USING DESIGN SENSITIVITY FOR STATISTICAL RESPONSE ANALYSIS

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Abstract

Statistical response analysis computes the statistical distribution of responses, given the distribution of design variable values. Variation in design variable values can result from manufacturing tolerances, and it can also be attributed to a level of uncertainty about the finite element input. Statistical distributions discussed herein are the standard deviation and variance.

Statistical response analysis can be accomplished in MSC/NASTRAN by using SOL 200 and DMAP alters in Version 67.5. This paper describes the theory, alters, and examples.

Introduction

Dimensions, material properties, and other physical properties have tolerances or variations that are generally not taken into account in structural analysis. For example, Young's modulus may be given as 30.0E6 psi for a particular grade of steel, yet if 100 specimens are sampled slight variations are found due to the manufacturing process.

In a statistical response analysis model parameters have a statistical distribution characterized by its mean, variance, and standard deviation. The computed response likewise has these same properties. In a "typical" structural analysis, the input values are analogous to the mean, and there is no variance or standard deviation. Similarly, the computed values represent the mean with no deviations.

The goal of statistical response analysis is to answer the question: "Given the variations in element and material properties, what are the variations in computed response?" This can be done in MSC/NASTRAN by using SOL 200 and DMAP alters.

Linear Statistical Model

A linear statistical model [Ref. 1] assumes that the structural response can be written as a Taylor's series:

$$\{r(p)\} = \{r(p_o)\} + [S]^T \{p-p_o\} \qquad [1]$$

where p denotes values of the perturbed design variables, p_o denotes values of the original design variables, $\{r(p)\}$ is a vector of response at perturbed values of the design variables, $\{r(p_o)\}$ is a vector of response at original values of the design variables, and [S] is the response sensitivity matrix ($S_{ij} = dr_i/dp_j$). The zero subscript refers to the mean or expected values.

Using this model, the covariance matrix of response is given by:

$$[COVR] = [S]^T [COVD] [S] \qquad [2]$$

where [COVR] is the covariance matrix of response (size n by n, for n responses), [COVD] is the covariance matrix of design variables (m by m, for m design variables), and [S] is the response sensitivity matrix (m by n).

Terms in [COVR] are:

$$[COVR] = \begin{bmatrix} VAR(1) & COV(1,2) & \dots & COV(1,n) \\ COV(2,1) & VAR(2) & \dots & COV(2,n) \\ \dots & \dots & \dots & \dots \\ COV(n,1) & COV(n,2) & \dots & VAR(n) \end{bmatrix} \qquad [3]$$

where COV(i,j) is the covariance between responses i and j and VAR(i) is the variance of the ith response.

Terms in [COVD] are:

$$[COVD] = \begin{bmatrix} VAR(1) & COV(1,2) & \dots & COV(1,m) \\ COV(2,1) & VAR(2) & \dots & COV(2,m) \\ \dots & \dots & \dots & \dots \\ COV(m,1) & COV(m,2) & \dots & VAR(m) \end{bmatrix} \qquad [4]$$

where COV(i,j) is the covariance between design variables i and j and VAR(i) is the variance of the ith design variable.

[COVD] is usually defined as a diagonal matrix; i.e., only variances are input and not covariances. This is analogous to saying that the design variables are uncorrelated. Off-diagonal terms in [COVD] represent correlation between design variables. One way to think of [COVD] is:

$$[COVD] = [STDVD]\,[CORR]\,[STDVD] \qquad [5]$$

where [STDVD] is a diagonal matrix of standard deviations (size m by m) and CORR is a matrix of correlation coefficients (m by m). Because the square of the standard deviation is the variance, terms in [STDVD] are the square of the diagonal terms in [COVD].

Terms in [CORR] are:

$$[CORR] = \begin{bmatrix} 1.0 & COR(1,2) & \dots & COR(1,m) \\ COR(2,1) & 1.0 & \dots & COR(2,m) \\ \dots & \dots & \dots & \dots \\ COR(m,1) & COR(m,2) & \dots & 1.0 \end{bmatrix} \qquad [6]$$

where COR(i,j) is the correlation between design variables i and j, with each term satisfying the relationship $0.0 \le COR(i,j) \le 1.0$.

For fully correlated design variables, all terms in [CORR] are 1.0. For uncorrelated design variables all off-diagonal terms are 0.0.

Note that the linear statistical model defined by Eqs. [1] and [2] is valid only for small changes in design variables (on the order of 5-10%, say, though this is governed by the linearity of the properties in the expansion region).

Implementation in MSC/NASTRAN Version 67.5

Statistical response analysis is implemented in MSC/NASTRAN via SOL 200 and a DMAP alter. Matrix [COVD] is input via DMI entries; [S] is computed in SOL 200 (and is called [DSCM2]); and [COVR] is computed via the alter. The alter and example files are supplied with Version 67.5; the alter is cova.v675. The alter is also listed in the appendix of this paper.

Input: Executive Control
Include 'cova.v675' (note: alter supplied with Version 67.5)

Case Control
None (other than standard design sensitivity requests)

Bulk Data

Three types of input are required:

1. Design sensitivity input (e.g., DESOBJ, DRESP1, DSCREEN, DCONSTR, DESVAR, DVPREL1)

2. PARAM,OPTEXIT,4 (to exit after sensitivity)

3. DMI entries for the design variable variances. This is a square matrix of size m by m, where m is the number of design variables. The name of this matrix is [COVD]. The order of the terms must match the order of the design variables (i.e., the 3,3 term in [COVD] is the third design variable).

Output: Five matrices are printed:

1. [DSCM2]--design sensitivity coefficient matrix (size m by n, where m is the number of design variables and n is the number of responses); computed in standard SOL 200

2. [COVD]--design variable covariances (m by m, square); user input

3. [COVR]--the response covariances (n by n, square, full); computed in the alter

4. [VAR]--diagonal terms of [COVR], giving the response variances (n by 1); computed in the alter

5. [SDEV]--square root of each term in [VAR], giving the standard deviations of responses (n by 1); computed in the alter

Example 1: Uncorrelated Variables

Consider the 10-bar truss shown on the next page. There are ten design variables (each one representing the area of a bar) and 37 responses (weight, plus eight displacements for each of two load subcases, plus ten stresses for each of two load subcases). In this example the design variables are assumed to be uncorrelated, so [COVD] is a diagonal matrix. The standard deviations of the bar areas range from 1 to 10% of the nominal value, averaging roughly 5% (actual values are listed in the input file in the appendix). This example is supplied with Version 67.5; the file is cov1.dat. The input file is shown beginning on page A-3.

