Identification of Critical Speeds of Rotors Attached to Flexible Supports

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ABSTRACT

It is common practice to include the gyroscopic terms for rigid rotors into the equations of motion for the calculation of critical speeds. This procedure works well for the case in which the rotors are connected directly to ground by elastic and/or damping elements. All calculated eigenfrequencies are critical speeds of the rotor. If additional degrees of freedom are included to model the actual support structure, then not all eigenfrequencies are critical speeds. Many of the calculated eigenfrequencies are simply modes of the support structure. The procedure presented allows critical speeds to be filtered from the set of eigenfrequencies calculated when the support structure is included in the analysis. The methodology is used to determine the critical speeds of the BRR 700 series aerojet engines.

INTRODUCTION

A rotor in an aerojet engine consists of a shaft connecting several large disks on which are mounted the compressor and turbine blades. The shaft is usually many times more flexible than the disks and allows the rotor to be idealize as a series of lumped masses located along the rotation axis connected by beams. Because the structural properties of the rotor are rotationally symmetric, i.e. the structural impedance presented to the stationary support is independent of the rotational position of the rotor, the idealized rotor can be added directly to the support structure for analysis.

In addition to the structural impedance of the rotor, there also exist additional inertial terms dependent upon the rotation speed. These inertial terms possess skew-symmetry and can greatly influence the critical speed of the rotor. By including these terms in the equations of motion for the coupled rotor/support structure, the critical speeds of a rotor can be directly determined through a complex eigenvalue analysis. Unfortunately, eigenfrequencies which are unrelated to the critical speeds are also calculated. The presence of these non-critical speed eigenvalues requires a technique to filter candidate modes from the set of calculated modes.

DESCRIPTION OF GYROSCOPIC TERMS

Gyroscopic effects significantly affect the dynamic behavior of rotating equipment and need to be included in analyses. The gyroscopic terms for a rigid rotor are dependent upon the rotor rotation speed and the angular velocity of motion perpendicular to the axis of rotation. The gyroscopic terms also possess a characteristic called *skew-symmetry*. Angular rotation perpendicular to the rotation axis produces a moment which is perpendicular to both the rotation axis and the angular motion. In matrix notation, these terms appear as a skew-symmetric matrix, as shown below for rotation about the z-axis.

$$\begin{Bmatrix} m_x \\ m_y \\ m_z \end{Bmatrix}_{gyroscopic} = \Omega \begin{bmatrix} 0 & I_P & 0 \\ I_P & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{Bmatrix} \text{ (for rotation about } z \text{ - axis)} \quad (1)$$

Physically, the gyroscopic terms can be viewed as promoting circular angular precession about the rotation axis. It is this effect which causes a toy top to precess if one tries to topple it. The larger the force used to topple the top (i.e., the larger the angular velocity) the faster the top will precess. As the top is precessing, the gyroscopic forces also provide the necessary moment required to prevent the top from falling. As the top precesses, the gyroscopic forces are in balance with those due to gravity (Figure 1).

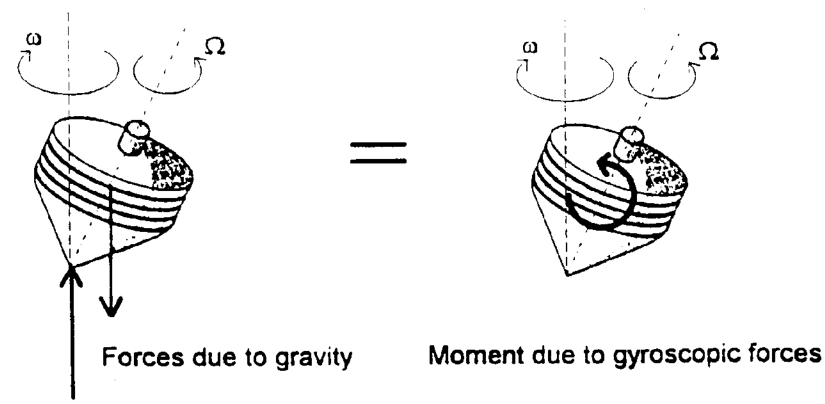

Figure 1. **Gyroscopic forces in balance with gravity forces**

If the top could maintain its rotation speed and no other outside forces acted on it, then the top could precess indefinitely.

