

Optimization Trial Analysis of A Journal/Thrust Bearing Structure

Takao Miki, Mitsuru Kondo, Fumio Mizuguchi
Mitsubishi Heavy Industries, Ltd.
Kobe, Japan
and
Yasuhisa Ogino
Ryoyu System Engineering, Ltd.
Kobe, Japan

ABSTRACT

Recently, due to the need to minimize structural weight and reduce material cost, several programs are offering optimization capabilities. An optimization capability has been added to MSC/NASTRAN in V66 and has been enhanced in V67. With V68, it will be also possible to optimize the shape of a structure.

This paper presents a trial analysis of optimization capability using the current version (V67) performed on a journal/thrust bearing structure. While supporting the static load and satisfying design constraints on stress and displacement, weight is minimized.

This trial analysis demonstrates the effectiveness of the optimization capability in MSC/NASTRAN in achieving satisfactory results while saving much of the designer time which is currently used in a manual iterative optimization procedure. Improvements such as easiness of use and shape optimization would help to put this capability to extensive use in design.

1. Introduction

Conventional general-purpose FEM structural analysis programs including MSC/NASTRAN have been used as a tool mainly for determining a response due to external force as a definite solution, and many efforts have been made to improve the solution accuracy.

In recent years, coupled with the improvement in technological capability in designing, there has been an increasing demand for lightweight structures and cost reduction of materials, and consequently a variety of studies have now been underway. However, the practice to date is such that a designer changes structure shapes and plate thicknesses based on his analysis results, and spends much of his time executing such a manual iterative optimization procedure, thus limiting the number of analyses to be conducted. In addition, when there are a variety of design parameters that can be changed, it is difficult to correctly grasp the effectiveness and correlation of such parameters. To what extent a specific design can be optimized is therefore questionable.

In order to meet the above demand, a sensitivity analysis based program for optimization of plate thickness, material constant, shape, weight, cost and other factors has recently made its debut in the market, and is now beginning to be used as a designing tool.

Also in MSC/NASTRAN, the optimization function has been added to V66 and subsequent versions for functional upgrading. For the purpose of checking its optimization function and usage, we have recently conducted static analyses to minimize structural weight using a simplified analytic model of a bearing structure of an actual machine, under the design constraints on stress and displacement using MSC/NASTRAN V67.

2. Object Structure, Analytic Model and Optimizatian Condition

The analysis is carried out on a simplified journal/thrust bearing structure supporting axial and radial force. The structure is shown in Figure 1. Its material is steel.

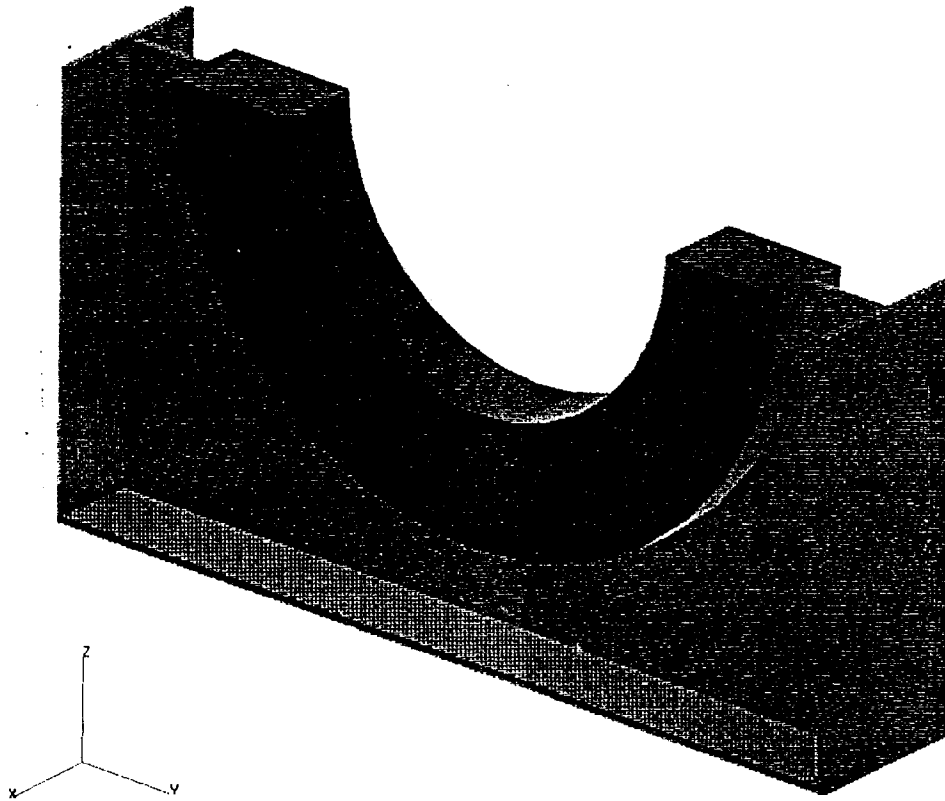


Figure 1 Simplified Bearing Structure Used in the Analysis

The analytic model was constructed using 8-node solid elements (HEXAs) and 4-node plate elements (QUAD4). The graphic plot of the FE mesh of this analytic model is shown in Figure 2. Number of nodes and number of elements are shown in Table 1. Material properties are shown in Table 2.