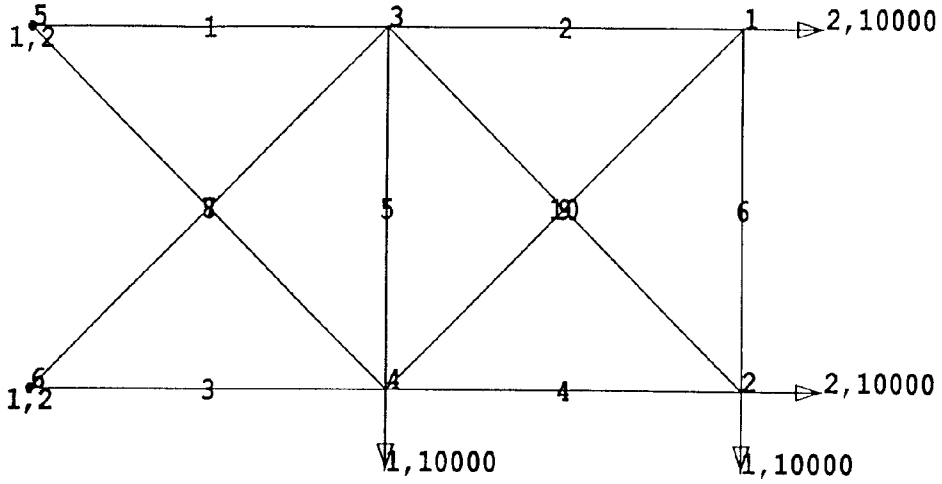

Figure 1: 10-Bar Truss for Examples 1 and 2.

Output is shown beginning on page A-7. Consider the fifth response, which is the T2 displacement of grid point 1 for the first load subcase (see the Correlation Table on page A-7 for the order of the responses). The value of this displacement is -0.759 in.; the resulting standard deviation is 0.014 in. (about 2%), given by the fifth row in [SDEV]. Consider the 37th response, the axial stress in bar 10 for the second load case. The value of this stress is 524.4 psi; the resulting standard deviation is 41.4 psi (about 8%), given by the 37th row in [SDEV].

Example 2: Fully Correlated Variables

This example is the same as the first, with the difference being that now all the design variables are fully correlated. "Fully correlated" means that whatever gives rise to variation in one design variable gives rise to a similar variation in the other design variables. [COVD] is now a full matrix, with each term computed via Eqs. [5] and [6] (note that all [CORR] terms are 1.0 in Eq. [6]). Matrix [COVD] input is shown on page A-11. This example is also supplied with Version 67.5; the file is cov2.dat.

Output is shown beginning on page A-11. Initial displacements, stresses, the Correlation Table, and the matrix [DSCM2] are the same as for the first example; matrices [COVD], [COVR], [VAR], and [SDEV] are different now. Consider again the fifth response, the T2 displacement of grid point 1 for the first load case. The value for this displacement is -0.759 in.; the resulting standard deviation is now 0.034 in. (contrast this with 0.014 from the uncorrelated example). Consider again the 37th response, the axial stress in bar 10 for the second load case. The value of this stress is 524.4 psi; the resulting standard deviation is now 30.9 psi (contrast this with 41.4 from the uncorrelated example).

Results Interpretation

The standard deviation is the most used statistical property in the statistical response analysis of structures. The statistical response is often written as the mean plus or minus the standard deviation. For instance, in the uncorrelated example the axial stress in bar 10 for the second load case is written as 524.4 ± 41.4 psi.

The standard deviation also gives a confidence level in the results. Assuming a Gaussian probability distribution, there is a 68% degree of confidence that the response falls within the bound given by the standard deviation. For the case in the previous paragraph, we have, then, a 68% degree of confidence that the stress is between 483.0 and 565.8 psi. There is a 95% confidence that the results are bounded by two standard deviations (i.e., between 441.6 and 607.2 psi), and there is a 99% confidence that the results are bounded by three standard deviations (i.e., between 400.2 and 648.6 psi).

Degree of Correlation Between Design Variables

The degree of cross-correlation between design variables ranges from 0 (no correlation) to 1 (fully correlated), with values in between denoting partial correlation. Assessment of the degree of design variable correlation is a topic for statistics courses, but in general the degree of correlation can be assessed by answering the question, "There is a statistical variation in design variable i due to some natural process or mechanism; to what degree does this process give rise to a similar variation in design variable j?" If the answer is "none," then there is <u>no</u> correlation between design variables i and j. If the answer is "some," then there is <u>some</u> correlation between the two design variables, the degree of which can be represented by a fractional number between 0 and 1.

This paper does not attempt to begin to address how to define the degree of correlation. However, if the degree can be defined then the method described herein can use it.

Summary

This paper has described a method to compute the standard deviation and variance of structural response, given the variation in design variables. This method uses SOL 200 and DMAP alters to compute the statistical distribution of responses. These statistical properties provide confidence levels associated with the computed responses.

