DATA RECOVERY AND MODEL REDUCTION METHODS FOR LARGE STRUCTURES

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ABSTRACT

This paper demonstrates important factors for the application of mode-superposition methods and component mode synthesis to transient response analyses of large structures. A theoretical review is presented and numerical results are evaluated for three case studies. Data recovery techniques based on the mode-superposition method are evaluated with respect to different types of force input, model reduction, model size, and computational resources. Cutoff frequency selection at the component- and system-level of component mode synthesis is discussed for accurate dynamic response calculations. This paper not only shows the theoretical differences between different data recovery methods, but also provides physical insights at each computational stage.

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INTRODUCTION

Transient response analyses for large structures widely utilize the mode-superposition method and component mode synthesis to calculate the dynamic load responses. Both techniques offer the advantage of reducing the computational resources necessary to handle large problems. Mode-superposition methods transform the equations of motion for a multi-degree of freedom (dof) system into a set of single dof problems which can easily be solved. In most cases, the size of the problem is also reduced by retaining only a subset of the normal modes. These methods vary in the recovery of response data from the modal solutions, which can significantly affect the results under certain loading conditions and reduction methods. Component mode synthesis (CMS) analyzes and reduces the structure in components before assembling them into a system model whose size becomes much smaller than that of the original model. Accuracy of the dynamic response results is dependent on the number of modes retained in the component- and system-level analyses.

Transient response analyses for the Space Station Freedom (SSF) have a unique combination of factors which lead to the use of these computational reduction techniques. Finite element models of the SSF structure are extremely complex and may contain millions of dofs if not reduced. The spacecraft can easily be broken into distinct components due to the nature of its assembly and the number of appendages such as photovoltaic (PV) arrays. Model reduction through CMS, in conjunction with mode-superposition methods, can effectively reduce computer resources. Dynamic characteristics and complexity of the force input as well as the SSF models vary for each analysis case. Thus, data recovery and model reduction methods must be selected on a case-by-case basis to improve accuracy and at the same time minimize computational effort.

Performance of the mode-superposition methods and component mode synthesis is closely related to the dynamic characteristics of the loading and structure. Improper selection of the data recovery methods can substantially under- or over-predict structural responses, or lead to computational inefficiency. Neglecting critical component modes can prevent a complete description of the dynamic behavior of the entire system. This paper will discuss the theoretical background of the methods before examining three case studies to illustrate key points of their implementation and to give physical insights at each computational stage.

THEORETICAL BACKGROUND

In this section, a brief theoretical background for data recovery and model reduction methods is presented to provide a basis for discussions in the following sections. A detailed discussion in this area can be found in References [1, 2, 3].

Mode Displacement Method

Consider an undamped linear n-dof dynamic system whose motion can be described by a set of n differential equations:

$$M\ddot{\boldsymbol{x}}(t) + K\boldsymbol{x}(t) = \boldsymbol{f}(t) \tag{1}$$

In general, these equations of motion are coupled such that simultaneous solution for n unknowns in n equations would be required to directly solve the problem.

By introducing the coordinate transformation, i.e. using the *mode-superposition method*

$$\boldsymbol{x}(t) = \Phi\boldsymbol{\eta}(t) = \sum_{i=1}^{n} \boldsymbol{\phi}_i \eta_i(t) \tag{2}$$

the equations of motion can be expressed in terms of principal coordinates, $\eta_i(t)$

$$\hat{M}\ddot{\boldsymbol{\eta}}(t) + \hat{K}\boldsymbol{\eta}(t) = \boldsymbol{p}(t) \tag{3}$$

where

$$\hat{M} = \Phi^T M \Phi, \quad \hat{K} = \Phi^T K \Phi, \quad \text{and} \quad \boldsymbol{p}(t) = \Phi^T \boldsymbol{f}(t) \tag{4}$$

Modal mass and stiffness matrices, $\hat{M}$ and $\hat{K}$ in Equation 4, become diagonal matrices due to the orthogonality condition. Therefore, the equations of motion for a n-dof system are transformed into a set of n decoupled, or single dof, differential equations.

Once the modal response time histories of $\eta_i(t)$ are calculated, the system response of $\boldsymbol{x}(t)$ can be obtained by mode superposition:

$$\boldsymbol{x}_D(t) = \sum_{i=1}^{m} \boldsymbol{\phi}_i \eta_i(t) \tag{5}$$

This process of reconstructing the physical coordinates from the modal coordinates is called *data recovery.* Equation 5 is called the *mode displacement method* and gives the exact solution when all n modes are included. However, transient response analyses for large structures often include only m ($\ll n$) modes to reduce computer time and storage, resulting in approximate solutions. If the number of modes, or cutoff frequency, is carefully chosen based on the dynamic characteristics of the structure and input force, effect of the truncated modes can be minimized.

The mode-superposition method can still be employed for a dynamic system with proportional damping which satisfies the orthogonal condition, i.e. the modal damping matrix is diagonal. For a system with general linear damping, the same procedure can be used after transforming the equations of motion from second-order into first-order differential equations.

Mode Acceleration Method

When only a part of the modes are included and input force is dominated by pseudo-static loading, the mode displacement method may fail to give an accurate solution. This phenomenon can be illustrated by assuming a system with the solution in the form of (Figure 1):

$$x_p(t) = A \sin \omega_q t + B \tag{6}$$

The mode displacement method requires many modes to represent the static contribution which is a step function, although dynamic contribution of the solution can be described by only one mode.