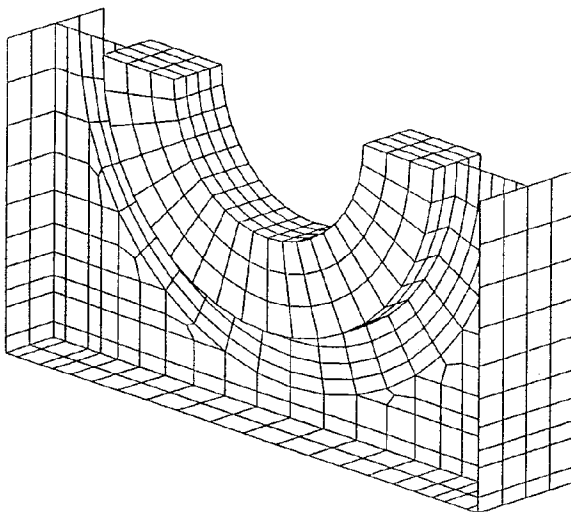


Table 1 Scale of Model

Number of Nodes	795
Number of Elements QUAD4	374
HEXA	240

Table 2 Material Property

Young's Modulus (kgf/mm ²)	21000.0
Poisson's Ratio	0.3

Figure 2 Graphic Plot of FE Mesh of Analytic Model

The bolted portions in the bottom are completely constrained, an evenly distributed vertical downward load of 10 tons is applied to area A in Figure 3 and an evenly distributed thrust load of 5 tons is applied to area B.

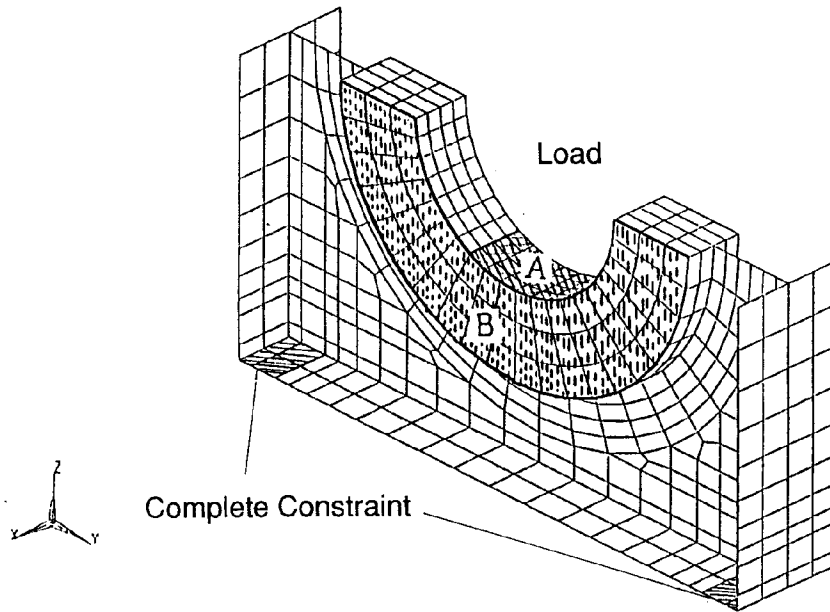


Figure 3 Conditions of Analysis

The purpose of optimization in this case is to minimize volume (weight). For this purpose plate thickness is allowed to be changed partially. However, stress and displacement must be kept less or equal to a constant allowable value for each.

Pigeonholing these details gives the following:

Objective function:

The purpose is to minimize the total volume while meeting constraint conditions.

Design variables:

As design variables, plate thicknesses of PSHELL ID 1 to 5 shown in Figure 4 are taken. All the five plate thicknesses have the same initial value of 10.0mm and can be varied in the range from 1 mm to 30 mm.

Constraint conditions:

- 1 The absolute value of the maximum principal stress in plate elements is to be kept less than or equal to 30 kgf/mm^2 .
- 2 Difference between displacements in Z direction at point A and at point B is to be kept less than or equal to 0.1mm. See Figure 4.

