A SUPEREFFICIENT, MSC/NASTRAN-INTERFACED COMPUTER CODE SYSTEM FOR DYNAMIC RESPONSE ANALYSIS OF NONPROPORTIONALLY DAMPED ELASTIC SYSTEMS

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ABSTRACT

With the emphasis on frequency response analysis case, development of the title computer code capability and application of the latter in evaluation of the computational efficiency of the MSC/NASTRAN code itself in the dynamic structural response analysis of nonproportionally damped elastic systems are made in this study. In this system, MSC/NASTRAN is used mainly for physical or modal structural (mass, damping, and stiffness) matrix assembling. The newly developed CMODEAN (Complex MOde/DEcoupling ANalysis) module uses the structural matrices as input for complex normal modes (state eigenmodes) calculation and equations of motion decoupling. Computational efficiency of CMODSTAN over MSC/NASTRAN for frequency response analysis of nonproportionally damped systems is demonstrated by an example problem with 225 dynamic degrees of freedom.

1. INTRODUCTION

There are a number of important classes of dynamic structural response analysis problems (e.g., passive and/or active damping enhanced, control/structure analysis problems) that must be treated as nonproportional damped elastic systems. Several efficient variants [1] of the standard state vector modal methods [2,3] for decoupling equations of motion have been formulated by the author. They were originally implemented into the baseline VAX version (version 0) of an MSC/NASTRAN-interfaced computer code system, CMODSTAN, for decoupling governing equations of motion and dynamic response analysis of nonproportionally damped elastic systems. Since then, a new version of the code has been created on Multiflow Trace 7/200 model minisupercomputer and a number of refinements/extensions have been made. As a result, the new version of CMODSTAN can be called superefficient compared with the frequency response analysis of the MSC/NASTRAN code [4].

Sections 2 briefly review the writer's previously formulated algorithms in [1] for decoupling equations of motion and dynamic response analysis. A brief description of the CMODSTAN code system is given in Section 3. A benchmarking study of the CMODSTAN code system is given in Section 4. Section 5 shows an example problem in which the widely used classical modal selection criteria may not always yield correct dynamic responses.

2. THEORETICAL FORMULATION

Consider a general elastic system governed by the physical (j=P) or modal (j=M) equations of motion

$$[M_j]\{u''_j\} + [C_j]\{u'_j\} + [K_j]\{u_j\} = \{P_j(t)\} \quad (j = \text{P or M}) \tag{1}$$

where

$[M_j]$, $[C_j]$, $[K_j]$	= $(N_j x N_j)$ symmetric physical (j=P) or modal (j=M) mass, damping, and stiffness matrices, respectively
$\{u_j\}$	= the displacement vector
$\{P_j(t)\}$	= the explicitly time-dependent applied force vector

( )'   = time (t) derivative

The subscript j (= P, M) will be dropped unless it is required for a clarity.

A. Matrix Equation of Motion Decoupling Techniques

(a) Case without Rigid-Body Components of Motion

To decouple this matrix equation of motion, it is first rewritten into the first order differential equation (called "state vector", {v} with dimension $N_v = 2N$) form

$$[A]\{v'\} = [B]\{v\} + \{F\} \tag{2}$$

There are several ways in which Eq. (1) can be recasted into Eq. (2). Table 1 gives six different sets of matrix/vector representations. In this table, [X] = [Z] is the state modal matrix of the adjoint (transpose) system of Eq. (2), $[\Omega_M]$ the diagonal natural modal frequency matrix of the corresponding undamped free vibration system, $i = (-1)^{1/2}$, bold quantities are the partitioned vectors or matrices, and subscript M and superscript T stands for "modal" and "transpose", respectively. Also shown in the last column of Table 1 is the $(N_q x N_q)$ complex participation factor matrix, [X], which results from decoupling the equations of motion discussed below.

