Application of Approximate Techniques in the Estimation of Eigenvalue Quality

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ABSTRACT

An eigenvalue quality estimate has been implemented in MSC/NASTRAN. The quality estimate is based on the eigenvalue difference from a lumped and consistent mass matrix formulation. This difference represents the error associated with the discretization of the finite element model. Normally two eigensolutions are required to compute the error estimate. However, several approximate solution techniques have been provided to efficiently compute the consistent mass matrix eigenvalues. The eigenvalue quality estimator has been implemented as a set of Direct Matrix Abstraction Programming (DMAP) alters to SOL 103 (SEMODES) of MSC/NASTRAN Version 67. Several numerical examples are provided to demonstrate the method.

INTRODUCTION

Finite element analysis of structures requires a discretization by finite elements. The discretization is accompanied by modeling error which is called the error in discretization. This error may be reduced by mesh refinement. Here the analyst seeks an adequate, but not excessive mesh in order to create a manageable problem size. An estimate of eigenvalue quality has been implemented in MSC/NASTRAN based on a difference in eigenvalues computed from a lumped and consistent mass matrix [1]. This difference represents the error in discretization and varies with each eigenvalue. The eigenvalue quality generally decreases with increasing eigenvalue.

Two eigensolutions are required to obtain the quality estimate. The initial eigensolution is obtained using a lumped mass matrix and the latter by a consistent mass matrix formulation. Since two eigensolutions are relatively expensive, various approximate solution techniques are provided to efficiently compute the consistent mass matrix eigenvalues. These approximate solution techniques are as follows: static mode reanalysis [2], Rayleigh's quotient [3], Timoshenko's quotient [3], and the inverse iteration quotient [3]. The capability for an exact consistent mass eigensolution is also provided for small problems. Numerical examples are given to demonstrate and validate the method.

PROBLEM DEFINITION

The eigenvalue percent difference is given by

$$e_i = \left(\frac{\lambda_L - \lambda_c}{\lambda_L + \lambda_c} \right)_i x\ 100.0\% \qquad \textbf{(1)}$$

where

λ_L represent the lumped mass matrix eigenvalues
λ_c represent the consistent mass matrix eigenvalues

Equation (1) gives a percent difference in lumped and consistent mass matrix eigenvalues for each eigenvalue extracted from the finite element model. The lumped mass matrix contains only diagonal terms, whereas the consistent mass matrix contains off-diagonal terms consistent with the element shape functions which define the stiffness matrix. It is well known that frequencies usually converge from below when the mass matrix in lumped and converge from above when the mass matrix is consistent. The lumped mass and consistent mass eigenvalues bound the exact eigenvalue from below and above for each mode. Equation (1) provides the analyst with the percent eigenvalue difference for each mode extracted. A significant difference between λ_L and λ_c suggests that the mesh is not adequate for that particular mode and should be refined. In order to utilize equation (1), the mass matrices must be derived from element densities and not concentrated masses.

IMPLEMENTATION IN MSC/NASTRAN

MSC/NASTRAN SOL 103 is used to obtain the lumped mass matrix eigenvalues. The consistent mass matrix is assembled in MSC/NASTRAN and various approximate eigensolution techniques may be employed at the user's discretion to obtain the consistent mass matrix eigenvalues. The following solution techniques have been programmed in DMAP:

(1) Static mode reanalysis method
(2) Rayleigh's quotient
(3) Timoshenko's quotient
(4) Inverse iteration quotient

The user may also chose the exact solution for small problems. The static mode reanalysis, Timoshenko's quotient, and inverse iteration quotient methods are set up for models without rigid body degrees of freedom only since they require a decomposition of the stiffness matrix. However, if rigid body modes are present the eigenvalues are identically zero for both mass formulations and the percent difference will not be defined since the numerator and denominator of equation (1) are zero.

