Updating MSC/NASTRAN Models to Match Test Data

Ken Blakely
The MacNeal-Schwendler Corporation
Los Angeles, CA

Because of modeling uncertainties, an MSC/NASTRAN model may not match test data acquired for the same structure. The Design Optimization capability, new in Version 66, can be used to update a model by minimizing the difference between computed results and test data. This paper shows how to use the equation capability in SOL 200 to update MSC/NASTRAN models, and shows application to a disk drive enclosure to match measured resonant frequencies.

Updating MSC/NASTRAN Models to Match Test Data

Introduction

Results obtained via finite element analyses do not always match those obtained from tests, often due to modeling uncertainties, particularly the boundary conditions. Even though test data contains uncertainties, most people feel that any difference between test and analysis is due solely to modeling uncertainties. This perception is reflected in the saying, "Everyone believes the test data except for the experimentalist, and no one believes the finite element model except for the analyst."

Once it is decided to update the model the refinement process proceeds in an inefficient or efficient manner. One inefficient way is by trial-and-error, i.e., simply changing one or more model parameters, rerunning the analysis, and comparing the new results to the test data. If the match is close enough, then stop the process. If it is not close enough, then change the parameters again and repeat the process. Obviously, this process is very inefficient for a large number of model parameters.

A less-inefficient way is to make a small change in each parameter and rerun the analysis, thereby creating a "sensitivity matrix" by hand. Then, proper choice of updated parameters is made in a sensible way by using the sensitivities. More efficient is using MSC/NASTRAN to compute the response sensitivities directly, either in SOL 200 (Design Optimization) or in SOLs 51, 53, and 55 (Design Sensitivity).

Most efficient is using SOL 200 to directly update the model, which is described in the remainder of this paper.

Model Updating via Bayesian Parameter Estimation

Ideally, a model updating procedure is both efficient and complete (complete in the sense that it uses all of the available information--test data, model parameters, and the experience and knowledge of both the experimentalist and analyst). An approach called *Bayesian parameter estimation* (Refs. 1-3) permits such a balanced model updating approach by incorporating uncertainties in both test data and in modeling parameters. In this approach each "piece" of test data has a confidence assigned to it, each modeling parameter has a confidence associated with it, and the test data as a whole can be weighed relative to the initial model as a whole. The Bayesian parameter estimation procedure is an iterative approach wherein an error is minimized. This error is written as:

$$E = wt*(\{RT\} - \{RA\})^T [WR] (\{RT\} - \{RA\}) + wp*(\{PF\} - \{PO\})^T [WP] (\{PF - PO\}) \quad [1]$$

where RT = responses from the test (i.e., test data)
RA = responses from the analysis
WR = weighting factors (confidences) for responses
PF = parameters of the final model
PO = parameters of the original model
WP = weighting factors (confidences) for the parameters
wt = scalar weighting for the test data as a whole
wp = scalar weighting for the model parameters as a whole

This equation is written in matrix form, since there are usually multiple responses and multiple parameters. This equation is solved in an iterative fashion to find PF, the final model parameters that minimize the error.

Model responses and test data can be static displacements, resonant frequencies and mode shapes, and element forces, stresses, and strains--any quantity that can be measured or computed with MSC/NASTRAN. Model parameters consist of beam areas and inertias and plate thicknesses--any quantity that can be entered on the MSC/NASTRAN property entries. Weighting matrices are usually diagonal, and are scaled such that they are divided by the square of the initial value. As a result, units for all computations become fractional changes rather than absolute changes, thereby avoiding numerical problems caused by a wide variance of units--an eigenvalue can be 10^6 or more, and a plate thickness can be much less than 1, for example.

One way to minimize Eq. 1 is to use SOL 200. The equation is written via the DEQATN entry and then the optimizer in MSC/NASTRAN is used to minimize the error. The minimum of the error gives us a model that matches the test data (making the experimentalist happy) while changing least from the initial model (making the analyst happy). This permits a balanced approach influenced by the "model updater's" choice of weightings.