Let the $(N_v x N_v)$ (complex) state modal matrix of Eq. (2) (with {F} = {0}) be

$$\{Y\} = \{Y_n\} = \left\{\begin{array}{c} \mathbf{Y}_{u'n} \\ \text{------} \\ \mathbf{Y}_{un} \end{array}\right\} = \left\{\begin{array}{c} \mathbf{Y}_{un}\mathbf{\Lambda} \\ \text{-------} \\ \mathbf{Y}_{un} \end{array}\right\} \tag{3}$$

where subscripts n = 1,..., 6 are the case number in Table 1 and $\mathbf{\Lambda} = [\Lambda]$ is the diagonal eigenvalue matrix of the damped system. Let

$$\{v(t)\} = [Y_n]\{q(t)\} \tag{4}$$

Table 1. Matrix and Vector Quantities in Eqs. (2) and [X] matrix in Eq. (5)

No.	Form.	{v}	{F}	[A]	[B]	[X]
1	A1	$\begin{Bmatrix} \mathbf{u}' \\ \mathbf{u} \end{Bmatrix}$	$\begin{Bmatrix} \mathbf{M}^{-1}\mathbf{P} \\ \mathbf{0} \end{Bmatrix}$	$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$	$\begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$	$[Y_1]^{-1}$
2	A2	same as A1				$[Z]^T$
3	B1	$\begin{Bmatrix} \mathbf{u}' \\ \mathbf{u} \end{Bmatrix}$	$\begin{Bmatrix} \mathbf{P} \\ \mathbf{0} \end{Bmatrix}$	$\begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix}$	$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix}$	$[Y_3]^T$
4	B2	$\begin{Bmatrix} \mathbf{u}' \\ \mathbf{u} \end{Bmatrix}$	$\begin{Bmatrix} \mathbf{0} \\ \mathbf{P} \end{Bmatrix}$	$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & -\mathbf{K} \end{bmatrix}$	$\begin{bmatrix} -\mathbf{C} & -\mathbf{K} \\ -\mathbf{K} & \mathbf{M} \end{bmatrix}$	$[Y_4]^T$
5	B3	$\begin{Bmatrix} \mathbf{u}_M' \\ i\Omega_M \mathbf{u}_M \end{Bmatrix}$	$\begin{Bmatrix} \mathbf{P}_M \\ \mathbf{0} \end{Bmatrix}$	$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$	$\begin{bmatrix} -\mathbf{C}_M & i\Omega_M \\ i\Omega_M & \mathbf{0} \end{bmatrix}$	$[Y_5]^T$
6	B4	same as B3				$\begin{Bmatrix} \mathbf{Y}_{u'1} \\ i\Omega_M \mathbf{Y}_{u1} \end{Bmatrix}^T$

where {q} is a new unknown modal coordinate vector. Substitution of Eq. (4) into Eq. (2) and premultiplication of the resulting equation with [X] in Table 1, there is obtained

$$\{q'\} = [A]\{q\} + [X]\{F\} \tag{5}$$

For detailed dynamic equation decoupling procedures and proof of orthogonality see [1-3] for the standard formulations, A1, A2 and B1, and [1] for the new B2-B4 formulations.

Some Remarks on Various Formulations - It should be noted that the A1 and A2 formulations are also applicable to nonconservatively loaded systems in which [C] and [K] matrices are nonsymmetric, i.e., they contain the antisymmetric nonconservative force matrix portions associated with gyroscopic and circulatory force components, respectively. Among various formulations given above, as will be demonstrated numerically later, the best one is the B4 formulation. This is because, unlike A1 or B1 or B2 formulation, no inversion of the complex modal matrix [Y] or complex normal modes solution of the two-matrix ([A] and [B]) system is required. This is particularly true if the free undamped vibrational modes are first used to reduce the physical equations of motion to the modal coordinates (in which $[M_M]$ and $[K_M]$ are diagonal matrices).