STATIC MODE REANALYSIS

To compute modes using static mode reanalysis, a static shape must be computed from the following equation:

$$\psi_i = [K]^{-1}(\lambda_i [\Delta M] [\phi_i]) \tag{2}$$

where

λ_i is the i^{th} eigenvalue
$[K]$ is the stiffness matrix
ϕ_i is the i^{th} eigenvector
ψ_i is the i^{th} static shape
$[\Delta M]$ is the mass difference matrix between the lumped mass and consistent mass formulations

The static shape is the response due to a loading which is derived from the difference in mass matrix formulations. Inherently, the static shape contains lower order as well as higher order modal information. Therefore it must be filtered of the lower order or retained modes. This is accomplished by the following:

$$\bar{\Psi} = \psi - \phi\,\phi^T[M]\,\psi \tag{3}$$

The filtered static shapes or residual static modes are now appended to the retained modes forming a new global approximation function. The consistent mass and stiffness matrices are pre and post multiplied by the global approximation function creating a reduced order eigenproblem which may be solved efficiently in MSC/NASTRAN. Reference [2] provides a complete derivation of the method.

RAYLEIGH'S QUOTIENT

Rayleigh's method for computing conservative system eigenvalues utilizes an assumed mode for harmonic motion and then equates the maximum kinetic energy to the maximum potential (strain) energy. For a discrete system, Rayleigh's quotient is defined by

$$RQ = \frac{\phi^T[K]\,\phi}{\phi^T[M_c]\,\phi} \tag{4}$$

where $[M_c]$ is the consistent mass matrix

TIMOSHENKO'S QUOTIENT

Timoshenko's quotient is given in reference [4] as

$$TQ = \frac{V^TBV}{V^TCV} \quad , \quad C=BA^{-1}B \tag{5}$$

In terms of present nomenclature,

$V = [\phi]$, $B = [M]$, $A = [K]$, and $C = [M][K]^{-1}[M]$

Timoshenko's quotient may now be expressed as a generalized Rayleigh quotient obtaining

$$TQ = \frac{\phi^T [M_c] \phi}{\chi^T [K] \chi} \tag{6}$$

where

$$\chi = [K]^{-1} [M_c] \phi \tag{7}$$

INVERSE ITERATION QUOTIENT

In inverse iteration, the eigenvalue at the r^{th} iteration is computed from the Rayleigh quotient

$$\lambda^{(r)} = \frac{(\phi^{(r)})^T [K] (\phi^{(r)})}{(\phi^{(r)})^T [M] (\phi^{(r)})} \tag{8}$$

with

$$[K] (\phi^{(r)}) = [M] (\phi^{(r-1)}) \tag{9}$$

specifically, for r=1,

$$\lambda^{(1)} = \frac{(\phi^{(1)})^T [K] (\phi^{(1)})}{(\phi^{(1)T} [M] (\phi^{(1)})} \tag{10}$$

and

$$(\phi^{(1)}) = [K]^{-1} [M] (\phi^{(0)}) \tag{11}$$

Now substitute equation (11) into equation (10) obtaining

$$\lambda^{(1)} = \frac{(\phi^{(0)})^T [M] [K]^{-1} [M] (\phi^{(0)})}{(\phi^{(0)})^T [M] [K]^{-1} [M] [K]^{-1} [M] (\phi^{(0)})} \tag{12}$$

or

$$IQ = \frac{\chi^T [K] \chi}{\chi^T [M_c] \chi} \tag{13}$$

with $(\phi) = (\phi^{(0)})$. Note that the eigenvalues obtained from the above methods are as follows

$$\lambda_{ic} \le IQ \le TQ \le RQ \tag{14}$$

with the static mode method eigenvalue closest to λ_{ic} (the exact consistent mass eigenvalue).