Reference

1. G. C. Hart and J. D. Collins, "The Treatment of Randomness in Finite Element Modeling," SAE Paper 700842, 1970.

Appendix: Input and Output Files

Alter for statistical analysis
cova.v675

```
$  ALTER FOR RESPONSE VARIANCES
$
COMPILE DESOPT, SOUIN=MSCSOU, NOLIST, NOREF
$
ALTER 450  $ VERSION 67.5
$
$-----------------------------------------------------------------------
$
$  COMPUTE VARIANCES IN RESPONSE DUE TO VARIANCES IN DESIGN VARIABLES
$
$  ALTER FOR VERSION 67.5, SOL 200
$
$  FORMULAS: COVR = (DSCM2)T * COVD * DSCM2     (T-TRANSPOSE)
$            VAR  = DIAGONAL TERMS OF COVR
$            SDEV = SQRT(VAR)
$
$  WHERE:    COVD  =  COVARIANCE MATRIX OF DESIGN VARIABLES
$                       (USER INPUT VIA DMI, IN ORDER OF DESIGN VARIABLES)
$            DSCM2 =  SENSITIVITY MATRIX
$                       (COMPUTED IN SOL 200)
$            COVR  =  COVARIANCE MATRIX OF RESPONSES
$                       (COMPUTED IN ALTER)
$                       (SEE CORRELATION TABLE FOR ORDER OF RESPONSES)
$                       (CORRELATION TABLE OUTPUT IN SOL 200)
$            VAR   =  VARIANCE OF RESPONSES
$                       (COMPUTED IN ALTER)
$                       (SEE CORRELATION TABLE FOR ORDER OF RESPONSES)
$            SDEV  =  STANDARD DEVIATION OF RESPONSES
$                       (COMPUTED IN ALTER)
$                       (SEE CORRELATION TABLE FOR ORDER OF RESPONSES)
$
$   NOTE:  MATRIX DEFINITIONS PRINTED  U N D E R  EACH MATRIX IN PRINTOUT
$
$
$   USER INPUT:
$
$          Design optimization entries
$          (make sure to use DSCREEN to retain all responses)
$
$          PARAM,OPTEXIT,4     $ Exit after sensitivity
$
$          PARAM,NASPRT,1      $ Data recovery at all cycles
$
$          COVD  (input as DMI)
$          (square matrix of size n by n, where n = no. design variables)
$
$          --- For uncorrelated design variables, the input is the
$              variance of each design variable; i.e.,
$
$               COVD(i,i) = STDEV(i)*STDEV(i)  where STDEV = standard deviation
$
$                Example:
```

```
$                DMI,COVD,0,1,1,0,,n,n  (n = no. of design variables)
$                DMI,COVD,1,1,v1        (v1 = variance of 1st des. variable)
$                DMI,COVD,2,2,v2        (v2 = variance of 2nd des. variable)
$                ...
$                DMI,COVD,n,n,vn        (vn = variance of nth des. variable)
$
$          --- For CORRELATED design variables, the input is given by
$
$               COVD(i,j) = STDEV(i)*STDEV(j)*P(i,j)
$
$                where P(i,j) = correlation coefficient (1>P>0)
$                             = 1.0 for fully correlated
$                             = 0.0 for uncorrelated (reduces to above case)
$                Note: for i =j,  P(i,j) = 1.0 for all cases
$
$               Example:
$                DMI,COVD,0,1,1,0,,n,n
$                DMI,COVD,1,1,s1s1,s1s2p12,s1s3p13,s1s4p14,s1s5p15,+cv1
$                +cv1,s1s6p16,.....
$                DMI,COVD,2,1,s2s1p21,s2s2,s2s3p23,........
$                ...
$                DMI,COVD,n,1,sns1pn1,....................
$
$                  where  si=STDEV(i), pij=P(i,j)
$
$
$    *******  NOTE!!  *******  NOTE!!  *******  NOTE!!  *******
$
$    Size of DMI is number of INDEPENDENT design variables
$    If have LINKing then must reduce size of DMI to correct size
$
$-----------------------------------------------------------------------
$
$  READ DMI INPUT -- COVD (DESIGN VARIABLE VARIANCES)
$
DMIIN  DMI,DMINDX/COVD,,,,,,,,,/S,N,YESCOVD/  $
$
$  MULTIPLY (DSCM2)TRANS * COVD = TMP1
$
MPYAD  DSCM2,COVD,/ TMP1 /1///$
$
$  MULTIPLY TMP1 * DSCM2 = COVR (RESPONSE COVARIANCES)
$
MPYAD  TMP1,DSCM2,/ COVR // $
$
$  PRINT TITLE AND COVD
$
MATPRN  COVD,,,,// $
MESSAGE  // 'VARIANCES OF DESIGN VARIABLES'/  $
MESSAGE  // 'USER INPUT, IN ORDER OF DESIGN VARIABLES'/ $
$
$  PRINT TITLE AND COVR
$
MATPRN  COVR,,,,// $
MESSAGE // 'RESPONSE COVARIANCE MATRIX'/ $
MESSAGE // 'SEE CORRELATION TABLE FOR ORDER OF RESPONSES'/ $
$
$  SET VAR AS VECTOR OF VARIANCES OF RESPONSES
```

```
$
DIAGONAL  COVR/VAR/'COLUMN'/ $
$
$  PRINT TITLE AND VAR
$
MATPRN  VAR,,,,// $
MESSAGE // 'VARIANCE OF RESPONSES (DIAGONAL TERMS OF COVR)'/ $
MESSAGE // 'SEE CORRELATION TABLE FOR ORDER OF RESPONSES'/ $
$
$  SET SDEV AS STANDARD DEVIATIONS OF RESPONSES
$
DIAGONAL  COVR/SDEV/'COLUMN'/0.5 $
$
$  PRINT TITLE AND SDEV
$
MATPRN  SDEV,,,,// $
MESSAGE // 'STANDARD DEVIATION OF RESPONSES'/ $
MESSAGE // 'SEE CORRELATION TABLE FOR ORDER OF RESPONSES'/ $
$
ENDALTER
```