(b) Case with Rigid-Body Components of Motion

In the presence of rigid-body components of motion, one superimposes the rigid-body motion with the nonrigid-body (elastic) parts given in the above formulations, i.e.,

$$\{u\} = [R]\{r\} + [Y_u]\{q\} \tag{6a}$$

where

[R], {r} = the rigid-body parts of eigenvector matrix and modal coordinate subvector

{r} = the rigid-body part of modal coordinate sub-vector

$[Y_{un}]$, {q} = the nonrigid-body (elastic) parts of eigenvector matrix and modal coordinate subvector (cf. Table 1) (with n=1-6).

It is readily shown that

$$\{r''\} = [R]^T\{P\} \quad ([R]^T[R] = [I]) \tag{6b}$$

B. Analytical Frequency Response Solution

With all governing equations decoupled using one of the formulations given above, calculation of frequency responses {u*} is reduced to a trivial operation of inverting a diagonal system matrix, i.e.,

$$\{u_A\} = [R]\{r_A\} + [Y_u]\{q_A\} \tag{7a}$$

$$\{r_A\} = [R]^T\{P_A\}/\omega_f^2 \tag{7b}$$

$$\{q_A\} = (i\omega[I] - [\Lambda])^{-1}[X]\{F_A\} \tag{7c}$$

where ω_f is the excitation frequency and quantities with subscript A (say, $\{g_A\}$) represent the amplitude quantities in the expression

$$\{g\} = \{g_A\}e^{i\omega_f t} \quad (\{g\} = \{u\}, \{r\}, \{q\}, \{P\}, \{F\}) \tag{7d}$$

3. CMODSTAN COMPUTER CODE SYSTEM

The forgoing algorithm sets have been implemented into a general purpose computer code system, CMODSTAN (Complex MODal STructural ANalysis) on the VAX 11/780, Micro VAX, and Multiflow Trace 7/200 computer systems. This system consists of the following modular codes:

1) MSC/NASTRAN - This computer code is used for generating/assembling the FE method-based system physical and modal structural stiffness, damping and mass matrices and also optionally used in postprocessing, such as curve plotting.

2) CMODEAN - This newly developed modular code is for extracting complex normal modes and decoupling the equations of motion. It uses EISPACK eigensolution subroutines [5] to perform the former task. It is capable of performing the decoupling task via any one of six formulations in Table 1.

3) FRESAN - This modular code is newly developed for frequency response analysis and curve plotting using the output data from CMODEAN.

4) TRESAN - To be developed for transient response analysis.

The FRESAN computer code module also possesses a capability of computing the resonant (absolute and relative maximum) frequency response values and/or response values at specified incremental frequency points. Because of this capability, a much larger incremental frequency size than that required for a MSC/NASTRAN computer run can be used to obtain adequate response results and curve plots so that significant computational time and cost can be saved. It also has an optional feature of returning to MSC/NASTRAN to perform transfer function plots. Figure 1 contains a macro flow chart of this system. Development of the transient response analysis module, TRESAN, has not yet been completed.

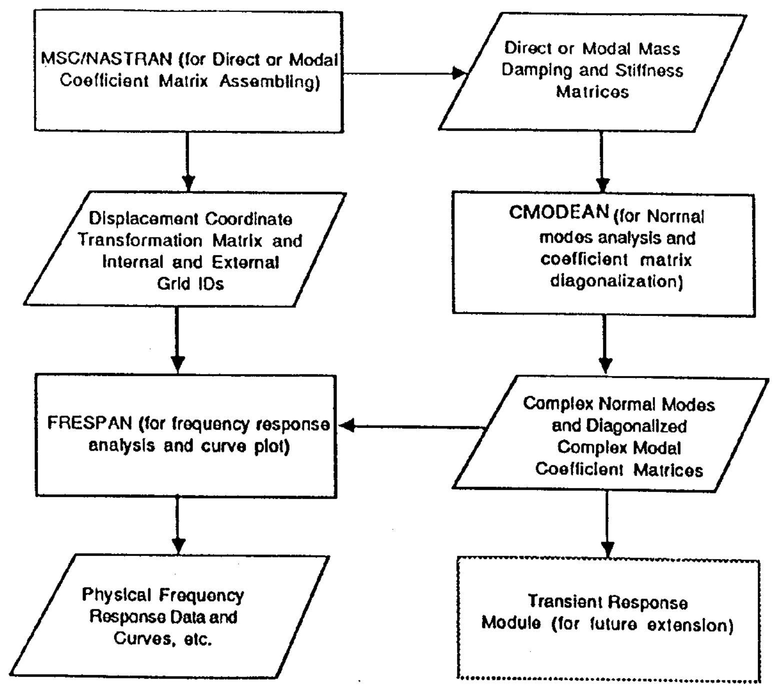

Fig. 1. Macro Flow Chart for the MSC/NASTRAN-Interfaced, Modular Type Code System, CMODSTAN

