FE MODEL REFINEMENT WITH ACTUAL FORCED RESPONSES OF AEROSPACE STRUCTURES

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Abstract

Formulation and computer algorithm for FE model refinement based on correlation with frequency response test results have been developed. The proposed approach derives from the direct frequency response formulation for FE analysis of dynamic systems. The computer algorithm has been effectively implemented in MSC/NASTRAN's DMAP language.

Introduction

FE models of structures have been widely used in aerospace industry for predicting dynamic responses under various 'flight' conditions. Instantaneous prediction of dynamic responses is an important input for automated flight control systems. It is clear that an effective control system requires precise prediction of dynamic responses of the structure under dynamic loads. Therefore, there is a need to develop an analytic capability for refining and validating FE models in order to achieve acceptable correlation between the predicted and the actual forced responses of the structures.

Conventionally, this process has been mostly tackled by an indirect approach [1,2] which involves the refinement of stiffness and mass matrices (instead the 'FE model') based on correlation of modal data. However, this kind of approach may achieve a limited success with incomplete modal data, as is always the case, and result in an uncertain answer to the problem with regard to the physical changes of the structure that an active control system is interested. In addition, the use of modal data for correlation involves two fundamental drawbacks. One as mentioned a priori is that complete modal data of the actual system is not available with today's dynamic testing technology which is likely to invalidate the method. Another drawback is due to the fact that actual modal properties of the structure are usually identified by a curve fitting process around peaks of frequency response curves at all available measurement points and dof [3]. This leads to an uncertainty in the quality of the modal data resulted as it is affected by many factors such as the method of curve fitting, structural damping, mode identification technique, and so on.

The paper presents an approach that is designed to circumvent the problems discussed. First, the correlation data for the structures are the frequency responses under known dynamic excitations. In doing so, the measured response data used could be more reliable than curve-fitted natural modes and, on the other hand, the correlation results should also satisfy the objectives of the conventional modal correlation. Second, correlations to the physical parameters of the FE models are directly calculated and can be a set of useful and meaningful data for control systems. The FE model refinement method is an iteration process that minimizes the difference between the predicted and the measured response of the structure. Nonlinear least squares method [4] is employed where the measured response is expanded as a Taylor series at each iteration on the assumption of local linearity. That is, at each iteration, FE analysis result and the associated design sensitivities of the response with respect to design variables are required for the updated FE model of the structure. Each design variable can be designated to link with one or a group of property value entries in the FE model.

However, there is a severe challenge in numerical difficulty when frequency response data.are used directly for correlation. This is due to the fact that the order of magnitude in difference between FE and real response can be very large and may cause the process to diverge or to result in unreasonable values. Therefore, this paper provides some suggestions to overcome the difficulty and presents the experience gained in implementing the approach into MSC/NASTRAN's DMAP language [5].

Direct Frequency Response Formulation

In frequency response analysis, the applied forces are harmonic and periodic with constant frequency, $f = \omega/2\pi$, at different points. Assuming the forced response of the structure is also harmonic with the same frequency, we can write down the dynamic response equation "directly" as

$$\left(-\omega^2[M] + i\omega[B] + [K]\right)\{u\} = \{P\} \tag{1}$$

where [M], [B], and [K] are the structural mass, damping, and stiffness matrices, respectively. The displacement vector {u} is the response corresponds to the constant force vector with the frequency ω. Both {u} and {P} may be complex, e.g.,

$$\{u\} = \{u\}_R + i\{u\}_I \tag{2}$$

Design Sensitivity Analysis

The dynamic equation for a system under constant frequency dynamic forces can be expressed as follows:

$$[D]\{u\} = \{f\} \tag{3}$$

where, for example,

$$[D] = (-\omega^2[M] + i\omega[C] + (1+ig)[K])$$

and g is the global structural damping coefficient.