The *mode acceleration method* [1] improves the accuracy of the mode-superposition method by solving static and dynamic contributions of the solution separately, i.e.

$$\begin{aligned}
\boldsymbol{x}_A(t) &= \left[K^{-1}\boldsymbol{f}(t)\right] + \left[-K^{-1}M\ddot{\boldsymbol{x}}_D(t)\right] \\
&= \underset{\text{static part}}{\left[K^{-1}\boldsymbol{f}(t)\right]} + \underset{\text{dynamic part}}{\left[-\sum_{i=1}^{m}\boldsymbol{\phi}_i\ddot{\eta}_i(t)\,/\,\omega_i^2\right]}
\end{aligned} \tag{7}$$

The pseudo-static part is calculated like other structural problems, at each time step, while the dynamic part is calculated by a regular mode-superposition method equivalent to the mode displacement method. Then the total system response is determined by combining the two solutions. For the previous example, the mode acceleration method requires only one mode to calculate an accurate solution regardless of the static part. It is noted that, however, this method gives the same results as the mode displacement method for the dynamic part of the total responses, thus it does not reduce the number of modes required to represent the dynamic contributions.

When the structure contains rigid body modes, the stiffness matrix of K in Equation 7 becomes singular, which prohibits inversion of the matrix. This problem can be resolved, for example, by a coordinate transformation which decouples Equation 1 into r-dof rigid and e-dof elastic dynamic equations, where $n = r + e$. The elastic dynamic response can now be determined because the corresponding system becomes constrained with respect to the selected rigid dofs, i.e. the corresponding stiffness matrix is non-singular.

Residual Flexibility Method

The mode acceleration method is more computationally expensive than the mode displacement method because the pseudo-static response must be calculated at each time step. An alternate data recovery method is the *residual flexibility method* [2] which adds the residual flexibility vectors to the mode displacement method, i.e.

$$\boldsymbol{x}_R(t) = \sum_{i=1}^{m}\boldsymbol{\phi}_i\eta_i(t) + \sum_{i=1}^{k}\boldsymbol{\psi}_i\zeta_i(t) \tag{8}$$

The residual flexibility vectors of $\boldsymbol{\psi}_i$ are static solutions to unit loading at the force input points and improve the accuracy of static contribution in the mode displacement method. It is an efficient method since the data recovery procedure is equivalent to the mode displacement method once a few static problems are solved. Since the static vectors are dependent on the force input and/or output points, the computational advantages may be diminished when numerous combinations of forcing functions and model configurations must be analyzed. Lanczos vectors have also been employed to improve the accuracy of static contribution to the solution [4].

Component Mode Synthesis

In component mode synthesis, once the system-level response time histories are calculated by the mode displacement or mode acceleration method, the solution for the unreduced component can be obtained by the transformation:

$$\boldsymbol{x}_{DD}(t) = H\boldsymbol{x}_{D}(t) \text{ and } \ddot{\boldsymbol{x}}_{DD}(t) = H\ddot{\boldsymbol{x}}_{D}(t) \tag{9}$$

or

$$\boldsymbol{x}_{AD}(t) = H\boldsymbol{x}_{A}(t) \text{ and } \ddot{\boldsymbol{x}}_{AD}(t) = H\ddot{\boldsymbol{x}}_{A}(t) \tag{10}$$

where H is a transformation matrix between the original component dofs, interior plus boundary coordinates, and the retained component dofs in the system-level analysis, boundary coordinates, for each component. These transformation matrices consist mainly of normal modes and constraint modes in the Craig-Bampton CMS method [5]. A constraint mode is defined by statically imposing a unit displacement on one physical boundary coordinate while all other boundary coordinates are constrained. The component normal modes are obtained by solving an eigenvalue problem for the component model represented by interior coordinates, resulting in fixed-interface normal modes. A general CMS approach uses mixed boundary normal modes.

When the system-level responses are calculated by the mode acceleration method, component responses can also be obtained by the same method to improve the accuracy. This is shown in its implicit form [3]:

$$\begin{aligned} \boldsymbol{x}_{AA}(t) \;=\; & \underbrace{\left[K_c^{-1}\boldsymbol{f}_c(t)\right]}_{\text{static part}} + \underbrace{\left[-K_c^{-1}M_c\ddot{\boldsymbol{x}}_{AD}(t)\right] + \left[\text{terms of } K_c,\, M_c,\, \boldsymbol{x}_{AD}(t), \text{ and } \ddot{\boldsymbol{x}}_{A}(t)\right]}_{\text{dynamic part}} \end{aligned} \tag{11}$$

Although this approach of using the mode acceleration method at both system and component levels improves the accuracy, the computer time and storage required for transient response analyses increases dramatically. The residual flexibility method can also be employed in data recovery for the component responses.

CASE STUDIES

Three case studies were conducted to illustrate the performance and computational effort of different data recovery and model reduction methods. A simple beam model was first employed to determine the impact of truncated modes and data recovery methods. The complex structure of the SSF SC-2 configuration was then used to evaluate the impact of model reduction (including component model reduction and mode truncation) as well as data recovery methods. Three different configurations of the SSF were used to assess the computer time and storage requirements. These study results were computed using MSC/NASTRAN on a Cray XMPEA-464 supercomputer, Direct Matrix Abstraction Programs (DMAPs), and in-house post-processing/database programs.

Free-Free Beam Model

A plane bending free-free beam model was employed to evaluate the data recovery methods in transient dynamic response analysis. Figure 2 shows a uniform 200 in. beam which has two rigid body modes and is modeled by 40 bar elements. Step forcing functions were used to maximize the static contribution anticipated in the solution. Free-free boundary conditions were imposed to require consideration of the rigid body modes. The beam was loaded with a 10 lb. load at the midpoint and two 5 lb. loads at each end to maintain static equilibrium. Three data recovery methods were evaluated, including mode displacement, mode acceleration, and residual flexibility methods. Dynamic responses were calculated using 3, 10, 15, 20, 30, 40, and 50 modes, with each including two rigid body modes. Modal contribution for the mode displacement method was also calculated.