Application of Bayesian Parameter Estimation to a Disk Drive Enclosure

Consider the disk drive enclosure shown in Figure 1. This MSC/NASTRAN model contains 1406 grid points and 1354 plate elements. The actual enclosure was tested by Structural Measurement Systems (Milpitas, CA) to determine its lowest resonant frequencies. Table 1 shows the measured and computed frequencies for the lowest four flexible resonant frequencies (the enclosure was tested in its unrestrained state). Computed mode shapes, plotted with MSC/XL, are shown in Figure 2.

Table 1: Measured and Computed Frequencies for Enclosure

Mode	Frequencies (Hz) Measured	Frequencies (Hz) Computed
1	345.7	233.7
2	1306.5	892.2
3	1566.9	1164.9
4	1677.7	1267.3

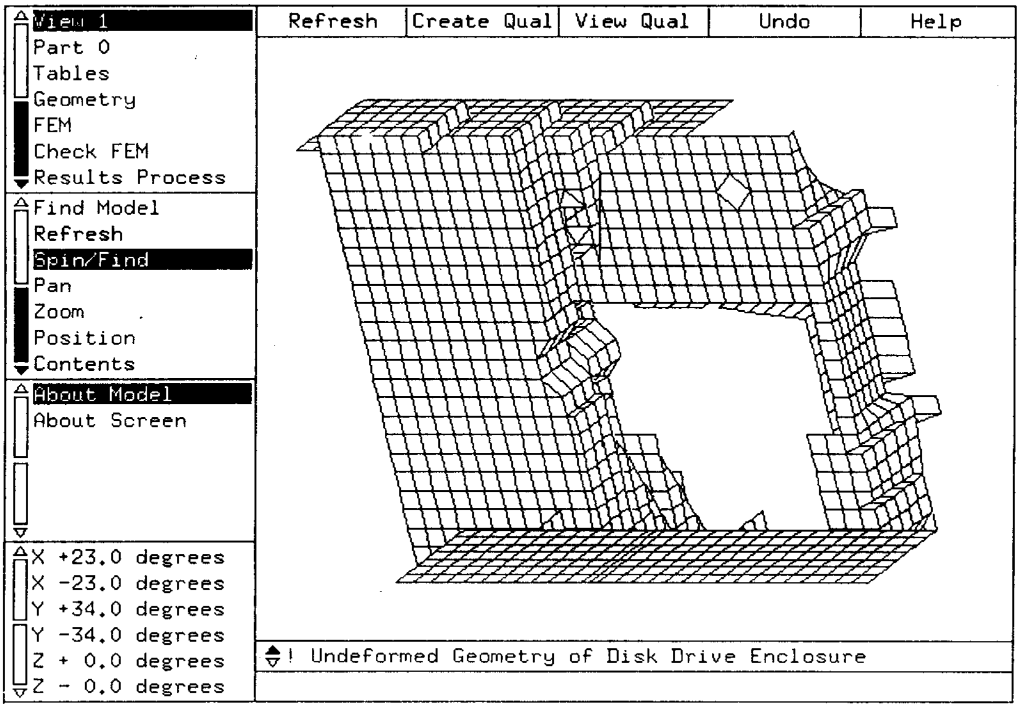

Figure 1: Undeformed Geometry of the Disk Drive Enclosure

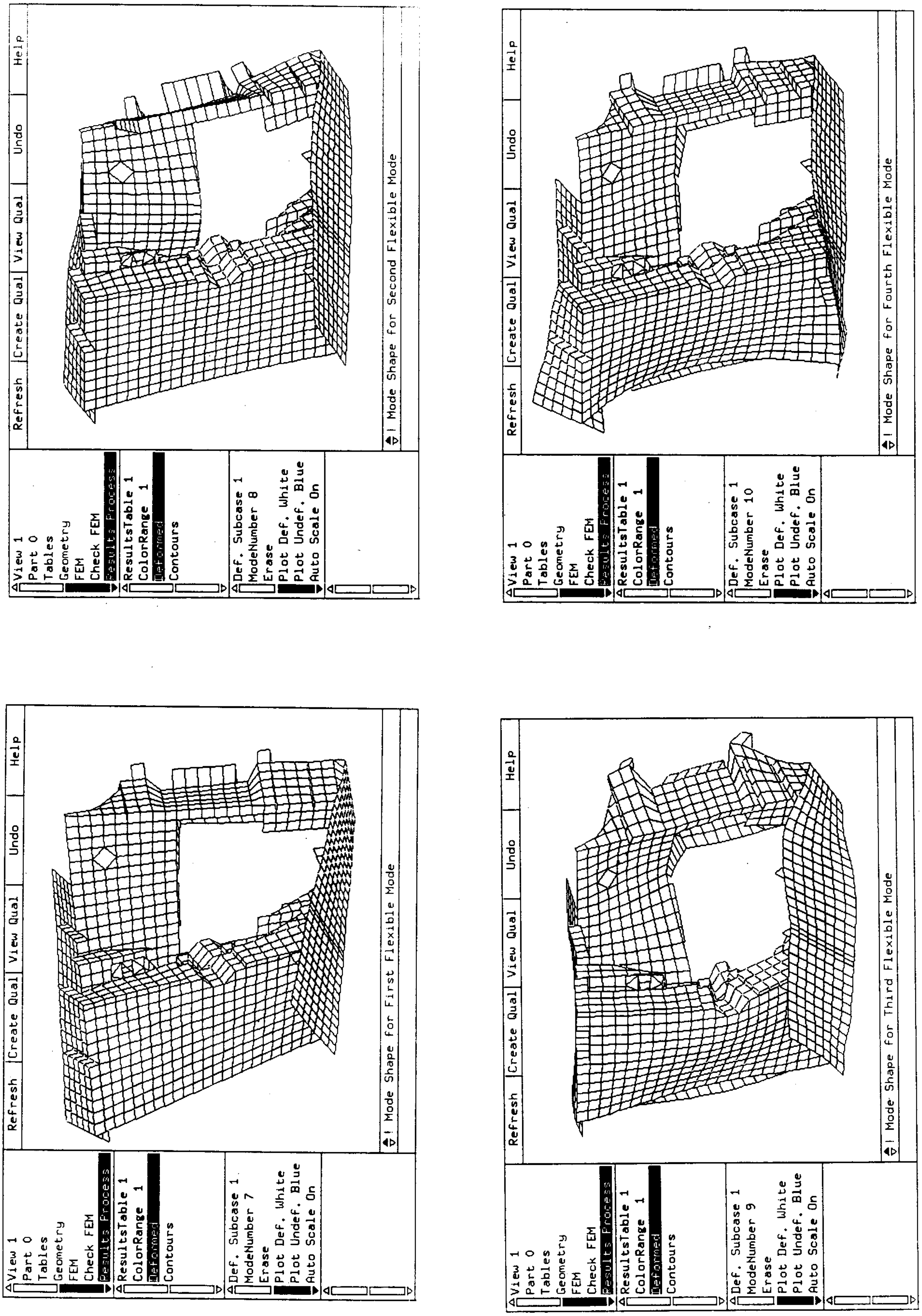

Figure 2: First Four Computed Flexible Modes

The model updating process was run in SOL 200 to update the model until its computed resonant frequencies matched those from the modal test. The initial model contained nine plate thicknesses, some of which were linked together to form four independent thicknesses that comprised all of the plates in the model. Table 2 shows the four independent design variables, along with their initial values and upper and lower bounds. There were uncertainties in these thicknesses because of the crude way used to measure them: a standard ruler was used to determine the thicknesses from the actual enclosure, leading to uncertainty as to the exact values (especially in areas hard to measure). Therefore, each of the four thicknesses had an initial value and some variation (represented by lower and upper bounds on the values, consistent with the uncertainty in the measurements). Since bending stiffness is proportional to the cube of the thickness, slight changes in the thickness make large changes in the stiffness.