The static mode reanalysis method approximates the eigenvalues more accurately than the other methods since the modification is incorporated into the eigenvectors. Timoshenko's and the inverse iteration quotient also recompute new eigenvectors. However, Rayleigh's quotient assumes the modified eigenvectors to be the same as the original system or base eigenvectors. All of the above approximate eigensolution techniques have been implemented as a set of DMAP alters to SOL 103 of MSC/NASTRAN Version 67. Reference [5] provides a description of the DMAP language. The user may choose any of the fore mentioned methods by inserting a PARAM,METHOD,XXXXXXX in the BULK DATA section of the input deck. Here XXXXXXX is defined to be:

```
XXXXXXX = STATICR   Static mode reanalysis
          ITERQ     Inverse iteration quotient
          TIMQ      Timoshenko's quotient
          RAYLYQ    Rayleigh's quotient
          EXACT     Exact solution
```

Note that the exact solution is the default method programmed in DMAP. Appendix 1 provides a complete listing of the DMAP alters required to compute the eigenvalue quality estimate. To insure the total weight of the consistent mass matrix, the grid point weight generator was turned on and printed in the .F06 file.

ANALYSIS

The eigenvalue quality estimator has been applied to the following MSC/NASTRAN finite element models

(1) cantilever beam (simple beam elements)
(2) cantilever frame (axial rod elements)
(3) cantilever plate (plate elements)
(4) large scale aircraft component (mixture of elements)

Each finite element model was run using all five methods for the modal quality estimate. The following is an example of the error output in the .F06 file for the cantilever beam model, Rayleigh's quotient, ten modes:

```
^^^MODAL SOLUTION QUALITY ESTIMATE
^^^RAYLEIGH QUOTIENT CONSISTENT MASS EIGENVALUES
^^^MODE NUMBER           1 HAS -2.396108E+00 PERCENT ERROR
^^^MODE NUMBER           2 HAS -3.990014E+00 PERCENT ERROR
^^^MODE NUMBER           3 HAS -5.387323E+00 PERCENT ERROR
^^^MODE NUMBER           4 HAS -6.725320E+00 PERCENT ERROR
^^^MODE NUMBER           5 HAS -8.022677E+00 PERCENT ERROR
^^^MODE NUMBER           6 HAS -1.026989E-01 PERCENT ERROR
^^^MODE NUMBER           7 HAS -9.408460E+00 PERCENT ERROR
^^^MODE NUMBER           8 HAS -1.176371E+01 PERCENT ERROR
^^^MODE NUMBER           9 HAS -1.602735E+01 PERCENT ERROR
^^^MODE NUMBER          10 HAS -2.331698E+01 PERCENT ERROR
```

Figure 1.0, 2.0, and 3.0 show the cantilever beam, frame, and plate models. The large scale aircraft component model is approximately 11,000 degrees of freedom. This model is used to demonstrate the efficiency of static mode reanalysis in determining eigenvalues.

Tables 1.0, 2.0, 3.0, and 4.0 present the eigenvalue percent differences computed by the four approximate methods compared to the percent differences computed with the exact consistent mass matrix eigenvalues for each of the models. Note that ten modes were extracted for each model.

DISCUSSION

In order to evaluate eigenvalue quality, a criteria must be defined to interpret the computed results. In general, an eigenvalue percent difference of less than 5% should be considered acceptable. However, the mesh should be refined for greater percent differences especially if those modes are to be utilized in other dynamic analyses. Table 1.0 presents the eigenvalue quality estimates for the cantilever beam of Figure 1.0. The cantilever beam model gives acceptable eigenvalues up to the fifth mode. Static mode reanalysis estimates the beam consistent mass eigenvalues very accurately. However, this problem becomes ill-conditioned at the addition of a tenth mode. Therefore only nine modes are extracted. The other approximate techniques show the correct trend but offer appreciable errors when compared to the exact percent differences. Table 2.0 shows the results of the cantilever frame frequency quality estimates. All ten modes of the frame model are within the criteria for acceptable eigenvalues. The approximate methods also perform well for the CROD element frame model. Table 3.0 presents the eigenvalue quality estimates for the cantilever plate model. All modes with the exception of five and ten are deemed "good" by the quality estimate. Again all of the approximate methods perform well in obtaining the consistent mass eigenvalues. Table 4.0 shows the frequency quality estimates of the large scale aircraft component model. The first six modes have less than a five percent difference. All of the approximate methods perform reasonably well with the exception of the last mode. The approximate eigensolution techniques efficiently extract eigenvalues from the finite element models. Significant CPU time savings can be realized on large scale models. Table 5.0 demonstrates the efficiency of the approximate methods.