Figure 3 illustrates the effect of truncated modes on both static and dynamic contributions to the total response. Figure 4 shows typical time history and modal contribution plots for shear force responses at the center of the beam. The mode acceleration method produced superior total response results to the mode displacement method in the case of force input with a large static contribution. The dynamic part of the total responses was comparable for both methods, since 15 modes were needed to accurately calculate it. However, the static part was drastically different, as the mode displacement method required 50 modes for an equivalent accuracy compared to the mode acceleration method result with no elastic modes. The modal contribution plot for the mode displacement method in Figure 4 supports the conclusion by showing significant contributions from the higher modes to represent the static part. The residual flexibility method was comparable to the mode acceleration method as it required 10 modes (2 rigid body modes, 5 elastic modes, and 3 residual vectors) to determine the static part of the response.

As shown in Figures 5 and 6, however, the mode displacement method produced accurate results for bending moment responses because the static part to the total response is not significantly greater than the dynamic part.

SSF SC-2 Configuration

Free-free beam study results were re-evaluated using a more complex structural model. Figure 7 shows a finite element model of the SSF SC-2 configuration which consists of three truss segments, two PV arrays, one Solar Array Rotary Joint (SARJ), and a heat rejection radiator. Each component was reduced by component mode synthesis primarily with fixed-interface normal modes and a few mixed boundary modes. The cutoff frequency varied between components. A 10-second pulse was applied at three truss points to maintain static equilibrium. This force input may not be realistic, but it served to demonstrate data recovery and model reduction issues on the complex structure. Only the mode displacement and mode acceleration data recovery methods were evaluated because the current DMAP for the residual flexibility method could not accommodate the mixed boundary conditions. Two different system-level cutoff frequencies of 5.0 Hz and 15.0 Hz were used to determine the effect of truncated modes on the accuracy. All three truss sections were included in the system model with and without reduction to evaluate the data recovery at the component-level, since MSC/NASTRAN implements the mode acceleration method only at the system-level.

Table 1 represents the maximum loads at several locations with the system cutoff frequency of 5.0 Hz. There are four cases with a combination of two different data recovery methods and two different truss reductions. Cases in Table 2 are equivalent to those in Table 1 except that the system cutoff frequency is 15.0 Hz. In general, there are significant differences between Case A results and corresponding Case B results, e.g. PV array mast base loads. This leads to a conclusion that system modes up to 15.0 Hz should be used to accommodate dynamic effects of the rectangular pulse regardless of data recovery methods. The impact of increasing system cutoff frequency is minimal considering 202 modes up to 15.0 Hz versus 166 modes up to 5.0 Hz.

The mode acceleration method was superior to the mode displacement method as shown by a substantial increase in the maximum loads even with the higher system cutoff frequency (Table 2). Truss reduction did not alter the results significantly (differences in cases with the mode acceleration method was not greater than those with mode displacement method). Therefore, using the mode acceleration method only at the system-level, but not at the component-level, was adequate for this case. It is noted that, for the truss interface axial loads, total responses were dominated by the static part as shown in Figures 8 and 9. As a result, different data recovery methods made more differences in the maximum loads than different cutoff frequencies. For the PV array mast base bending moment that was dominated by its dynamic part, however, a higher cutoff frequency of 15.0 Hz was necessary as shown in Figures 10 and 11.

Computer Resources

The computer resources for the mode displacement and mode acceleration methods were compared using SC-2, SC-7, and SC-17 configurations of the SSF. These configurations provide a variety of problem sizes ranging from 21,000 to 70,000 dofs. A relatively simple forcing function which simulates an astronaut pushoff during extravehicular activity was used for all cases. CPU time, I/O time, and database size were determined for the eigenvalue analysis, modal response analysis, and data recovery parts of the computation (Table 3).

The eigenvalue analyses used comparable levels of computer resources for both data recovery methods, as expected. However, the modal response analysis and data recovery showed substantial increases in computer resources used by the mode acceleration method. The differences became more significant, especially for the data recovery part, as the size of the model increases.

CONCLUSIONS

Issues surrounding the transient response analysis of large structures were addressed including data recovery and model reduction. Theoretical background was briefly provided for the mode displacement, mode acceleration, and residual flexibility methods. Three case studies were presented to demonstrate the important factors for the application of the mode-superposition method and component mode synthesis.

The mode acceleration data recovery method provided superior results to the mode displacement method in the case of force input with a large static contribution. However, the mode acceleration method required substantially more computer resources. It is noted that the number of modes retained in mode superposition should be determined by the frequency contents of the forcing functions regardless of data recovery methods. When forcing functions have large static loading contributions, additional modes are required for the mode displacement method to obtain accurate results. Choice of data recovery methods for acceptable accuracy and computational efficiency is highly dependent on the nature of forcing functions and models to be analyzed. In component mode synthesis, using the mode acceleration method only at the system-level was adequate with a careful selection on the type and number of component modes. The residual flexibility data recovery method showed promise in achieving accuracy equivalent to the mode acceleration method, but with fewer computer resources comparable to the mode displacement method.