For the case of constant damping and constant {f}, the first derivative of the dynamic equation with respect to design variable x yields:

$$\left[\frac{\partial D}{\partial x}\right]\{u\} + [D]\left\{\frac{\partial u}{\partial x}\right\} = 0 \tag{4}$$

Thus,

$$\left\{\frac{\partial u}{\partial x}\right\} = -[D]^{-1}\left[\frac{\partial D}{\partial x}\right]\{u\} \tag{5}$$

where

$$\left[\frac{\partial D}{\partial x}\right] = -\omega^2\left[\frac{\partial M}{\partial x}\right] + (1+ig)\left[\frac{\partial K}{\partial x}\right] \tag{6}$$

Correlation Methodology

For a constant force vector, the experimental results can be expressed using the first order Taylor's series:

$$\{u_e\} = \{u_a\} + \left\{\frac{\partial u_a}{\partial x}\right\}\Delta x + \{\varepsilon\} \tag{7}$$

where $\{u_e\}$ and $\{u_a\}$ represent the experimental and analysis displacement vectors, respectively. $\{\varepsilon\}$ accounts for the errors involved. For clarity, only a single design variable, x, is used here and in the following derivation of equations.

From above, we obtain

$$\{\varepsilon\} = \{u_e\} - \{u_a\} - \left\{\frac{\partial u_a}{\partial x}\right\}\Delta x \tag{8}$$

Then, by applying the least squares method, we want to

$$\textit{minimize}\ \Phi = \{\varepsilon\}^T\{\bar{\varepsilon}\} \tag{9}$$

where $\{\bar{\varepsilon}\}$ is the conjugate of $\{\varepsilon\}$

The necessary condition for the minimum of Φ is:

$$\left(\left\{\frac{\partial u_a}{\partial x}\right\}^T\left\{\frac{\partial \bar{u}_a}{\partial x}\right\} + \left\{\frac{\partial \bar{u}_a}{\partial x}\right\}^T\left\{\frac{\partial u_a}{\partial x}\right\}\right)\Delta x - \left(\left\{\frac{\partial u_a}{\partial x}\right\}^T\left\{\Delta \bar{u}\right\} + \left\{\frac{\partial \bar{u}_a}{\partial x}\right\}^T\left\{\Delta u\right\}\right) = 0 \qquad (10)$$

where

$$\left\{\Delta u\right\} = \left\{u_e\right\} - \left\{u_a\right\} \qquad (11)$$

Removing the imaginary terms, Eq.(10) can be rewritten as:

$$2\left(\left\{\frac{\partial u_a}{\partial x}\right\}^T\left\{\frac{\partial \bar{u}_a}{\partial x}\right\}\right)_R \Delta x - 2\left(\left\{\frac{\partial u_a}{\partial x}\right\}^T\left\{\Delta \bar{u}\right\}\right)_R = 0$$

Thus, we obtain

$$\Delta x = \frac{\left(\left\{\frac{\partial u_a}{\partial x}\right\}^T\left\{\Delta \bar{u}\right\}\right)_R}{\left(\left\{\frac{\partial u_a}{\partial x}\right\}^T\left\{\frac{\partial \bar{u}_a}{\partial x}\right\}\right)_R} \qquad (12)$$

This solution can be easily expanded for the multiple design variables, $\{x\}$, cases. Let

$$[T] = \left[\frac{\partial\{u_a\}}{\partial\{x\}}\right] = [T]_R + i[T]_I \qquad (13)$$

then

$$\left\{\Delta x\right\} = \left([T]^T[T]\right)_R^{-1}\left([T]\{\Delta \bar{u}\}\right)_R \qquad (14)$$

Computational Algorithm

The iterating procedure of the method can be stated as follows:

(1) Start with an initial FE model. Set iteration counter k=0

(2) Perform frequency response analysis to determine $\{u_a\}_k$

(3) Compute design sensitivity matrix $[T]_k$ and $\{\Delta u\}_k$

(4) Set

$$\left\{\Delta x\right\}_{k+1} = \left(\left([T]^T[T]\right)_R^{-1}\left([T]\{\Delta \bar{u}\}\right)_R\right)_k \qquad (15)$$

(5) Update FE model.

(6) Check for the convergence criterion.
   a. stop the procedure if it is met.
   b. continue the procedure if it is not met.

(7) Set k=k+1 and go to step (2).

Numerical Scaling

For the practical correlation with forced response data, it frequently involves dealing with a large number of response vectors corresponding to various excitation frequencies. It is the nature of the vibration in a structure that the response level varies with excitation frequencies. It is possible that the magnitudes of the responses, e.g., $|\{u(\omega)\}|$, for different frequencies may differ by a large extent. This, sometimes, causes a severe numerical instability problem in the correlation process.