With this view, it is easier to understand the contribution of the gyroscopic terms in determining critical speeds. Using the above example, the rotor takes the place of the top and the support structure replaces gravity as an opposing force. For a given rotor speed, there exists a precession rate at which the gyroscopic and opposing forces are in equilibrium. A critical speed is a unique situation in which the precession rate and rotor speed coincide. For a perfectly balanced rotor with no exterior forces acting on the rotor-support system, a rotor precessing at a critical speed is just a mathematical idealization, similar the a single degree-of-freedom system vibrating at its natural frequency. Unfortunately, rotors are never perfectly balanced and outside forces do act on the system. Similar to exciting a single degree-of-freedom system at its resonant frequency, a small rotor imbalance or a cyclic force at the rotation speed can result in large amplitude responses. For this reason, critical speeds of rotating equipment are important.

IDENTIFICATION OF CRITICAL SPEEDS

Before a critical speed can be identified, a definition for the description of a critical speed must be developed. The usual definition of a critical speed is: a rotational speed of the rotor which coincides with an eigenfrequency of the rotor-support structure. This is the definition used in the development of Campbell diagrams. In a Campbell diagram, eigenfrequencies are calculated for given rotor speeds and plotted on the ordinate of a graph (y-position) with the given rotor speed as the abscissa (x-position). Eigenfrequencies with similar modes shapes are connected with lines which follow the change in eigenfrequency with a change in rotor speed. An additional diagonal line, starting at the origin and having a slope of one, is drawn on the graph. This line represents the rotation speed of the rotor. The intersections of the diagonal line and the lines connecting the eigenfrequencies are critical speeds.

Because of the skew-symmetry of the gyroscopic inertial terms, modes which contain rotor motion split into two types:

-modes with an eigenfrequency which increases with rotor rotational speed
-modes with an eigenfrequency which decreases with rotor rotational speed.

Both types of modes exhibit rotor motion which is circular about the neutral rotation axis. In modes with an increasing eigenfrequency., the circular motion is in the same direction as the rotor rotation. These modes are called *forward* critical speeds. In modes with a decreasing eigenfrequency, the circular motion is in the opposite direction of the rotor rotation. These modes are called *backward* critical speeds.

A rotor imbalance acts as a force synchronous with the rotor speed. Therefore, only forward critical speeds are excited by imbalance forces. Backward critical speeds can be excited by a rotor rubbing against the casing. Since rotor imbalance is much more common than rotor rubbing, only forward critical speeds are of interest. Backward critical speeds are of interest only under special circumstances.

An alternate way of determining critical speeds is to include the gyroscopic inertial terms directly in the equations of motion and perform a complex eigenvalue analysis. The gyroscopic terms are added in a manner such that the rotor rotation speed and eigenfrequency are identical. All modal displacements, including any motion of the rotor about the neutral rotation axis, are at the same frequency as the rotor rotation speed.

If a rotor is connected by scalar elements directly to ground, then all calculated eigenfrequencies are critical speeds. If a model of the support structure is included in the analysis, then the critical speed are intermixed with modes of the rotor-support structure. Identification of the critical speeds is not easily done by examination of mode shapes. A filter which identifies which modes, among all the calculated modes, are possible critical speeds is required. One such filter is described in the following section.