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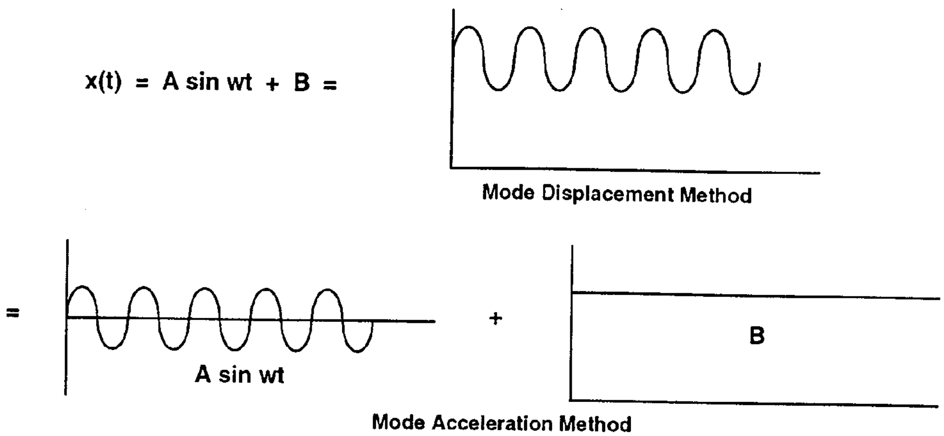

Figure 1. Mode Displacement vs. Mode Acceleration Method.

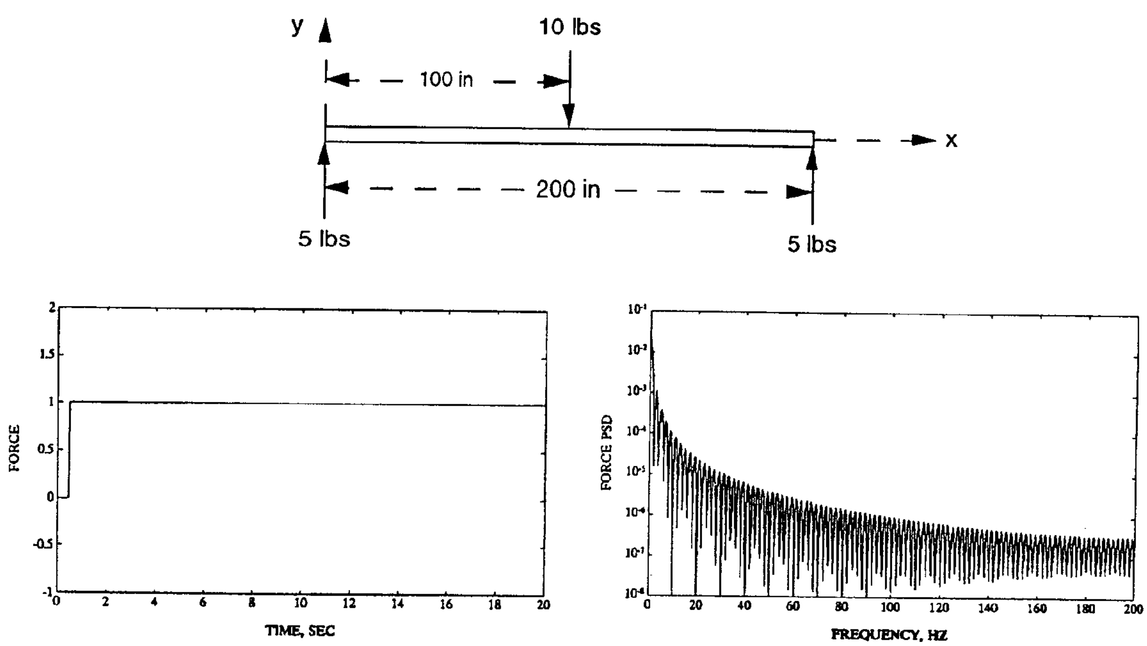

Figure 2. A Free-Free Beam Model and Forcing Functions.

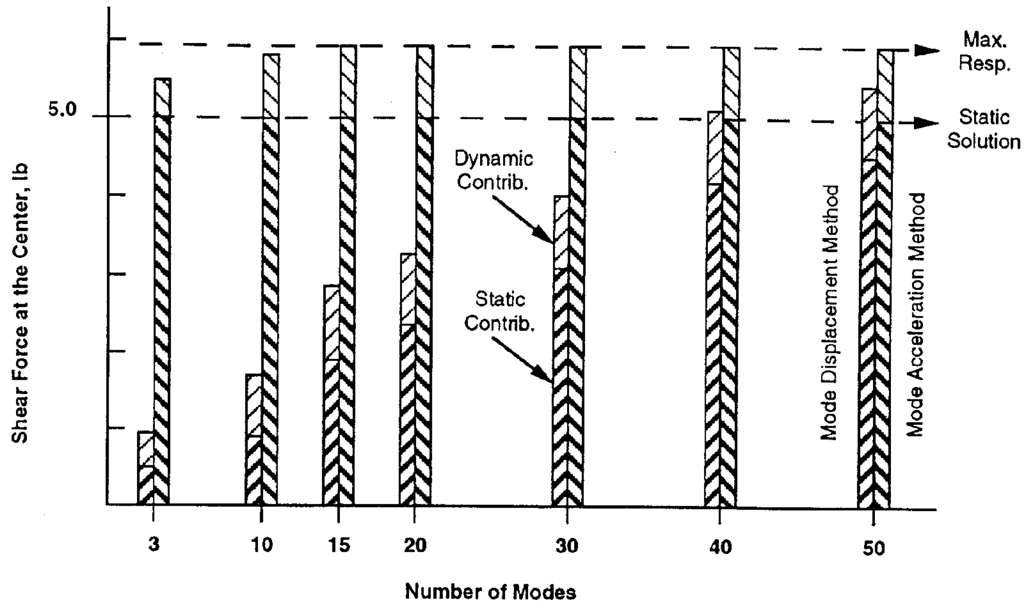

Figure 3. Static and Dynamic Parts of Shear Force Responses.