In order to overcome the numerical difficulties involved, it is necessary to introduce numerical scaling for all the response vectors used in the correlation process. Let [U] be matrix with each column being a response vector of frequency ω, i.e., $\{u(\omega)\}$ and, likewise, [Ue] and [Ua] are matrices associated with test and analysis vectors, respectively. Since [Ue] stays constant throughout the entire process it is reasonable to first normalize [Ue] such that each column of [Ue] is unitary. This means that the normalized [Ue] is

$$[\tilde{U}_e] = [U_e][\Gamma] \tag{16}$$

and

$$[\Gamma] = \text{Diag}([U_e]^T[\bar{U}_e])^{-1/2} \tag{17}$$

is a diagonal matrix with each diagonal term associates with the magnitude of the corresponding column in [Ue]. Similarly, [Ua] should be normalized as

$$[\tilde{U}_a] = [U_a][\Gamma] \tag{18}$$

with a constant scaling matrix $[\Gamma]$ for the entire process.

Programming Aspects

The computer program for calculating design sensitivities and the design changes for the forced response test-analysis correlation is written in MSC/NASTRAN's DMAP language. The program has been developed based on the available modules existing in the version 65 of MSC/NASTRAN.

<u>Design Sensitivities</u>

The determination of design sensitivities for forced response follows the same approach as those design sensitivity analysis solution sequences provided by MSC. This means that the design sensitivity information is determined partially in an approximated sense. For instance, Eq.(5) is approximated by

$$\left\{\frac{\delta u}{\delta x}\right\} = -[D]^{-1}\left[\frac{\delta D}{\delta x}\right]\{u\} \tag{19}$$

where $\{\delta u/\delta x\}$ approximates $\{\partial u/\partial x\}$ with δx being a small perturbation of x. It should be noted that the only approximated item on the right hand side of Eq.(19) is

$$\left[\frac{\delta D}{\delta x}\right] = \frac{1}{\delta x}\left(-\omega^2[\delta M] + (1+ig)[\delta K]\right) \tag{20}$$

where $[\delta M]$ and $[\delta K]$ are perturbation of matrices [M] and [K], respectively, due to δx.

To directly compute and store $[\delta M]$ and $[\delta K]$ can be an inefficient (or even be restricted by the available memory space limitation) way to begin in calculating the sensitivities with respect to multi-variables, i.e., $\{x\}$. The approach adopted in MSC/NASTRAN's DSVG1 module is effective for most applications. This approach computes directly the resultant changes in the grid point equilibrium as $[\delta M]\{u\}$ and $[\delta K]\{u\}$ at element level and then assembles to g-set vectors.

At the first glance of Eq.(19), it seems to involve merely the solution of linear equations. However, the fact is far more complicated than the first impression. Firstly, the matrix [D] is a function of the excitation frequency w; it needs to be decomposed as many time as there are ω's. Secondly, since all the terms in Eq.(19) are, in general, complex and the equation may not be solved efficiently by conventional methods used in handling real value problems. Fortunately, since Eq.(19) shares almost the same format as that of the original equation for solving forced response, i. e., Eq.(3), it can be solved in a similar manner as the solution for forced response analysis. The functional module FRRD1 used in solution sequence 68 of MSC/NASTRAN is the key module in solving frequency response problem of Eq.(3). With a careful arrangement of all the 'force vectors', i.e., $[\delta D]\{u\}$, with respect to excitation frequencies, FRRD1 module can be used to solve for $\{\delta u\}$ in Eq.(19) without any modification. This approach has shown to achieve a much better efficiency than the conventional methods do.

The Least Squares Solution

After all the sensitivity information has been determined, we rearrange it into the appropriate format to fit the least squares equation (Eq.(14)) for solving the design changes. The sensitivity matrix [T] in Eq.(14) is so arranged that its columns correspond to design variables and its rows correspond to response dof's with various excitation frequencies.

Since the solution of Eq.(14), $\{\Delta x\}$, is real, it is logical to alter the originally complex equation into a real value equation. There are many reasonable ways to achieve this. However, we found that Eq.(14) to be the best among all the possible approaches. Having turned the least squares equation into real values, the remaining tasks in the correlation process can be handled in exactly the same way as we do for real valued data, e.g., natural frequencies and mode shapes.

