

Structural Analysis of an 85 ft. Free Standing Tower*

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ABSTRACT

An 85' high rectangular truss type antenna support tower is currently in use. The tower bracing scheme is typical of and similar to other towers used for this application. Tower members were designed using conventional analytical techniques as presented in design codes such as AISC, ANSI/A58.1 and ANSI/EIA-222. Resulting stress analysis showed factors of safety for all members to be satisfactory and capable of sustaining design loads. Since a conventional tower bracing scheme was used, the overall buckling capacity of this tower was assumed to be adequate. Recently a new antenna configuration proposed for use required additional structural analysis of this tower. The resulting analysis included an overall buckling analysis utilizing the MSC/NASTRAN program with the buckling solution sequence. The buckling analysis revealed that the tower was incapable of withstanding the design loads for either the original or the proposed antenna configurations. In conclusion, it should also be noted that the results of this study suggest that other towers currently in use may also be inadequately designed and subject to potential failures.

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INTRODUCTION

A large flat rotating antenna has been mounted atop a steel support tower. The antenna frontal envelope is approximately 5 meters by 10 meters (Figure 1). The face of the bare antenna is approximately 61% solid. The existing design and operation of the antenna (configuration 1) has its long axis horizontal and its short axis vertical. The base of the antenna is set 85 feet above grade. This has been done by mounting the antenna at the center of the top of an 85 foot free standing rectangular tower.

Future operation of the antenna will orient the axes perpendicular to their present configuration. The long axis will be vertical and the short axis will be horizontal. This orientation has been referred to as configuration 2. Refer to Figure 2 for a sketch of the two configurations. The antenna is set on a rotary mount with limited overturning moment resistance. The antenna will still be required to rotate. In order to mitigate overturning effects due to wind drag forces the center of the antenna will remain at the same elevation as in configuration 1. In support of this, an interface structure and counterweights have been added to the design.

The new interface structure and counterweights will add approximately 10,000 lb. to the existing design. The antenna will be offset from the center of rotation by approximately 10 feet.

In order to insure that the integrity of the antenna would not be compromised under configuration 2, a re-analysis of the existing design was performed. This report will present this re-analysis.

SCOPE

The scope of this report will present a re-analysis of the free standing tower for the present environment using allowable stresses based on unbraced lengths and compared to those developed by the tower manufacturer. Then a buckling analysis will be performed on the overall tower and the results compared.

ENVIRONMENTAL CONDITIONS

The environments associated with the various sites of antenna operation are listed below in Table 1. Wind speeds are taken from a combination of data accumulated from measurements as well as data taken from ANSI¹. Ice loads are developed based on Tattelman². Moments and torques are calculated using procedures developed earlier⁵.

Table 1
Environmental Conditions

Site	Elevation above MSL (ft)	Height above ground (ft)	Wind Speed¹ (mph)	Ice Thickness (in)
Site I	100	100	88	1.0
Site II	0	100	131	0.8
Site III	8,000	90	133	1.0
Site IV	1,500	30	100	0.

¹ Referenced to MSL

LOADINGS

Under survival wind speeds the antenna will be allowed to free wheel to mitigate the torque effects⁵ anticipated as a result of varying angle of wind attack. Under service wind loads, the antenna drive will be required to resist wind torques as well as moments. The bare antenna is approximately 61% solid. When coated with 1" of ice, the antenna solidity increases to approximately 83%. The antenna will not be operated when covered with ice.

This results in the set of loading combinations below:

Survival Loading

1. Survival wind without ice
 $W_s + G < 1.33Y$
2. Survival wind plus ice
 $.667(W_s + I) + G < 1.33Y$

Service Loading

3. Ice load plus service wind
 $I + W_p + G < Y$

4. Service wind

$$W_o + M_o + T_o + G < Y$$

where W_s = drag from survival wind of 133 mph
 W_p = drag from service wind of 60 mph
 W_o = drag from service wind on bare antenna
 T_o = moment from service wind on bare antenna
 M_o = torque from service wind on bare antenna
 I = effects from 1" ice
 G = gravity loads
 Y = yield allowable

When applied to the antenna these effects result in the following forces at the geometric center of the antenna:

W_s = 23,835 lb. (antenna 61% solid)
 W_s = 24,324 lb. (antenna 83% solid)
 W_p = 5,985 lb. wind drag force (antenna 83% solid)
 W_o = 5,865 lb. wind drag force
 T_o = 23,142 ft-lb maximum wind torque at angle of wind
 M_o = 6,839 ft-lb maximum wind moment at elevation
 I = 17,000 lb.

The weight of the bare antenna and pedestal assembly is 38,000 lb.

TOWER DESIGN

The tower is a rectangular steel conventionally framed structure (Figure 3). Figures 4 and 5 show pictures of similarly framed towers. Most members of this tower are structural angles or channels (Figure 6). Sections are connected by bolting through one leg of the angle or through the web of the channel. For analysis purposes, the connections are all considered to act as pins except where members are continuous. All members have been sized to maintain the unbraced lengths within bracing criteria presented by EIA-2223.

The tower consists of four main continuous corner members which extend from the base of the tower to the top. These are 85' in length with a moment splice 50' above the base support. The bottom 50' is L6x6x5/8 structural steel angle. The top 35' is L6x6x1/2 structural steel angle. Horizontal bracing members are supplied at 25', 50' and 75' as well as the top. These members are 2Ls4x3-1/2x1/4 long legs back to back. Diagonal bracing is then provided in the side planes of the tower from the midpoint of this horizontal to the connection of the corner angle with the next horizontal down. These braces are L4x4x3/8. Additional diagonal bracing is placed in the plane of the horizontal members to connect their midpoints together. These members are L3-1/2x3-1/2x1/4. Secondary bracing (horizontal and diagonal) is placed in their vertical planes of the truss between the corner and the diagonal to decrease the unbraced length of the diagonal and corner angles.

Finally, additional secondary bracing (horizontals only) is used to connect the braced points of each pair of two main vertical diagonal braces that frame to a common corner member. All secondary bracing is L2-1/2x2-1/2x5/16.