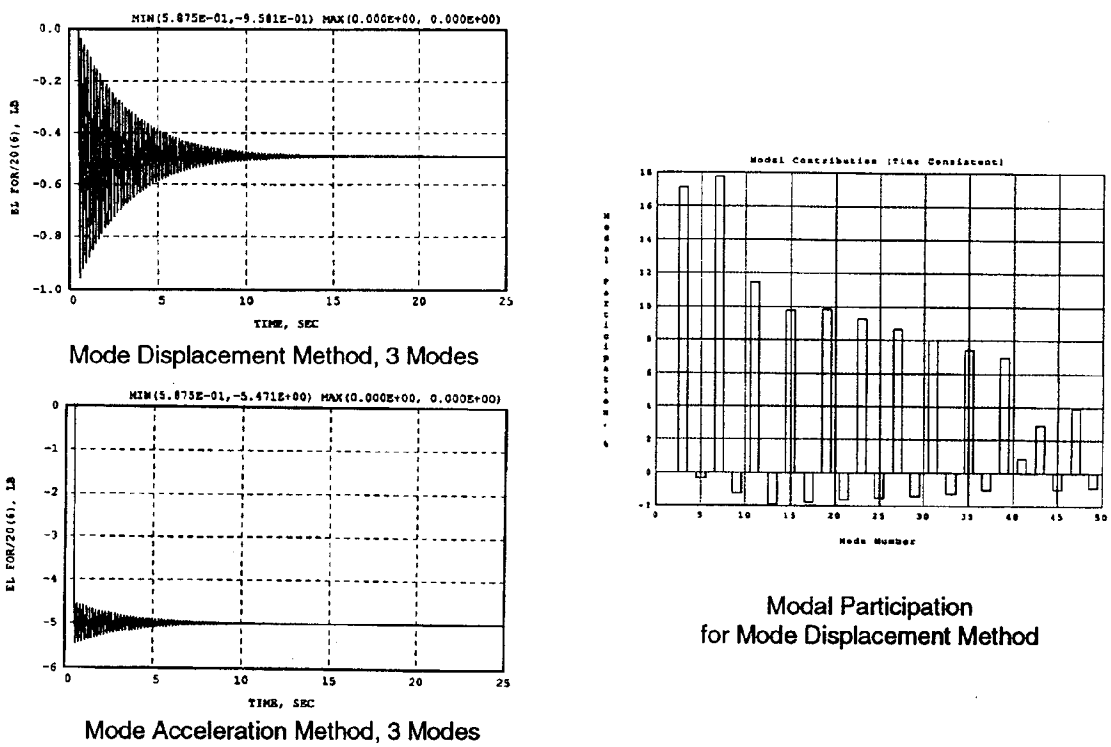

Figure 4. Shear Force Time History and Modal Contribution Plots.

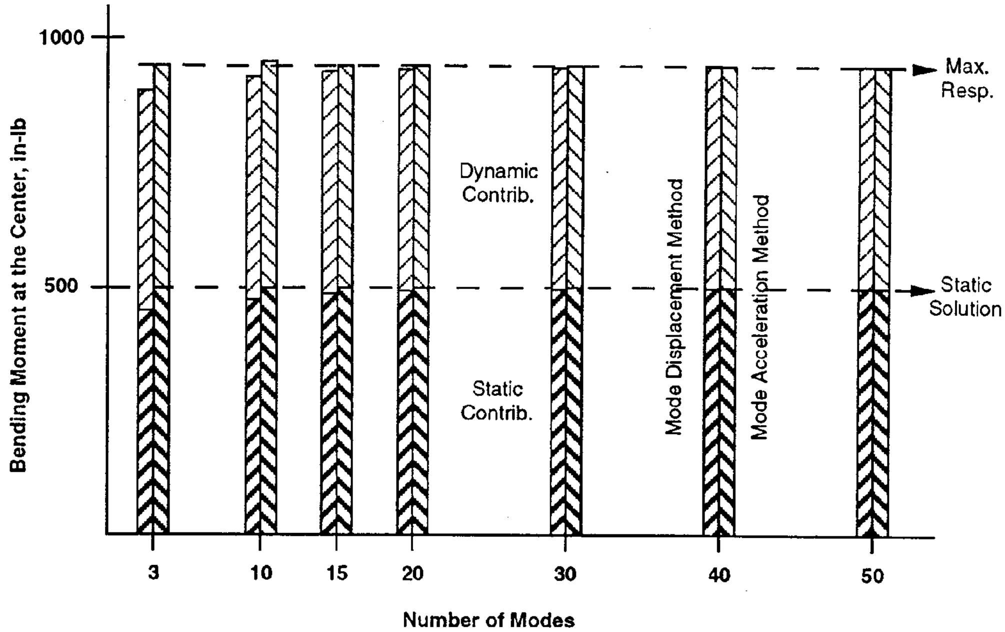

Figure 5. Static and Dynamic Parts of Bending Moment Responses.