CONCLUSIONS

Currently many industries require the use of finite element models to predict the behavior of actual structures. It is of tantamount importance that these models be constructed with high quality. The error of discretization must be kept to a minimum. This paper presents a simple eigenvalue quality estimator that provides MSC/NASTRAN user's with a gauge to measure the quality of their finite element model's modal characteristics with respect to discretization error. Future work should also include eigenvector error calculations, since mode shapes are also important. Both errors may also be used to adaptively refine the model mesh to define higher quality dynamic finite element models.

REFERENCES

[1] Cook, R. D., "Error Estimators for Eigenvalues Computed from Discretized Models of Vibrating Structures," AIAA Journal, Vol. 29, No. 9, September 1991, pp. 1527-1529.

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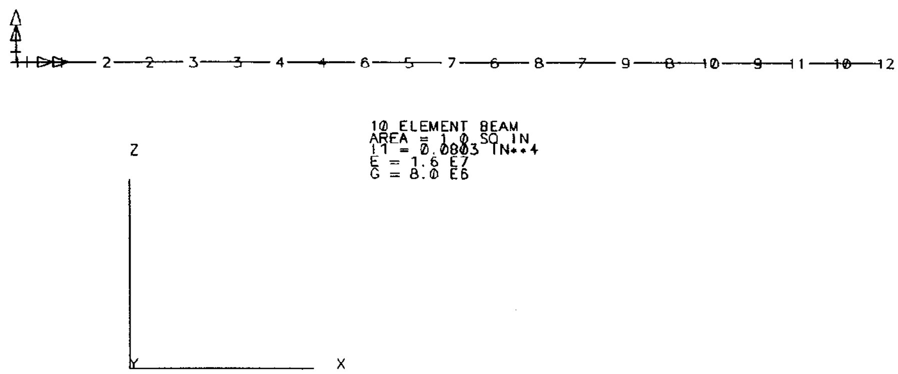
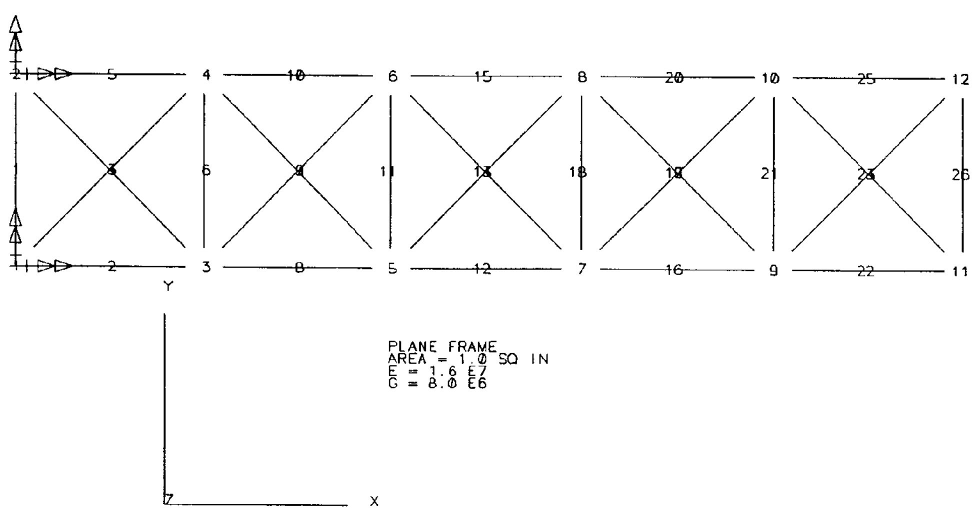
[3] Wang, B. P., Pilkey, W. D., "Eigenvalue Reanalysis of Locally Modified Structures Using a Generalized Rayleigh's Method," AIAA Journal, Vol. 24, No. 6, June 1986, pp. 983-990.