Table 2: Independent Design Variables

Design Variable No.	PSHELL No.	Initial Value (in.)	Bounds (in.)	
			Lower	Upper
1	1	0.08	0.05	0.125
2	4	0.12	0.05	0.20
3	8	0.10	0.05	0.15
4	50	0.20	0.10	0.30

MSC/NASTRAN's Design Optimization capability was used to update the model. An equation was written via the DEQATN entry to specify the error to be minimized. This equation was essentially the same as Eq. 1 except that the thicknesses and resonant frequencies were divided by the initial values to produce fractional changes. Test data as a whole (wt) was weighted at 1.0 and the initial model as a whole (wp) was weighted at 0.001, or one one-hundredth as much. In addition, each resonant frequency was weighted equally (1.0) to one another and each of the four independent model parameters was also weighted equally (1.0) to one another.

The Executive Control, Case Control, and selected Bulk Data entries are listed in the appendix for reference. The front of the appendix shows the relationship of some of the key Bulk Data entries. Note especially the relationship of the arguments of DEQATN and the terms of DRESP2--they must be in the same order.

As in the case of running Design Optimization for minimizing weight, MSC/NASTRAN's optimizer uses response sensitivities and approximate analyses to select new (better) parameters at each iteration in the process. This iterative cycle continues automatically until convergence is achieved by a lack of change in the parameters or the objective function between consecutive iterations.

Results of the model updating process are summarized in Table 3. The overall process required eight analyses to converge (the initial analysis plus seven optimization iterations). The first, second, and fourth frequencies were closely matched, while the third frequency was not as close. The first design variable (pertaining to PSHELL 1) reached its upper bound (0.125 in.), while the rest of the design variables stayed well within their limits.

Table 3: Comparison of Resonant Frequencies and Model Parameters for Enclosure

Mode	Frequencies (Hz)		
	Measured	Computed (initial model)	Computed (revised model)
1	345.7	233.7	346.1
2	1306.5	892.2	1295.8
3	1566.9	1164.9	1460.6
4	1677.7	1267.3	1694.6

Model Parameter	Initial Value (in.)	Final Value (in.)
1	0.08	0.125
2	0.12	0.076
3	0.10	0.079
4	0.20	0.236

Consequently, another set of runs was made, this time increasing the upper bound of design variable 1 from 0.125 to 0.150 in. The results are summarized in Table 4. The third frequency is now improved and the others are a little worse. Note that the final design variables in this run are vastly different than those of the previous run. This occurs because the process has been started from scratch with the initial model instead of restarting from the final model of the previous case.

Table 4: Comparison of Results for Upper Bound of Variable Increased to 0.150

Mode	Frequencies (Hz)		
	Measured	Computed (initial model)	Computed (revised model)
1	345.7	233.7	350.8
2	1306.5	892.2	1273.9
3	1566.9	1164.9	1503.1
4	1677.7	1267.3	1705.9

Model Parameter	Initial Value (in.)	Final Value (in.)
1	0.08	0.136
2	0.12	0.141
3	0.10	0.099
4	0.20	0.118

What happens if we weight the test and analysis equally; i.e., what if wt=wp=1.0? These results are shown in Table 5. As expected, the frequencies are not matched as well as in the above cases, and the model parameters are closer to the initial values. Choosing appropriate weightings is an art and not a science. The implications and guidelines for the weightings selection are presented in Ref. 4.