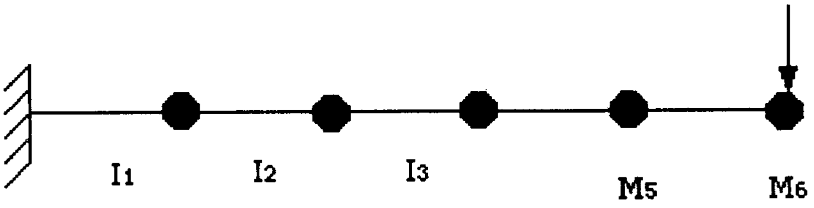
Numerical Example

A simple cantilever beam was used for demonstration purpose. The finite element model consists of 5 beam elements and lumped mass elements at nodes. Principal area moments of inertia in the vertical bending plane of the three close-to-wall elements, i.e., I_1, I_2, I_3, and two close-to-tip masses, i.e, M_5 and M_6, were designated as design variables. There were a total of five design variables and their baseline values as well as the perturbed values for generating mock test data are shown in Fig.1.

Assuming a frequency-dependent vertical force was applied at the tip of the beam. The magnitudes of vertical displacements at all grid points corresponding to ten selected excitation frequencies were used as correlation data. These ten excitation frequencies are: 14., 17., 80., 88., 189., 197., 296., 304., 377., and 385. Hz.

For the large perturbation in design variables (-40% from their baseline values), the correlation algorithm performed well and converged to the target result for most cases. Fig.2 shows an overlaid frequency response plot for curves corresponding to beam-tip responses generated by FE models of the baseline, after 2nd iteration, after 4th iteration, and the target. By a visual inspection, a typical frequency response curve converged to the corresponding target curve at or around the 8th iteration. Table 1 shows the natural frequencies of the baseline and target models.

Since, numerically, the discrepancies between the responses of the baseline and target models are very large, it becomes necessary to impose step size constraints for every design change at the end of every iteration. A test was performed with various constraint values on step sizes to investigate their effect on convergence rate. Table 2 lists some of the iteration results with the application of three different step size limits in every iteration. It should be observed that all cases converged to the target values within 15 iterations and the 15% bound on the fractional change of every design variable achieved a better rate of convergence than the others do. Table 3 shows the results for another test case where every condition stayed the same as before except that all target response values were rounded off to two significant digits as input to the program through DMI cards. It is interesting to note that for at least one step size bound (20%) the result did not converge.



BASELINE

$I_1 = I_2 = I_3 = 3.0$
$M_5 = M_6 = 5.0$
$(E = 10.x10^6)$*
$(I_4 = I_5 = 3.0)$*
$(M_2 = M_3 = M_4 = 5.0)$*
* values unchanged