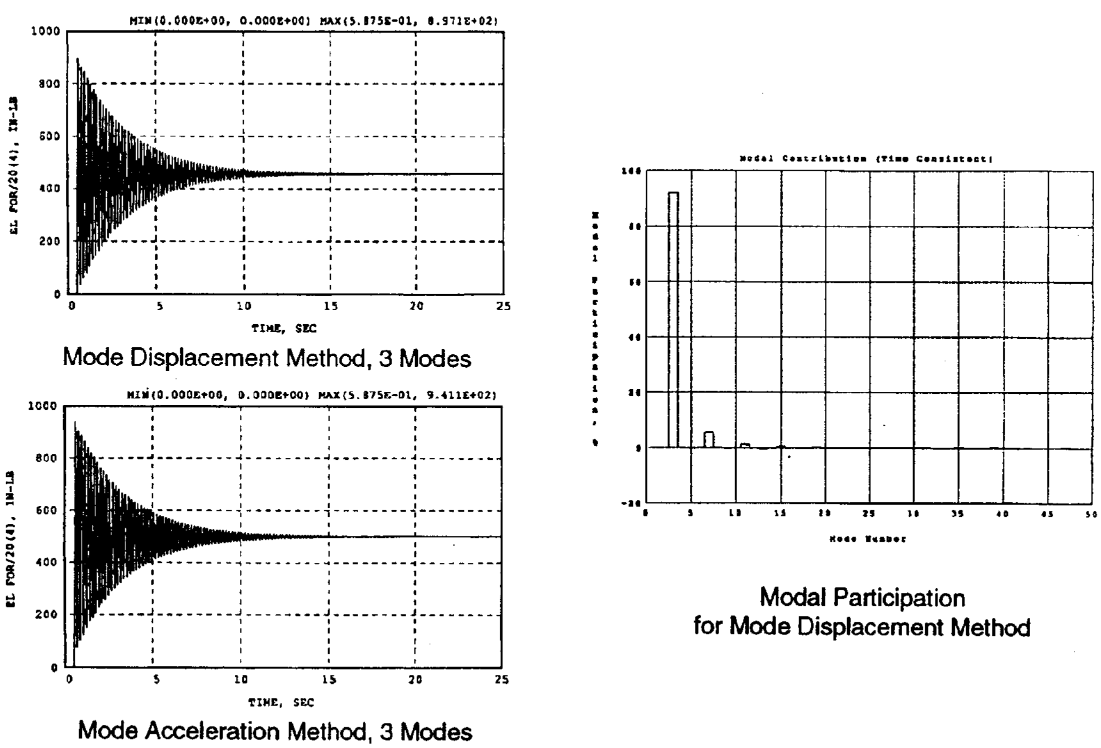

Figure 6. Bending Moment Time History and Modal Contribution Plots.

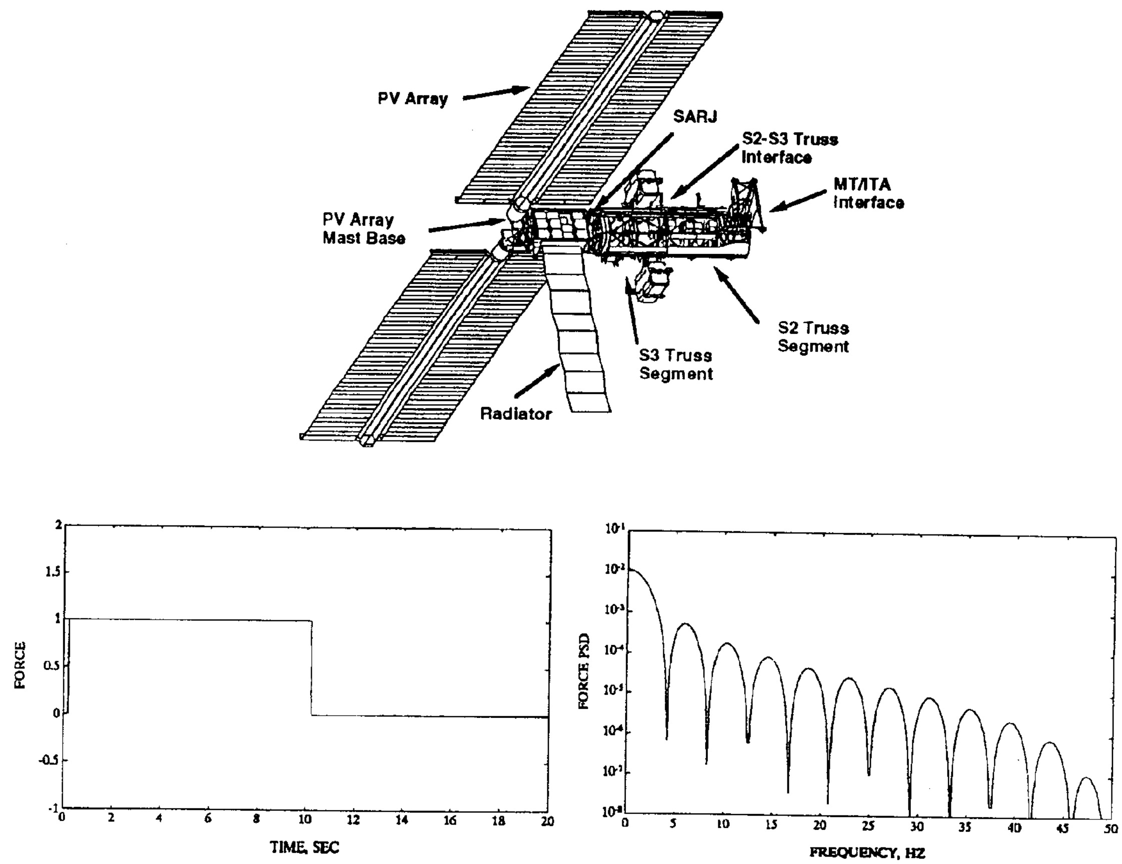

Figure 7. The SSF SC-2 Configuration and Forcing Functions.