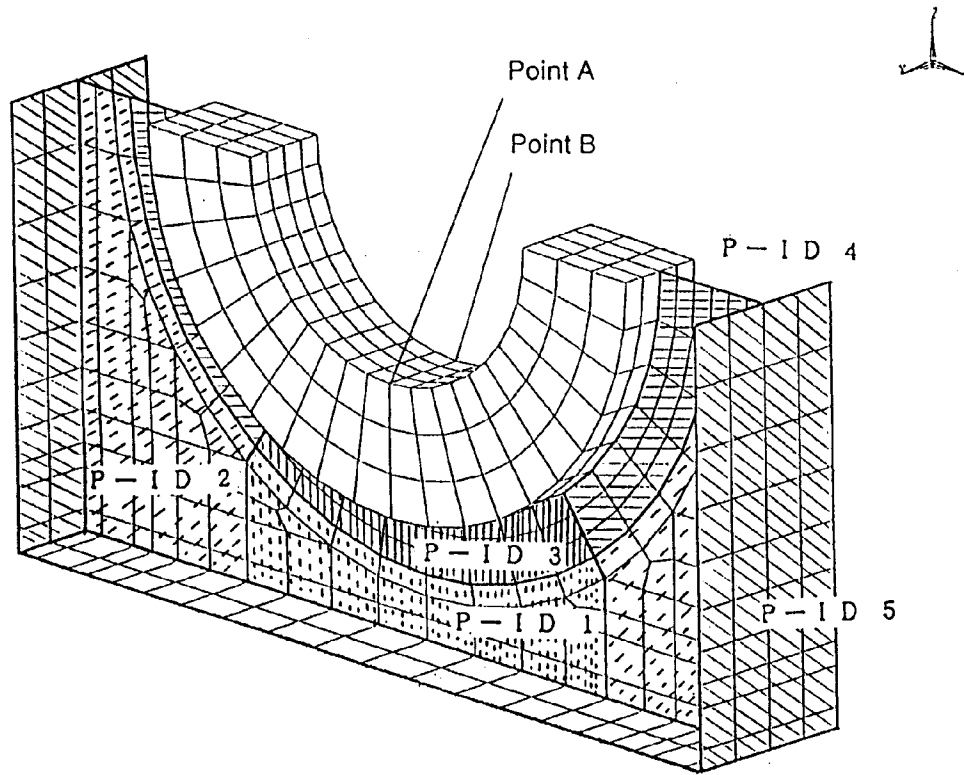


Figure 4 Position of Design Variable

3. Input Data for Optimization

The input data used in analysis is shown in Table 3. In the following, entries related to optimization are explained.

Section for objective function setting:

Taking volume as an objective function is instructed in the 4th field of DRESP 1[1] entry, and finding its minimum value is instructed in the 5th field of DESOBJ entry having the same No. as that for the 2nd field of DRESP 1 entry.

Section for design variable setting:

The type of property entry is instructed in the 3rd field of DVPREL 1 entry, property identification number of PSHELL entry is instructed in the 4th field, and taking the 4th field of this PSHELL entry, i.e. the plate thickness, as a design variable is instructed in the 5th field.

In DESVAR entry, the initial value of the said plate thickness is shown in the 4th field, and the variable range is shown in the 5th and 6th fields.

Section for setting of constraint condition 1 :

In DRESP 1 entry, property identification number (1 to 5 in the present analysis) is specified in the 9th field. In the 5th field it is specified that these are PSHELL IDs. Instruction that the constraint condition is given by stress, is in the 4th field. The stress component is shown in 7th field. Here, 7 is the maximum principal stress in Z1 plane, 8 is the minimum principal stress in Z1 plane, 15 is the maximum principal stress in Z2 plane, and 16 is the minimum principal stress in Z2 plane. These stress component numbers are tabulated in Table 4.

The range of allowable stress is shown in the DCONSTR entry having the same No. as that for the 2nd field of DRESP 1 entry.

Section for setting of constraint condition 2 :

In this problem, the difference between displacements at node No.370 and at node No.34 is the constraint condition 2, and is given as a displacement at node No.795 (dummy node) using a MPC relationship.

The MPC data is prepared according to the expression:

$$w_{795} = w_{370} - w_{34}$$

This constraint condition may also be specified using the DRESP 2 entry. However, in this analysis MPC entry has been used.

In DRESP 1 entry, it is specified that the displacement in the 3rd degree of freedom at node No.795 is chosen as the constraint condition.

In the DCONSTR entry having the same No. as that for the 2nd field of DRESP 1 entry, the range of allowable displacement is shown.

Parameter setting :

The 3rd field of DOPTPRM entry specifies the printed output during optimization phase, and its 4th field shows the maximum number of design cycles to be performed. Other parameters are kept with their default values.

Table 3 Partial Table of Input Data

```

ID X X
TIME 100
SOL 200
CEND
$
TITLE = STATIC ANALYSIS OF BEARING 1992/09/01
MAXLINES = 1000000
$
DISP      (      ,PRINT) = ALL
SPCFORCES (      ,PRINT) = ALL
STRESS    (      ,PRINT) = ALL
ECHO      = UNSORT
$
SPC      = 1000
MPC      = 200
$
SUBCASE 1
SUBT     = CASE 1
LOAD     = 100
PARAM,APPC,STATICS
$-----+
$ Optimization of Elastic Stress Analysis is instructed +
$-----+
$
BEGIN BULK
PARAM AUTOSPC YES
PARAM GRDPNT 0
$
$
$-----+
$ Section for Objective Function setting +
$-----+
$
DESOBJ    20      V      MIN
DRESP1    20      V VOLUME
$-----+
$ ID 20 on the 2nd field of DESOBJ correspond to that on DRESP1 +
$-----+
$
$
$-----+
$ Section for Design Variable setting +
$-----+
$
$-----+
$ Initial Lower Upper +
$ value bound bound +
$-----+
DESVAR    1      A1    10.0  1.0  30.0
$-----+
$-----+
DVPREL1   10 PSHELL  1      4
+DV1      1      1.0
$-----+
$ ID 1 on the 2nd field of DESVAR correspond to that on DVPREL1 +
$-----+
$
DESVAR    2      A2    10.0  1.0  30.0
DVPREL1   11 PSHELL  2      4
+DV2      2      1.0
$-----+
$
$
$
$
$
DESVAR    5      A2    10.0  1.0  30.0
DVPREL1   14 PSHELL  5      4
+DV5      5      1.0
$-----+
$