Input file for uncorrelated design variables cov1.dat

```
ID TRUSS, DESOPT
TIME 30
$
SOL 200
$
INCLUDE 'cova.v675'
$
CEND
TITLE =    TRUSS EXAMPLE FOR SIZING OPTIMIZATION
SUBTITLE =     DIFFERENT VARIANCES
DISPLACEMENT = ALL
OLOAD = ALL
STRESS = ALL
SPC   = 1
$
PARAM,APPC,STATICS
$
SUBCASE = 1
 LOAD = 1000
 LABEL = VERTICAL LOAD
SUBCASE = 2
 LOAD = 2000
 LABEL = HORIZONTAL LOAD
$
$
BEGIN BULK
$
$PARAM,POST,0
$
MAT1,   1, 1.000E7, ,0.3 ,0.1 ,0.0     ,0.0
PROD ,   1,   1 ,5.0    ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,   1,   1,   5,   3
PROD ,   2,   1 ,5.0     ,0.0     ,0.0     ,0.0     ,        ,        ,
```

```
CROD ,   2,   2,   3,   1
PROD ,   3,   1 ,5.0    ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,   3,   3,   6,   4
PROD ,   4,   1 ,5.0    ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,   4,   4,   4,   2
PROD ,   5,   1 ,5.0     ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,   5,   5,   3,   4
PROD ,   6,   1 ,5.0     ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,   6,   6,   1,   2
PROD ,   7,   1 ,5.0     ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,   7,   7,   5,   4
PROD ,   8,   1 ,5.0     ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,   8,   8,   6,   3
PROD ,   9,   1 ,5.0    ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,   9,   9,   3,   2
PROD ,  10,   1 ,5.0     ,0.0     ,0.0     ,0.0     ,        ,        ,
CROD ,  10,  10,   4,   1
FORCE ,  1,   2,,1.      ,0.0     ,-10000. ,0.0
FORCE ,  1,   4,,1.      ,0.0     ,-10000. ,0.0
FORCE ,  2,   1,,1.      ,10000. ,0.0 ,0.0
FORCE ,  2,   2,,1.      ,10000. ,0.0 ,0.0
SPC,1,   5,   1,0.0
SPC,1,   5,   2,0.0
SPC,1,   6,   1,0.0
SPC,1,   6,   2,0.0
LOAD,1000,1.0,1.0,1
LOAD,2000,1.0,1.0,2
GRID,   1,,720.    ,360.    ,0.0     ,,  3456
GRID,   2,,720.    ,0.0     ,0.0     ,,  3456
GRID,   3,,360.    ,360.    ,0.0     ,,  3456
GRID,   4,,360.    ,0.0     ,0.0     ,,  3456
GRID,   5,,0.0     ,360.    ,0.0     ,,  3456
GRID,   6,,0.0     ,0.0     ,0.0     ,,  3456
PARAM,AUTOSPC,YES
$
$-----------------------------------------------------------------------
$
$  DMI INPUT FOR COVARIANCE MATRIX OF DESIGN VARIABLES
$
$   IN THIS EXAMPLE, STD. DEVS. RANGE FROM 1 TO 10%
$    PROD(I) HAS STD. DEV. OF I% (I.E., 1 = 1%, 2 = 2%, ETC.)
$
$    EACH ROD AREA = 5.0, SO:
$      10% IS 0.5; VARIANCE IS (STD. DEV.)**2, OR 0.5**2 = 0.25
$       9% IS 0.45; VAR. = 0.2025
$       8% IS 0.4; VAR. = 0.16
$       7% IS 0.35; VAR. = 0.1125
$       6% IS 0.3; VAR. = 0.09
$       5% IS 0.25; VAR. = 0.0625
$       4% IS 0.2; VAR. = 0.04
$       3% IS 0.15; VAR. = 0.0225
$       2% IS 0.10; VAR. = 0.01
$       1% IS 0.05; VAR. = 0.0025
$
DMI,COVD,0,1,1,0,,10,10
DMI,COVD,1,1,0.0025
DMI,COVD,2,2,0.01
DMI,COVD,3,3,0.0225
```

```
DMI,COVD,4,4,0.04
DMI,COVD,5,5,0.0625
DMI,COVD,6,6,0.09
DMI,COVD,7,7,0.1125
DMI,COVD,8,8,0.16
DMI,COVD,9,9,0.2025
DMI,COVD,10,10,0.25
$
$-----------------------------------------------------------------------
$
$  DESIGN OPTIMIZATION INPUT
$
$  EXIT AFTER SENSITIVITY ANALYSIS
$
PARAM,OPTEXIT,4
$
$  SET PARAMETER TO PERFORM DATA RECOVERY AT ALL DESIGN CYCLES
$
PARAM,NASPRT,1
$
$  RETAIN ALL RESPONSES
$
DSCREEN,999,DISP,-1.0E7,1000
DSCREEN,998,STRESS,-1.0E7,1000
$
$  DEFINE OBJECTIVE FUNCTION
$
DESOBJ,11,WEIGHT
$
$  DEFINE RESPONSES:  WEIGHT, DISPLS, STRESS
$
DRESP1,11,WEIGHT,WEIGHT
DRESP1,101,X1,DISP,,,1,,1
DRESP1,102,X2,DISP,,,1,,2
DRESP1,103,X3,DISP,,,1,,3
DRESP1,104,X4,DISP,,,1,,4
DRESP1,201,Y1,DISP,,,2,,1
DRESP1,202,Y2,DISP,,,2,,2
DRESP1,203,Y3,DISP,,,2,,3
DRESP1,204,Y4,DISP,,,2,,4
DRESP1,301,S1,STRESS,PROD,,2,,1
DRESP1,302,S2,STRESS,PROD,,2,,2
DRESP1,303,S3,STRESS,PROD,,2,,3
DRESP1,304,S4,STRESS,PROD,,2,,4
DRESP1,305,S5,STRESS,PROD,,2,,5
DRESP1,306,S6,STRESS,PROD,,2,,6
DRESP1,307,S7,STRESS,PROD,,2,,7
DRESP1,308,S8,STRESS,PROD,,2,,8
DRESP1,309,S9,STRESS,PROD,,2,,9
DRESP1,310,S10,STRESS,PROD,,2,,10
$
$  STRESS CONSTRAINTS
$
DCONSTR,301,ALL,-3000.,3000.
DCONSTR,302,ALL,-3000.,3000.
DCONSTR,303,ALL,-3000.,3000.
DCONSTR,304,ALL,-3000.,3000.
DCONSTR,305,ALL,-3000.,3000.
```

```
DCONSTR,306,ALL,-3000.,3000.
DCONSTR,307,ALL,-3000.,3000.
DCONSTR,308,ALL,-3000.,3000.
DCONSTR,309,ALL,-3000.,3000.
DCONSTR,310,ALL,-3000.,3000.
$
$  DISPLACEMENT CONSTRAINTS
$
DCONSTR,101,ALL,-1.0,1.0
DCONSTR,102,ALL,-1.0,1.0
DCONSTR,103,ALL,-1.0,1.0
DCONSTR,104,ALL,-1.0,1.0
DCONSTR,201,ALL,-1.0,1.0
DCONSTR,202,ALL,-1.0,1.0
DCONSTR,203,ALL,-1.0,1.0
DCONSTR,204,ALL,-1.0,1.0
$
$  SIZING DESIGN VARIABLES:  ROD AREAS
$
DESVAR,1,A1,5.0,0.01,10.0
DESVAR,2,A2,5.0,0.01,10.0
DESVAR,3,A3,5.0,0.01,10.0
DESVAR,4,A4,5.0,0.01,10.0
DESVAR,5,A5,5.0,0.01,10.0
DESVAR,6,A6,5.0,0.01,10.0
DESVAR,7,A7,5.0,0.01,10.0
DESVAR,8,A8,5.0,0.01,10.0
DESVAR,9,A9,5.0,0.01,10.0
DESVAR,10,A10,5.0,0.01,10.0
$
$  RELATE SIZING DESIGN VARIABLES TO PROPERTIES
$
DVPREL1,1,PROD,1,4,,,,,+DV1
+DV1,1,1.0
DVPREL1,2,PROD,2,4,,,,,+DV2
+DV2,2,1.0
DVPREL1,3,PROD,3,4,,,,,+DV3
+DV3,3,1.0
DVPREL1,4,PROD,4,4,,,,,+DV4
+DV4,4,1.0
DVPREL1,5,PROD,5,4,,,,,+DV5
+DV5,5,1.0
DVPREL1,6,PROD,6,4,,,,,+DV6
+DV6,6,1.0
DVPREL1,7,PROD,7,4,,,,,+DV7
+DV7,7,1.0
DVPREL1,8,PROD,8,4,,,,,+DV8
+DV8,8,1.0
DVPREL1,9,PROD,9,4,,,,,+DV9
+DV9,9,1.0
DVPREL1,10,PROD,10,4,,,,,+DV10
+DV10,10,1.0
$
ENDDATA
```