Table 2
Tower Member Capacities
(Unfactored for wind loads)

Member	Use	Elevation	Unbraced Length (ft)	r min (in)	Compression Capacity (ksi)
L6x6x5/8	Corner	0' to 25'	8.40	1.18	14.74
L6x6x5/8	Corner	25' to 50'	8.40	1.18	14.74
L6x6x1/2	Corner	50' to 75'	8.40	1.18	14.74
L6x6x1/2	Corner	75' to 85'	10.09	1.18	12.64
L4x4x3/8	Diagonal	0' to 25'	7.14	.788	11.85
L4x4x3/8	Diagonal	25' to 50'	7.38	.788	11.35
L4x4x3/8	Diagonal	50' to 75'	6.95	.788	12.21
L4x4x3/8	Diagonal	75' to 85'	8.75	.788	8.41
2L4x3-1/2	Horizontal	25'	9.40	1.27	14.34
2L4x3-1/2	Horizontal	50'	7.04	1.27	16.79
C10x15.3	Horizontal	75'	4.69	.713	15.48
C10x15.3	Horizontal	85'	7.50	.713	9.38
L3-1/2x3-1/2	Hor. Diag.	25'	11.76	.694	3.61*
L3-1/2x3-1/2	Hor. Diag.	50'	8.87	.694	6.35

* $F_{cr} = 1.037 \times 10^6 / (l/r_{min})^2$ (units of length in ft.)

There are also two platforms on the tower, one at the top at 85' called the antenna platform. The other is at 75' and denoted the working platform. The platforms do not include any horizontal bracing.

Limiting member unbraced lengths are presented in the tower design and restated here. From these, the member capacities are developed. These are consistent with those presented by the manufacturer in the original design.

STRUCTURAL ANALYSIS

All structural analysis of the tower has been performed with MSC/NASTRAN. The model is described below. The entire assembly (tower, mounting structure and antenna) has been evaluated. The tower alone will be addressed.

Structural Model

The model of the tower is shown as Figure 7. This model consists of 142 nodes, 297 CBEAM elements and 840 degrees of freedom. All members have been represented with CBEAM elements. Orientation of the principal axes is represented with the product of inertia. No intermediate nodes have been provided, all nodes are at the ends of the beams. All main and secondary bracing is included in the analysis. As stated before, all connections are considered pinned.

ANALYSIS

Linear structural analysis has been performed for the load combinations previously provided. The loading is applied as a set of forces at the interface with the antenna rotational mechanism.

The loads from load combinations 1A, 1B, 2A and 2B are applied to the tower at azimuth angles 0, 30, 45 and 60 degrees (0° is taken along -x as defined in Figure 7). A linear stress analysis is performed using the MSC/NASTRAN model. The results of these analyses are listed in Table 3.

From the results of the stress analysis, a limiting set of stresses is extracted for comparison to the factored stress allowables. These are presented in Table 4.

Table 3
Stress Analysis Results

Load Comb.	Member	Use	Angle of wind from -X axis (deg.)			
			0	30	45	60
			Calc. Stress (ksi)	Calc. Stress (ksi)	Calc. Stress (ksi)	Calc. Stress (ksi)
1A	L6x6x5/8	Corner	15.3	15.9	16.5	15.0
	L6x6x1/2	Corner	9.2	10.1	10.0	9.5
	L6x6x1/2	Corner	5.9	6.5	6.6	7.4
	L4x4x3/8	Ver. Brace	12.3	10.8	9.2	8.6
	2L4x3-1/2	Horiz.	2.6	3.0	3.6	3.7
	L3-1/2x3-1/2	Hor. Brace	0.2	0.1	0.1	0.1
	L4x4x3/8	Ver. Diag.	5.6	6.4	6.3	5.9
1B	L6x6x5/8	Corner	17.6	18.2	17.6	17.0
	L6x6x1/2	Corner	11.2	10.4	9.0	7.6
	L6x6x1/2	Corner	4.1	5.1	5.2	5.2
	L4x4x3/8	Ver. Brace	14.4	12.6	10.9	9.9
	2L4x3-1/2	Horiz.	2.7	3.6	3.9	4.0
	L3-1/2x3-1/2	Hor. Brace	0.2	0.1	0.1	0.1
	L4x4x3/8	Ver. Diag.	7.3	6.7	6.7	5.7
2A	L6x6x5/8	Corner	9.9	12.1	12.2	11.8
	L6x6x1/2	Corner	9.0	10.6	10.8	10.6
	L6x6x1/2	Corner	6.3	7.8	8.0	7.8
	L4x4x3/8	Ver. Brace	6.7	6.9	6.6	6.6
	2L4x3-1/2	Horiz.	2.0	2.2	2.4	2.6
	L3-1/2x3-1/2	Hor. Brace	0.1	0.1	0.1	0.1
	L4x4x3/8	Ver. Diag.	3.4	4.2	4.7	4.9
2B	L6x6x5/8	Corner	7.2	7.7	7.8	6.8
	L6x6x1/2	Corner	4.3	4.7	4.6	4.5
	L6x6x1/2	Corner	2.6	3.0	3.0	3.0
	L4x4x3/8	Ver. Brace	6.7	6.9	6.5	5.8
	2L4x3-1/2	Horiz.	1.3	1.9	2.0	2.0
	L3-1/2x3-1/2	Hor. Brace	0.2	0.2	0.2	0.2
	L4x4x3/8	Ver. Diag.	4.1	4.8	4.8	4.5

Table 4
Limiting State of Stress/Angle by Load Combination

Member	Use	Unfactored Stress Allowables (ksi)	Limit Stress (ksi)/Angle*			
			Load Combination			
			1A	1B	2A	2B
L6x6x5/8	Corner	14.47	16.5/c	18.2/c	12.2/c	7.8/c
L6x6x1/2	Corner	14.47	10.1/b	11.2/a	10.8/c	4.7/b
L6x6x1/2	Corner	12.64	7/4/d	5.2/c	8.0/c	3.0/c
L4x4x3/8	Ver. Brace	11.35	12.3/a	14.4/a	6.9/b	6.9/b
2L4x3-1/2	Horiz.	14.34	3.7/d	4.0/d	2.6/d	2.0/c
L3-1/2x3-1/2	Hor. Brace	3.61	0.2/a	0.2/a	0.1/a	0.2/a
L4x4x3/8	Ver. Diag.	8.41	6.4/b	7.3/a	4.7/c	4.8/b

*a = 0°, b = 30°, c = 45°, d = 60°

Note: Unfactored Stress Allowables will be increased by 1.33 against wind load (Comb. 1A, 1B).

Based on the data taken from Table 4 above, a limiting set of factors of safety for all members can be derived for each load combination. This is listed in Table 5 below.

Table 5
Factors of Safety
(Based on Stress Analysis)

Load Combination	F.S.	Limiting Member	Angle of Load
1A	1.19	Corner	45
1B	1.05	Vertical Brace	0
2A	1.21	Corner	45
2B	1.64	Vertical Brace	0

Based on the above, the limiting load combination appears to be load combination 1B with the load applied at an angle of 0°. In addition, the limiting factor of safety is 1.05 taken from the stress results of vertical bracing.

BUCKLING ANALYSIS

Based on all data taken from the above analysis, the tower appears to be adequate. However, as a check on the overall margin, a buckling analysis was performed on the tower for the same set loadings. Since loading 1B appears to be the most limiting, this was initially chosen for use with the evaluation. The loading was applied at the same set of angles used in the stress analysis.