Cases		A1	A2	A3	A4
Truss Component Reduction		Yes	No	Yes	No
System DOFs		522	4327	522	4327
System Cutoff Frequency, Hz		5	5	5	5
Number of System Eigenvalues		166	166	166	166
Data Recovery Method		Mode Disp.	Mode Disp.	Mode Accel.	Mode Accel.
SARJ	Axial Loads, lb	65.6	65.8	52.1	51.8
S2-S3 Truss Interface	Axial Loads, lb	**205**	**205**	**825**	**825**
MT/ITA Interface	Axial Loads, lb	250	249	143	143
Truss Segment	Axial Loads, lb	182	177	1053	858
	Bending Moment, in-lb	2679	2669	2524	1797
PV Mast Base	Bending Moment, in-lb	**1156**	**1133**	**1155**	**1133**

Table 1. Maximum Loads with a 5.0 Hz System Cutoff Frequency.

Cases		B1	B2	B3	B4
Truss Component Reduction		Yes	No	Yes	No
System DOFs		522	4327	522	4327
System Cutoff Frequency, Hz		15	15	15	15
Number of System Eigenvalues		202	202	202	202
Data Recovery Method		Mode Disp.	Mode Disp.	Mode Accel.	Mode Accel.
SARJ	Axial Loads, lb	94.9	97.4	80.3	85.3
S2-S3 Truss Interface	Axial Loads, lb	**510**	**520**	**923**	**912**
MT/ITA Interface	Axial Loads, lb	215	220	169	167
Truss Segment	Axial Loads, lb	505	425	1094	889
	Bending Moment, in-lb	3889	3655	2764	2492
PV Mast Base	Bending Moment, in-lb	**3947**	**4285**	**3966**	**4268**

Table 2. Maximum Loads with a 15.0 Hz System Cutoff Frequency.

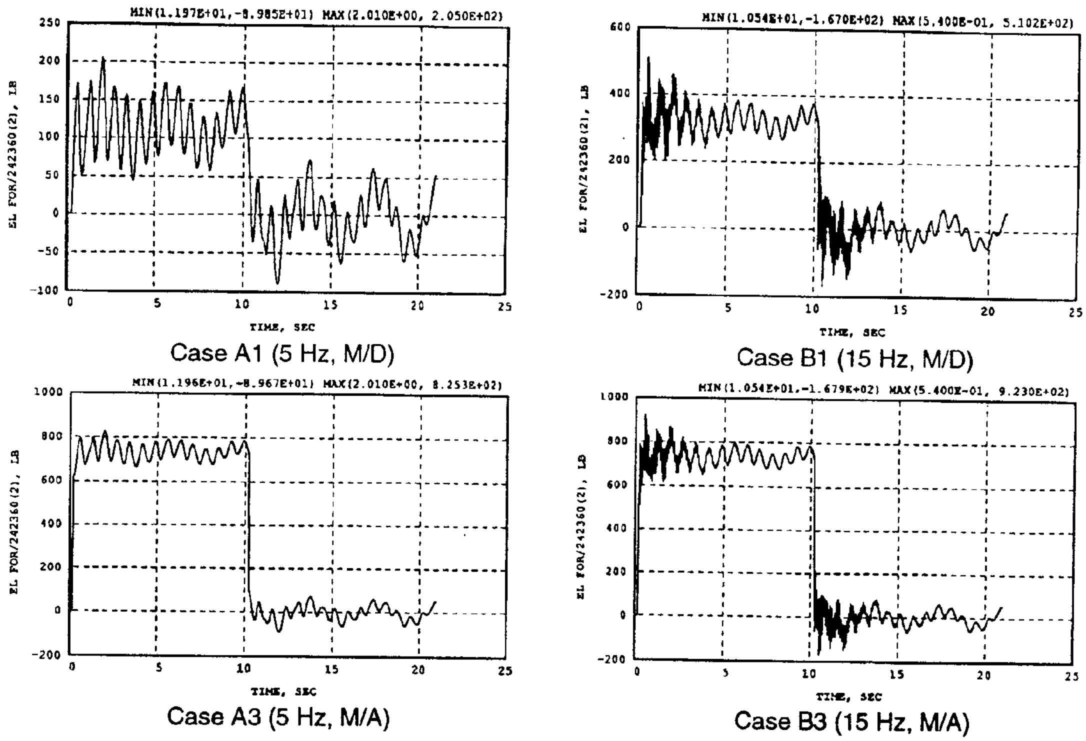

Figure 8. Time History Plots for Truss Interface Axial Loads.

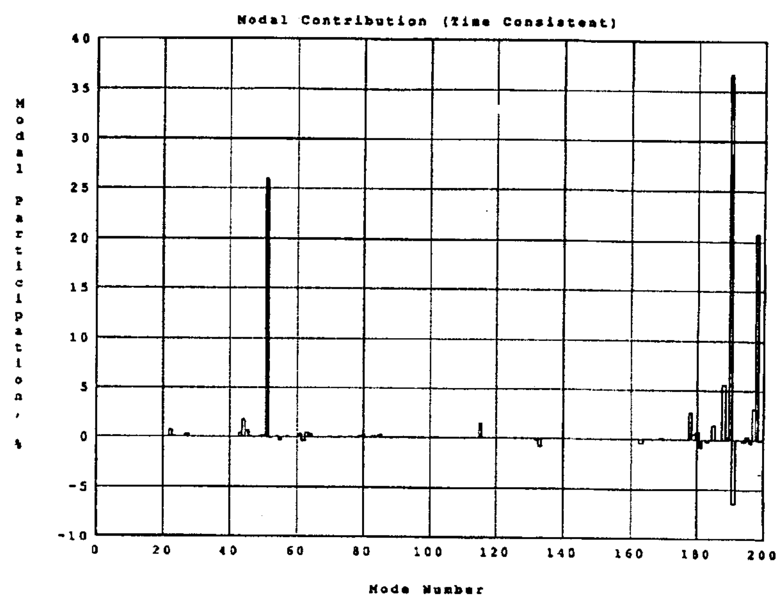

Figure 9. The Modal Contribution Plot for Truss Interface Axial Loads.