Output from uncorrelated design variable example cov1.f06

VERTICAL LOAD SUBCASE = 1

D I S P L A C E M E N T V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.695525E-01	-7.590253E-01	0.0	0.0	0.0	0.0
2	G	-1.904475E-01	-7.879150E-01	0.0	0.0	0.0	0.0
3	G	1.406628E-01	-3.348705E-01	0.0	0.0	0.0	0.0
4	G	-1.473372E-01	-3.604230E-01	0.0	0.0	0.0	0.0
5	G	0.0	0.0	0.0	0.0	0.0	0.0
6	G	0.0	0.0	0.0	0.0	0.0	0.0

HORIZONTAL LOAD SUBCASE = 2

D I S P L A C E M E N T V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.155589E-01	-6.674419E-03	0.0	0.0	0.0	0.0
2	G	1.155589E-01	6.674419E-03	0.0	0.0	0.0	0.0
3	G	5.690778E-02	-1.422053E-02	0.0	0.0	0.0	0.0
4	G	5.690778E-02	1.422053E-02	0.0	0.0	0.0	0.0
5	G	0.0	0.0	0.0	0.0	0.0	0.0
6	G	0.0	0.0	0.0	0.0	0.0	0.0

VERTICAL LOAD SUBCASE = 1

S T R E S S E S I N R O D E L E M E N T S (C R O D)

ELEMENT ID.	AXIAL STRESS	SAFETY MARGIN	TORSIONAL STRESS	SAFETY MARGIN	ELEMENT ID.	AXIAL STRESS	SAFETY MARGIN	TORSIONAL STRESS	SAFETY MARGIN
1	3.907300E+03		0.0		2	8.024926E+02		0.0	
3	-4.092700E+03		0.0		4	-1.197507E+03		0.0	
5	7.097924E+02		0.0		6	8.024926E+02		0.0	
7	2.959525E+03		0.0		8	-2.697329E+03		0.0	
9	1.693531E+03		0.0		10	-1.134896E+03		0.0	

HORIZONTAL LOAD SUBCASE = 2

S T R E S S E S I N R O D E L E M E N T S (C R O D)

ELEMENT ID.	AXIAL STRESS	SAFETY MARGIN	TORSIONAL STRESS	SAFETY MARGIN	ELEMENT ID.	AXIAL STRESS	SAFETY MARGIN	TORSIONAL STRESS	SAFETY MARGIN
1	1.580772E+03		0.0		2	1.629199E+03		0.0	
3	1.580772E+03		0.0		4	1.629199E+03		0.0	
5	-7.900294E+02		0.0		6	-3.708011E+02		0.0	
7	5.928785E+02		0.0		8	5.928785E+02		0.0	
9	5.243918E+02		0.0		10	5.243918E+02		0.0	

----- CORRELATION TABLE BETWEEN SENSITIVITY MATRIX COLUMNS -----
----- AND THE DRESP1 RESPONSES ASSOCIATED WITH THESE COLUMNS -----