```

```

$
$-----+
$ Section for setting of Constraint Condition 1 +
$-----+
$
$-----+
$              Lower   Upper
$              bound   bound
$-----+
DCONSTR      21      ALL  -30.0   30.0
$-----+
$              Stress item ----- P-ID ---
DRESP1       21      S1  STRESS  PSHELL              7              1
$-----+
$ ID 21 on the 2nd field of DCONSTR correspond to that on DRESP1 +
$-----+
DCONSTR      22      ALL  -30.0   30.0
DRESP1       22      S2  STRESS  PSHELL              8              1
DCONSTR      23      ALL  -30.0   30.0
DRESP1       23      S3  STRESS  PSHELL             15              1
DCONSTR      24      ALL  -30.0   30.0
DRESP1       24      S4  STRESS  PSHELL             16              1
$
$
$
$
$
$
DCONSTR      41      ALL  -30.0   30.0
DRESP1       41      S17 STRESS  PSHELL              7              5
DCONSTR      42      ALL  -30.0   30.0
DRESP1       42      S18 STRESS  PSHELL              8              5
DCONSTR      43      ALL  -30.0   30.0
DRESP1       43      S19 STRESS  PSHELL             15              5
DCONSTR      44      ALL  -30.0   30.0
DRESP1       44      S20 STRESS  PSHELL             16              5
$
$-----+
$ Section for setting of Constraint Condition 2 +
$-----+
$
$---- Dummy node ----
GRID         795              0.00   0.00   0.00
$
MPC          200      795      3      1.0      34      3      1.0      +MPC
+MPC                  370      3      -1.0
$
$-----+
$              Lower   Upper
$              bound   bound
$-----+
DCONSTR      45      ALL  -0.10   0.10
$-----+
$              Disp. Comp. ---- Dummy node ---
DRESP1       45      D3      DISP              3              795
$-----+
$ ID 45 on the 2nd field of DCONSTR correspond to that on DRESP1 +
$-----+
$
$-----+
$              Max. number
$              of design cycles
$-----+
DOPTPRM      5      10
+DOP
$-----+
$-----<PSHELL>-----+
PSHELL       1      1      10.0      1      1
PSHELL       2      1      10.0      1      1
PSHELL       3      1      10.0      1      1
PSHELL       4      1      10.0      1      1
PSHELL       5      1      10.0      1      1
PSHELL       6      1      15.0      1      1
PSHELL       7      1      10.0      1      1
PSOLID       8      1              0

```


9

Table 4 Partial Table of Stress Component

Element Name (Code)	Real Stresses or Strains		Complex Stresses or Strains		
	Item Code	Item	Item Code	Item	Real/Mag. or Imag./Phase
QUAD4 (33) (Cont.)	6	θ Shear Angle at Z1	6 ¹	Normal y at Z1	IP
	7	Major principal at Z1	7 ¹	Shear xy at Z1	RM
	8	Minor principal at Z1	8 ¹	Shear xy at Z1	IP
	9	von Mises or Maximum Shear at Z1		Z2 = Fibre Distance 2	
		Z2 = Fibre Distance 2	10 ¹	Normal x at Z2	RM
	11 ¹	Normal x at Z2	11 ¹	Normal x at Z2	IP
	12 ¹	Normal y at Z2	12 ¹	Normal y at Z2	RM
	13 ¹	Shear xy at Z2	13 ¹	Normal y at Z2	IP
	14	θ Shear Angle at Z2	14	Shear xy at Z2	RM
	15	Major principal at Z2	15	Shear xy at Z2	IP
	16	Minor principal at Z2			
	17	von Mises or Maximum Shear at Z2			
QUAD4 ³ (95)	2	Lamina Number		Undefined	
	3	Normal-1			
	4	Normal-2			
	5	Shear-12			
	6	Shear-1Z			
	7	Shear-2Z			
	8	θ Shear Angle			
	9	Major principal			
	10	Minor principal			
	11	von Mises or Maximum Shear			
QUAD8 (64)	5 ¹	Normal x at Z1	5 ¹	Normal x at Z1	RM
	6 ¹	Normal y at Z1	6 ¹	Normal x at Z1	IP
	7 ¹	Shear xy at Z1	7 ¹	Normal y at Z1	RM
	8	θ Shear Angle at Z1	8 ¹	Normal y at Z1	IP
	9	Major principal at Z1	9 ¹	Shear xy at Z1	RM
	10	Minor principal at Z1	10 ¹	Shear xy at Z1	IP
	11	von Mises or Maximum Shear at Z1	12 ¹	Normal x at Z2	RM

4. Results of Analysis

The results of the present analysis are as follows:

Table 5 is a part of MSC/NASTRAN output list showing the aspect that the configuration is being optimized by repeated design cycles. Table 6 has been rearranged from Table 5 to give more intelligible data.

According to Table 6, the initial principal stress with maximum absolute value of -49.2 kgf/mm^2 is reduced to -29.9 kgf/mm^2 in the final configuration. Also, the initial difference between displacements at points A and B of 0.6867 mm is reduced to -0.0974 mm in the final configuration. Both of those meet the constraint condition.

The volume of the final configuration could be reduced by about 11% from that of the initial configuration.

Figure 5 and Figure 6 show respectively the deformation and the principal stress contour before optimization, while Figure 7 and Figure 8 show the deformation and the principal stress contour after optimization.

When comparing the figures of principal stress contour before and after optimization (Figure 6 and Figure 8), it can be seen that stress has been leveled in the configuration after optimization, while high stresses arose locally in the configuration before optimization.