CRITICAL SPEED FILTER

To determine whether the mode exhibits circular motion about the neutral rotation axis, an error fit procedure is used. Basis vector which describe forward and backward circular motion of the rotor about the neutral rotation axis are used in the procedure. The error is defined as the following least-mean-squared fit:

$$\{error\} = \{u\} - [W]\{a\} \tag{2}$$

where $\{u\}$ is the modal displacement vector

$[W]$ is a matrix of basis vectors

$\{a\}$ is the coefficient vector for the basis vectors

Minimizing the error with respect to the coefficient vector, $\{a\}$, results in

$$\{a\} = \left([W^*]^T[W]\right)^{-1}[W^*]^T\{u\} \tag{3}$$

Forward whirl at a rotor point is defined as circular motion in the direction of rotation.

$$\text{forward whirl} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 1 \\ -i \\ 0 \end{Bmatrix} \text{ (for rotation about } z \text{ - axis)} \tag{4}$$

Backward whirl motion is defined as circular motion opposite the direction of rotation.

$$\text{backward whirl} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 1 \\ i \\ 0 \end{Bmatrix} \text{ (for rotation about } z \text{ - axis)} \tag{5}$$

The basis vectors for the fitting procedure contain both forward and backward whirl motion vectors for each point on the rotor. This more general definition allows the identification of *mixed* critical speed modes, in which the rotor may exhibit both 'forward' and 'backward' motion simultaneously.

After the components of forward and backward motion of each mode are determined, the proportional kinetic and strain energies can be calculated. Modes with high proportional 'forward' or 'backward' kinetic or strain energy are deemed to be critical speeds.

EXAMPLE APPLICATION

The procedure described above is implemented in MSC/NASTRAN through a DMAP alter and applied to a baseline model of the BRR 700 series aerojet engine. The engine consists of a casing and two rotors spinning independently at different rates. An efficient approach to the dynamic analysis of the engine is to divide the engine into these three substructures: 1) engine casing, 2) low pressure rotor, and 3) high pressure rotor. Coupling between the rotors and the casing is performed using appropriate bearing stiffnesses and dampings (Figure 2 shows the bearing stiffness and damping connections).

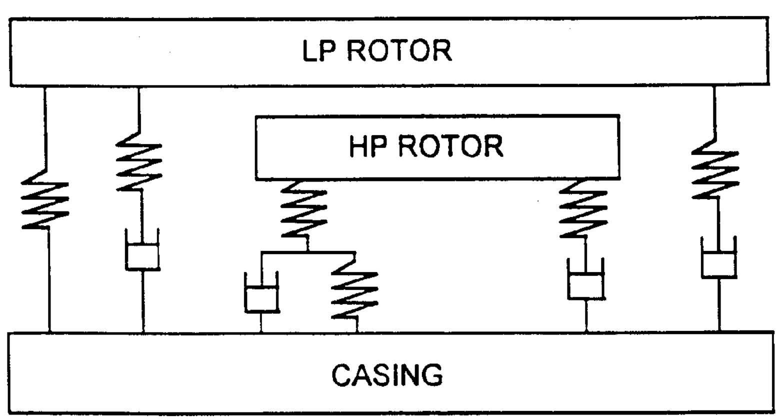

Figure 2. **Substructures of a Dual Rotor Aerojet Engine**

Each substructure was defined as a superelement and combined as a single-level superelement structure. Both rotors were modeled using three dimensional elements and reduced statically to points along the rotors' axis. The casing was reduced using component modes, with boundary points at the bearing interfaces.

Rotor frequencies in the engine running range are called 'critical' when the major part of the proportional strain energy is in the forward whirl motion. By calculating proportional strain energies for each substructure (superelement), the engineer is able to separate rotor frequencies from system frequencies. Only rotor frequencies with 'high' proportional strain energy need to be investigated further.

If a rotor is free of forward whirls with high proportional strain energies between 0 and 120% of take-off speed, then the engine is considered free of critical speeds. Backward whirls may occur, but will not be excited by rotor imbalance. Figure 3 shows the proportional strain energy of the low pressure rotor for the calculated frequencies in the operating range. Only a few frequencies in the operating range have significant forward strain energy.