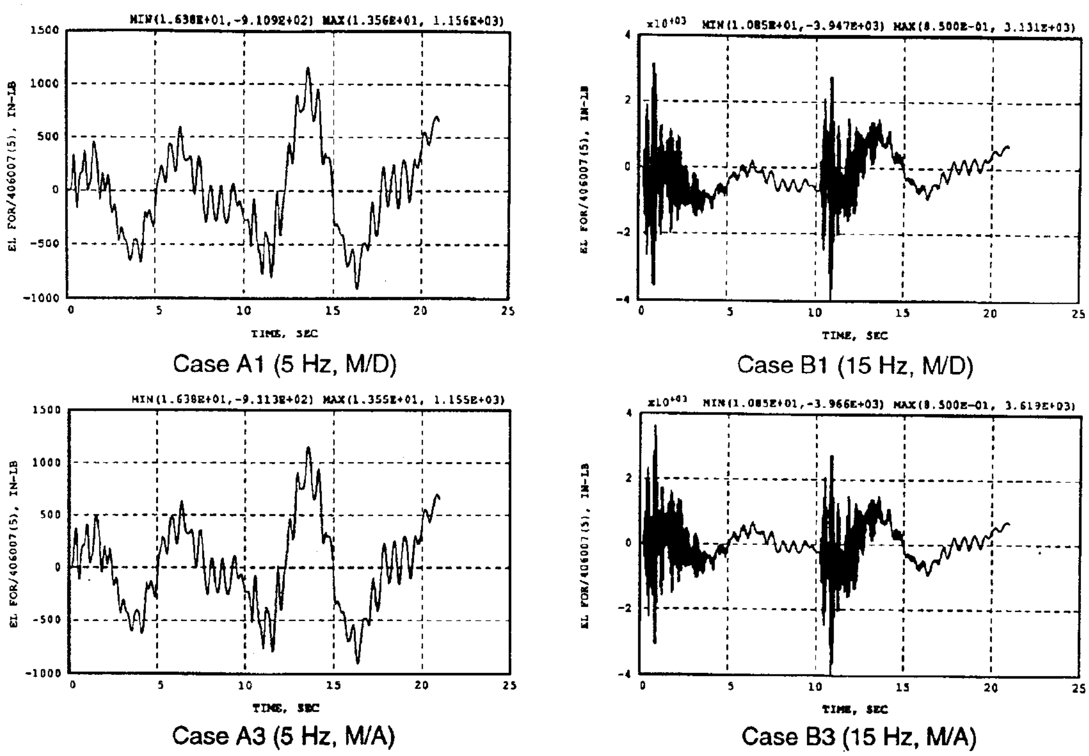

Figure 10. Time History Plots for PV Array Mast Base Bending Moment.

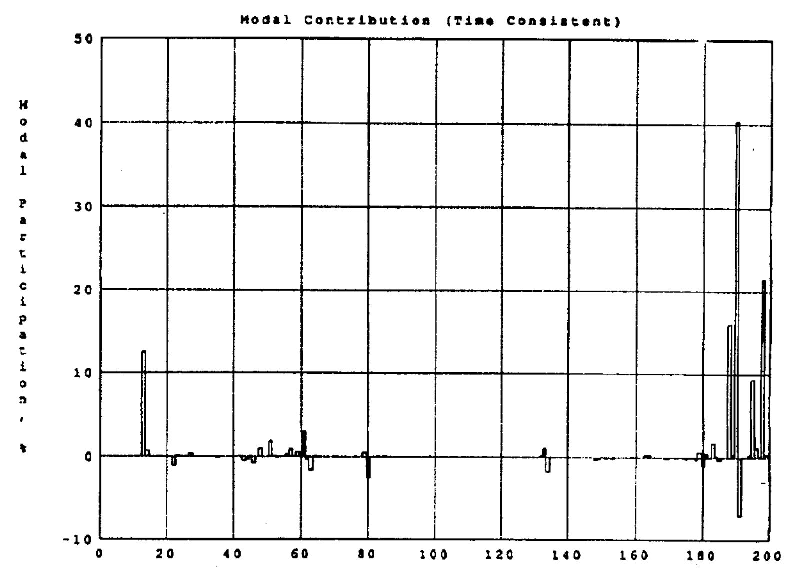

Figure 11. The Modal Contribution Plot for PV Array Mast Base Bending Moment.

Configuration	Procedure	CPU Time, Sec. MD / MA	I/O Time, Sec. MD / MA	Database, Mbytes MD / MA
SC-2	Eigenvalue Analysis	140 / 141	111 / 111	46.3 / 46.3
	Response Analysis	25 / 63	80 / 155	64.5 / 124
	Data Recovery	1059 / 1675	929 / 2098	n.a.
SC-7	Eigenvalue Analysis	288 / 286	210 / 212	75.2 / 76.5
	Response Analysis	33 / 106	125 / 255	106 / 217
	Data Recovery	1735 / 3031	1663 / 3855	n.a.
SC-17	Eigenvalue Analysis	860 / 857	695 / 700	157 / 160
	Response Analysis	82 / 367	291 / 645	251 / 507
	Data Recovery	1586 / 3720	2271 / 4465	n.a.

MD / MA: Mode Displacement Method / Mode Acceleration Method

Table 3. Computer Resources for SSF Transient Response Analyses.