MSC/NASTRAN performs buckling analysis by solving the eigenvalue equation⁷.

$$[K_{aa} + \omega K_{aa}^a] (u) = 0$$

where K_{aa} = the stiffness matrix
 K_{aa}^a = the differential stiffness matrix
 ω = the eigenvalue

Differential stiffness applies to linear terms in the equation of motion of an elastic body that arise from simultaneous consideration of large nonlinear motions and applied loads. The theory of differential stiffness is predicated on the assumption that the loading systems remain fixed in direction and magnitude and move with their points of application during motion of the system.

The matrix, K_{aa}^a is a direct function of the loading, and can therefore, be rewritten as $K_{aa}^a = F \cdot K_{bb}^a$, where the term F is the loading.

Inspection of the eigenvalue equation shows the eigenvalue can be expressed as:

$$\omega = K_{aa} / (F \cdot K_{bb}^a) \text{ or,}$$

$$\omega F = K_{aa} / K_{bb}^a$$

Thus the eigenvalue is the factor which when applied to the loading will cause buckling of the structure. This can be restated as the eigenvalue is the factor of safety for the loading against theoretical buckling.

The same mathematical model developed for the stress analysis was used for the buckling analysis. The results of the buckling analysis for load combination 1B are listed in Table 6.

Table 6
Buckling Eigenvalues Load Combination 1B

Direction of Loading	Eigenvalue
0	0.918
30	0.747
45	0.744
60	0.789

From this analysis, it is seen that the factor of safety against theoretical buckling is actually less than 1.0. Further, the results taken from Table 6 are not an adequate comparison with Table 5. A factor of safety is applied (in this case by AISC)⁴ against the theoretical yield stress to calculate the allowable stresses presented in Table 2. Using the CRC Basic Column Curve presented in Tall⁷ this factor of safety against unfactored yield in compression can be shown to be 1.667. This factor of safety can be shown to be approximately 1.25 against yield stress factored for wind. It is proposed that to provide consistency between this analysis and AISC, the eigenvalues be reduced by the factor 1/1.25 (0.80), resulting in:

Table 7
Factored Buckling Eigenvalues Load Combination 1B

Direction of Loading	Unfactored Eigenvalue	Factored Eigenvalue	Factor of Safety
0	0.918	0.734	0.73
30	0.747	0.598	0.60
45	0.744	0.595	0.60
60	0.789	0.631	0.63

Comparison of the results from Table 7 with those from Table 5 shows that the stress analysis does not predict failure nor does it identify the critical loading direction.

Results from the stress analysis (Table 5) show the design to be limited (by the lowest factor of safety) when the loading is applied at 0° (parallel to the short axis of the tower). At this limiting angle of load application, the tower is shown to be adequate with a factor of safety 1.05.

It has already been shown that the factored eigenvalue is equal to the factor of safety against design allowable. The results taken from the buckling analysis (Table 7) now show the design to be limited (again by the lowest factor of safety) when the load is applied between 30° and 45° . Under this limiting angle of load application, when buckling is evaluated, the tower is shown to be inadequate to sustain design loads. The limiting factor of safety is 0.6. Consequently, the tower is actually underdesigned by 40%.

The reasons behind this are clear when the phenomenon is understood. Consider the section of the tower shown in Figure 8. This is taken from the MSC/NASTRAN model of the tower between 50 feet and 75 feet. The largest forces are applied to the corner members when the loading is at an angle with the x axis (30° to 45°). This is consistent with the location of the limiting eigenvalues from the buckling analysis. Therefore, it could be expected that the bracing would be most important when the load was applied in this direction. However, the largest forces on individual braces occur when the load is applied coplanar with the main bracing (along the x or y axis). The reality, however, is that the braces actually work as a unit. Therefore, the total force applied to the corner assembly must be considered. When the loading is coplanar with a brace, two units resist the load. When the load is applied at an angle only one unit is effective. Figure 9 is a plot of the buckled shape.

The problem with the design comes from the fact that the main bracing members are well braced with redundants in the planes of the sides of the tower (along the x and y axes). However, in the plane of the pairs of bracing members, which is where the loading is most critical, they are not as well supported. And to make matters worse, these members are unbraced about their weak axes.

Since it appears that the most severe wind loading occurs with the wind around 30° , analyses of the remaining load sets was performed for the wind along this axis. The results of this investigation are listed below.

Table 8
Buckling Analysis of All Loads with Wind at 30°
(As Built Tower)

Load Combination	Unfactored Eigenvalue	Factored Eigenvalue	Factor of Safety
1A	1.085	0.868	0.87
1B	0.747	0.598	0.60
2A	2.996	2.397	2.40
2B	2.867	2.294	2.29

Thus it can be concluded from the evaluation that load combination 1B is limiting when the wind is at 30° to the x axis. It can be seen from the results of Table 8 that load combination 1A also overloads the tower.

RESOLUTION

The resolution to the problem is to add additional secondary braces between the pairs of main vertical braces at each corner of the tower. These secondaries will be diagonal. They will be sized to maintain their effective length ratio (kl/r) below 250.

These braces were included in the model at the upper bay of the top section tower (50' to 75') and a re-analysis was performed. The eigenvalues were developed for the wind applied at 30° for all of the load combinations. The results are presented below in Table 9.

Table 9
Buckling Analysis of All Loads with Wind at 30°
(Modified Tower - Upper Section Braced)

Load Combination	Unfactored Eigenvalue	Factored Eigenvalue
1A	1.542	1.234
1B	1.085	0.868
2A	4.782	3.826
2B	2.930	2.344

From the results, the theoretical buckling capacity of the tower is beyond that required to prevent buckling. However, as stated before, the design capacity of the tower is still too low. This can be increased by adding additional bracing at the next section down (25' to 50').

Secondary braces were added to the top bay of the upper section (50' to 75') then the top bays of the upper two sections (25' to 50' and 50' to 75') and then the top bays of all three sections (0' to 25', 25' to 50', 50' to 75'). The results of buckling analysis along with a corresponding theoretical wind speed and design wind speed for Load Combination 1B applied at 30° are listed in Table 10.

Table 10
Effect of Further Modification

Additional Braces	Unfactored Eigenvalue	Theoretical Wind Speed (mph)	Factored Eigenvalue	Design Wind Speed
None	0.747	115	.598	103
50' to 75'	1.085	139	.868	124
25' to 75'	1.249	149	1.000	133
0' to 75'	1.663	172	1.330	153

Note: Wind speed is calculated at the antenna. The design is based on a wind speed of 133 mph.