[4] Ku, A. B., "Upper and Lower Bound Eigenvalues of Conservative Discrete Systems," Journal of Sound and Vibration, Vol 53, No. 2, 1977, pp. 183-187.

[5] MSC/NASTRAN User's Manual, Version 67, Vol. II, The MacNeal Schwendler Corp., Los Angeles, California, August 1991.



<u>Figure 1.0</u>   <u>A Cantilever Beam Model</u>



<u>Figure 2.0</u>   <u>A Cantilever Frame Model</u>

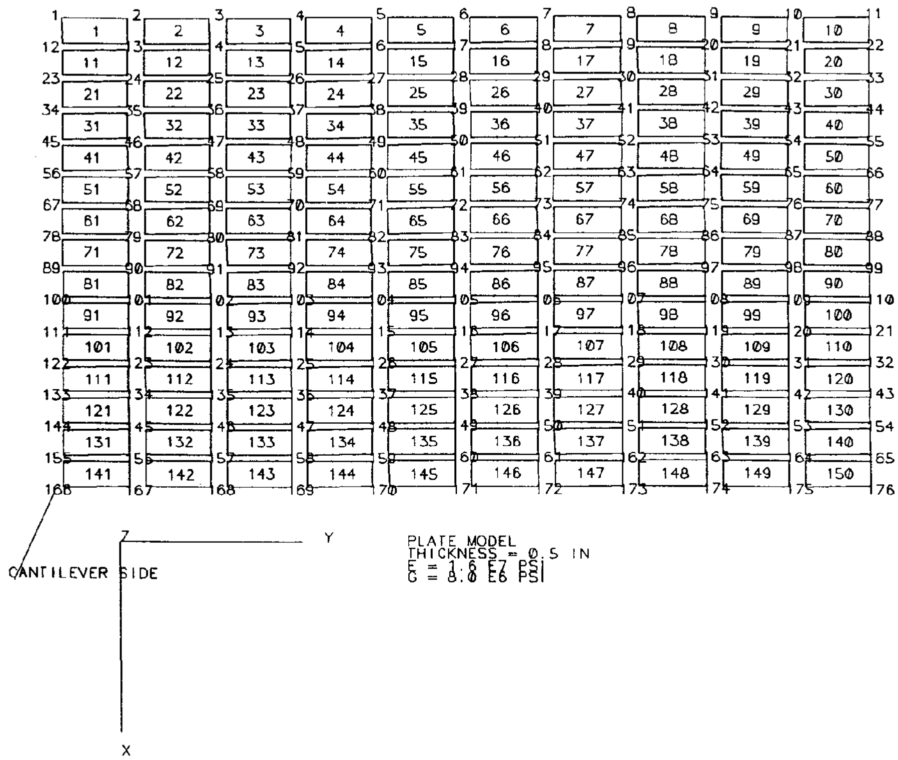


Figure 3.0 A Cantilever Plate Model

Table 1.0: Beam Eigenvalue Percent Quality Estimates

Mode No.	Exact	Static	Iteration	Timoshenko	Rayleigh
1	-.46	-.46	-2.25	-2.28	-2.40
2	-1.58	-1.58	-3.82	-3.86	-3.99
3	-2.61	-2.61	-5.21	-5.25	-5.39
4	-3.72	-3.72	-6.47	-6.51	-6.73
5	-5.00	-5.00	-7.65	-7.70	-8.02
6	-0.10	-0.10	-0.10	-0.10	-0.10
7	-6.64	-6.64	-8.62	-8.70	-9.41
8	-9.10	-9.10	-10.35	-10.46	-11.76
9	-13.20	-13.28	-13.07	-13.49	-16.03
10	-20.08	--.--	-19.65	-20.45	-23.32