Table 5: Comparison of Results for wt=wp=1.0

Mode	Frequencies (Hz) Measured	Computed (initial model)	Computed (revised model)
1	345.7	233.7	322.0
2	1306.5	892.2	1204.2
3	1566.9	1164.9	1420.6
4	1677.7	1267.3	1562.7

Model Parameter	Initial Value (in.)	Final Value (in.)
1	0.08	0.116
2	0.12	0.116
3	0.10	0.101
4	0.20	0.206

For each of the above cases, the third frequency was not matched very well. An additional run was then made in which the third frequency was weighted 100 times more than the other frequencies (and wt=1.0 and wp=0.001). After the refinement process, the final frequencies were 349.8, 1309.3, 1466.3, and 1728.5 Hz--which were still not a great match to the third frequency. An inspection of the response sensitivities revealed the cause: the third mode was less than one-tenth as sensitive to the model parameters as were the other frequencies. In addition, the parameters had reasonable move limits (the lower and upper bounds on the values). If the parameters had a larger range of values, then perhaps the third frequency would have been matched better.

So, which of these models is best? Since "best" is the model that gives a minimum error, "best" depends upon the equation that defines the error in the first place. Different equations--as well as a different choice of model parameters or different upper and lower bounds--give rise to different models. The user needs to get input from the experimentalist regarding test data and their confidences input from the analyst regarding model parameters and their confidences. In addition, the user needs to weigh the test data relative to the initial model as a whole. The rest is a "turn the crank" process as far as the number-crunching is concerned, though the model refiner needs to assess the final results to ensure that they are both meaningful and realistic.

Perhaps it is better not to think in terms of "best," but rather to think in terms of "better." For all of the cases shown herein, a model was obtained that was better than the initial one. An analyst can expect reasonable performance from the results obtained from any of the better models since at least these models matched test data.

What is missing from this analysis, however, is perhaps the most important constraint of all: the overall weight of the enclosure. Obviously, the final model should match the enclosure's weight. Unfortunately, this weight was not measured, and the enclosure was lost upon its return to SMS. (This is typical of actual testing: there is little time in which to test, and by the time the test window has passed--and someone decides he really needed to measure at least one more thing--the test article is unavailable. This is a lesson for good test planning!) The weight of the initial and final models differed by some 30 percent; which one, if either, was close to the correct weight will never be known. At any rate, in an ideal case the weight constraint could have been included in the error equation. Stating this another way: acquire as much data as possible.

In the examples above the parameters spanned the entire model; i.e., other than material properties, which are not considered as design variables in SOL 200, the element properties uniquely defined the model. But in many cases there may be numerous parameters, many of which will not be candidates for updating because they are known "exactly," at least known far better than the others. Therefore, only a subset would be considered for updating. Proper choice of this subset is key to: (1) getting a close match to the test data; and (2) getting a realistic model in the process. One way to decide on this subset is to include: (1) only the parameters that are not certain; and (2) the parameters that contribute the most to the response sensitivity. The latter condition can be met by running SOL 200 to obtain the sensitivities for all parameters first and then basing the choice of parameters on those results. One area with a large degree of uncertainty as well as a sensitive response is that of boundary conditions, especially for civil-type structures. Since there is no truly fixed support, the amount of base flexibility can have a large impact on at least the lowest few modes.

And, finally, what of the model updating process itself? In the process described herein the paramaters were modified. But what if the whole model is really wrong? What if there are nonlinearities or other aspects of the actual structure that are not modeled? Then simply changing parameters--while that might give a reasonable match to the test data--may give an unrealistic model. Also, the test must span the frequency range or load paths that will be simulated via the model. For example, if the enclosure will be analyzed for a 50-g shock pulse, then simply matching the lowest few frequencies may not be enough to give confidence in the drop simulation results.

Summary

This paper has shown application of MSC/NASTRAN's Design Optimization capability to update a model to match test data. The approach was illustrated for matching measured resonant frequencies, though it is equally valid for matching static displacements. The key is the error function, which defines what the model refiner accepts as a "best"--or at least "better"--model.

References

1. Collins, J.D., Hart, G.C., Hasselman, T.K., and Kennedy, B., "Statistical Identification of Structures," *AIAA Journal*, Vol. 12, No. 2, February 1974.