From the results of the stress analysis presented previously (Table 5), after the tower is adequately braced, the limiting stress occurring within the vertical bracing members limits the design. Since loading 1B limits the design and loading 1B is driven predominantly by survival wind loads (133 mph measured at the antenna), the wind speed that the tower is capable of withstanding can be estimated as the square root of the factor of safety (which for buckling is the eigenvalue). The limiting design wind speed based on stress analysis alone is then determined to be (based on a factor of safety of 1.05) 136 mph.

As stated before, loading 1B limits the design and is driven predominantly by wind force. The tower total sail area and drag are essentially the same for both antenna configurations. Therefore, the tower without modifications, is incapable of withstanding survival wind forces under configuration 1 as well.

By adding braces at top bays of the top two sections the tower design buckling capacity will be increased to accommodate a design wind speed of 133 mph.

The tower would not be expected to buckle under design loads once the additional secondary braces were added to the upper section. However, to maintain consistency of safety factors with the present design, the braces should also be added to the second bay as well. Addition of any additional bracing would not be prudent since the capacity of the tower will then be limited by local member capacities as well as buckling.

RESULTS

The tower is inadequate to sustain the design wind loads under either antenna configuration. However, with the addition of a modification to the tower (four secondary braces added between pairs of main vertical bracing at elevation 75' and four more between main verticals

at elevation 50'), the tower will be adequate to sustain the design wind loads within the design buckling capacity.

The design capacity of the tower without the proposed change is a 103 mph design wind speed measured at the antenna. The theoretical capacity against failure is 115 mph.

The design capacity of the tower with the proposed change is a 133 mph wind speed measured at the antenna. The theoretical capacity is 149 mph against failure.

The required design capacity is for a 133 mph wind speed measured at the antenna.

CONCLUSION

There are several conclusions to this analysis.

1. This tower as designed is inadequate to sustain the design wind speed (133 mph calculated at the antenna). Therefore, additional secondary bracing should be added to the tower. This will increase the calculated capacity of the tower to accommodate the 133 mph wind speed (at the antenna).
2. If stress analysis alone is used to evaluate structural capacity, then the adequacy of the bracing scheme may not be known. This problem occurred because it was assumed that the bracing was adequate. This assumption was never verified.

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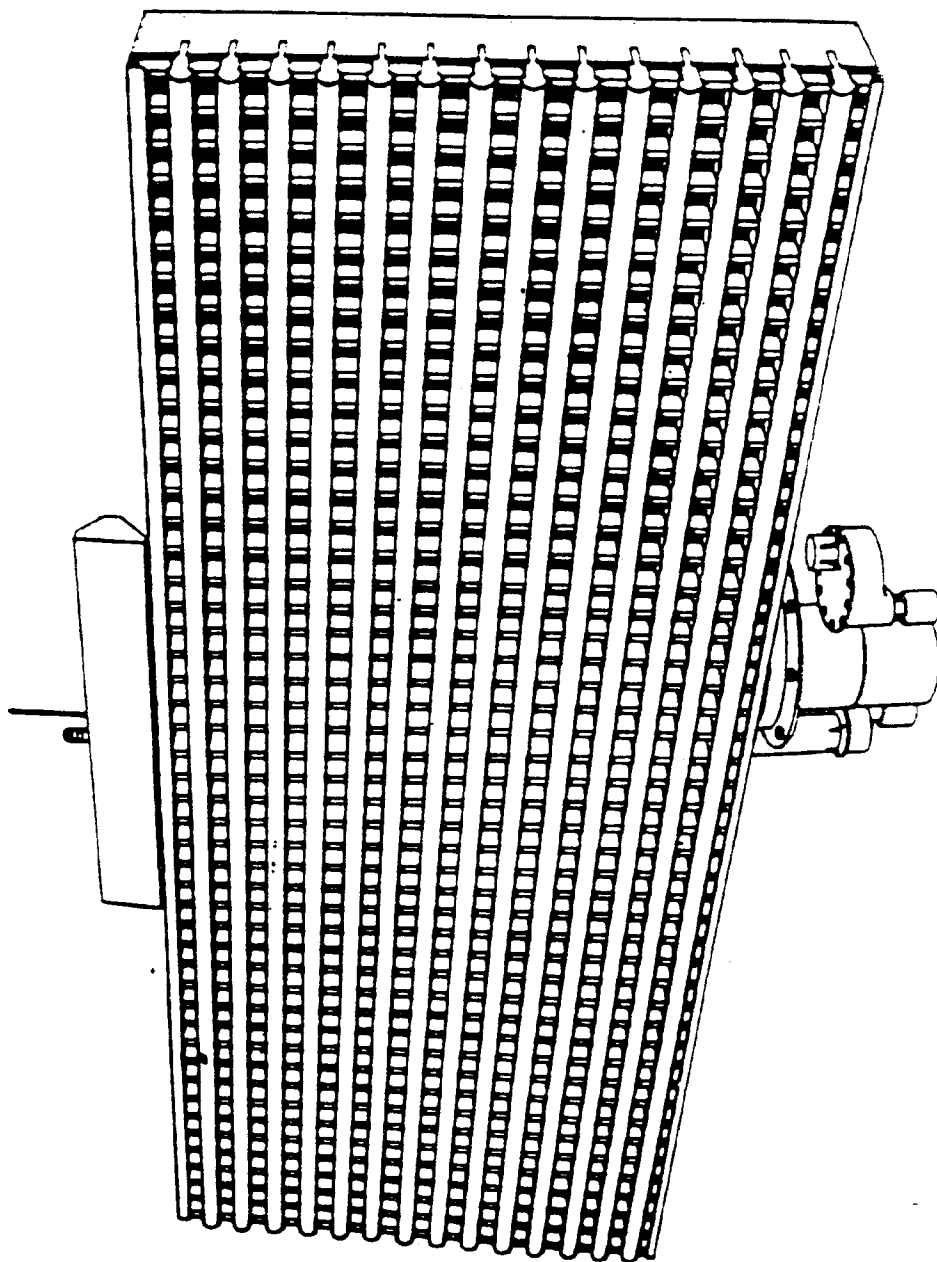


Figure 1. Antenna.