Table 2.0: Frame Eigenvalue Percent Quality Estimates

Mode No.	Exact	Static	Iteration	Timoshenko	Rayleigh
1	-0.01	-0.01	-0.01	-0.01	-0.01
2	-0.16	-0.16	-0.16	-0.16	-0.16
3	-0.19	-0.19	-0.19	-0.19	-0.19
4	-0.84	-0.84	-0.84	-0.84	-0.84
5	-2.05	-2.05	-2.05	-2.05	-2.05
6	-1.50	-1.50	-1.50	-1.50	-1.50
7	-3.48	-3.48	-3.48	-3.48	-3.48
8	-4.76	-4.76	-4.77	-4.77	-4.77
9	-2.92	-2.92	-2.92	-2.92	-2.92
10	-3.10	-3.10	-3.10	-3.10	-3.10

Table 3.0: Plate Eigenvalue Percent Quality Estimates

Mode No.	Exact	Static	Iteration	Timoshenko	Rayleigh
1	-0.38	-0.38	-0.40	-0.40	-0.40
2	-0.67	-0.67	-0.67	-0.67	-0.67
3	-2.65	-2.65	-2.76	-2.76	-2.77
4	-2.55	-2.55	-2.57	-2.57	-2.57
5	-6.24	-6.24	-6.47	-6.47	-6.49
6	-2.02	-2.02	-2.02	-2.02	-2.02
7	-4.57	-4.57	-6.41	-6.42	-6.47
8	-2.20	-2.20	-3.51	-3.51	-3.52
9	-3.69	-3.69	-3.69	-3.69	-3.70
10	-11.30	-11.30	-11.26	-11.30	-11.44

Table 4.0: Large Scale Model Eigenvalue Percent Quality Estimates

Mode No.	Exact	Static	Iteration	Timoshenko	Rayleigh
1	-0.13	-0.13	-0.13	-0.13	-0.13
2	-0.19	-0.19	-0.19	-0.19	-0.19
3	-0.66	-0.66	-0.67	-0.67	-0.67
4	-0.35	-0.35	-0.38	-0.38	-0.38
5	-2.13	-2.13	-2.53	-2.54	-2.65
6	-2.35	-2.35	-2.50	-2.53	-2.66
7	-5.27	-5.27	-6.43	-6.52	-7.01
8	-6.28	-6.30	-5.65	-5.82	-6.45
9	-9.66	-10.23	-10.42	-11.01	-12.47
10	-15.07	-21.60	-21.65	-21.72	-22.03

Table 5.0: Finite Element Model CPU Comparisons (sec.)

Model	Exact	Static	Iteration	Timoshenko	Rayleigh
Beam	43.06	18.19*	41.73	41.48	41.20
Frame	18.33	17.97	14.89	15.27	14.68
Plate	32.39	28.14	24.06	24.01	23.81
Large	576.45	359.59	364.31	370.11	325.71

* Only nine modes extracted using static mode reanalysis.