2. Isenberg, J., "Progressing from Least Squares to Bayesian Identification," ASME Winter Annual Meeting, New York City, New York, December 1979.

3. Blakely, K.D., and Dobbs, M.W., "Modification and Refinement of Large Dynamic Structural Models: Efficient Algorithms and Computer Applications," *Proceedings of the First International Modal Analysis Conference*, Orlando, Florida, November 1982.

4. Blakely, K.D., and Walton, W.B., "Selection of Measurement and Parameter Uncertainties for Finite Element Model Revision," *Proceedings of the Second International Modal Analysis Conference*, Orlando, Florida, February 1984.

Appendix
Executive Control, Case Control, and Selected Bulk Data Entries

The following pages show the Executive Control, Case Control, and selected Bulk Data entries for the first application of model updating for the enclosure. The Bulk Data entries show that:

DESOBJ references DRESP2 (700)
DRESP2 references DEQATN (100)
DRESP2 references DESVAR numbers (1, 4, 8, 50 -- the four independent variables)
DRESP2 references DTABLE entries (1-14)
DRESP2 references DRESP1 numbers (101-104)

DESVAR contains design variables (initial value, lower and upper bounds)
DTABLE contains terms (test data, weightings)
DRESP1 contains computed responses

The order of the terms on DRESP2 must match the order of the arguments on DEQATN. MSC/NASTRAN assumes that they are in the same order--you must verify that they are.