The computer used in the present analysis is a CRAY X-MP, and the computing time used in optimization (9 design cycles) was 300 seconds, which is about 7.3 times longer than the 41 seconds of computing time used in one-cycle run of elastic analysis for the initial configuration.

Table 5 Extracts from NASTRAN Output List

*
* D E S I G N C Y C L E 1 *
*

* * * * * INITIAL DESIGN TO APPROXIMATE OPTIMIZATION PROBLEM * * * * *

(1) DESIGN VARIABLES AT THE INITIAL DESIGN

THE INITIAL DESIGN OF THIS CYCLE IS IDENTICAL TO USERS INPUT DESIGN

(2) DESIGN OBJECTIVE FUNCTION VALUE = 2.149796E+07

***** FINAL DESIGN FROM APPROXIMATE OPTIMIZATION PROBLEM *****

(3) RETAINED RESPONSES AT THE FINAL DESIGN

D E S I G N C Y C L E 1

----- RETAINED WEIGHT, VOLUME RESPONSES -----

RESPONSE ID	RESPONSE LABEL	LOWER BOUND	APPROX. VALUE	UPPER BOUND
WEIGHT				
VOLUME	20 V	-1.000000E+36	2.174829E+07	1.000000E+36

STATIC ANALYSIS OF BEARING 1992/09/01

MARCH 16, 1993 MSC/NASTRAN 9/10/91 PAGE 88

Difference in displacement between
at Point A and at Point B obtained
in the 1st cycle of optimization

D E S I G N C Y C L E 1 L O A D C A S E 1
----- RETAINED DISPLACEMENT RESPONSES AT THE FINAL DESIGN -----

RESPONSE ID	RESPONSE LABEL	LOWER BOUND	APPROX. VALUE	UPPER BOUND	GRID/ELEMENT ID	COMPONENT ID
45 D3		-1.000000E-01	1.088868E-01	1.000000E-01	795	3

STATIC ANALYSIS OF BEARING 1992/09/01

MARCH 16, 1993 MSC/NASTRAN 9/10/91 PAGE 89

Principal stress values obtained
in the 1st cycle of optimization

DESIGN CYCLE 1 LOAD CASE = 1 in the 1st cycle of optimization

----- RETAINED STRESS RESPONSES AT THE INITIAL DESIGN -----

RESPONSE ID	RESPONSE LABEL	LOWER BOUND	RESPONSE VALUE	UPPER BOUND	GRID/ELEMENT ID	COMPONENT ID
26	S6	-3.000000E+01	-1.753156E+01	3.000000E+01	263	8
26	S6	-3.000000E+01	-1.519445E+01	3.000000E+01	419	8
26	S6	-3.000000E+01	-1.519439E+01	3.000000E+01	422	8
26	S6	-3.000000E+01	-2.168683E+01	3.000000E+01	431	8
26	S6	-3.000000E+01	-2.168683E+01	3.000000E+01	434	8
26	S6	-3.000000E+01	-2.853584E+01	3.000000E+01	443	8
26	S6	-3.000000E+01	-2.853585E+01	3.000000E+01	446	8
26	S6	-3.000000E+01	-1.753149E+01	3.000000E+01	278	8
27	S7	-3.000000E+01	2.150070E+01	3.000000E+01	431	15
27	S7	-3.000000E+01	2.150071E+01	3.000000E+01	434	15
27	S7	-3.000000E+01	2.850895E+01	3.000000E+01	443	15
27	S7	-3.000000E+01	2.850896E+01	3.000000E+01	446	15
37	SG	-3.000000E+01	2.834548E+01	3.000000E+01	439	7
37	SG	-3.000000E+01	4.880206E+01	3.000000E+01	454	7
37	SG	-3.000000E+01	4.880213E+01	3.000000E+01	453	7
37	SG	-3.000000E+01	2.834542E+01	3.000000E+01	440	7
37	SG	-3.000000E+01	4.213310E+01	3.000000E+01	441	7
37	SG	-3.000000E+01	3.167237E+01	3.000000E+01	452	7
37	SG	-3.000000E+01	3.167243E+01	3.000000E+01	451	7
37	SG	-3.000000E+01	4.213290E+01	3.000000E+01	442	7
37	SG	-3.000000E+01	3.010360E+01	3.000000E+01	430	7
37	SG	-3.000000E+01	3.010300E+01	3.000000E+01	429	7
40	SJ	-3.000000E+01	-2.873178E+01	3.000000E+01	439	16
40	SJ	-3.000000E+01	-4.925974E+01	3.000000E+01	454	16
40	SJ	-3.000000E+01	-4.925983E+01	3.000000E+01	453	16
40	SJ	-3.000000E+01	-2.873175E+01	3.000000E+01	440	16
40	SJ	-3.000000E+01	-4.273191E+01	3.000000E+01	441	16
40	SJ	-3.000000E+01	-3.191923E+01	3.000000E+01	452	16
40	SJ	-3.000000E+01	-3.191929E+01	3.000000E+01	451	16
40	SJ	-3.000000E+01	-4.273172E+01	3.000000E+01	442	16
40	SJ	-3.000000E+01	-3.138595E+01	3.000000E+01	430	16
40	SJ	-3.000000E+01	-3.138533E+01	3.000000E+01	429	16
41	SK	-3.000000E+01	1.707713E+01	3.000000E+01	538	7