APPENDIX 1 - EIGENVALUE QUALITY ESTIMATE DMAP

```
NASTRAN SPARSE=25
ID SPC,QUALITY ESTIMATE
TIME 50000
DIAG 8
SOL 103
COMPILE SEMODES SOUIN=MSCSOU NOLIST NOREF
ALTER 36 $
TYPE      PARM,,CHAR7,Y,METHOD='EXACT  ' $
TYPE      PARM,,I,Y,COUPMASS,GRDPNT $
TYPE      PARM,,I,N,NMODES $
FILE      LAMACM=OVRWRT/LAMAC=OVRWRT $
EMG       EST,CSTMS,MPTS,DIT,GEOM2S,,,/
          KELM1,KDICT1,MELM1,MDICT1,BELM1,BDICT1/
          S,N,NOKGGX/S,N,NOMGG/S,N,NOBGG/S,N,NOK4GG/S,N,HNNLK
          /1///////////K6ROT $
EMA       GPECT,MDICT1,MELM1,BGPDTS,SILS,CSTMS/
          MJJC,/-1/V,Y,WTMASS=1.0 $
IF (GRDPNT >=0) THEN $
   GPWG  BGPDTS,CSTMS,EQEXINS,MJJC/OGPWG1/GRDPNT/WTMASS $
   OFP   OGPWG1// $
ENDIF $
EQUIVX MJJC/MNNC/NOMSET $
IF (NOMSET >=0) THEN $
  MCE2  USET,GM,MJJC,,,/MNNC,,, $
ENDIF $
EQUIVX MNNC/MFFC/NOSSET $
IF (NOSSET >=0) THEN $
  UPARTN USET,MNNC/MFFC,,,/'N'/'F'/'S' $
ENDIF $
EQUIVX MFFC/MAAC/NOOSET $
IF (NOOSET >=0) THEN $
  UPARTN USET,MFFC/MAAC,,,/'F'/'A'/'O' $
ENDIF $
PARAML  PHA//'TRAILER'/1/S,N,NMODES $
$
$$ STATIC MODE REANALYSIS METHOD
$
IF (METHOD='STATICR') THEN $
  ADD    MMAA,MAAC/DELM/-1.0 $
  MPYAD  DELM,PHA,/DMPHI/0 $
  MPYAD  PHA,MKAA,/LAMBD1/1 $
  MPYAD  LAMBD1,PHA,/LAMBD/0 $
  MPYAD  DMPHI,LAMBD,/PA/0 $
  DCMP   USET,SILS,EQEXINS,MKAA/LLL,/-1/0/BAILOUT/
      MAXRATIO/'F'/1.E-20/DECOMP//////S,N,SING/
      S,N,NBRCHG/S,N,ERR $
  FBS   LLL,,PA/PSI/-1 $
  SMPYAD PHA,PHA,MMAA,PSI,,PSI/PSIR/4/-1/1/0//1/// $
  APPEND PHA,PSIR/PHIN/1 $
  SMPYAD PHIN,MKAA,PHIN,,,/KHHN/3////1////6 $
  SMPYAD PHIN,MAAC,PHIN,,,/MHHN/3////1////6 $
  REIGL  KHHN,MHHN,DYNAMICS,CASECC,,,,/LAMAC,
      PHIQ,MI,EIGVMAT,/'MODES'/S,N,NEIGV/NSKIP $
  OFP   LAMAC// $
  MESSAGE  //'MODAL SOLUTION ERROR ESTIMATE' $
  MESSAGE  //'STATIC MODE REANALYSIS CONSISTENT MASS EIGENVALUES' $
  CALL ERROR LAMA,LAMAC//NMODES/METHOD $
ELSE IF (METHOD='ITERQ  ') THEN $
$
$$ ITERATION QUOTIENT
```

```
$
  DCMP   USET,SILS,EQEXINS,MKAA/LLL1,/-1/0/BAILOUT/
      MAXRATIO/'F'/1.E-20/DECOMP/////S,N,SING/
      S,N,NBRCHG/S,N,ERR $
  MPYAD  MAAC,PHA,/ILOAD $
  FBS    LLL1,,ILOAD/IY/-1 $
  SMPYAD IY,MKAA,IY,,,/IK/3////1////6 $