4. NUMERICAL VERIFICATION AND BENCHMARKING STUDY

A. Statement of the Benchmarking Example Problem

Consider a free-free system composed of two box subsystems that are joined together via viscoelastic connectors (Fig. 2). Each box structure is fabricated from aluminum honeycomb panels, which are reinforced with aluminum ribs at its edges and three-axis center line locations of both top and bottom panels. An additional cantilever member is a graphite composite tube. All masses are lumped at 75 grid points so there are 225 translational dynamic degrees of freedom.

In addition to the viscous damping induced by the viscoelastic connectors, it is assumed that 0.5% critical modal damping is also presented in the system. The source of sinusoidal disturbances is located at the geometrical center point of the left top box (box A).

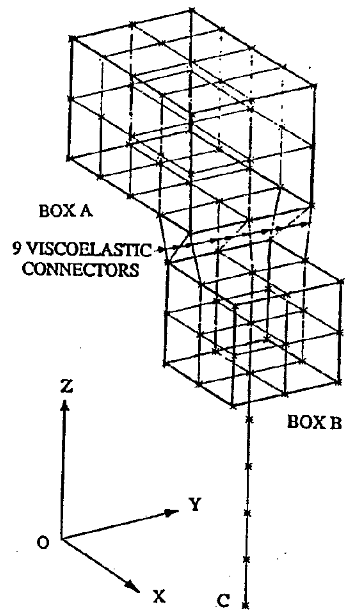

Fig. 2. Viscoelastic Connector Joined, Two-Box Type Dynamic Structural System

B. Numerical Verification of CMODSTAN Code Algorithms and Solution Accuracy

The CMODSTAN code system was used in solving both the complex modes and frequency response problems of several non-proportionally damped systems, including that of the benchmarking problem (Fig. 2) described above. This was accomplished by comparing the numerical results of several example problems (ranging from $N_M = 36$ to 225 dynamic or modal degrees of freedom systems with non-proportional damping), with those using "Superelement Modal Frequency Response" solution sequence (SOL 71) of MSC/NASTRAN.

Figure 3 shows the CMODSTAN and MSC/NASTRAN generated frequency response transfer function results for an over-damped case, designated as the damping level 3 (DL-3) case, using the entire 225 modal vectors.

Almost exact correlation of results are seen, except for the first peak response. This relatively large deviation of the MSC/NASTRAN result from the "exact" CMODSTAN result occurred due to use of an inadequate (too large) frequency incremental step size in MSC/NASTRAN.