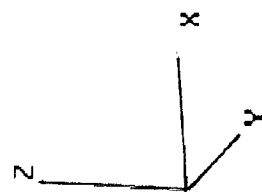
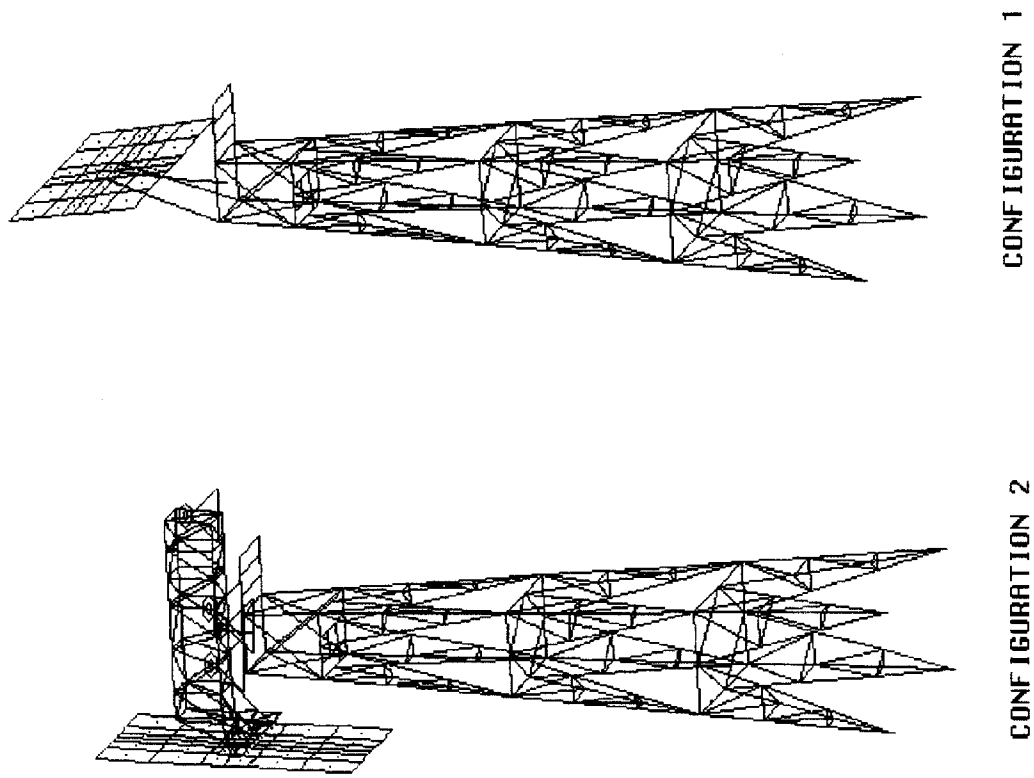


Figure 2. Antenna configurations.

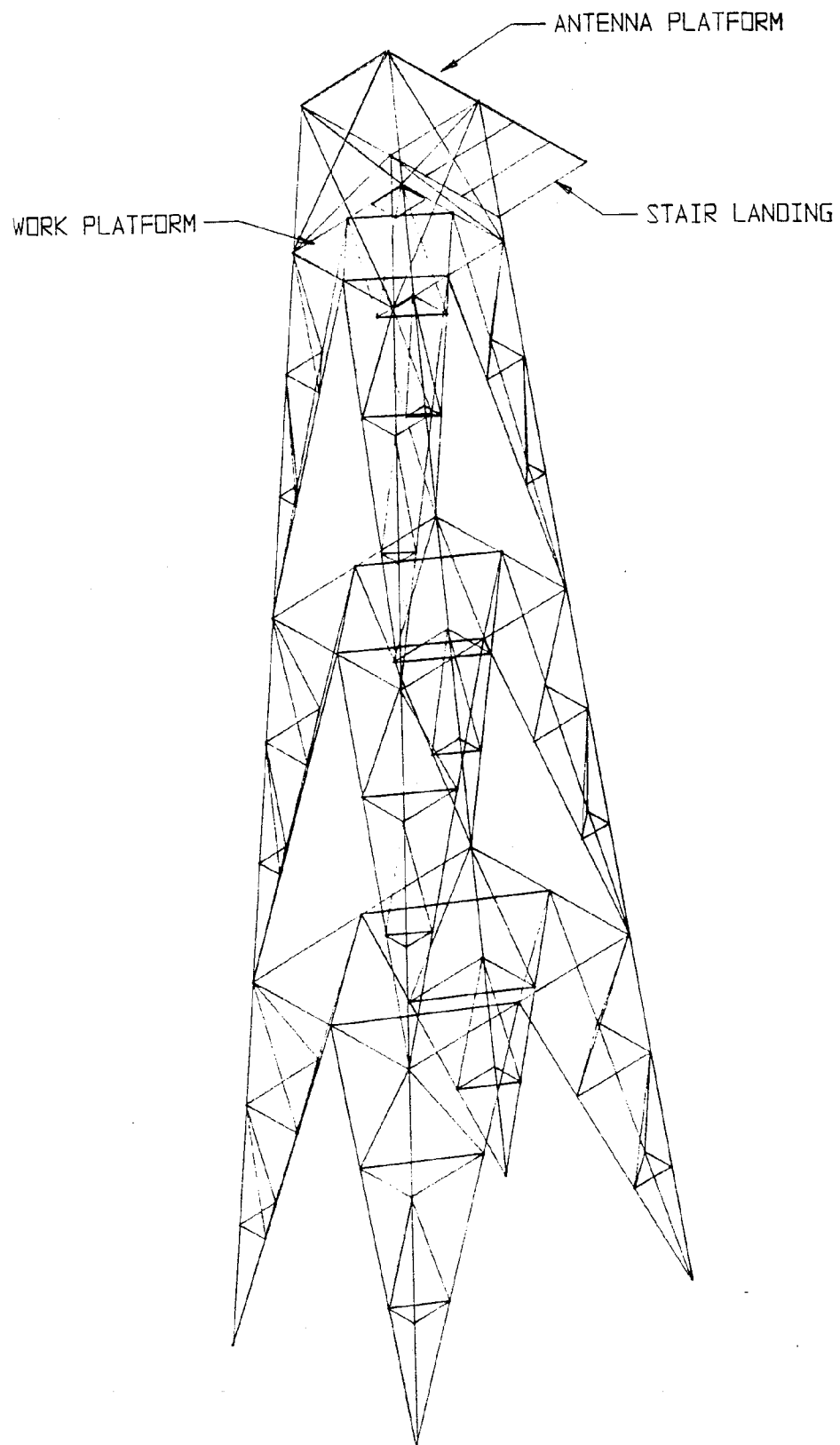


Figure 3. Isometric of tower.

