ (10), (14)):

$$N = p \cdot R \cdot \cos\alpha \qquad (16)$$

which violate membrane equation (1). They vary with mesh cosα for circle and constant pressure. The tangential equilibrium is satisfied nevertheless with equ (14):

$$0 = -(p1 \cdot R1 \cdot \sin^2\alpha 1 + N1 \cdot \cos\alpha 1) + (p2 \cdot R2 \cdot \sin^2\alpha 2 + N2 \cdot \cos\alpha 2) \qquad (17)$$

For a cylindrical vessel of constant radius, constant pressure, and constant mesh distance equation (15) results in constant but lower hoop forces.

The line or surface load input device in pressure vessel applications is thus limited to a specific mesh distance relation:

$$\cos\alpha = const/p \cdot R \qquad (18)$$

5. Conclusions

Faceted modelling is currently the only reliable technique for FE analysis of curved shells. Equilibrium between membrane and bending forces is critical in curved shells to escape the membrane locking syndrome, or excess flexibility in pressure cases.

The error of hoop forces resulting from line loads on beam elements is as such quite small for the circular arc. That the hoop forces are not constant is a more severe problem, particularly for shells, and when the correct solution is not known.

The equations given for pressure nodal loads apply to faceted curved beam or cylindrical shell modeling. They are valid for variable mesh, pressure, and radius. Correct hoop force equilibrium is satisfied, resulting in constant and precise hoop forces in the tests.

A corresponding formulation for faceted models of double curved shells would be very usefull.

Curved beam and shell elements must include equilibrium of membrane and bending, as well as membrane forces balancing applied pressure. Much remains to be done for both cylindrical and doubly curved shell elements.

References

1. W.Fluegge. Stresses in Shells. Ch. 3.1.1. Springer Verlag New York Inc.1966

2. G.W.Haggenmacher & R.S.Lahey. Practical Aspects of the Finite Element Method. Paper to the Second World Congress on F.E.M, Bournemouth 1978. Editor John Robinson.

3. G.W.Haggenmacher & R.S.Lahey. Diagnostics in Finite Element Analysis. First Chautauqua on Finite Element Modeling. Harwichport,MA, 1980. Editor J.H.Conaway.