COLUMN NO.	DRESP1 ENTRY ID	RESPONSE TYPE	GRID/ELM ID	COMPONENT NO.	SUB CASE	FREQ/ TIME
1	11	WEIGHT	0	0	1	
2	202	DISP	2	2	1	
3	203	DISP	3	2	1	
4	204	DISP	4	2	1	
5	201	DISP	1	2	1	

```
                 6          104          DISP          4              1          1
                 7          103          DISP          3              1          1
                 8          102          DISP          2              1          1
                 9          101          DISP          1              1          1
                10          309        STRESS          9              2          1
                11          308        STRESS          8              2          1
                12          307        STRESS          7              2          1
                13          306        STRESS          6              2          1
                14          305        STRESS          5              2          1
                15          304        STRESS          4              2          1
                16          303        STRESS          3              2          1
                17          302        STRESS          2              2          1
                18          301        STRESS          1              2          1
                19          310        STRESS         10              2          1
                20          202          DISP          2              2          2
                21          203          DISP          3              2          2
                22          204          DISP          4              2          2
                23          201          DISP          1              2          2
                24          104          DISP          4              1          2
                25          103          DISP          3              1          2
                26          102          DISP          2              1          2
                27          101          DISP          1              1          2
                28          309        STRESS          9              2          2
                29          308        STRESS          8              2          2
                30          307        STRESS          7              2          2
                31          306        STRESS          6              2          2
                32          305        STRESS          5              2          2
                33          304        STRESS          4              2          2
                34          303        STRESS          3              2          2
                35          302        STRESS          2              2          2
                36          301        STRESS          1              2          2
                37          310        STRESS         10              2          2
^^^ DMAP INFORMATION MESSAGE 9029 (DESOPT) -    DESIGN SENSITIVITY COEFFICIENT MATRIX FOR DIRECT
    AND SYNTHETIC RESPONSES - GRADIENTS OF RESPONSES    WITH RESPECT TO INDEPENDENT DESIGN VARIABLES

      MATRIX DSCM2     (GINO NAME 101 ) IS A REAL         37 COLUMN X     10 ROW RECTANG  MATRIX.
COLUMN       1       ROWS       1 THRU      10     ------------------------------------------------
ROW
     1)    3.6000E+01  3.6000E+01  3.6000E+01  3.6000E+01  3.6000E+01  3.6000E+01  5.0912E+01  5.0912E+01  5.0912E+01  5.0912E+01
COLUMN       2       ROWS       1 THRU      10     ------------------------------------------------
ROW
     1)    4.2369E-02  2.5856E-03  4.4023E-02  4.7624E-03 -2.3723E-04  2.5856E-03  2.1050E-02  1.9656E-02  1.3472E-02  7.3147E-03
...
COLUMN      37       ROWS       1 THRU      10     ------------------------------------------------
ROW
     1)    5.4138E+00 -4.8296E+01  5.4138E+00 -4.8296E+01  2.0714E+01  1.0992E+01 -4.0601E+00 -4.0601E+00  3.1090E+01 -7.3788E+01
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =      10
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.


      MATRIX COVD      (GINO NAME 101 ) IS A DB  PREC     10 COLUMN X     10 ROW SQUARE   MATRIX.
COLUMN       1       ROWS       1 THRU       1     ------------------------------------------------
ROW
     1)    2.5000D-03
COLUMN       2       ROWS       2 THRU       2     ------------------------------------------------
ROW
     2)    1.0000D-02
COLUMN       3       ROWS       3 THRU       3     ------------------------------------------------
```

```
ROW
     3)    2.2500D-02
COLUMN      4      ROWS      4 THRU      4     ------------------------------------------------
ROW
     4)    4.0000D-02
COLUMN      5      ROWS      5 THRU      5     ------------------------------------------------
ROW
     5)    6.2500D-02
COLUMN      6      ROWS      6 THRU      6     ------------------------------------------------
ROW
     6)    9.0000D-02
COLUMN      7      ROWS      7 THRU      7     ------------------------------------------------
ROW
     7)    1.1250D-01
COLUMN      8      ROWS      8 THRU      8     ------------------------------------------------
ROW
     8)    1.6000D-01
COLUMN      9      ROWS      9 THRU      9     ------------------------------------------------
ROW
     9)    2.0250D-01
COLUMN     10      ROWS     10 THRU     10     ------------------------------------------------
ROW
    10)    2.5000D-01
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =       1
THE DENSITY OF THIS MATRIX IS  10.00 PERCENT.
^^^VARIANCES OF DESIGN VARIABLES
^^^USER INPUT, IN ORDER OF DESIGN VARIABLES

      MATRIX COVR     (GINO NAME 101 ) IS A DB  PREC     37 COLUMN X     37 ROW SQUARE   MATRIX.
COLUMN      1      ROWS      1 THRU     37     ------------------------------------------------
ROW
     1)    2.1740D+03  5.6778D-01  2.8988D-01  2.9703D-01  5.4585D-01  3.1461D-02  5.0569D-03  4.0658D-02 -2.3957D-04 -3.3653D+03
    11)    4.0963D+03 -3.6883D+03 -6.0950D+02 -1.9811D+02  2.5535D+02  8.7373D+02 -1.4725D+02  1.4040D+02  3.0156D+03 -3.4004D-02
    21)   -2.2441D-02 -6.4001D-03 -4.2157D-02 -2.1254D-02 -1.3060D-02 -5.4945D-02 -3.4081D-02 -4.2118D+02 -4.9308D+02 -2.0633D+02
    31)   -2.2644D+02 -4.4556D+02 -9.3598D+02 -5.9045D+02 -5.8407D+02 -3.6281D+02 -6.7480D+02
...
COLUMN     37      ROWS      1 THRU     37     ------------------------------------------------
ROW
     1)   -6.7480D+02 -7.4764D-02 -4.3670D-02  2.9362D-03 -1.2540D-01  8.5058D-03  4.5361D-03 -5.3067D-02 -3.7590D-02 -3.6806D+02
    11)   -5.4358D+02  7.7219D+01 -1.4064D+03 -1.2948D+03 -1.7103D+03  2.3627D+02 -1.1701D+03  1.2599D+02 -2.4227D+03  9.6794D-02
    21)    6.5553D-03 -6.1412D-03  1.0499D-01 -1.6098D-03 -3.7740D-04  2.6606D-02  1.0842D-02 -8.7853D+02  8.5801D+01  6.2934D+01
    31)    2.2768D+02  3.5268D+02  7.8380D+02 -4.4708D+01  3.1170D+02 -1.0476D+01  1.7164D+03
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =      37
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
^^^RESPONSE COVARIANCE MATRIX
^^^SEE CORRELATION TABLE FOR ORDER OF RESPONSES

      MATRIX VAR      (GINO NAME 101 ) IS A DB  PREC      1 COLUMN X     37 ROW RECTANG  MATRIX.
COLUMN      1      ROWS      1 THRU     37     ------------------------------------------------
ROW
     1)    2.1740D+03  2.1147D-04  1.1979D-04  1.1775D-04  2.0576D-04  2.1402D-05  7.9491D-06  2.4724D-05  8.6948D-06  1.2866D+04
    11)    2.5695D+04  2.3275D+04  3.5362D+03  6.0544D+03  3.5173D+03  1.6514D+04  1.9074D+03  6.1333D+03  8.6514D+03  2.0695D-05
    21)    6.2814D-06  6.1901D-06  2.0598D-05  2.5759D-06  5.9094D-07  7.4061D-06  1.9957D-06  1.5037D+03  1.2909D+03  1.0647D+03
    31)    6.8968D+02  1.4216D+03  3.6552D+03  1.9877D+03  1.1377D+03  4.5601D+02  1.7164D+03
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =      37
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
^^^VARIANCE OF RESPONSES (DIAGONAL TERMS OF COVR)
^^^SEE CORRELATION TABLE FOR ORDER OF RESPONSES
```

```
     MATRIX SDEV     (GINO NAME 101 ) IS A DB  PREC      1 COLUMN X     37 ROW RECTANG  MATRIX.
COLUMN      1       ROWS      1 THRU     37     ------------------------------------------------
ROW
    1)    4.6627D+01  1.4542D-02  1.0945D-02  1.0851D-02  1.4344D-02  4.6262D-03  2.8194D-03  4.9723D-03  2.9487D-03  1.1343D+02
   11)    1.6030D+02  1.5256D+02  5.9466D+01  7.7810D+01  5.9307D+01  1.2851D+02  4.3674D+01  7.8315D+01  9.3013D+01  4.5491D-03
   21)    2.5063D-03  2.4880D-03  4.5385D-03  1.6050D-03  7.6873D-04  2.7214D-03  1.4127D-03  3.8778D+01  3.5929D+01  3.2630D+01
   31)    2.6262D+01  3.7704D+01  6.0458D+01  4.4583D+01  3.3729D+01  2.1354D+01  4.1430D+01
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =      37
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
^^^STANDARD DEVIATION OF RESPONSES
^^^SEE CORRELATION TABLE FOR ORDER OF RESPONSES
```

```
*******************************************************
S U M M A R Y   O F   I T E R A T I O N   H I S T O R Y
*******************************************************

 NUMBER OF FINITE ELEMENT ANALYSES COMPLETED          1
 NUMBER OF OPTIMIZATIONS W.R.T. APPROXIMATE MODELS    0
```