SUMMARY OF ITERATION HISTORY

(HARD CONVERGENCE ACHIEVED)
(SOFT CONVERGENCE ACHIEVED)

NUMBER OF FINITE ELEMENT ANALYSES COMPLETED 10
NUMBER OF OPTIMIZATIONS W.R.T. APPROXIMATE MODELS 9

OBJECTIVE FUNCTION HISTORY

ITERATION NUMBER	OPTIMAL WITH RESPECT TO APPROXIMATION	EXACT EVALUATION BY COMPLETE ANALYSIS	FRACTIONAL ERROR OF APPROXIMATION	MAXIMUM VALUE OF CONSTRAINTS
INITIAL		0.214980E+08		0.586675E+01
1	0.217483E+08	0.217483E+08	-0.164440E-13	0.152335E+01
2	0.215613E+08	0.215613E+08	0.000000E+00	0.410304E+00
3	0.205554E+08	0.205554E+08	0.115988E-13	-0.934431E-01
4	0.198777E+08	0.198777E+08	0.239885E-13	-0.257664E-01
5	0.196116E+08	0.196116E+08	0.607852E-14	-0.284313E-02
6	0.194542E+08	0.194542E+08	0.122554E-13	-0.254844E-02
7	0.192400E+08	0.192400E+08	0.123918E-13	-0.709677E-02
8	0.191235E+08	0.191235E+08	0.623367E-14	-0.458091E-03
9	0.191235E+08	0.191235E+08	0.000000E+00	-0.458091E-03

DESIGN VARIABLE HISTORY

DV. ID.	INITIAL	1	2	3	4	5	6	7
1	0.1000E+02	0.6706E+01	0.5363E+01	0.4291E+01	0.3433E+01	0.2746E+01	0.3174E+01	0.3788E+01
2	0.1000E+02	0.1455E+02	0.1524E+02	0.1219E+02	0.9756E+01	0.9214E+01	0.8675E+01	0.7473E+01
3	0.1000E+02	0.7103E+01	0.5683E+01	0.4545E+01	0.3637E+01	0.4364E+01	0.5237E+01	0.6285E+01
4	0.1000E+02	0.8031E+01	0.6425E+01	0.5140E+01	0.4112E+01	0.3290E+01	0.2632E+01	0.2105E+01
5	0.1000E+02	0.1353E+02	0.1624E+02	0.1342E+02	0.1350E+02	0.1346E+02	0.1357E+02	0.1383E+02

DV. ID.	8	9	10	11	12	13	14	15
1	0.3534E+01	0.3534E+01						
2	0.7387E+01	0.7387E+01						
3	0.6208E+01	0.6208E+01						
4	0.1684E+01	0.1684E+01						
5	0.1380E+02	0.1380E+02						

*** DMAP INFORMATION MESSAGE 9030 (DESOPT) - RUN TERMINATED DUE TO HARD CONVERGENCE AT ITERATION NO. = 9
*** END OF JOB ***

Table 6 Summary of Results of Analysis

	Initial Configuration	design cycle 1	design cycle 2	design cycle 3	design cycle 4	design cycle 5	design cycle 6	design cycle 7	design cycle 8	Final Configuration
Volume (m ³)	2.150E+7	2.175E+7	2.156E+7	2.055E+7	1.989E+7	1.961E+7	1.945E+7	1.924E+7	1.912E+7	1.912E+7
Plate Thickness (mm)										
P-ID 1	10.00	6.71	5.36	4.29	3.43	2.75	3.17	3.79	3.53	3.53
P-ID 2	10.00	14.55	15.24	12.19	9.76	9.21	8.68	7.47	7.39	7.39
P-ID 3	10.00	7.10	5.68	4.55	3.64	4.36	5.24	6.29	6.21	6.21
P-ID 4	10.00	8.03	6.43	5.14	4.11	3.29	2.63	2.11	1.68	1.68
P-ID 5	10.00	13.53	16.24	13.42	13.50	13.46	13.57	13.83	13.80	13.80
Principal stress with maximum absolute value (kgf/mm ²)	-49.26	-25.13	-20.16	-27.20	-29.23	-29.91	-29.92	-29.78	-29.99	-29.99
Difference in Displacement bet. at Point A and at Point B (mm)	0.6867	0.2523	0.1410	0.0739	-0.0702	-0.0939	-0.0766	-0.0523	-0.0974	-0.0974