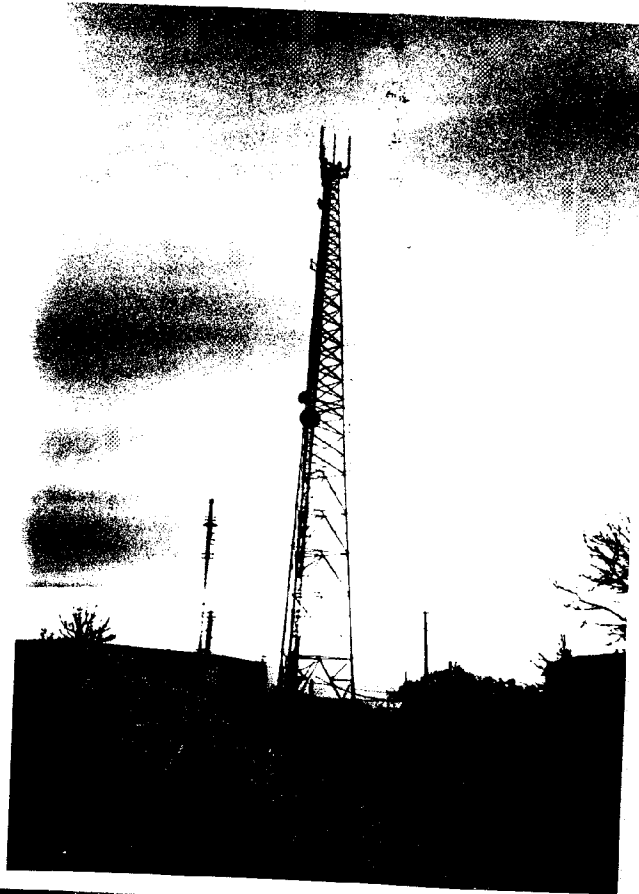


Figure 4. 180' free standing triangular Andrew tower owned by Cellular One near Tampa, FL framed with structural angles.

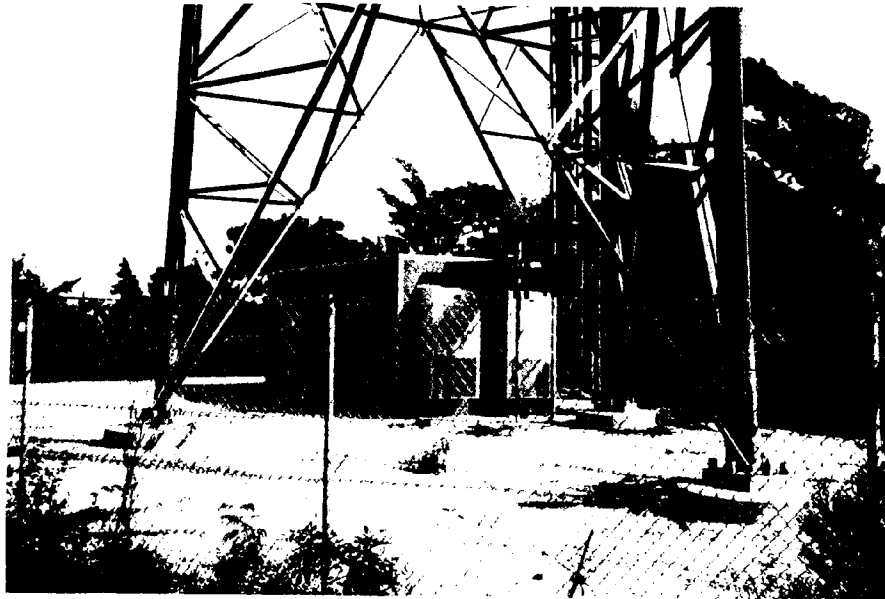
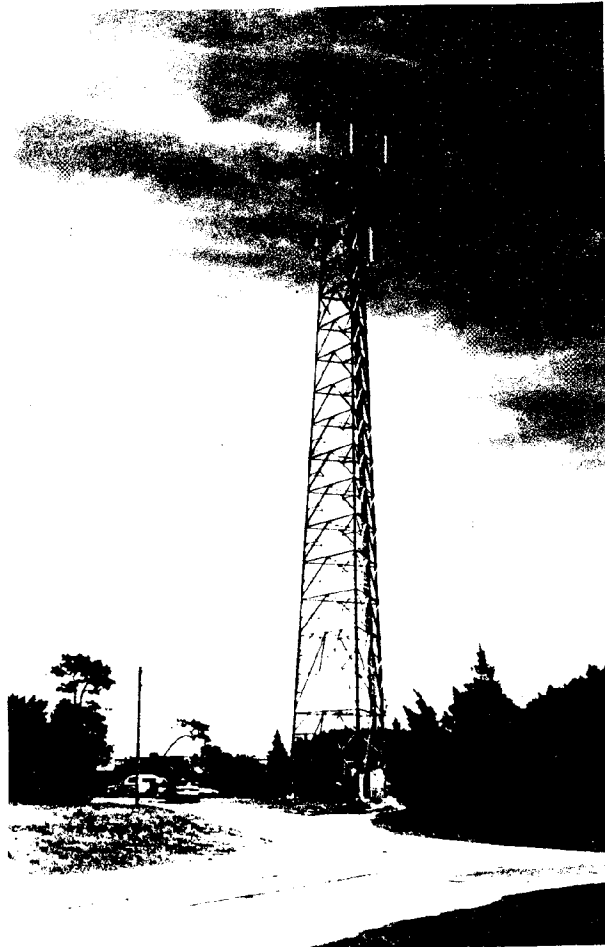


Figure 5. 295' free standing triangular Rohn tower owned by Union Electric Co. near St. Louis, MO framed with round structural tubes.