  SMPYAD IY,MAAC,IY,,,/IM/3////1////6 $
  SOLVE  IM,IK/LAMACM $
  DIAGONAL LAMACM/LAMAC $
  MATPRN LAMAC// $
  MESSAGE  //'MODAL SOLUTION ERROR ESTIMATE' $
  MESSAGE  //'ITERATION QUOTIENT CONSISTENT MASS EIGENVALUES' $
  CALL   ERROR LAMA,LAMAC//NMODES/METHOD $
ELSE IF (METHOD='TIMQ   ') THEN $
$
$$ TIMOSHENKO'S QUOTIENT
$
  DCMP   USET,SILS,EQEXINS,MKAA/LLL2,/-1/0/BAILOUT/
      MAXRATIO/'F'/1.E-20/DECOMP/////S,N,SING/
      S,N,NBRCHG/S,N,ERR $
  MPYAD  MAAC,PHA,/TLOAD $
  FBS    LLL2,,TLOAD/TY/-1 $
  SMPYAD TY,MKAA,TY,,,/TK/3////1////6 $
  SMPYAD PHA,MAAC,PHA,,,/TM/3////1////6 $
  SOLVE  TK,TM/LAMACM $
  DIAGONAL LAMACM/LAMAC $
  MATPRN LAMAC// $
  MESSAGE  //'MODAL SOLUTION ERROR ESTIMATE' $
  MESSAGE  //'TIMOSHENKO QUOTIENT CONSISTENT MASS EIGENVALUES' $
  CALL   ERROR LAMA,LAMAC//NMODES/METHOD $
ELSE IF (METHOD='RAYLYQ ') THEN $
$
$$ RAYLEIGH'S QUOTIENT
$
  SMPYAD PHA,MKAA,PHA,,,/RK/3////1////6 $
  SMPYAD PHA,MAAC,PHA,,,/RM/3////1////6 $
  SOLVE  RM,RK/LAMACM $
  DIAGONAL LAMACM/LAMAC $
  MATPRN LAMAC// $
  MESSAGE  //'MODAL SOLUTION ERROR ESTIMATE' $
  MESSAGE  //'RAYLEIGH QUOTIENT CONSISTENT MASS EIGENVALUES' $
  CALL   ERROR LAMA,LAMAC//NMODES/METHOD $
ELSE IF (METHOD='EXACT  ') THEN $
$
$$ EXACT
$
  REIGL  MKAA,MAAC,DYNAMICS,CASES,,MR,DM,USET/LAMAC,
      PHIQ,MI,EIGVMAT,/'MODES'/S,N,NEIGV/NSKIP $
  OFP    LAMAC// $
  MESSAGE  //'MODAL SOLUTION ERROR ESTIMATE' $
  MESSAGE  //'EXACT CONSISTENT MASS EIGENVALUES' $
  CALL ERROR LAMA,LAMAC//NMODES/METHOD $
ENDIF $
EXIT $
COMPILE ERROR
SUBDMAP ERROR LAMA,LAMAC//NMODES/METHOD $
TYPE    PARM,,CHAR7,Y,METHOD='EXACT  ' $
TYPE    PARM,,I,N,NMODES $
TYPE    PARM,,I,Y,MODE,W $
TYPE    PARM,,RS,N,LMDL,LMDC,ERROR $
W=-4
```

```
MODE=0
DO WHILE(MODE<NMODES) $
  MODE=MODE+1 $
  W=W+7 $
  PARAML  LAMA//'DTI'/2/W/S,N,LMDL $
  IF (METHOD='STATICR' OR METHOD='EXACT  ')THEN  $
    PARAML  LAMAC//'DTI'/2/W/S,N,LMDC $
  ELSE $
    PARAML  LAMAC//'DMI'/1/MODE/S,N,LMDC $
  ENDIF $
  ERROR=((LMDL-LMDC)/(LMDL+LMDC))*100.
  MESSAGE //'MODE NUMBER'/MODE/' HAS'/ERROR/' PERCENT ERROR' $
ENDDO $
RETURN $
END $
ENDALTER $
```