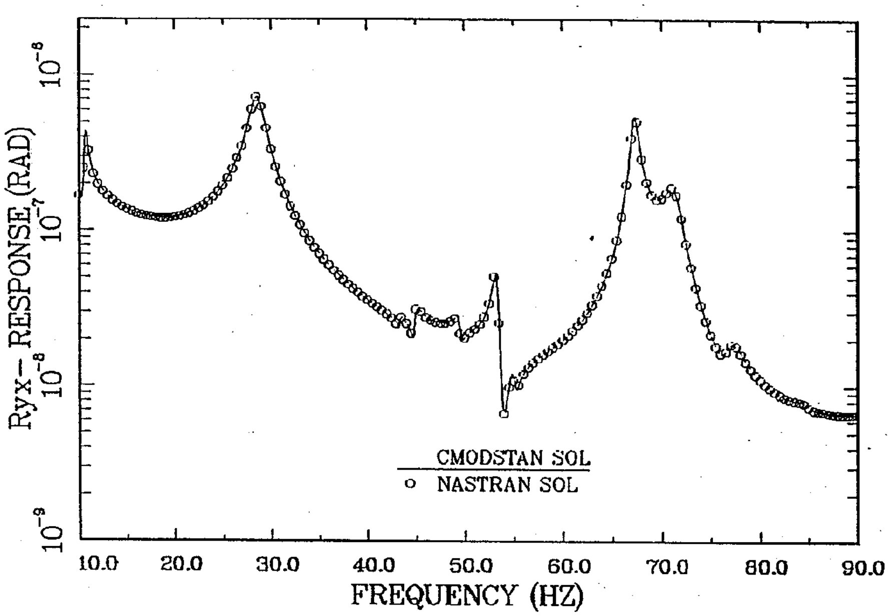

Fig. 3. Correlation of CMODSTAN- and MSC/NASTRAN-Based Frequency Response Analysis Results

C. Computational Efficiency Benchmarking Study Results

(a) Complex Normal State Modes Analysis/EOM Decoupling Operation Case

The CMODEAN module code of the CMODSTAN code system was used in extracting complex modes and decoupling (system coefficient matrix diagonalization) operations of the modal equations of motion for the example problem. In all cases, the physical and modal dynamic systems used are heavily and nonproportionally damped. The modal structural matrices were generated through use of MSC/NASTRAN.

Table 2 lists the CPU time results for performing the complex modal extraction and decoupling of the modal equations of motion. It is seen that the newly formulated computational algorithm set, B4 in Table 2, is computationally most efficient. It is roughly two and four times as efficient as the standard A1 and B1 algorithm sets (formulation), respectively. It is also the

Table 2: VAX 11/780 Model Computer CPU Time Required by Complex Normal Mode Calculations and Dynamic Equation Decoupling Operations

Formulation No.	VAX 11/780 CPU Time (min.)				
	30 MODF	60 MODF	100 MDOF	150 MDOF	200 MODF
A1	0.78	4.36	-	-	-
A2	0.82	5.43	-	-	-
B1	1.54	9.92	62.89	-	-
B2	1.33	9.16	-	-	-
B3	25.73	-	-	-	-
B4	0.47	2.78	14.30	43.73	112.29
B4	Multiflow Trace 7/200 CPU Time for the 225 MDOF case: 7.3 min.*				

Notes:

1. Here all N_M = MDOF listed correspond to non-ridged body degrees of freedom. Actual degrees of freedom used in the frequency response analysis are whatever is listed plus 6.
2. Various appropriate EISPACK subroutines were used in complex eigenmodes extractions.
3. All CPU times given here correspond to an over-damped system case. Somewhat smaller CPU times are required for the corresponding dynamic system with a moderately or lightly damped condition, e.g., 36.93 minutes were also obtained for 94 and 194 MODF cases, respectively, using B1 algorithm on an IBM 3084 model computer.

* In comparison, MSC/NASTRAN required 22.7 and 22.4 min. to obtain complex eigensolution alone using the SOL 67 and SOL 70 solution sequeces, respectively on the Multiflow system.

most economical in core space usage because it only needs to store two ($2N_M$ x $2N_M$) (N_M = number of modal degrees-of-freedom) expanded matrices, [B] and [Y_6], as opposed to requiring storage of three ($2N_M$ x $2N_M$) matrices, [A], [B] and [Y_4], for using the B1 and B2 formulations. It was also found that MSC/NASTRAN required three times the CPU time in just obtaining the complex eigensolution alone compared with the CMODEAN code in performing both the complex eigen analysis and equation decoupling operation for the 225 MDOF system on the Multiflow system.