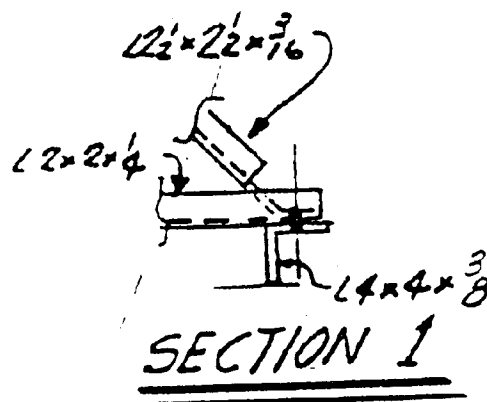
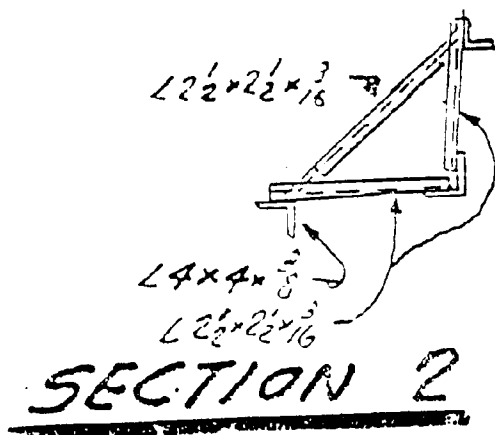
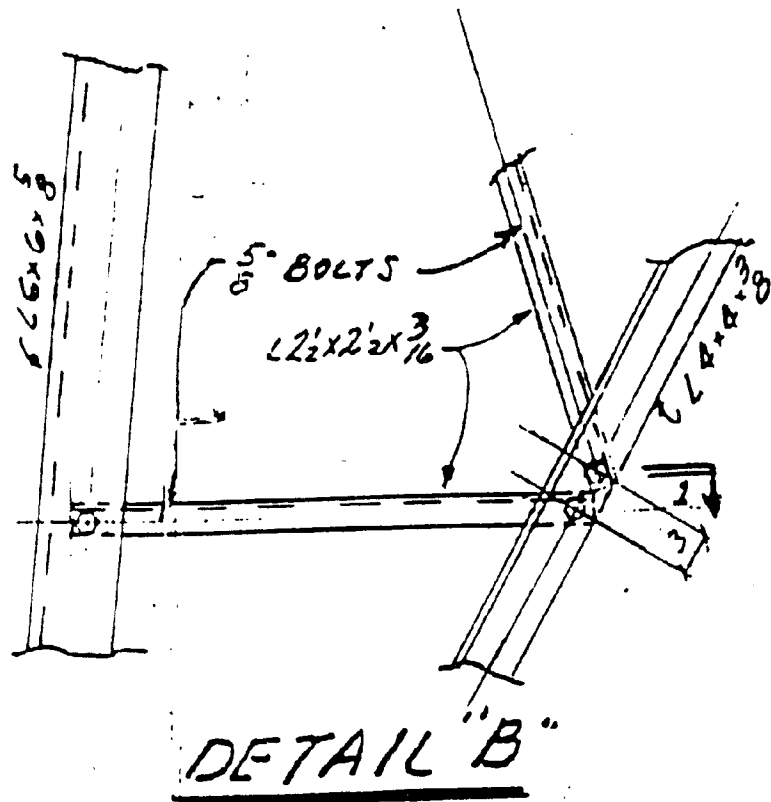
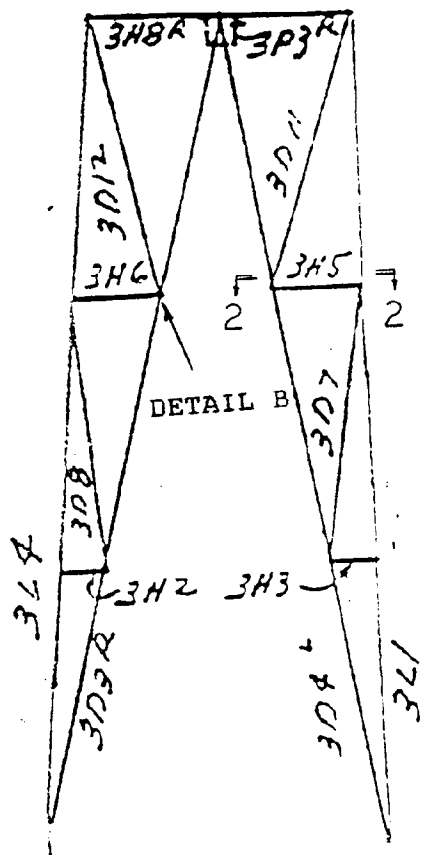


Figure 6. Tower details.

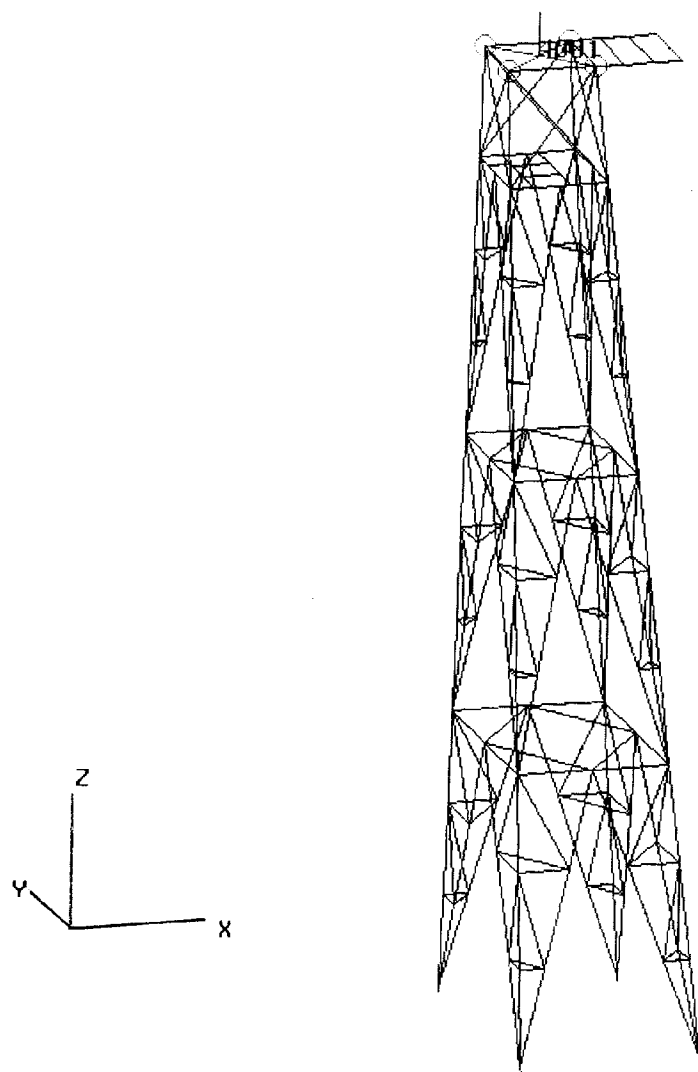


Figure 7. MSC/NASTRAN model of tower.