OBJECTIVE FUNCTION HISTORY

ITERATION NUMBER	OPTIMAL WITH RESPECT TO APPROXIMATION	EXACT EVALUATION BY COMPLETE ANALYSIS	FRACTIONAL ERROR OF APPROXIMATION	MAXIMUM VALUE OF CONSTRAINTS
INITIAL		0.209823E+04		0.364233E+00

DESIGN VARIABLE HISTORY

DV. ID.	INITIAL	1	2	3	4	5	6	7
1	0.5000E+01							
2	0.5000E+01							
3	0.5000E+01							
4	0.5000E+01							
5	0.5000E+01							
6	0.5000E+01							
7	0.5000E+01							
8	0.5000E+01							
9	0.5000E+01							
10	0.5000E+01							

```
^^^ DMAP INFORMATION MESSAGE 9030 (DESOPT) -    RUN TERMINATED DUE TO OPTEXIT=          4
```

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Input file for fully correlated design variables cov2.dat

```
...
BEGIN BULK
$-----------------------------------------------------------------------
$
$  DMI INPUT FOR COVARIANCE MATRIX OF DESIGN VARIABLES
$
$      EACH ROD AREA = 5.0, SO A STANDARD DEVIATION OF X% IS:
$        10% IS 0.5, 9% IS 0.45, 8% IS 0.4, 7% IS 0.35, 6% IS 0.3,
$        5% IS 0.25, 4% IS 0.2, 3% IS 0.15, 2% IS 0.10, 1% IS 0.05
$
$     COVD(I,J) = SDEV(I)*SDEV(J)*P(I,J)
$
$         SDEV = STANDARD DEVIATION OF INPUT
$         P(I,J) = CORRELATION COEFFICIENT
$                = 0.0 FOR UNCORRELATED
$                = 1.0 FOR FULLY CORRELATED
$
$    IN THIS EXAMPLE, DESIGN VARIABLES ARE FULLY CORRELATED
$
DMI,COVD,0,1,1,0,,10,10
DMI,COVD,1,1,0.0025,0.005,0.0075,0.01,0.0125,+CV1
+CV1,0.015,0.0175,0.02,0.0225,0.025
DMI,COVD,2,1,0.005,0.01,0.0150,0.02,0.025,+CV2
+CV2,0.030,0.035,0.040,0.045,0.050
DMI,COVD,3,1,0.0075,0.015,0.0225,0.030,0.0375,+CV3
+CV3,0.045,0.0525,0.060,0.0675,0.075
DMI,COVD,4,1,0.010,0.020,0.030,0.040,0.050,+CV4
+CV4,0.060,0.070,0.080,0.090,0.10
DMI,COVD,5,1,0.0125,0.025,0.0375,0.05,0.0625,+CV5
+CV5,0.075,0.0875,0.10,0.1125,0.125
DMI,COVD,6,1,0.015,0.03,0.045,0.06,0.075,+CV6
+CV6,0.09,0.105,0.12,0.135,0.15
DMI,COVD,7,1,0.0175,0.035,0.0525,0.07,0.0875,+CV7
+CV7,0.105,0.1225,0.14,0.1575,0.175
DMI,COVD,8,1,0.020,0.040,0.060,0.080,0.10,+CV8
+CV8,0.12,0.14,0.16,0.18,0.20
DMI,COVD,9,1,0.0225,0.045,0.0675,0.09,0.1125,+CV9
+CV9,0.135,0.1575,0.18,0.2025,0.225
DMI,COVD,10,1,0.025,0.05,0.075,0.1,0.125,+CV10
+CV10,0.15,0.175,0.2,0.225,0.25
...
ENDDATA
```