Among the various formulations, the worst case from both computational time and core storage viewpoints belongs to the newly formulated B3 algorithm subroutines because coefficient matrix [B] is complex. However, this type of formulation should be better than any other type if [K_M] (modal stiffness) is inherently complex-valued, e.g., in the case that structural damping is present, or a complex component mode synthesis technique is used in assembling system the coefficient matrices of the equations of motion [6]. As can be predicted, the CPU times based on the A2 algorithm (formulation) are approximately twice those based on the B4 algorithm because of repeated calculations of complex eigenvalues and eigenvectors for the latter case. For the general case (i.e., coefficient matrices are non-symmetric), the computational algorithm set for the A1 formulation is predictably better than that of the A2 formulation.

(b) Frequency Response Analysis Case

Table 3 shows the CPU time results for the frequency response portion alone and overall response analysis using the successive modal transformation approach in which the real modal displacement method (MDM) is first used to transform the physical system to a modal system and the latter solved using the equation decoupling/modal superposition techniques. Also shown are the corresponding MSC/NASTRAN CPU time results on both Multiflow and VAX systems using the conventional real modal transformation approach (SOL 71). The speed-up factors for CMODSTAN versus MSC/NASTRAN are seen to be more than 1000 and 72 for the frequencyresponse calculation and overall response analysis, respectively, on Multiflow. An additional significant (several time) increase in the speedup factors of CMODSTAN code over

Table 3. Multiflow Trace 7/200 and VAX 11/780 CPU Times for Frequency Response Calculations of a Nonproportionally Damped 225 DOF Dynamic System

computer code or code system	total and breakdowns of cpu times consumed in various computational modules/elements				cpu time ratio (vs. cmodstan2's)	
	msc/nast. phys. & modal struct. matrix calc.	cmodean eigensol./EOM decoupling	freq. resp. sol. (time/points)	tot. time for 2111 pts.	freq. resp. calc. only	entire analysis
multiflow msc/nast.	78 s	-	3888.4/20 = 19.35 s/pt	11.35 hrs.	1036	72
multiflow[1] cmodstan1	80 s	451 s	21.08/2111 = 0.00998 s/pt	0.153 hr.	0.53	0.97
multiflow[1] cmodstan2	80 s	451 s	39.5/2111 = 0.0187 s/pt	0.158 hr.	1	1
vax msc/nast.	1605 s	-	1101.87/4 = 275.47 s/pt	162 hrs.	14,730	1022

1. The CMODSTAN1 case calculated a single (fifth) response component only while the CMODSTAN2 and MSC/NASTRAN cases calculated all 6 displacement response components at the cantilever tip point.

MSC/NASTRAN can be easily realized in solving a real world problem because of ability of CMODSTAN and the inability of MSC/NASTRAN to exactly pinpoint and calculate the peak (relative maximum or minimum) response quantities, respectively. Without such ability/capability, a (or a set of) sufficiently small excitation frequency step size(s) is usually needed by MSC/NASTRAN to obtain adequate response solution.

It should be noted that the speed-up factors of CMODSTAN versus MSC/NASTRAN shown in Table 3 will increase or decrease according to whether the number of frequency response solution points are increased or decreased from the 2111 points used to cover undamped eigenfrequency range (10 to 6450 Hz). The 2111 excitationfrequency points were determined automatically by CMODSTAN to include all natural eigenfrequency points (i.e., candidate peak response points) and ten equally spaced frequency steps between each pair of two successive natural frequencies.

C. MDM Reduced-Order Method Based Solution Convergence Pattern vs. No. (N_M) of Lowest Modes Used in Dynamic Analysis of the Heavily/Overly, Nonproportionally Damped System

Also studied were the convergence patterns of the MDM reduced-order method with respect to as variations in damping level (DL) and number (N_M) of the lowest undamped vibration modal vectors (reduced basis vectors). Figures 4A and 4B show the results for an over-damped (DL=3) condition with critical damping ratio of approximately 50. It is seen that steep jumps in peak frequency responses occur as N_M is increased from 213 to 214, and also from 215 to 216. The total jump in response from N_M = 213 to 216 is 165%. A similar jump (or steep response increase) was also observed for a critical damping ratio of approximately 5. To obtain a correct (convergent) solution for the peak responses, N_M = 216 out of a total of 225 modal vectors were required as the reduced basis vectors. Even if one uses the conventional cutoff frequency criterion of mode selection that is 3 times the excitation frequency of 134 Hz (i.e. 400 Hz), the number of modal vectors selected as the reduced basic vectors would be 136. The corresponding two peak responses are seen from Fig. 4A to be only approximately 40% of the "exact" value obtained by using the entire 225 modal vectors. Similarly, the conventional mode participation factor criterion of mode selection would also fail to include all important eigenvectors (such as the one associated with mode No. 216) required for obtaining an adequate