	DRESP2			
DEQATN	DESVAR	DTABLE	DRESP1	Description
T1	1			PSHELL 1
T4	4			PSHELL 4
T8	8			PSHELL 8
T50	50			PSHELL 50
E1T		1		Eigenvalue 7 (flex. mode 1)
E2T		2		Eigenvalue 8 (flex. mode 2)
E3T		3		Eigenvalue 9 (flex. mode 3)
E4T		4		Eigenvalue 10 (flex. mode 4)
FACT		5		Weighting for test (wt)
FACP		6		Weighting for params. (wp)
W1		7		Weighting for param. 1 (T1)
W4		8		Weighting for param. 2 (T4)
W8		9		Weighting for param. 3 (T8)
W50		10		Weighting for param. 4 (T50)
X1		11		Weighting for test e-value 7
X2		12		Weighting for test e-value 8
X3		13		Weighting for test e-value 9
X4		14		Weighting for test e-value 10
E1			101	Computed e-value 7
E2			102	Computed e-value 8
E3			103	Computed e-value 9
E4			104	Computed e-value 10

```
ID KB, MSC
$
$ MAKE EQATN GO DOWN
$    AND DIVIDE BY ORIGINAL VALUE FOR THICKNESS
$
TIME 9999
DIAG 8,19,49
SOL 200
CEND
TITLE = DISK DRIVE MODAL ANALYSIS
SUBTITLE =    FACP 0.001, FACT 1
SEALL=ALL
PARAM,APPC,MODES
METHOD=10
SPC=9999
ECHO=UNSORT
BEGIN BULK
$
EIGR,10,SINV,0.1,2000.,,,,,+EIG1
+EIG1,MASS
$
PSHELL,1,1,.08,1
PSHELL,2,1,.08,1
PSHELL,3,1,.08,1
PSHELL,4,1,.12,1
PSHELL,7,1,.08,1
PSHELL,8,1,.10,1
PSHELL,9,1,.12,1
PSHELL,300,1,.08,1
PSHELL,50,1,.20,1
PSHELL,201,1,.20,1
$
$
```

```
$DESIGN OPTIMIZATION ENTRIES
$
DOPTPRM,,,8
$
DESOBJ,700,BAYES,,MIN
$
$  EQUATION:  FACP=PARAMS WEIGHTING,   FACT=TEST WEIGHTING
$             T*=THICKNESSES, E1*T=TEST FREQS,  E1*=COMPUTED FREQS
$
$234567890123456789012345678901234567890123456789012345678901234567890
$
DEQATN  100     F(T1,T4,T8,T50,                                  +DEQ1
+DEQ1   E1T,E2T,E3T,E4T,FACT,FACP,                               +DEQ11
+DEQ11  W1,W4,W8,W50,X1,X2,X3,X4,E1,E2,E3,E4) =                  +DEQ2
+DEQ2   FACP*( W1* ((T1-.08)/.08)**2 +                           +DEQ3
+DEQ3   W4* ((T4-.12)/.12)**2 +                                  +DEQ31
+DEQ31  W8* ((T8-.10)/.10)**2 +                                  +DEQ32
+DEQ32  W50* ((T50-.20)/.20)**2 )  +                             +DEQ33
+DEQ33  FACT*( X1* ((E1T-E1)/E1T)**2 +                           +DEQ6
+DEQ6   X2* ((E2T-E2)/E2T)**2  +                                 +DEQ7
+DEQ7   X3* ((E3T-E3)/E3T)**2  +                                 +DEQ8
+DEQ8   X4* ((E4T-E4)/E4T)**2 )
$
DRESP2,700,FREQ1,100,,,,,,+DRE21
+DRE21,DESVAR,1,4,8,50,,,,+DRE23
+DRE23,DTABLE,1,2,3,4,5,6,7,+DRE24
+DRE24,,8,9,10,11,12,13,14,+DRE25
+DRE25,DRESP1,101,102,103,104
$
DESVAR,1,PS1,.08,.05,.125
DESVAR,2,PS2,.08,.05,.125
DESVAR,3,PS3,.08,.05,.125
DESVAR,7,PS7,.08,.05,.125
DESVAR,300,PS3010,.08,.05,.125
DESVAR,4,PS4,.12,.05,.20
DESVAR,9,PS9,.12,.05,.20
DESVAR,8,PS8,.10,.05,.15
DESVAR,50,PS50,.20,.10,.30
DESVAR,201,PS201,.20,.10,.30
```

```
$
DLINK,2,2,,1.,1,1.
DLINK,3,3,,1.,1,1.
DLINK,7,7,,1.,1,1.
DLINK,300,300,,1.,1,1.
DLINK,9,9,,1.,4,1.
DLINK,201,201,,1.,50,1.
$
DRESP1,101,FREQ1,EIGN,,,7
DRESP1,102,FREQ1,EIGN,,,8
DRESP1,103,FREQ1,EIGN,,,9
DRESP1,104,FREQ1,EIGN,,,10
$
DTABLE,100,E1T,4.718E6,E2T,67.387E6,E3T,96.926E6,,+DT1
+DT1,,E4T,111.12E6,FACT,1.0,FACP,0.001,,+DT2
+DT2,,W1,1.0,W4,1.0,W8,1.0,,+DT3
+DT3,,W50,1.0,X1,1.0,X2,1.0,,+DT4
+DT4,,X3,1.0,X4,1.0
$
DVPREL1,1,PSHELL,1,4,,,,,+DVP1
+DVP1,1,1.
DVPREL1,2,PSHELL,2,4,,,,,+DVP2
+DVP2,2,1.
DVPREL1,3,PSHELL,3,4,,,,,+DVP3
+DVP3,3,1.
DVPREL1,4,PSHELL,4,4,,,,,+DVP4
+DVP4,4,1.
DVPREL1,7,PSHELL,7,4,,,,,+DVP7
+DVP7,7,1.
DVPREL1,8,PSHELL,8,4,,,,,+DVP8
+DVP8,8,1.
DVPREL1,9,PSHELL,9,4,,,,,+DVP9
+DVP9,9,1.
DVPREL1,50,PSHELL,50,4,,,,,+DVP50
+DVP50,50,1.
DVPREL1,201,PSHELL,201,4,,,,,+DVP201
+DVP201,201,1.
DVPREL1,300,PSHELL,300,4,,,,,+DVP300
+DVP300,300,1.
```