PERTURBED

$I_1 = I_2 = I_3 = 1.8$
$M_5 = M_6 = 3.0$

Fig.1. Baseline and Perturbed Design Variables

Table 1.
Natural Frequencies (Hz) of Baseline and Target Models

MODE NO.	1	2	3	4	5
BASELINE	15.6	83.7	193.1	300.2	380.6
TARGET	14.7	73.1	177.2	257.9	354.3

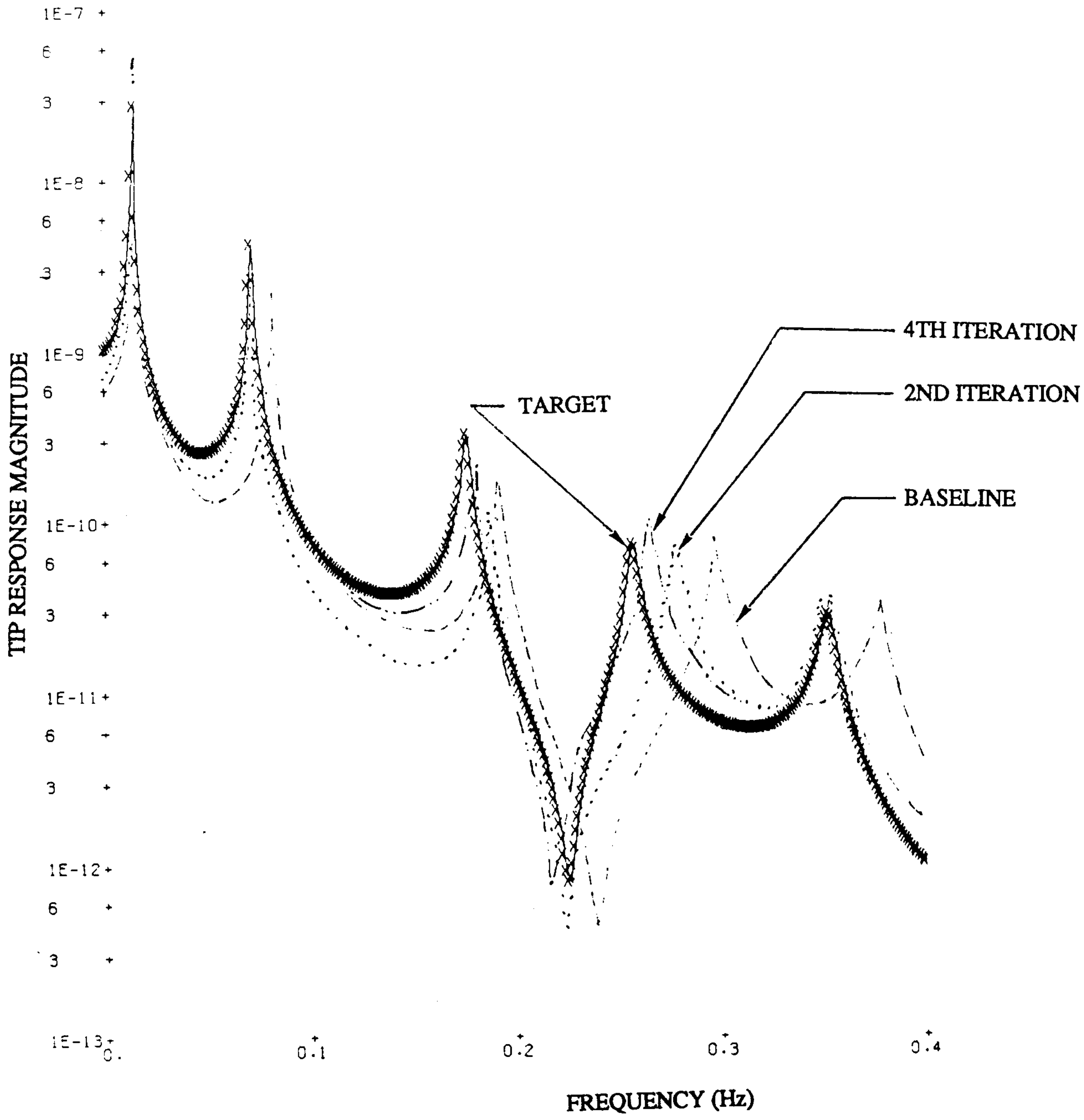


Fig.2. Comparative Iteration FRF Curves

Table 2.
Fractional Design Changes with Accurate Target Response Data

ITERATION NO.	DV NO.	FRACTIONAL CHANGE		
		20% BOUND	15% BOUND	10% BOUND
10	1	-0.4623	-0.4122	-0.4094
	2	-0.3047	-0.3946	-0.2580
	3	-0.3732	-0.3943	-0.3720
	4	-0.5044	-0.4314	-0.5312
	5	-0.3875	-0.4017	-0.3165
15	1	-0.4008	-0.4002	-0.3995
	2	-0.3998	-0.3999	-0.4002
	3	-0.3997	-0.4002	-0.3999
	4	-0.3999	-0.4000	-0.4002
	5	-0.4001	-0.3998	-0.3999

Table 3.
Fractional Design Changes with Rounding Off Target Response Data

ITERATION NO.	DV NO.	FRACTIONAL CHANGE		
		20% BOUND	15% BOUND	10% BOUND
10	1	-0.7492	-0.4111	-0.4113
	2	+1.2015	-0.3955	-0.2438
	3	-0.2969	-0.3943	-0.3750
	4	-0.7752	-0.4298	-0.5312
	5	-0.5347	-0.4016	-0.3239
15	1	-0.7511	-0.3997	-0.4005
	2	+1.4347	-0.4001	-0.4002
	3	-0.1928	-0.4003	-0.4002
	4	-0.4406	-0.3997	-0.4002
	5	-0.0131	-0.3994	-0.3999

Concluding Remarks

A procedure to refine FE model based on test/analysis correlation with forced response data has been developed and implemented in MSC/NASTRAN's DMAP language. The effectiveness of the procedure has been demonstrated with a test problem where the procedure correctly identified all the predefined target values for all design parameters. The convergence rate was found to be reasonable. It can be observed from the results presented that the response curve converges to the target in less than eight iterations and a comparable rate for that of most design variables.

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