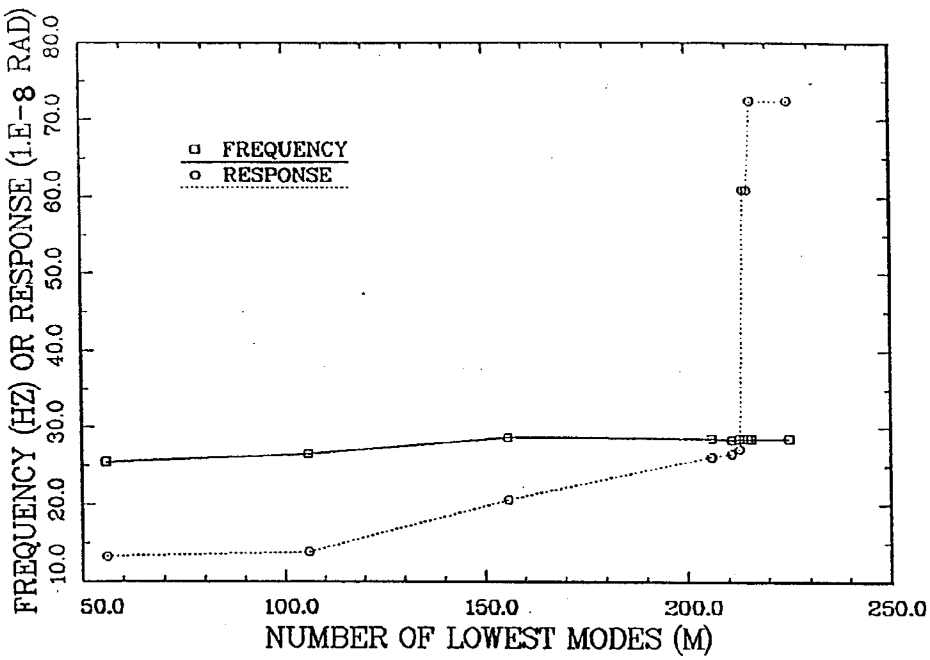

Fig. 4A. R_{yx} Peak Frequency Response Solution and Corresponding Excitation Frequency vs. Number (N_M) of the Lowest Modes (DL=3 Case)