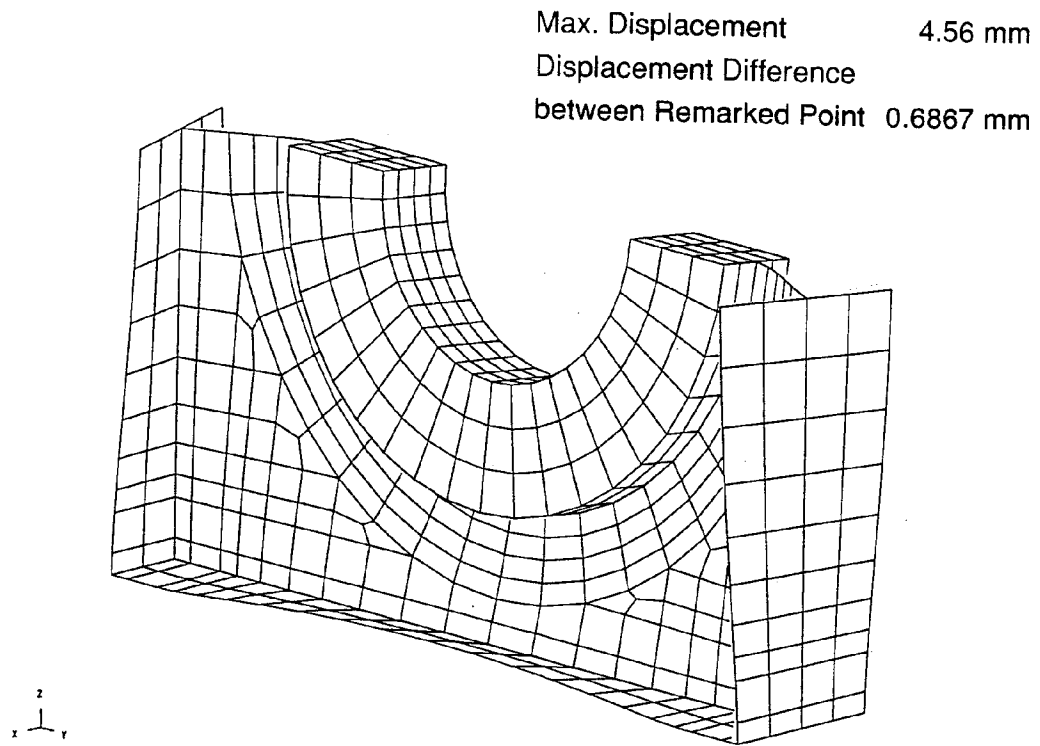


Figure 5 Overall Deformation before Design Optimization

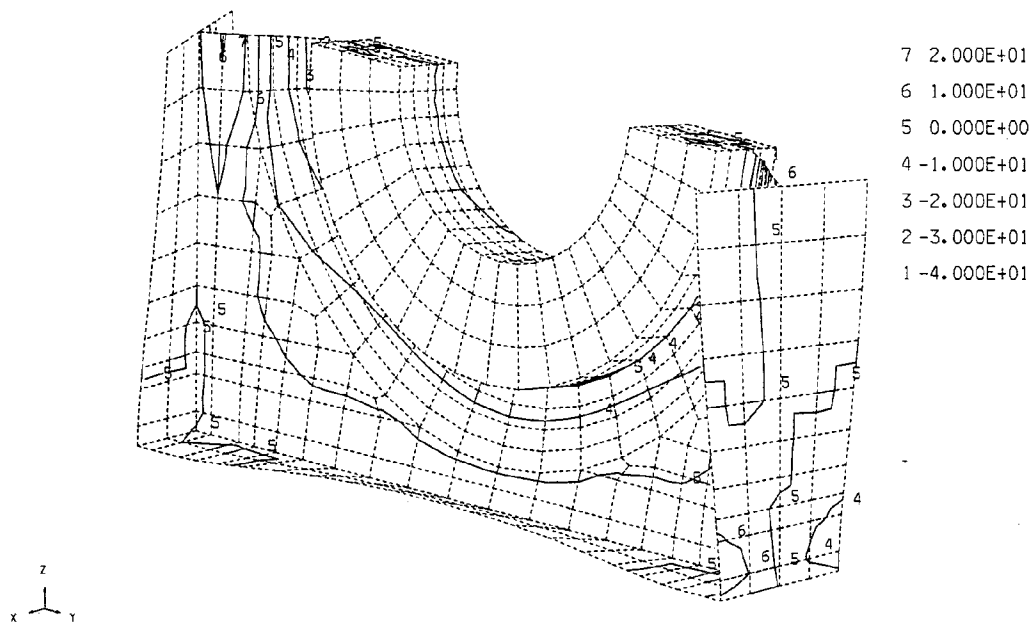


Figure 6 Principal Stress Contour before Design Optimization

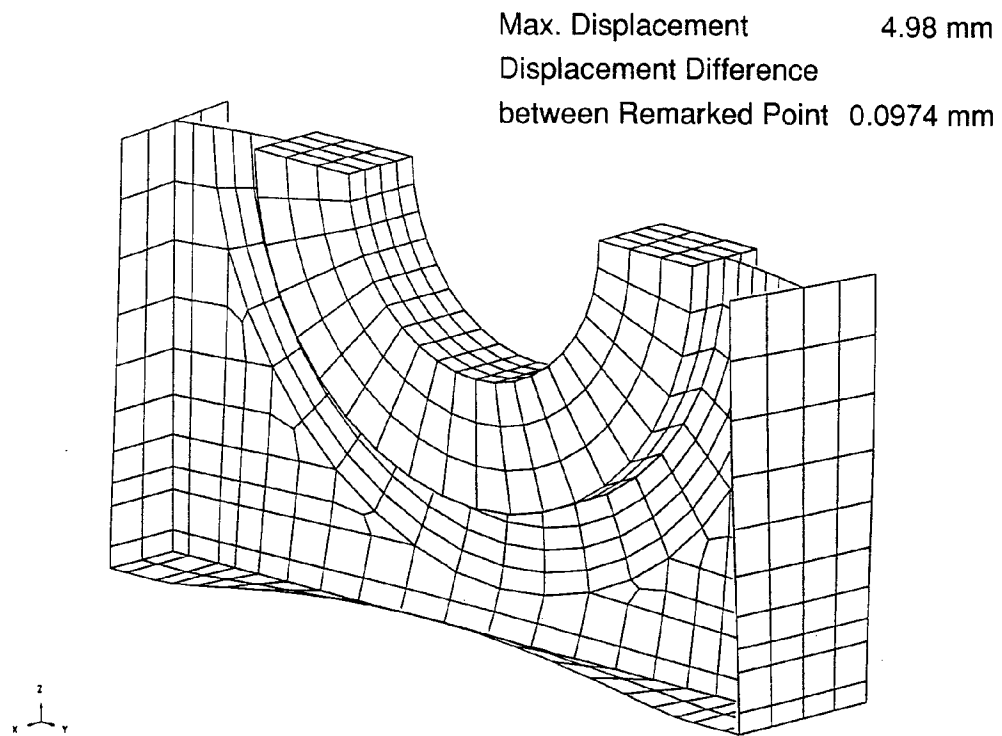


Figure 7 Overall Deformation after Design Optimization

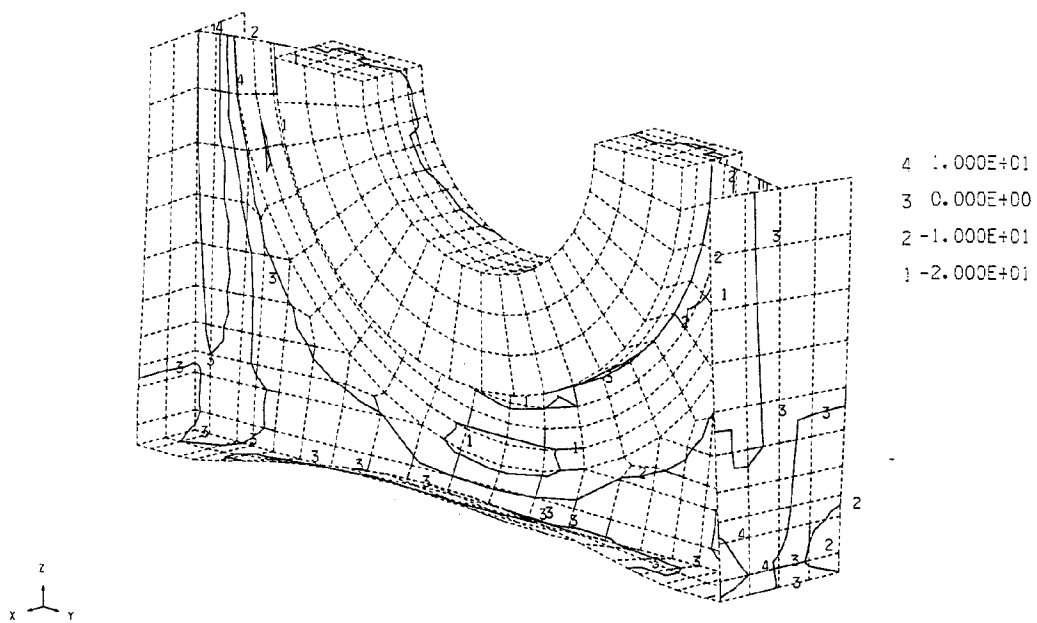


Figure 8 Principal Stress Contour after Design Optimization

5. Conclusions

In this trial run of Sol 200 (structural optimization capability) of MSC/NASTRAN V67, the following has been found.

In conventional design work, a configuration meeting the constraint conditions is found out, mainly by repeatedly making design changes based on examining the results of structural analysis and the experience and engineering sense of designers. It has been difficult to optimize a structure by reducing the objective function while meeting the constraint conditions.

However, when this optimization capability was used, it was possible to find out a configuration meeting the set conditions, in a computing time only about 7.3 times as long as that in conventional static analysis, by only adding optimization data to the data prepared for conventional structural analysis.

The objective function (volume) could be reduced by about 11% from its initial value, by adjusting the design variable (plate thickness in 5 different parts) so as to meet the constraint conditions (stress and displacement).

It was found that even optimization of a structure by taking only the plate thickness as a design variable was a strong tool for increasing the efficiency of design work, when the structure could be modeled with shell elements.

Though the optimization capability of MSC/NASTRAN can be a strong tool in design work, the constitution of input data is complicated and hard to understand such that it is difficult for designers to have this capability at their command. Though the current input method which have a large degree of freedom for setting the conditions of optimization is necessary, a simpler input method enabling designers to easily set these conditions is further required. For example, it will be very convenient to provide a simple input generator.

In the optimization capability of V67, plate thickness of shell elements and property data of bar elements (except BEND element) can be set as design variables. However, configurations can not be varied by changing the coordinates of nodes. Designers are manually performing optimization of configurations by transfer of nodes, and the increase of efficiency of this work is hoped for.

The authors express their wish that further improvements will be made on the points mentioned to make MSC/NASTRAN a stronger and more effective tool for design and analysis.

6. Acknowledgements

The authors would like to express their gratitude to Dr. S. Rashed and Mr. E. Sato of MSC Japan for their advice on executing this trial run.

7. References

- [1] *MSC/NASTRAN User's Manual*, Version 67, The MecNeal-Schwendler Corporation, Los Angeles, CA, August 1991.