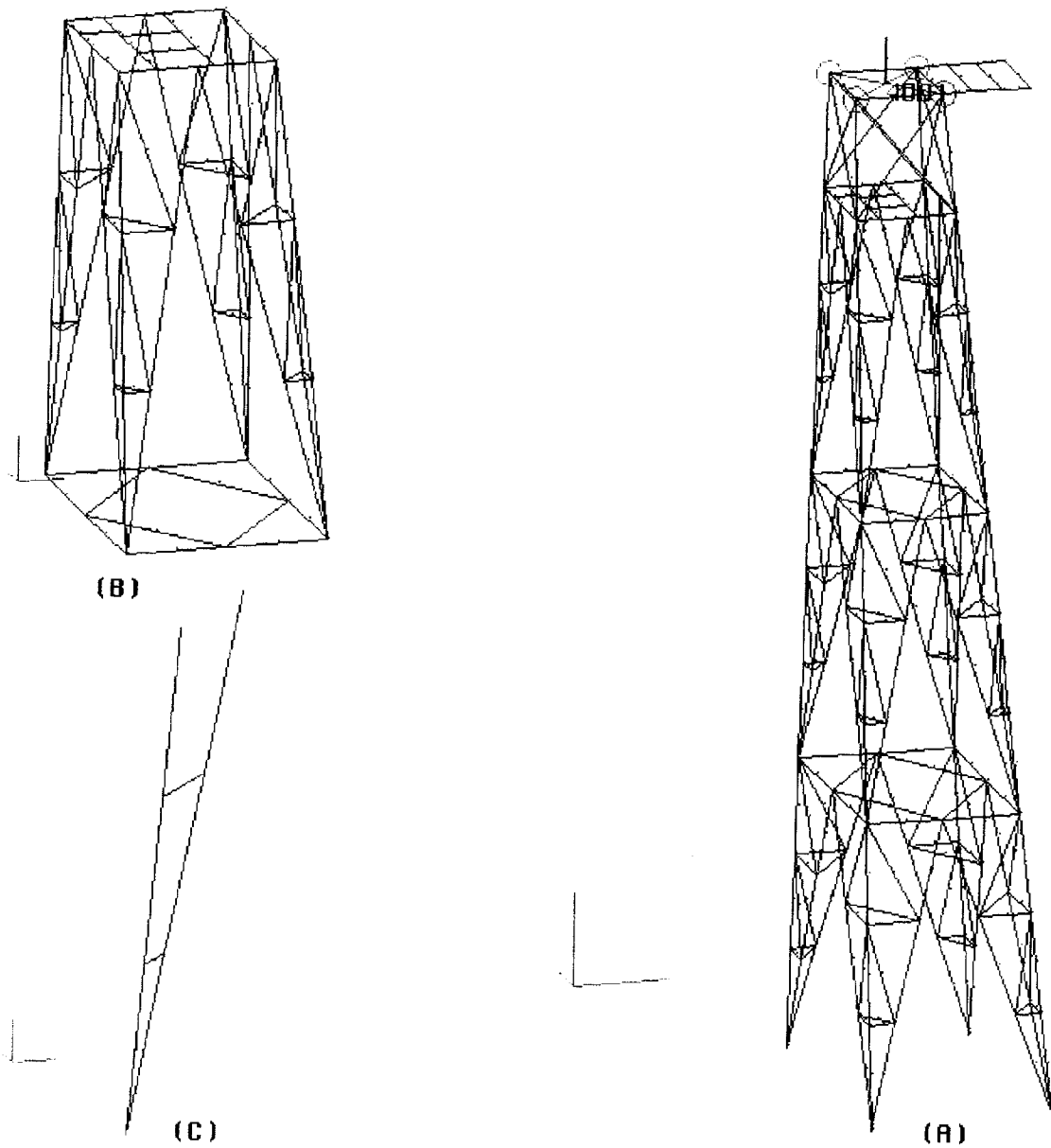


Figure 8. (a) 85' free standing tower. (b) Tower section between elevations 50' to 75'. (c) Main corner bracing, taken from tower section between elevation 50' to 75'.

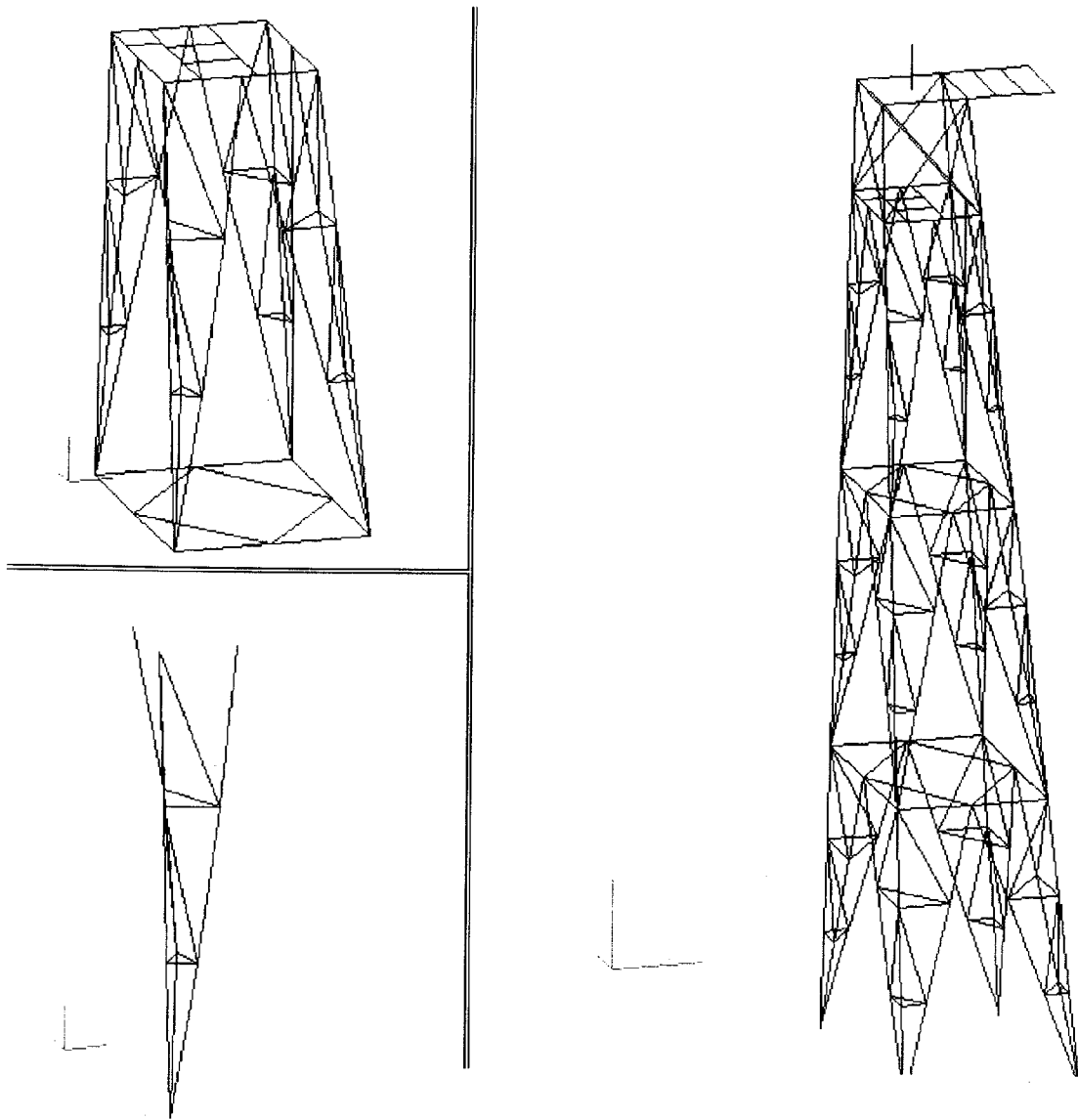


Figure 9. Buckled shape (eigenvector).