Output from fully correlated design variable example cov2.f06

```
      MATRIX COVD     (GINO NAME 101 ) IS A DB  PREC     10 COLUMN X     10 ROW SQUARE   MATRIX.
COLUMN      1       ROWS      1 THRU     10     ------------------------------------------------
ROW
    1)    2.5000D-03  5.0000D-03  7.5000D-03  1.0000D-02  1.2500D-02  1.5000D-02  1.7500D-02  2.0000D-02  2.2500D-02  2.5000D-02
COLUMN      2       ROWS      1 THRU     10     ------------------------------------------------
ROW
```

```
     1)    5.0000D-03  1.0000D-02  1.5000D-02  2.0000D-02  2.5000D-02  3.0000D-02  3.5000D-02  4.0000D-02  4.5000D-02  5.0000D-02
COLUMN      3       ROWS      1 THRU     10      ------------------------------------------------
ROW
     1)    7.5000D-03  1.5000D-02  2.2500D-02  3.0000D-02  3.7500D-02  4.5000D-02  5.2500D-02  6.0000D-02  6.7500D-02  7.5000D-02
COLUMN      4       ROWS      1 THRU     10      ------------------------------------------------
ROW
     1)    1.0000D-02  2.0000D-02  3.0000D-02  4.0000D-02  5.0000D-02  6.0000D-02  7.0000D-02  8.0000D-02  9.0000D-02  1.0000D-01
COLUMN      5       ROWS      1 THRU     10      ------------------------------------------------
ROW
     1)    1.2500D-02  2.5000D-02  3.7500D-02  5.0000D-02  6.2500D-02  7.5000D-02  8.7500D-02  1.0000D-01  1.1250D-01  1.2500D-01
COLUMN      6       ROWS      1 THRU     10      ------------------------------------------------
ROW
     1)    1.5000D-02  3.0000D-02  4.5000D-02  6.0000D-02  7.5000D-02  9.0000D-02  1.0500D-01  1.2000D-01  1.3500D-01  1.5000D-01
COLUMN      7       ROWS      1 THRU     10      ------------------------------------------------
ROW
     1)    1.7500D-02  3.5000D-02  5.2500D-02  7.0000D-02  8.7500D-02  1.0500D-01  1.2250D-01  1.4000D-01  1.5750D-01  1.7500D-01
COLUMN      8       ROWS      1 THRU     10      ------------------------------------------------
ROW
     1)    2.0000D-02  4.0000D-02  6.0000D-02  8.0000D-02  1.0000D-01  1.2000D-01  1.4000D-01  1.6000D-01  1.8000D-01  2.0000D-01
COLUMN      9       ROWS      1 THRU     10      ------------------------------------------------
ROW
     1)    2.2500D-02  4.5000D-02  6.7500D-02  9.0000D-02  1.1250D-01  1.3500D-01  1.5750D-01  1.8000D-01  2.0250D-01  2.2500D-01
COLUMN     10       ROWS      1 THRU     10      ------------------------------------------------
ROW
     1)    2.5000D-02  5.0000D-02  7.5000D-02  1.0000D-01  1.2500D-01  1.5000D-01  1.7500D-01  2.0000D-01  2.2500D-01  2.5000D-01
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =      10
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
^^^VARIANCES OF DESIGN VARIABLES
^^^USER INPUT, IN ORDER OF DESIGN VARIABLES


      MATRIX COVR     (GINO NAME 101 ) IS A DB  PREC     37 COLUMN X     37 ROW SQUARE   MATRIX.
COLUMN      1       ROWS      1 THRU     37      ------------------------------------------------
ROW
     1)    1.5463D+04  4.4267D+00  2.1636D+00  2.3479D+00  4.2008D+00  5.3488D-01 -1.8972D-01  7.3877D-01 -2.7207D-01 -1.8539D+04
    11)    2.7414D+04 -2.5180D+04 -6.2803D+03 -5.1172D+03  5.6631D+03  1.4856D+04 -2.2887D+03 -5.2702D+03  1.4527D+04 -3.5092D-01
    21)   -7.8927D-02 -1.1569D-01 -3.1957D-01 -2.8411D-01 -1.4258D-01 -6.4406D-01 -3.5667D-01 -3.1876D+03 -3.0767D+03 -2.3394D+03
    31)    8.7091D+02  1.0212D+03 -9.9997D+03 -7.8922D+03 -5.9478D+03 -3.9608D+03 -3.8397D+03
...
COLUMN     37       ROWS      1 THRU     37      ------------------------------------------------
ROW
     1)   -3.8397D+03 -1.0992D+00 -5.3725D-01 -5.8302D-01 -1.0431D+00 -1.3282D-01  4.7110D-02 -1.8345D-01  6.7560D-02  4.6034D+03
    11)   -6.8074D+03  6.2526D+03  1.5595D+03  1.2707D+03 -1.4062D+03 -3.6890D+03  5.6831D+02  1.3087D+03 -3.6073D+03  8.7139D-02
    21)    1.9599D-02  2.8727D-02  7.9355D-02  7.0549D-02  3.5405D-02  1.5993D-01  8.8566D-02  7.9154D+02  7.6399D+02  5.8092D+02
    31)   -2.1626D+02 -2.5358D+02  2.4831D+03  1.9598D+03  1.4769D+03  9.8354D+02  9.5346D+02
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =      37
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
^^^RESPONSE COVARIANCE MATRIX
^^^SEE CORRELATION TABLE FOR ORDER OF RESPONSES

      MATRIX VAR      (GINO NAME 101 ) IS A DB  PREC      1 COLUMN X     37 ROW RECTANG  MATRIX.
COLUMN      1       ROWS      1 THRU     37      ------------------------------------------------
ROW
     1)    1.5463D+04  1.2673D-03  3.0273D-04  3.5650D-04  1.1412D-03  1.8502D-05  2.3277D-06  3.5296D-05  4.7872D-06  2.2226D+04
    11)    4.8603D+04  4.1004D+04  2.5507D+03  1.6934D+03  2.0740D+03  1.4273D+04  3.3874D+02  1.7962D+03  1.3648D+04  7.9638D-06
    21)    4.0286D-07  8.6549D-07  6.6045D-06  5.2201D-06  1.3147D-06  2.6827D-05  8.2268D-06  6.5712D+02  6.1216D+02  3.5394D+02
    31)    4.9052D+01  6.7444D+01  6.4666D+03  4.0282D+03  2.2878D+03  1.0146D+03  9.5346D+02
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =      37
```

```
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
^^^VARIANCE OF RESPONSES (DIAGONAL TERMS OF COVR)
^^^SEE CORRELATION TABLE FOR ORDER OF RESPONSES

      MATRIX SDEV     (GINO NAME 101 ) IS A DB  PREC      1 COLUMN X     37 ROW RECTANG  MATRIX.
COLUMN       1       ROWS       1 THRU     37     ---------------------------------------------------
ROW
     1)    1.2435D+02  3.5599D-02  1.7399D-02  1.8881D-02  3.3782D-02  4.3014D-03  1.5257D-03  5.9410D-03  2.1880D-03  1.4908D+02
    11)    2.2046D+02  2.0249D+02  5.0505D+01  4.1152D+01  4.5542D+01  1.1947D+02  1.8405D+01  4.2382D+01  1.1682D+02  2.8220D-03
    21)    6.3472D-04  9.3032D-04  2.5699D-03  2.2847D-03  1.1466D-03  5.1794D-03  2.8682D-03  2.5634D+01  2.4742D+01  1.8813D+01
    31)    7.0037D+00  8.2124D+00  8.0415D+01  6.3468D+01  4.7831D+01  3.1852D+01  3.0878D+01
THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN =      37
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
^^^STANDARD DEVIATION OF RESPONSES
^^^SEE CORRELATION TABLE FOR ORDER OF RESPONSES
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