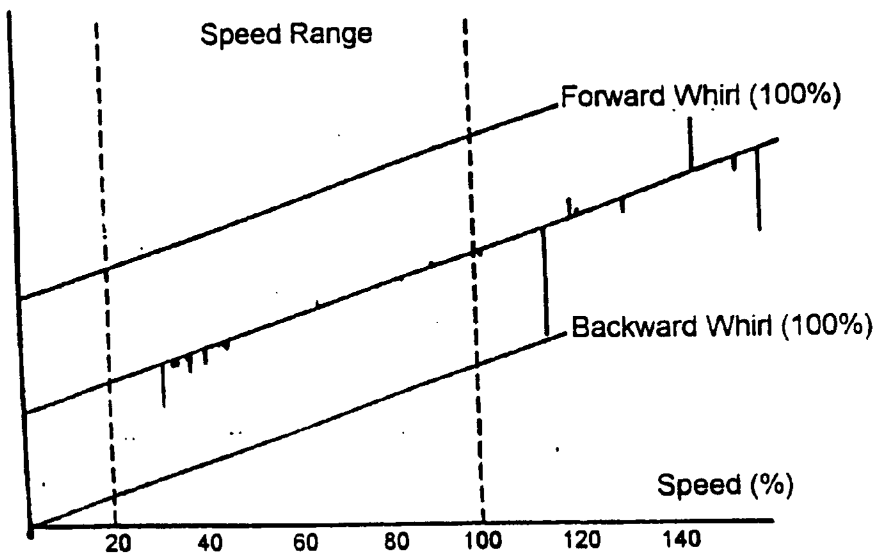

Figure 3. **Low Pressure Rotor Strain Energy Distribution Across Speed Range**

A critical rotor frequency occurs at 140% speed of the low pressure rotor, with all its proportional strain energy in the forward motion. Its deformed bending shape, including the high pressure rotor and casing, was animated using the general purpose program PATRAN (Figure 4). The complex animation procedure makes possible the visualization of forward and backward whirl motion.

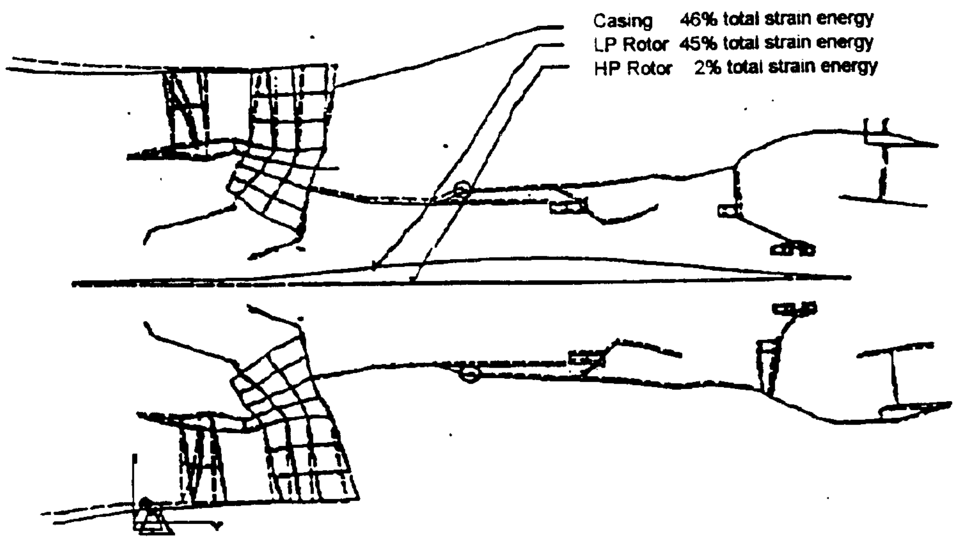

Figure 4. **Deformed Shape of the System Structure for a Critical LP Rotor Frequency**

CONCLUSION

The procedure described in this paper allows critical speeds of rotating structures to be determined directly using a complex eigenvalue analysis. In addition to calculating critical speeds, it also allows the user to determine whether they are forward or backward whirl speeds. Since forward whirl critical speeds can be excited by rotor imbalance, the ability to distinguish between forward and backward whirl is important in the design of all rotating structures. The methodology was applied to a BRR 700 series engine to determine whether there existed forward critical speeds in the engine operating range.

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