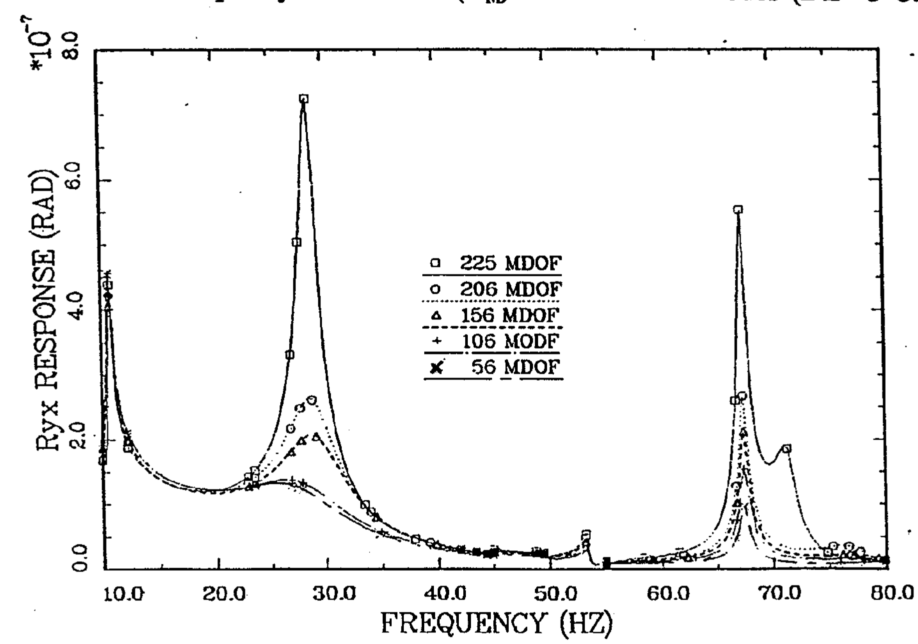

Fig. 4B. Solution Convergence Pattern of the MDM Reduced-Order Method Based R_{yx} Frequency Responses Versus No. (N_M) of the Lowest Free Undamped Vibration Modes Used solution. Use of the mode acceleration method (MAM) in conjunction with the conventional mode selection criteria will not solve this problem either because it does not tie the mode selection procedure to damping values.

It should be noted that a heavily or over-damped system is not uncommon in designs where passive and/or active damping measures are used to suppress excessive vibrations. As a matter of fact, the present development work was initiated originally to deal with the design analysis problem of an actual mechanical damper with a maximum critical damping ratio as high as 50. This is on the same order as that of the damping level 3 case studied here. Therefore, within the framework of conventional reduced modal basis vector method, there is a definite need to modify or reformulate the associated mode selection criteria (if it can be done) so that the modified (reformulated) criteria would be capable of also selecting those higher undamped modes associated with the actual, heavily damped modes. Such a mode selection criterion is being formulated and preliminary numerical test results show its capablility in selecting all critical modes while drastically reducing the number of modes required for generating correct (convergent) eigensolution and frequency response (and, thus, transient response) results.

An alternate approach is to use a higher order method (such as the force derivative method [7,8]) that include damping effects on the reduced-order modal vectors.

5. CONCLUDING REMARKS

Six formulations/algorithms for equations of motion (EOM) decoupling and frequency response analysis of nonproportionally damped systems were presented. The implementation of these formulations into an MSC/NASTRAN-interfaced general-purpose computer code system, CMODSTAN, was also described. CMODSTAN uses the MSC/NASTRAN code as a module to calculate the finite element structural and (real) modal structural (mass, damping, and stiffness) matrices and two newly developed Fortran codes, CMODEAN and FRESAN, for complex normal modes analysis/EOM decoupling and frequency analysis, respectively. The CMODSTAN code system was used in assessment of the relative computational efficiency of the six formulations/algorithm sets. One (B4) of the three algorithms (B2-B4) formulated by the author for decoupling equations of motion (including the required complex normal modes analysis) was shown to be computationally four times as efficient as the standard (B1) algorithm while requiring only a two-third of computer memory.

The superefficiency of the CMODSTAN code system versus MSC/NASTRAN in both complex modes and frequency response analyses was demonstrated using a nonproportionally damped elastic system with 225 dynamic degrees of freedom. The speed-up factors were found to be three for the complex modal extraction portion, 1000 for the frequency response calculational portion, and 70 for the overall (entire) frequency response analysis. In addition, the mode displacement (reduced-order) method (MDM) based complex eigensolution and frequency response convergence patterns were studied as functions of both excitation frequency and number of the lowest free undamped vibration modes used. It was found that the MDM based solution of the example problem, in conjunction with using the conventional mode selection criterion, failed to yield an adequate solution for at least the critically or overly damped cases studied. In the latter case, the 216 lowest modes of the 225 model were required to obtain an adequate frequency response solution. Use of the mode acceleration method (MAM) will not accerate solution convergence either. Therefore, in using MDM or MAM based dynamic structural analysis computer codes, such as MSC/NASTRAN, caution should be exercised for dynamic analysis of a heavily or overly damped systems. Here, formulation of a new mode selection criterion is needed. Alternately, use of a higher order modal reduced-order method should be made. Initiation of extending CMODSTAN to include such capabilities are being made.

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